A. Energy Band Diagrams

Electron energy, $E$

Vacuum level

$2p$

$2p$

$2s$

$1s$

ATOM

SOLID

In a metal, the various energy bands overlap to give a single band of energies that is only partially full of electrons. There are states with energies up to the vacuum level where the electron is free.

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3-1 Semiconductor Concepts and Energy Bands

(a) A simplified two dimensional view of a region of the Si crystal showing covalent bonds. (b) The energy band diagram of electrons in the Si crystal at absolute zero of temperature.

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Electron in CB: $m_e^*$
Hole in CB: $m_h^*$

(a) A photon with an energy greater than $E_g$ can excite an electron from the VB to the CB.
(b) Each line between Si-Si atoms is a valence electron in a bond. When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created.

$\Delta E_p = eV$

$\frac{f(E_p)}{2}$ Electric work input or output per $e^-$

Electron concentration in CB

$$n = \int_{E_v}^{E_v + x} g_{CB}(E) f(E) dE$$

$$(E_c - E_v) \gg k_B T \Rightarrow f(E) = \exp\left(-\frac{(E-E_v)}{E_p}\right)$$

Boltzmann Statistics

Non-generate Semiconductor

$$(E_c - E_v) \ll k_B T$$

$$n = N_c \exp\left(-\frac{E_c - E_v}{k_B T}\right)$$

Effective DOS at the CB edge

$$N_e = 2 \pi m_e^* k_B T / h^2$$

Hole concentration in CB

$$p = N_v \exp\left[-\frac{(E_F - E_p)}{k_B T}\right]$$

Effective DOS at the VB edge

$$N_v = 2 \pi m_h^* k_B T / h^2$$

$*E_p$ determines the electron and hole concentration.

Intrinsic semiconductor (pure crystal), $n = p$

$$E_{Fi} = E_v + \frac{1}{2} E_g - \frac{1}{2} kT \ln\left(\frac{N_c}{N_v}\right)$$

Typically, $N_c \sim N_v$ and $E_{Fi} = E_v + \frac{1}{2} E_g$

Mass Action Law

$$np = N_c N_v \exp(-\frac{E_g}{k_B T}) = n_i^2$$

$n = p = n_i$ (intrinsic concentration)

Probability of finding $e^-$

Fermi-Dirac function

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_{Fi}}{k_B T}\right)}$$

Density of States (DOS)

$$g(E) \propto (E - E_c)^{3/2}$$

Energy band diagram.

Density of states (number of states per unit energy per unit volume).

Fermi-Dirac probability function (probability of occupancy of a state).

The product of $g(E)$ and $f(E)$ is the energy density of electrons in the CB (number of electrons per unit energy per unit volume). The area under $n(E)$ vs. $E$ is the electron concentration.
C. Extrinsic Semiconductors

n-type semiconductor: add pentavalent impurity.
Donor impurity: e.g. As in Si, \( \rightarrow As^+ + e^- \)

\[
N_d > n_i \quad n = N_d, \quad p = \frac{n_i^2}{n}
\]

Semiconductor conductivity

\[
\sigma = en\mu_e + ep\mu_h
\]  \( (8) \)

n-type conductivity

\[
\sigma = eN_d\mu_e + e\left(\frac{n_i^2}{N_d}\right)\mu_h \approx eN_d\mu_e
\]  \( (9) \)

Electrons: majority carrier; Holes: minority carrier

\[ n = n_i, \quad p = \frac{p_i}{n} \]

P-type semiconductor: add trivalent impurity

Acceptor impurity: e.g. B in Si, \( \rightarrow B^- + h^+ \)

\[
N_a > n_i \quad p = N_a, \quad n = \frac{n_i^2}{p}
\]

p-type conductivity

\[
\sigma \approx eN_a\mu_h
\]  \( (10) \)

Electrons: minority carrier; Holes: majority carrier

\[ n = \frac{n_i}{p}, \quad p = n_i \]

Energy band diagrams for (a) intrinsic (b) n-type and (c) p-type semiconductors. In all cases, \( np = n_i^2 \). Note that donor and acceptor energy levels are not shown.

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D. Compensation Doping

Doping of semiconductor with both donors and acceptors to control the properties.

