公共選擇

- Social/public choice: process of social/collective decision-making

- Preference aggrgation mechanism:

  1. Social decision rule: collective ranking R of all alternatives A
     – Aggregation of individual preference $\{R_i\}$
     – Indv Ranking $\{R_i\}$ in, Social Ranking R out.
     (Eg) beauty contest, ice skating

  2. Social choice function (SCF): single choice
     – Indv Ranking $\{R_i\}$ in, Social Choice $a \in A$ out.
     (Eg) political election, travel destination choice

- Saari [1988] story: choice of drink in department meeting

15 voters	1st	2nd	3rd
6	Milk	Juice	Beer
5	Beer	Juice	Milk
4	Juice	Beer	Milk

  – "Milk" chosen initially as most favored (M6 : B5 : J4)
  – "Beer" served in meeting for lack of Milk
  – But people found that "Juice" (10) is preferred to "Beer" (5)
  – Further: "Milk" least favored by pairwise comparision (J9:M6, B9:M6)

- Unanimity rule (一致決)

  1. Wicksell [1896]: consistent with Pareto criterion
     ▷ Bill passed must make everyone better off!

2. Problems:
   – (Theory) Social ranking not "complete"! Agreement rarely reached!
   – (Reality) Distribution/jealousy issue not considered.
     ▷ Some may prefer non-Paretian situation.
   – (Reality) Everyone has veto power, transaction costs high

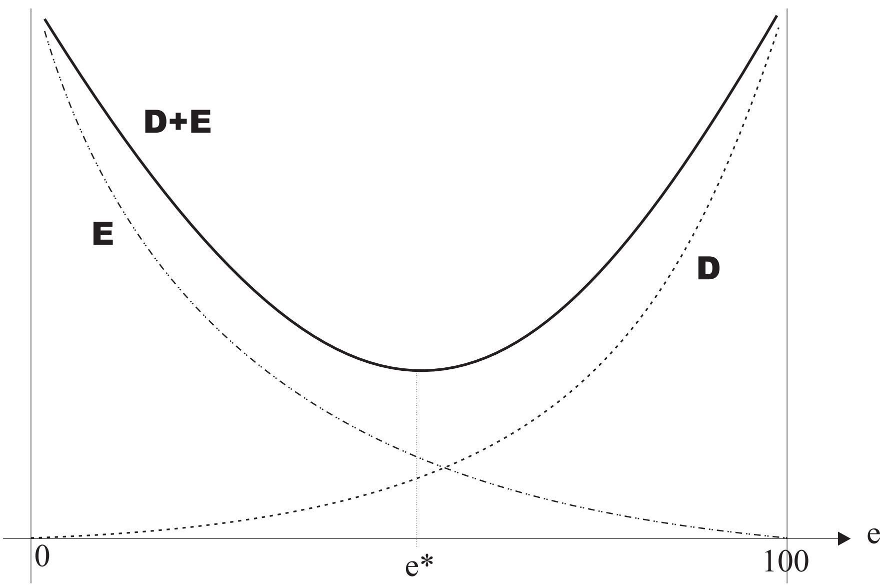
3. Unanimity with compensation: buying votes is illegal?

- Majority voting (多數決)

1. Relative majority (相對多數): $\eta\%$ ($\geq$ 50%)
   – Miminal total social costs [Buchanan-Tullock 1962]:

$$\min_{\eta} D + E$$

   – External costs (外部成本) E: damages imposed on minority
   – Decision costs (交易成本) D: costs for reaching decisions



2. Condorcet winner:
   – Binary agenda (pairwise comparision) for 3 or more options.
   – Winner against all other candidates.

3. Plurality rule: [Hindriks-Myles, 2006, MIT press, p.319]
   - Simultaneous majority voting for 3 or more candidates.
   - Condorcet winner may not be selected:

(9 voters)	1st	2nd	3rd
2	A	B	C
3	B	A	C
4	C	A	B

   ▷ C is the Plurality winner
   ▷ A is the Condorcet winner.

   - Strategic behavior[1]

4. May's Theorem: [Hindriks-Myles, 2006, MIT press, p.306]
   With only 2 options, only majority rule can satisfy:
   (a) Anonymity: symmetry among all voters (treated equally).
   (b) Neutrality: symmetry among all candidates.
   (c) Decisiveness: a winner will always be picked.
   (d) Positive responsiveness: more votes, more likely to win. □

5. Voting paradox [Condorcet 1785]:

Ranking	1st	2nd	3rd
Voter 1	A	B	C
Voter 2	B	C	A
Voter 3	C	A	B

   - Voting cycles:

$$A \succ_{1,3} B \succ_{1,2} C \succ_{2,3} A$$

   ▷ Outcome subject to "agenda manipulation"

   - Single-peaked preferences (單峰偏好) [Black]: Figure 1
   ▷ Applicable only to 1-dim

[1]For example, people may vote for 2nd choice, if they feel their top choice has no chance to win.

