Chapter 3
Reconstruction of Index Profiles of Fiber Gratings from Reflection Spectra

3.1 Introduction

Fiber gratings fabricated with ultraviolet (UV) laser exposure on photosensitive optical fiber have been widely used for optical filtering [3.1][3.2], sensing [3.3], dispersion compensation [3.4][3.5], etc. Bragg reflection characteristics of a fiber grating, including amplitude and phase, are controlled by its longitudinal refractive-index distribution, i.e., the local grating period and the depth of effective index modulation. Therefore, the retrieval of the longitudinal index distribution is crucially important for controlling the characteristics of a fiber grating. Several studies have been conducted for this purpose. The technique of side-diffraction [3.6] has been used to determine the
index profile. However, this technique suffers from the disadvantage of the strict alignment of the probe laser beam and the lack of the information of background index change and grating chirp. Also, the spatial resolution is limited by the size of the probe beam. The technique of side-scattering can be used to measure the grating chirp [3.7]. However, the spatial resolution is limited by the detection system. Later, an algorithm was proposed to retrieve the index profiles by processing the data from optical coherence-domain reflectometry (OCDR) [3.8]. However, no experimental work was reported based on the proposed method. Besides, it is difficult to obtain the required phase information from the OCDR measurement. Meanwhile, although the phase information can be obtained from reflectivity data through the Wiener-Lee transforms without direct measurement [3.9][3.10], it is feasible only for simple grating structures. Therefore, without direct measurement of phase, it is difficult to retrieve the index profiles of gratings with apodization, chirping or dc-component structures.

In this chapter, we report our results of retrieving the index profiles of fiber gratings based on a novel technique. In the technique, we measure both intensity and phase of Bragg reflection from the fiber grating with an interferometric setup. Then, with a simple algorithm, we can retrieve index profiles of fiber gratings with any apodization, chirping or dc structures. Although our basic theory is the same as that in [3.8], we provide more
concrete discussions on the required procedures for retrieving the apodization, chirping and dc structures. Meanwhile, our experimental implementation demonstrates the feasibility of this algorithm. Two fiber grating samples were tested for demonstrating the capability of the technique.

In Section 3.2 of this chapter, we discuss the theoretical background of the technique. Then, in Section 3.3 the procedure for reconstruction refractive index distribution is described. The reconstruction results of the two fiber gratings are shown in Section 3.4. Finally, discussions and conclusion are given in Section 3.5.

### 3.2 THEORETICAL ANALYSIS

For fiber gratings fabricated with UV laser, the index profile can be described by

\[ n(z) = n_o + \delta n(z) \left\{ h + \cos \left[ \frac{2\pi}{\Lambda_o} z + \phi(z) \right] \right\}, \quad (3.1) \]

which is derived formerly as Eq. (2.5), where \( n_o \) is the effective refractive index of the fiber before UV exposure, \( \delta n(z) \) is the envelope of index change, \( h \) is the background component of index change (\( h \cdot \delta n(z) \) is the dc-component of the index change), \( \Lambda_o \) is the nominal grating period, and \( \phi(z) \) describes grating chirp. For a linearly chirped grating, the phase term has the form
\( \phi(z) = \frac{-\pi}{\Lambda_o} \alpha \cdot z^2 \) \hspace{1cm} (3.2)

(\( \alpha = 0 \) for a uniform grating). If we define the complex amplitudes \( P(z,v) \) and \( Q(z,v) \) for the forward propagating wave field \( A(z,v) \) and the backward propagating wave field \( B(z,v) \) in the grating as

\[ A(z,v) = P(z,v) \cdot \exp[-j\sigma(z)z] \] \hspace{1cm} (3.3)

and

\[ B(z,v) = Q(z,v) \cdot \exp[j\sigma(z)z], \] \hspace{1cm} (3.4)

respectively, where

\[ \sigma(z) = \frac{2\pi}{\lambda} \cdot \delta n(z) \cdot h \] \hspace{1cm} (3.5)

is a slowly-varying propagation constant corresponding to the dc component of the index change, the coupled-mode equation can be reduced to [3.11]

\[ \frac{dP}{dz} = \bar{\kappa}^s \cdot Q \cdot \exp(j2\pi v z), \] \hspace{1cm} (3.6)

and

\[ \frac{dQ}{dz} = \bar{\kappa} \cdot P \cdot \exp(-j2\pi v z). \] \hspace{1cm} (3.7)

Here,

\[ v = \frac{\beta}{\pi} = \frac{1}{\Lambda_o} = 2n_o \left( \frac{1}{\lambda} - \frac{1}{\lambda_{Bo}} \right) \] \hspace{1cm} (3.8)

is the frequency detuning, \( \beta = \frac{2\pi \cdot n_o}{\lambda} \) is the background propagation constant, and \( \lambda_{Bo} = 2n_o\Lambda_o \) is the central Bragg wavelength. Meanwhile, the complex
coupling coefficient $\tilde{\kappa}(z)$, which contains the index change information, is given by

$$\tilde{\kappa}(z) = j \frac{\pi}{\lambda} \delta n(z) \cdot \exp\{-j[2\sigma(z)z + \phi(z)]\}.$$  \hspace{1cm} (3.9)