Electrons (form donors) recombine with holes (from acceptors)

\[
\begin{align*}
    n &= N_d - N_a \gg n_i \\
    p &= N_a - N_d \gg n_i \\
    \frac{n^2}{n} &= p \\
    n &= \frac{n_i^2}{p}
\end{align*}
\]

p-type --> n-type n-type --> p-type

E. Degenerate and Non-degenerate Semiconductors

Non-degenerate semiconductor: \( n \ll N_v \) and \( p \ll N_c \)

The electron statistics ~the Boltzmann statistics (4)
Pauli exclusion principle can be neglected.

Degenerate semiconductor: \( n > N_v \) or \( p > N_c \)

The electron statistics ~the Fermi-Dirac statistics (1) Pauli exclusion principle becomes important.

More metal-like than semiconductor-like.

This band overlap with CB. \( E_F \) is within CB or above \( E_c \)

(a) Degenerate n-type semiconductor. Large number of donors form a band that overlaps the CB. (b) Degenerate p-type semiconductor.

F. Energy Band Diagrams in an Applied Field

Energy band diagram of an n-type semiconductor connected to a voltage supply of \( V \) volts. The whole energy diagram fills because the electron now has an electrostatic potential energy as well.

Example 3.1.1 半導體中的費米能階

一 n 型半導體被均勻的掺入 \( 10^{18} \) 個鋅原子，計算相對於本質費米能量 \( E_F \) 的費米能階位置，上述的砂樣品，更進一步地掺入 \( 2 \times 10^{17} \) 個硼原子，計算在室溫下相對於本質費米能 \( E_F \) 及上述所提相對於 n 型例子費米能量的費米能階位置。

\[
(5) \quad N_d = 10^{18} \quad n \quad \text{型紡錘} \quad \text{因 \( N_d \gg n_i = 1.45 \times 10^{10} \) }
\]

對於本質砂，

\[
N_E = N_d \exp\left(-\frac{(E_c - E_F)}{k_B T}\right)
\]

然而對於有著砂的砂

\[
N_E = N_d \exp\left(-\frac{(E_F - E_{F0})}{k_B T}\right) = N_d
\]

其中 \( E_{F0} \) 是本質和 n 型砂的費米能量，將兩運算式相除，

\[
\frac{N_d}{n_i} = \exp\left[\frac{(E_{F0} - E_F)}{k_B T}\right]
\]
**3-2 Direct and Indirect Bandgap Semiconductors: E-k Diagrams**

**Example 3.1.2 Transmission Coefficient**

In a general uniform doping of $10^{16} \text{cm}^{-3}$, and (P) atoms (the P-type semiconductor), the transmission coefficient is given by

\[ E_{p_0} - E_{n_0} = k\ln (n_d / n_i) = (0.0259 \text{ eV}) \ln (10^{16} / 1.5 \times 10^{10}) = 0.348 \text{ eV} \]

When semiconductors, such as germanium, are doped to form a crystal, the band structure of the semiconductor is modified by the presence of the impurities. The band gap, which is the energy difference between the valence and conduction bands, is affected by the doping process. For this reason, the energy difference

\[ E_{p_0} - E_{n_0} = k\ln (n_d / n_i) = (0.0259 \text{ eV}) \ln (10^{16} / 1.5 \times 10^{10}) \]

is also affected. The energy gap for these semiconductors is typically in the range of 0.1 to 1.5 eV.

The electron potential energy $V(x)$ inside the crystal is periodic with the same periodicity as that of the crystal, $a$. Far away outside the crystal, by choice, $V = 0$ (the electron is free and $PE = 0$).
The electron in an infinite potential well of width \( L \)

\[
E_n = \left( \frac{n\pi}{L} \right)^2 \quad \text{for } n = 1, 2, 3, \ldots
\]

\( E \) increases parabolically with \( k \).

The nearly free electron model of a metal.

The electron in a crystal

Schrodinger Equation

\[
\frac{d^2 \psi}{dx^2} + \frac{2m_e}{\hbar^2} \left( E - V(x) \right) \psi = 0
\]

Periodic Potential

\( V(x) = V(x + ma) \); \( m = 1, 2, 3, L \)

Bloch Wavefunctions

\[
\psi_k(x) = U_k(x) \exp(jkx)
\]

\( U_k(x) \rightarrow \text{periodic function depend on } V(x) \)

\( e^{jks} \cdot (e^{-jP/k}) \rightarrow \text{traveling wave} \)

**E-k Diagram**

States in CB,

States in VB.

\( e^- \text{ at } 0^\circ\text{K, all fill the lower states} \)

\( > 0^\circ\text{K, from VB to CB} \).