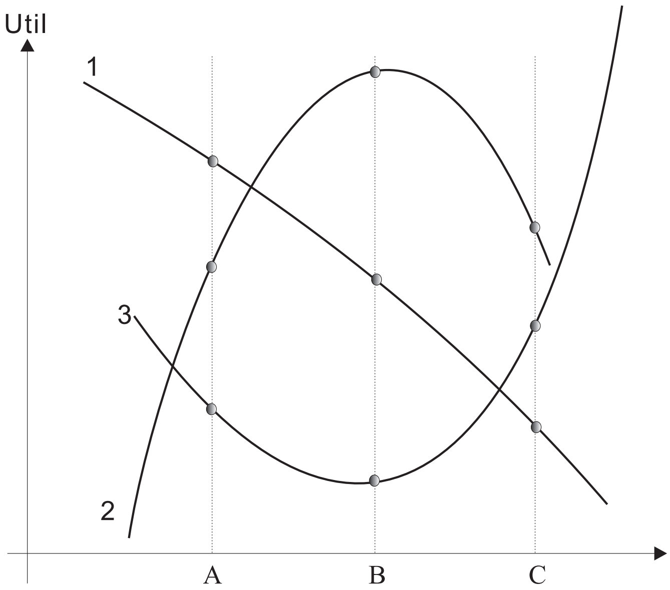


Figure 1: 1-dim preference: $A \succ B \succ C \succ A$

– Single-crossing preferences (單次交叉) [Hindriks-Myles, 2006, MIT, pp.310]

∗ Def: On a 1-dim line, for 2 voters $a < b$, and 2 options $x < y$: if

$$U^a(y) > U^a(x) \quad \Rightarrow \quad U^b(y) > U^b(x)$$

and

$$U^b(x) > U^b(y) \quad \Rightarrow \quad U^a(x) > U^a(y) \quad \square$$

∗ If voter preferences satisfy single-crossing, then there is no cycle.

∗ Condorcet winner is preferred option of the median voter M.[2]

– Cycle probability 1-2%; not detectable when it arises!

– 2-dim voting cycle Figure 2

$$A \succ_{1,3} C \succ_{2,3} B \succ_{1,2} A$$

(eg) 3 people dividing $1: no Condorcet winner!

Round	A	B	C
#1	1/3	1/3	1/3
#2	1/2	1/2	0
#3	2/3	0	1/3
#4	0	1/2	1/2
...			

6. Independence from Irrelevant Alternatives (IIA) may be violated

– (Example)

#voters / ranking	1st	2nd	3rd
9	A	B	C
4	B	C	A
6	C	B	A

– With all 3 candidates: (A9 : B4 : C6) ⇒ A elected

– If C drops out: (A9 : B10) ⇒ B elected

– Need IIA to avoid sabotage (攪局) ![3]

7. Outcome may be Pareto inferior!

[2]Because, for any 2 options $x < y$, if M prefers x, then all voters to his left will also prefer x. If M prefers y, then all voters to his right must also prefer y. □

[3]For example: Taipei city mayor election 1998, Presidential election 2000.

