1. Alice’s utility of consuming \( x \) units of food and \( y \) units of clothing can be express as \( u_a(x, y) = \ln(x) + \ln(y) \). She has $100 to spend on these two goods. Answer the following questions.

(a) What is Alice’s marginal rate of substitution of food in terms of clothing when she buys \( x \) units of food and \( y \) units of clothing?

(b) When the price of food is \( p_x = 2 \) and the price of clothing is \( p_y = 5 \), how much food and clothing does she consume?

(c) What is Alice’s demand function for food when the price of clothing is fixed at \( p_y = 5 \)?

(d) Is food a normal good for Alice?

2. Alice eats vegetable (\( X \)) and “all other food” (\( Y \)). She always spends $1,000 on food every month. Her utility from consuming food can be expressed as \( u(x, y) = 80\sqrt{x} + y \), where \( x \) and \( y \) are quantities of vegetable and “all other food”, respectively.

(a) How much vegetable does Alice consume in a month when the price of vegetable is $10 and the price of “all other food” is $1? How much does she spend on all other food?

(b) Suppose a typhoon causes several damage on vegetable farms. How much vegetable does Alice consume in a month when the price of vegetable rises from $10 to $40 and the price of “all other food” remains at $1?

(c) How much is the substitution effect and income effect of the price change in (b) on the consumption of vegetable? (Please use Hicks’ definition on the decomposition.)

(d) Continue from (b). Suppose many people are dissatisfied with the effect caused by the higher vegetable price. Therefore, the government wants to choose one of the following two policies. Which one does the government spend more money? Why?

i. Subsidize Alice $30 on buying each unit of vegetable.
ii. Give Alice a cash subsidy. The amount is chosen wisely so that the happiness of Alice is unchanged by the price increase in vegetable.

3. Frank eats only two kinds of food: vegetable \((X)\) and rice \((Y)\). He always spends $150 on food every month. Her utility from consuming food can be expressed as \(u(x, y) = \sqrt{x} + \sqrt{y}\), where \(x\) and \(y\) are quantities of vegetable and rice, respectively. The price of vegetable is $2, and the price of rice is $1.

(a) What is Frank’s income elasticity of the demand for vegetable at the current prices?

(b) Are vegetable and rice substitutes or complements for Frank? Why?