**3-3 pn Junction Principles**

The \( E-k \) diagram of a direct bandgap semiconductor such as GaAs. The \( E-k \) curve consists of many discrete points with each point corresponding to a possible state, wavefunction \( \psi_k(x) \), that is allowed to exist in the crystal. The points are so close that we normally draw the \( E-k \) relationship as a continuous curve. In the energy range \( E_i \) to \( E_e \) there are no points (\( \psi_k(x) \) solutions).
### Properties of the pn junction.

#### Depletion widths

\[ dE/dx = p_n(x)/\varepsilon \quad \text{Build-in field} \]

\[ E_o = -eN_d W_p = eN_n W_p \]

\[ E = -dV/dx \quad \text{Build-in potential} \quad V_o = -E_o W_o = eN_n N_d W_o^2 / 2\varepsilon (N_n + N_d) \]

By Boltzmann statistics

\[ \frac{n_2}{n_1} = \exp\left[-\frac{(E_x - E)}{k_B T}\right], \text{where } E \text{ is potential energy.} \]

Minority/majority

\[ n_{po} / n_{no} = \exp(-eV_0 / k_B T) \]

\[ p_{po} / p_{po} = \exp(-eV_0 / k_B T) \]

Build-in potential

\[ V_o = k_B T \frac{\ln(n_{no} / n_{po})}{e} \]

\[ V_o = k_B T \frac{\ln(P_{po} / P_{no})}{e} \]

\[ V_o = k_B T \frac{\ln\left(N_n N_d / n_i^2\right)}{e} \]

#### Forward Bias

Voltage drop across SCL or depletion region \( W \)

\( \rightarrow \) Reduce the build-in potential against diffusion

\( \rightarrow \) Result in the injection of excess minority carrier and small increase in the majority carrier

\[ p_n(0) = p_{po} \exp \left[ -e(V_o - V) / k_B T \right] \]

\[ p_p(0) = p_{po} \exp \left[ eV / k_B T \right] \]

\[ n_p(0) = n_{po} \exp \left[ eV / k_B T \right] \]

Excess minority carrier concentration

\[ \Delta p_n(x') = \Delta p_n(0) \exp(-x' / L_h) \]

Hole diffusion length

\[ L_h = \sqrt{D_h \tau_h} \]

\( \tau_h \): mean hole recomb. lifetime
The total current anywhere in the device is constant. Just outside the depletion region it is due to the diffusion of minority carriers.

\[ J = J_{\text{elec}} + J_{\text{hole}} \]

Assume electron and hole currents don't change across the SCL. The total current density

At \( x' = 0 \)

\[ J_{D,\text{hole}} = \left( \frac{eD_n}{L_n} + \frac{eD_p}{L_p} \right) n_0^2 \exp \left( \frac{eV}{k_B T} \right) - 1 \]

Shockley diode equation

\[ J_{\text{so}} \] (reverse saturation current density)

reverse bias \(-V_r \) \((> k_B T / e = 25mV)\)

\[ J = -J_{\text{so}} \]

Forward biased \( p-n \) junction and the injection of carriers and their recombination in the SCL.

\[ J = \frac{dQ}{dt} \]

Electrons recombine in \( W_p \) and holes recombine in \( W_n \).

\[ J_{\text{recom}} = \frac{1}{\tau_e} n_M p_M \]

Recombination Current

\( \tau_{h,e} \): mean hole (electron) recombination time in \( W_{n,p} \)

\[ p_M = n_i \exp \left( \frac{eV}{2k_B T} \right) \]

\[ J_{\text{recom}} = \frac{e}{2} \left( \frac{W_p}{\tau_e} + \frac{W_n}{\tau_h} \right) \exp \left( \frac{eV}{2k_B T} \right) \]

Diode Equation

\[ I = I_o \exp \left( \frac{eV}{\eta k_B T} \right) - 1 \]

Diode ideality factor

\( \eta = 1, \text{diffusion controlled} \)

\( \eta = 2, \text{SCL recomb. controlled} \)

Some of the minority carriers will recombine in the depletion region.