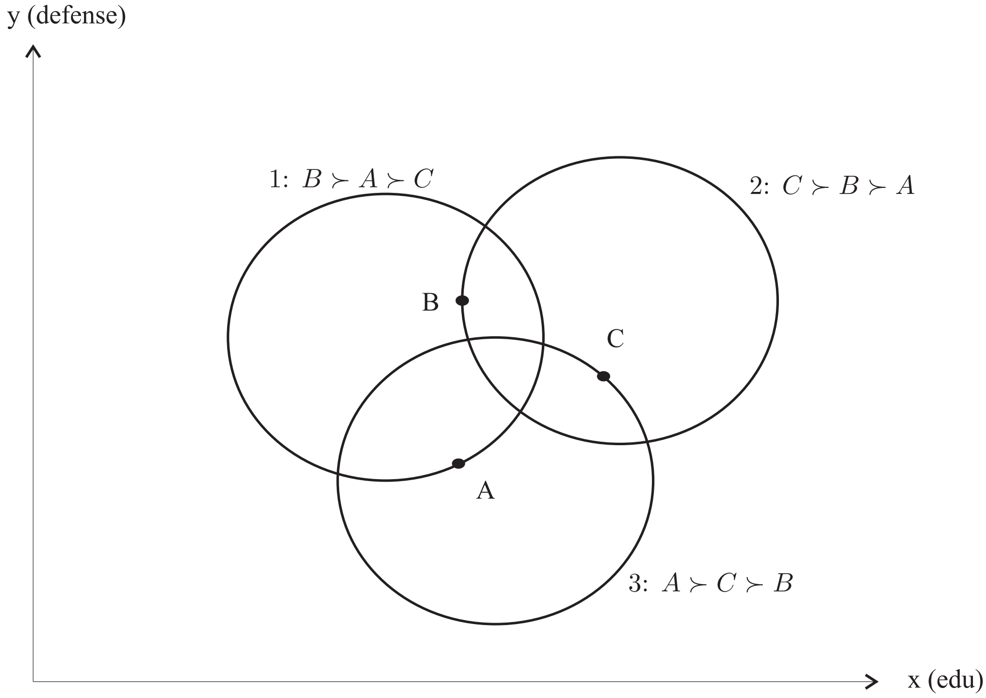


Figure 2: 2-dim preference: $A \succ C \succ B \succ A$

Ranking	1st	2nd	3rd	4th	5th	6th	7th
Voter 1	A	B	C	D	E	F	G
Voter 2	C	D	A	F	G	B	E
Voter 3	D	A	G	B	C	E	F

▷ Possible outcome: $A \to D \to C \to B \to G \to F \to E$

▷ E is Pareto inferior to (A, B, C, D) !

8. Voter preference intensity not considered:

   ▷ Logrolling (選票互換): vote trading/exchange

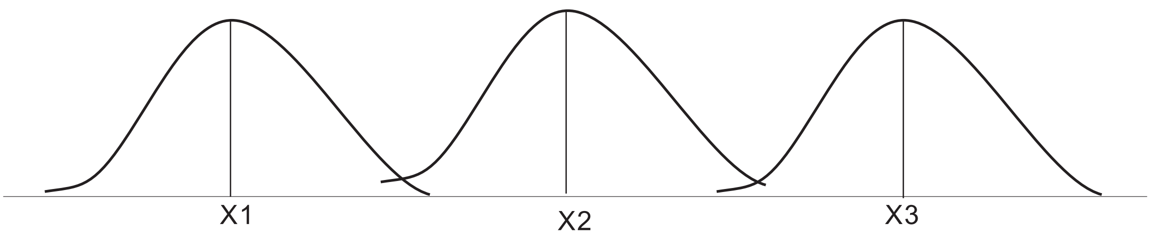
   – (Yes) Voter intensity revealed: compromise means efficiency!

(Project)	A	B	C	NetValue	M.V.	logrolling
Hospital	200	-50	-55	95	n	y (1,2)
Library	-40	150	-30	80	n	y (1,2), (2,3)
Park	-120	-60	400	220	n	y (2,3)

– (No) Special-interest gains may outweight general losses!

(Project)	A	B	C	NetValue	M.V.	logrolling
Hospital	200	-110	-105	-15	n	y (1,2)
Library	-40	150	-120	-10	n	y (1,2), (2,3)
Park	-270	-140	400	-10	n	y (2,3)

9. 64% mojority rule [Caplin-Nalibuff, Econometrica 1988]
   – In k-dim elections, incumbent can garantee only:
   $$\sigma_k = \left(\frac{k}{k+1}\right)^k$$
   ▷ For example: $\sigma_1 = 1/2$, $\sigma_2 = 4/9$
   – In real-life elections, a challenger will get at least:
   $$\sigma_\infty = \lim_{k\to\infty}\left[1-\left(\frac{k}{k+1}\right)^k\right] = 1-\frac{1}{e} \approx 64\% \quad \square$$
10. Median Voter Theorem (中值選民定理) [Holcombe pp.175–76; Hyman p.165]
    – M.V. outcome reflects preference of the median voter:



▷ X_2 chosen by majority

– Outcome usually inefficient!

- Borda count (包達計數法)

1. Counting pocedure: choose one with lowest count ⇒ no cycles

#voters	Keynes	Becker	Chair
10 Macro	1	2	3
10 Micro	2	1	3
1 Chair	2	3	1
Rank / Score	1(32)	2(33)	3(61)

▷ May set rank values to reflect relative weights (eg, 1,2,3,10,...)
▷ Similar to pairwise comparision: win (+1), lose (-1), tie (0) [Copeland rule]

2. Problems:

– Strategic manipulation:
(eg) 10 Micros now claim [Chair as 2nd, Keynes as 3rd]

#voters	Keynes	Becker	Chair
10 Macro	1	2	3
10 Micro	3	1	2
1 Chair	2	3	1
Rank / Score	2(42)	1(33)	3(51)

– IIA violated: different outcomes w/w.o. chair

#voters	Keynes	Becker
10 Macro	1	2
10 Micro	2	1
1 Chair	1	2
Rank / Score	1(31)	2(32)

- Arrow's Impossibility Theorem [1951] (不可能定理):
  1. Axiomatic approach
  2. No social decision rule can guarantee satisfaction of the following:
     - Universality (全域性): Voters may have any preference patterns.
     - Consistency (一致性): social preference is transitive, no cycle.
     - Pareto axiom
     - IIA (Independence of Irrelevant Alternatives)
     - Non-dictatorship
  3. Use of cardinal social welfare functions: measurement problem.
  4. Satherswaite Theorem: strategy-proofness required (instead of IIA).