Combination of Eqs. (3.2) and (3.3) leads to a relation between the complex reflection spectrum and the complex coupling coefficient as

$$r(0,\nu) = -\int_0^L \tilde{\kappa}(z) \left[ 1 + \int_0^z \tilde{\kappa}^*(z)r(z',\nu) \exp(j2\pi\nu z')dz' \right] \exp(-j2\pi\nu z)dz,$$

where $r(z,\nu) = Q(z,\nu)/P(0,\nu)$ is the normalized backward propagating wave field and $r(0,\nu)$ is the complex reflection coefficient measured at the input end of the fiber grating. Equation (3.10) shows that the complex reflection spectrum is the Fourier transform of the quantity inside the curve brackets on the right hand side of the equation. After taking the inverse Fourier transform of Eq. (3.10), we can obtain an iterative relation as

$$\tilde{\kappa}_{m+1}(z) = -\mathcal{F}^{-1}\{r(0,\nu)\} - \mathcal{F}^{-1}\{p_m(\nu)\},$$  \hspace{1cm} (3.11)

where

$$p_m(\nu) = \int_0^L \tilde{\kappa}_m(z) \cdot \int_0^z \tilde{\kappa}_m^*(z')r_m(z',\nu) \exp[j2\pi\nu(z' - z)]dz'dz.$$  \hspace{1cm} (3.12)

Meanwhile, $\mathcal{F}$ denotes the Fourier transformation operator and the subscript $m = 0, 1, 2, \cdots$ is the iteration number. The normalized backward propagating field $r_m(z,\nu)$ is calculated through Eqs. (3.6) and (3.7) with $\tilde{\kappa}(z) = \tilde{\kappa}_m(z)$ at
each iteration. Equation (3.11) describes the algorithm for reconstructing the complex coupling coefficient $\tilde{\kappa}(z)$ from the complex reflection spectrum. The initial conditions in the algorithm are set to be $\tilde{\kappa}_0(z) = 0$ and $r_0(z,\nu) = 0$. Hence, with the procedure described above, we can obtain $\tilde{\kappa}(z)$ from the measured complex reflection spectrum from which the index profile of the fiber grating can be consequently obtained through Eq. (3.9).

### 3.3 Reconstruction Procedures

Figure 3.1 shows the experimental setup for measuring the amplitude and phase of the Bragg reflection coefficient. The system was a balanced Michelson interferometer with a test arm connected to the fiber grating to be characterized and a free-space reference arm in which signal was retro-reflected by a gold-coated mirror. A tunable laser diode with a wavelength resolution of 0.001 nm was used as the light source. We measured the fiber
Bragg reflection, interference, and reference intensities, respectively, by appropriately blocking light with the two attenuators. The Bragg reflection, interference, and reference intensities have the following relationships:

\[ |r| = \sqrt{I_{\text{reflect}}} \]  

\[ I_{\text{inter}} = I_{\text{ref}} + I_{\text{reflect}} + 2\sqrt{I_{\text{ref}}I_{\text{reflect}}} \cos(\phi_r) \]  

where \( I_{\text{reflect}} \), \( I_{\text{inter}} \), and \( I_{\text{ref}} \) are the Bragg reflection, interference, and reference intensities, respectively, normalized by the input intensity. Meanwhile, \(|r|\) and \(\phi_r\) are the amplitude and phase of the complex reflection coefficient at the position \( z = 0 \). By measuring the spectra of \( I_{\text{reflect}} \), \( I_{\text{inter}} \), and \( I_{\text{ref}} \), the phase part of the complex reflection coefficient can be retrieved through Eqs. (3.13) and (3.14) with a suitable curve fitting as described in [3.12]. In curve fitting, the uncertain sign of phase may cause ambiguous index distribution. However, causality, which requires the existence of grating in the region \( z \geq 0 \), can help us to choose the correct result.
After we have the complex reflection coefficient \( r(z, v) \), Eq. (3.11) is used to reconstruct the complex coupling coefficient \( \tilde{\kappa}(z) \). After we have \( \tilde{\kappa}(z) \), we can first obtain \( \delta n(z) \) by taking the absolute values of both sides of Eq. (3.9), i.e.,

\[
\delta n(z) = |\tilde{\kappa}(z)| \frac{\lambda}{\pi}.
\] (3.15)

Note that the dc component of index change and grating chirp are mixed in the phase of the complex coupling coefficient. We can separate these two quantities with the following assumptions: 1) the period of the fiber grating is either uniform or linearly chirped. 2) \( \delta n(z) \) is not a linear function. With the assumptions, the first-order derivative of the phase of the complex coupling coefficient can be a linear combination of the dc component of index change and grating chirp with the two parameters \( h \) and \( \alpha \) as

\[
\frac{d\phi_{\tilde{\kappa}}(z)}{dz} = \frac{d \arg(\tilde{\kappa}(z))}{dz} = -h \cdot \frac{2\pi}{\lambda} \delta n(z) + \frac{2\pi}{\Lambda_o} \alpha \cdot z.
\] (3.16)

Then, we can use linear programming method to fit Eq. (3.16) for separating these two quantities. Figure 3.2 shows the procedures of the retrieving algorithm.