Log (carrier concentration)
C. Reversed Bias

Reverse biased $pn$ junction. (a) Minority carrier profiles and the origin of the reverse current. (b) Hole $PE$ across the junction under reverse bias

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Shockley diode equation (12)

$J = -J_n = \left( \frac{eD_i}{L_i} + \frac{eD_n}{L_n} \right) n_i^2$

"-": reverse direction

EHP thermal generation in SCL

$J_{gen} = \frac{eWn_i}{\tau_g}$

$\tau_g$: mean thermal generation time

Total Reverse Current

$J_{rev} = \left( \frac{eD_i}{L_i} + \frac{eD_n}{L_n} \right) n_i^2 + \frac{eWn_i}{\tau_g}$ (17)

$T > 238K$

$\ln(I_{rev}) \propto \frac{1}{V} \geq 0.63 eV - E_i = 0.66 eV \rightarrow I_{rev} \propto n_i^2$

$T < 238K$

$\ln(I_{rev}) \propto \frac{1}{V} \leq 0.33 eV - \frac{1}{2} E_i \rightarrow I_{rev} \propto n_i$

Reverse diode current (A) at $V = -5 V$

Reverse diode current in a Ge $pn$ junction as a function of temperature in a $I$ vs. $1/T$ plot. Above 238 K, $I_{rev}$ is controlled by $n_i^2$ and below 238 K it is controlled by $n_i$. The vertical axis is a logarithmic scale with actual current values. (From D. Scansen and S.O. Kasap, *Chalk J. Physics* 70: 1070-1075, 1992)

$\nu$
D. Depletion Layer Capacitance

Depletion Layer Capacitance

\[ C_{dep} = \frac{\text{d}Q}{\text{d}V} \]  \hspace{1cm} (18)

SCL width and voltage

\[ W = \left[ \frac{2\varepsilon (N_a + N_d)(V_o - V)}{eN_a N_d} \right]^{1/2} \]  \hspace{1cm} (19)

\[ C_{dep} = \frac{\varepsilon A}{W} \left[ \frac{A}{(V_o - V)^{1/2}} \right]^{1/2} \left[ e\varepsilon (N_a N_d)/2(N_a + N_d) \right]^{1/2} \]  \hspace{1cm} (20)

E. Recombination Lifetime

Forward bias \( n_p = n_{po} + \Delta n_p \) minority

\( p_p = p_{po} + \Delta n_p \) (\( \Delta p_p = \Delta n_p \)) majority

\[ \partial \Delta n_p / \partial t = -BN_p p_p + G_{thermal} \]  \hspace{1cm} (21)

Rate of change due to recombination

\[ \frac{\partial \Delta n_p}{\partial t} = -B(n_p p_p - n_{po} p_{po}) \]  \hspace{1cm} (22)

Excess minority carrier recombination time (lifetime)

\[ \frac{\partial \Delta n_p}{\partial t} = -\frac{\Delta n_p}{\tau_e} \]  \hspace{1cm} (23)

In weak injection, \( \Delta n_p \ll p_{po} \rightarrow n_p = \Delta n_p, p_p = p_{po} = N_a \)

\[ \frac{\partial \Delta n_p}{\partial t} = -BN_p \Delta n_p \]  \hspace{1cm} (24)

\[ \tau_e = 1/BN_a \]  \hspace{1cm} (25)

In strong injection, \( \Delta n_p \gg p_{po} \)

\[ \frac{\partial \Delta n_p}{\partial t} = B\Delta p_p \Delta n_p = B(\Delta n_p)^2 \]  \hspace{1cm} (26)

例題 3.3.1 直接能隙 pn 接面

一對稱的 GaAs 接面，其截面積 \( A = 1 \text{mm}^2 \) ，有下列性質： \( N_a \) (\( p \)-側稀釋) = \( N_d \) (\( n \)-側稀釋) = \( 10^{17} \text{cm}^3 \)； \( B = 7.21 \times 10^{-16} \text{m}^2 \cdot \text{s} \)； \( n_i = 1.8 \times 10^{12} \text{cm}^3 \)； \( \varepsilon = 13.2 \)； \( \mu_s \) (在 \( n \)-側) = \( 250 \text{cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1} \)； \( \mu_s \) (在 \( p \)-側) = \( 5000 \text{cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1} \)。擴散係數可經由愛因斯坦關係，使傳遷移速率有關：

\[ D_p = \mu_s k_B T/e, D_n = \mu_s k_B T/e \]

加於二極體的順差偏壓為 \( 1 \text{V} \) ，求在 \( 300 \text{K} \) 時，由於少數載子擴散所造成的二極體電流何為？假設是直接復合。假如乏區中，平均少數載子復合生命期的數量級 \( \sim 10^{-5} \text{s} \)，估計電流中復合份量大小？

假設注入，我們能容易地分別由中性 \( p \) 及 \( n \) 區的電子和電離復合來計算復合時間 \( \tau_p \) 及 \( \tau_n \)，使用 S.I. 單位及 \( k_B T/e = 0.02585 \text{V} \)。對一對稱的元件，

\[ \tau_n = \tau_p = \frac{1}{BN_n} = \frac{1}{(7.21 \times 10^{-16} \text{m}^2 \cdot \text{s})(1 \times 10^{-13} \text{m}^3)} = 1.39 \times 10^{-9} \text{s} \]

\[ L_p = (D_p \tau_p)^{1/2} = [(6.46 \times 10^{-4} \text{m}^2 \cdot \text{s})(1.39 \times 10^{-8} \text{s})]^{1/2} = 3.00 \times 10^{-4} \text{m} \]

\[ L_n = (D_n \tau_n)^{1/2} = [(1.29 \times 10^{-2} \text{m}^2 \cdot \text{s})(1.39 \times 10^{-8} \text{s})]^{1/2} = 1.34 \times 10^{-4} \text{m} \]

由於在中性區擴散的逆向飽和電流為

\[ I_{so} = A \left( \frac{D_p}{L_p N_x} + \frac{D_n}{L_n N_p} \right) e n_i^2 \]

\[ = (10^{-4}) \left[ \frac{6.46 \times 10^{-4}}{(3.00 \times 10^{-10})(10^{15})} + \frac{1.29 \times 10^{-2}}{(1.34 \times 10^{-5})(10^{15})} \right] (1.6 \times 10^{-19})(1.8 \times 10^{15})^2 \]

\[ = 6.13 \times 10^{-21} \text{A} \]
### 3-4 The pn Junction Band Diagram

Energy band diagrams for a pn junction under (a) open circuit, (b) forward bias and (c) reverse bias conditions. (d) Thermal generation of electron hole pairs in the depletion region results in a small reverse current.

---

$I_{\text{diff}} = I_m \exp \left( \frac{eV}{k_BT} \right) = (6.13 \times 10^{-23} \text{ A}) \exp \left[ \frac{1.0 \text{ V}}{0.02585 \text{ V}} \right] = 3.9 \times 10^{-4} \text{ A}

內建電壓 $V_o$ 可得為

$V_o = \frac{k_BT}{e} \ln \left( \frac{N_p N_n}{n_i^2} \right) = (0.02585) \ln \left[ \frac{10^{21}}{(1.8 \times 10^{17})^2} \right] = 1.28 \text{ V}$

空乏層寬度 $W$ 為

$W = \left[ 2e(N_p + N_n)(V_o - V) \right]^{1/2} \frac{eN_p N_n}{eN_p N_n} = \left[ 2(13.2)(8.85 \times 10^{-12} \text{ Fm}^{-1})(10^{21} + 10^{23} \text{ m}^{-3})(1.28 - 1 \text{ V}) \right]^{1/2} = 9.0 \times 10^{-4} \text{ m} \text{ 或 } 0.090 \mu \text{m}$

對一對稱二極體，$W_p = W_n = \frac{1}{2} W$，並取 $\tau_n = \tau_p = 10 \text{ ns}$ 时,

$I_m = \frac{Aen}{2} \left[ \frac{W}{\tau_n + \tau_p} \right] = \frac{Aen}{2} \left[ \frac{W}{\tau_n} \right] = \frac{10^{-4}(1.6 \times 10^{20})(1.8 \times 10^{17})}{2} = 9.0 \times 10^{-8} \text{ A} = 1.3 \times 10^{-12} \text{ A}$

所以

$I_{\text{rec}} = I_m \exp \left( \frac{eV}{2k_BT} \right) = (1.3 \times 10^{-12} \text{ A}) \exp \left[ \frac{1.0 \text{ V}}{2(0.02585 \text{ V})} \right] = 3.3 \times 10^{-4} \text{ A}$

在這個例子，擴散和複合分量大約為相同數量級大小。
3-5 Light Emitting Diodes

**A. Principles**

LED: pn junction diode
direct band gap semiconductor, e.g. GaAs
EHP recombination ‡ photon emission (injection electroluminescence) ‡ spontaneous emission in active region (SCL & within $L_e$ in p region)

---

![Diagram of LED](image)

(a) The energy band diagram of a $p$-$n$-$n^+$ (heavily $n$-type doped) junction without any bias. Built-in potential $V_{bi}$ prevents electrons from diffusing from $n^+$ to $p$ side. (b) The applied bias reduces $V_{bi}$ and thereby allows electrons to diffuse, be injected, into the $p$-side.