### 3.4 Reconstruction Results

In our experiment, we measured two fiber gratings with different index
profiles. First, we measured a fiber grating fabricated with a chirped phase mask (chirp of the Bragg wavelength is 0.02 nm/mm). Apodization was assumed due to the nearly-Gaussian profile of the used UV laser beam. The grating length is about 6 mm.

Figure 3.3 shows the measured values of $I_{\text{reflect}}$, $I_{\text{inter}}$, and $I_{\text{ref}}$, respectively, of this grating. Figure 3.4 shows the retrieved amplitude and phase of the reflection coefficient (the continuous curve in the lower portion is drawn to fit the data points of filled circles). After several iterations of the reconstructing algorithm, we could obtain satisfactory result of the reconstructed complex coupling coefficient. Figure 3.5 shows the amplitude and phase of the reconstructed coupling coefficient of this grating, as functions of position in the grating. The reconstructed amplitude of the coupling coefficient after each iteration is also shown. The curve at the top represents the result after the last iteration. The result implies a nearly-$\exp(-|z|^3)$-apodized profile of index change.

Figure 3.6 shows the reconstructed profiles of the envelop of index change and the local Bragg wavelength along the fiber grating. The dc component of the index change was calculated as $h = 1.03$. The results agree very well with the fabrication conditions.
Fig. 3.3 Measured values of $I_{\text{reflect}}$, $I_{\text{inter}}$, and $I_{\text{ref}}$ respectively, of the chirped grating.

Fig. 3.4 Retrieved amplitude and phase of the reflection coefficient of the chirped grating.
Then, we consider a fiber grating made with a uniform phase mask. To demonstrate the capability of our reconstruction technique, we purposely fabricated the grating with a UV laser of two peaks in beam profile. Such a laser beam came from an excimer laser, which oscillated with a higher-order transverse mode. Therefore, we expect a depression of index modulation along the grating. Figure 3.7 shows the measured values of $I_{\text{reflect}}$, $I_{\text{inter}}$, and $I_{\text{ref}}$ of this grating, respectively. We can see that the measured reflection spectrum is
Fig. 3.7 Measured values of $I_{\text{reflect}}$, $I_{\text{inter}}$, and $I_{\text{ref}}$, respectively, of the uniform grating.

Fig. 3.8 Retrieved amplitude and phase of the reflection coefficient of the uniform grating.
asymmetric. This is contrary to what we expected from a uniform grating, however, is consistent with the expected index-modulation depression. Figure 3.8 shows the retrieved amplitude and phase of the reflection coefficient. Figure 3.9 shows the amplitude and phase of the reconstructed coupling coefficient, as functions of position in the grating.

The result indicates a depression of the UV laser beam profile and of the index modulation, as expected. Figure 3.10 shows the reconstructed profiles of
the envelop of index change and the local Bragg wavelength along the fiber grating. The constant Bragg wavelength near 1540.7 nm is consistent with the period of the phase mask. We can clearly see the index-modulation depression near \( z = 3 \) mm, which agrees well with the laser beam profile. The dc component of the index change in this fiber grating was calculated as \( h = 1 \).

### 3.5 Discussions

The reconstruction algorithm is based on the coupled-mode equations and is simplified by using the Fourier transformation. Due to the inherent characteristics of the Fourier transformation, the spatial resolution of the reconstructed index profile with this algorithm is limited by the measured spectral bandwidth. The spatial resolution \( \Delta z \) can be approximately expressed as

\[
\Delta z = \frac{0.441 \lambda_B^2}{\Delta \lambda}.
\]  

(3.17)

Here, \( \Delta \lambda \) is the spectral bandwidth of measurement and \( \lambda_B \) is the Bragg wavelength. Equation (3.17) implies that the wider spectral bandwidth we measure, the higher resolution of the index profile we can reconstruct. For example, when the spectral bandwidth of measurement is 4 nm (as the case in Fig. 3.3), the spatial resolution of the reconstructed index profile is approximately 0.26 mm.
The used algorithm fails when there is an abrupt change in the index profile or Bragg wavelength. In such a case, an additional measurement from the other side of the fiber grating can help to solve this problem [3.8].

3.6 Summary

In summary, we have demonstrated a novel technique for retrieving the refractive-index profile of a fiber grating with any well-behaved chirp, apodization and dc structure. Two purposely designed fiber gratings were used to demonstrate the capability of this technique. Currently, we are applying this method for in situ studying the evolution of the index profiles of fiber gratings during the fabrication process.

References to Chapter 3