Recombination around the junction and within the diffusion length of the electrons in the $p$-side leads to photon emission.

---

**B. Device Structures**

$n^+$ region: heavily doped, electron injection
photo-ionization or reflection

$p$ region: light emitting, narrow (a few microns) to avoid photon absorption

Diffused junction planar LED (b)
Substrate+$n$ region: lattice matching
Lattice mismatch ‡ radiationless EHP recomb.

---

A schematic illustration of typical planar surface emitting LED devices. (a) $n^+$ is formed by donor diffusion into the substrate. (b) $n$-$p$ layer is grown epitaxially on an $n^+$ substrate. (c) $p$ region is then formed by donor diffusion into the epitaxial layer.

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3-6 LED Materials

Direct bandgap:

\[ \text{GaAs}_x \text{P}_y \quad (0 \leq y < 0.45, \quad 870 \text{ nm} > \lambda > 630) \]
\[ \text{Al}_{1-x} \text{Ga}_x \text{As} \quad (0 \leq x < 0.43, \quad 870 \text{ nm} > \lambda > 640) \]
\[ \text{In}_{0.58} \text{Ga}_{0.42} \text{As} \quad (0.05 \leq x < 0.17) \quad \text{(High intensity)} \]
\[ \text{In}_{x} \text{Ga}_{1-x} \text{As} \quad (\text{Optical communication}) \]
\[ \text{GaN} \quad (E_g = 3.4 \text{ eV}) \]

Indirect Bandgap:

\[ \text{GaAs}_x \text{P}_y \quad (y > 0.45), \text{doped with} \ N \ \text{fl donor type localized energy level} \]
\[ \text{SiC}, \text{doped with} \ Al \ \text{fl acceptor type localized energy level} \]

(a) GaAs\(_x\)P\(_y\) (y < 0.45)
(b) N doped GaP
(c) Al doped SiC

(a) Photon emission in a direct bandgap semiconductor. (b) GaP is an indirect bandgap semiconductor. When doped with nitrogen there is an electron trap at \(E_N\). Direct recombination between a trapped electron at \(E_N\) and a hole emits a photon. (c) In Al doped SiC, EHP recombination is through an acceptor level like \(E_A\).

Free space wavelength coverage by different LED materials from the visible spectrum to the infrared including wavelengths used in optical communications. Hatched region and dashed lines are indirect \(E_g\) materials.

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---

### Table 3.1: LED Materials

<table>
<thead>
<tr>
<th>LED Material</th>
<th>Base</th>
<th>(D) or (I) (nm)</th>
<th>(\eta_{\text{external}}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs</td>
<td>GaAs</td>
<td>870 - 900</td>
<td>10</td>
</tr>
<tr>
<td>Al(<em>{0.43})Ga(</em>{0.51})As</td>
<td>GaAs</td>
<td>640 - 870</td>
<td>5 - 20</td>
</tr>
<tr>
<td>In(<em>{0.2})Ga(</em>{0.8})As</td>
<td>InP</td>
<td>1 - 1.6 (\mu)m</td>
<td>&gt;10</td>
</tr>
<tr>
<td>In(_{0.7})GaN</td>
<td>GaN or SiC</td>
<td>430 - 460</td>
<td>2</td>
</tr>
<tr>
<td>Saphire</td>
<td>Saphire</td>
<td>500 - 530</td>
<td>3</td>
</tr>
<tr>
<td>SiC</td>
<td>Si ; SiC</td>
<td>460 - 470</td>
<td>0.02</td>
</tr>
<tr>
<td>Al(<em>{0.55})Ga(</em>{0.45})As</td>
<td>GaAs</td>
<td>590 - 630</td>
<td>1 - 10</td>
</tr>
<tr>
<td>GaAs(_x)P(_y) (y &lt; 0.45)</td>
<td>GaAs</td>
<td>630 - 870</td>
<td>&lt;1</td>
</tr>
<tr>
<td>GaAs(_x)P(_y) (y &gt; 0.45) (N) or (Zn)</td>
<td>GaP</td>
<td>560 - 700</td>
<td>&lt;1</td>
</tr>
<tr>
<td>GaP (Zn-O doping)</td>
<td>GaP</td>
<td>700</td>
<td>2 - 3</td>
</tr>
<tr>
<td>GaP (N)</td>
<td>GaP</td>
<td>565</td>
<td>&lt;1</td>
</tr>
</tbody>
</table>

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3-7 Heterojunction High Intensity LEDs

Homojunction: narrow p region ‡ EHP recombine through surface defects
long p region ‡ re-absorption of photons increases
Heterojunction: a junction between two diff. $E_g$ semiconductors

Double heterojunction device:
Thin p GaAs layer
p AlGaAs: a confining layer
$\Delta E_c$: potential energy barrier against electron in CB (p-GaAs)
to CB of p-AlGaAs
Photon will not be absorbed with the wide bandgap material AlGaAs.

3-8 LED Characteristics

Wavelengths of photon emission depends on the $e$ and $h^+$ distribution.
The min. photon energy = $E_g$
The max. emission at photon energy $\sim E_c + k_BT = E_c + 0.5k_BT + 0.5k_BT$
The linewidth $\sim 2.5-3k_BT$
For heavily doped semiconductor ‡ narrow impurity band overlaps CB
‡ the minimum photon energy $<E_g$
**Example: GaAsP LED (center at 655 nm)**

Spectrum: less asymmetric than ideal spectrum

Linewidth ~24 nm (2.7kT)

The output light power is not linear with LED current as in (b)

The turn-on or the cut-in voltage \( \sim 1.5 \text{V} \) (depends on material, and increases as \( E_p \) increases e.g. blue 3.5~4.5V, yellow ~2V)

(a) Typical output spectrum (relative intensity vs wavelength) from a red GaAsP LED.

(b) Typical output light power vs. forward current.

(c) Typical I-V characteristics of a red LED. The turn-on voltage is around 1.5V.

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\[
\Delta \lambda = \frac{hc}{E_p} \Delta E_{ph}
\]

我們可給定輸出頻譜的能量寬度 \( \Delta E_{ph} = \Delta (h\nu) = 3kT \), 則利用後者並以 \( \lambda \) 取代 \( E_{ph} \)，我們找到，

\[
\Delta \lambda = \frac{\lambda^2}{c} 3kT
\]

因此在

\[
\lambda = 870 \text{nm}, \quad \Delta \lambda = 47 \text{nm},
\lambda = 1300 \text{nm}, \quad \Delta \lambda = 105 \text{nm},
\lambda = 1550 \text{nm}, \quad \Delta \lambda = 149 \text{nm}.
\]

這些線寬為典型值，而精確值取決於 LED 架構。

---

### 例題 3.8.1 LED 輸出光譜

給定相對強度的寬度對 LED 光子能量頻譜，典型地在 \( \sim 3kT \) 左右，則輸出頻譜中的線寬 \( \Delta \omega_2 \)，以波長表示時為何？

若我們注意到發射波長 \( \lambda \) 和光子能量 \( E_{ph} \) 的關係為

\[
\lambda = c / \nu = \frac{hc}{E_{ph}}
\]

假如我們對 \( \lambda \) 微分，相對於光子能量 \( E_{ph} \)，我們可以得到

\[
\frac{d\lambda}{dE_{ph}} = -\frac{hc}{E_{ph}^2}
\]

我們可以微分表示小改變或間隔（或 \( \Delta \)），亦即 \( \Delta \lambda / \Delta E_{ph} = |d\lambda / dE_{ph}| \)，

則

\[
\text{Example: GaAsP LED (center at 655 nm)}
\]

Spectrum: less asymmetric than ideal spectrum

Linewidth ~24 nm (2.7kT)

The output light power is not linear with LED current as in (b)

The turn-on or the cut-in voltage \( \sim 1.5 \text{V} \) (depends on material, and increases as \( E_p \) increases e.g. blue 3.5~4.5V, yellow ~2V)

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(c) Typical I-V characteristics of a red LED. The turn-on voltage is around 1.5V.

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\Delta \lambda = \frac{\lambda^2}{c} 3kT
\]

因此在

\[
\lambda = 870 \text{nm}, \quad \Delta \lambda = 47 \text{nm},
\lambda = 1300 \text{nm}, \quad \Delta \lambda = 105 \text{nm},
\lambda = 1550 \text{nm}, \quad \Delta \lambda = 149 \text{nm}.
\]

這些線寬為典型值，而精確值取決於 LED 架構。

---

### 例題 3.8.2 LED 輸出波長變動

考慮一 GaAs LED，GaAs 隨著在 300 K 是 1.42 eV，其隨溫度變動（減少），可表示為 \( dE_g / dT = -4.5 \times 10^{-3} \text{eV K}^{-1} \)，假如溫度改變 10°C 時，則發射波長變動為何？

忽略 \( kT \) 項，並取 \( \lambda = hc / E_g \)，我們可得

\[
\frac{d\lambda}{dT} = \frac{hc}{E_g^2} \frac{dE_g}{dT} = \frac{hc}{E_g^2} \left( \frac{6.626 \times 10^{-34}}{(1.42 \times 1.6 \times 10^{-19})^2} \right) (45 \times 10^{-3} \times 1.6 \times 10^{-19})
\]

所以

\[
\frac{d\lambda}{dT} = 2.77 \times 10^{-10} \text{m K}^{-1} \quad \text{或} \quad 0.277 \text{nm K}^{-1}
\]

對 \( \Delta T = 10 \degree \text{C} \) 波長變動 \( \Delta \lambda \) 為

\[
\Delta \lambda = (d\lambda / dT) \Delta T = (0.277 \text{nm K}^{-1}) (10 \text{K}) = 2.8 \text{nm}
\]

因 \( E_g \) 隨溫度而減少，波長隨溫度而增加，在這計算所得的變動量，典型的是在 10% 以內，這是引述於資料書籍中的 GaAs LED。
3-9 LED for Optical Fiber Communications

Short haul communication: use LED
simpler to drive, more economic, longer lifetime, wider o/p spectrum

Long haul and wide bandwidth communication: use LD
narrow linewidth, high o/p power, higher signal BW capacity

Surface emitting LED (SLED)
Edge emitting LED (ELED)

(a) Surface emitting LED
(b) Edge emitting LED

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例題 3.8.3 InGaAsP 在 InP 基板上

三元合金 $\text{In}_{x}\text{Ga}_{y}\text{As}_{1-y}$，成長在 InP 晶體基板上，是合適的商用半導體材料，
可用來作為紅外線波長 LED 及雷射二極體的應用，這元件需要 InGaAsP 樣格匹配
於 InP 晶體基板，以避免晶體缺陷，出現在 InGaAsP 層，這將需要 $y = 2.2x$，三元合金
能隙 $E_g$ 以 eV 爲單位，由經驗關係可表示為，

$$E_g = 1.35 - 0.72y + 0.12y^2; \quad 0 \leq y \leq 0.47$$

計算發射峰值在 1.3 μm 波長下，InGaAsP 三元合金的組成為何？

我們首先注意到，我們需要得到對感興趣波長的能隙 $E_g$ ，在發射峰值下，光子
能量為 $\hbar c/\lambda = E_g + k_B T$，則表為電子伏特，

$$E_g = \frac{\hbar c}{\varepsilon} - \frac{k_B T}{e}$$

而在 $\lambda = 1.3 \times 10^{-4}$ m，取 $T = 300$ K，

$$E_g = \left(3 \times 10^8\right)\left(6.626 \times 10^{-34}\right)\left(1.6 \times 10^{-19}\right)\left(1.3 \times 10^{-5}\right)$$

則 InGaAsP 必有 $y$ 滿足

$$0.928 = 1.35 - 0.72y + 0.12y^2$$

在計算機上解這二次方程得 $y = 0.66$，則 $x = 0.66/2.2 = 0.3$，最後三元合金為

$\text{In}_{0.7}\text{Ga}_{0.3}\text{As}_{0.66}\text{P}_{0.34}$.
**Coupling of SLED to a fiber**

**Burrus type device**

(a) Light is coupled from a surface emitting LED into a multimode fiber using an index matching epoxy. The fiber is bonded to the LED structure.

(b) A microlens focuses diverging light from a surface emitting LED into a multimode optical fiber.

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**Coupling of ELED to a fiber**

Light from an edge emitting LED is coupled into a fiber typically by using a lens or a GRIN rod lens.

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o/p spectra from SLED and ELED are not necessarily the same

Diff. doping levels, self-absorption guided along active layer in ELED

Typ. linewidth of ELED < linewidth of SLED

**Edge emitting LED: greater light intensity, more collimated beam**

InGaAs: active region
InGaAsP: confining layer
InP: cladding layer

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