Social Choice — Spatial Models

1 Individual Preference R^i on Space X

Def 1: Better/worse sets

(1) Upper contour set: $P_i(x) \equiv \{y \in X \mid {}_yP^i{}_x\}$, $R_i(x) \equiv \{y \in X \mid {}_yR^i{}_x\}$

(2) Lower contour set: $\tilde{P}_i(x) \equiv \{y \in X \mid {}_xP^i{}_y\}$, $\tilde{R}_i(x) \equiv \{y \in X \mid {}_xR^i{}_y\}$

(3) Indifference set: $I_i(x) \equiv \{y \in X \mid {}_yI^i{}_x\}$

Def 2 (Continuity) For individual preference R^i on domain X:

(1) R^i is upper continuous (`UC`) iff $\forall\, x \in X$, $P_i(x)$ is open [or $\tilde{R}_i(x)$ is closed]

(2) R^i is lower continuous (`LC`) iff $\forall\, x \in X$, $\tilde{P}_i(x)$ is open [or $R_i(x)$ is closed]

(3) R^i is continuous iff it is both `UC` and `LC`.

Condition F: R^i such that, for any *finite* set $S \subseteq X$, $\exists\, x \in X$: ${}_xR^i{}_y$, $\forall\, y \in S$.

Thm (Fan) If R^i is `LC`, then: R^i satisfies condition F iff $M(R^i, S) \neq \emptyset$, $\forall\, S \subseteq X$. ■

$\triangleright$ $M(R, S)$ is defined for *any* sets, including *infinite* sets![1]

Def 3: Convex combination:

$$z \;=\; \lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n, \quad \lambda_i \geq 0, \quad \sum \lambda_i = 1$$

(1) Convex set: any convex combination of elements in S is also in S.

(2) Convex hull: $\mathrm{Hull}(S) \equiv$ minimal convex set containing S.

Def 4 (Convexity) For individual preference R^i on convex set X:

(1) R^i is strictly convex iff:

$${}_xR^i{}_y \implies {}_{(\lambda x + [1-\lambda] y)}P^i{}_y, \ \forall\, \lambda \in (0, 1)$$

(2) R^i is semi-convex iff:

$$\forall\, x \in X, \quad x \notin \ \mathrm{Hull}(P_i(x))$$

Lmm: If individual preference R^i is strictly convex, then:

(1) it is semi-convex. [Pf: $x \in \mathrm{Hull}(P_i(x)) \Rightarrow {}_xP^i{}_x$ $\not\rightsquigarrow$]

(2) both $R_i(x)$ and $P_i(x)$ are convex sets for any $x \in X$.

(3) indifference set $I_i(x)$ cannot be a thick stripe.

(4) if $M(R^i, S) \neq \emptyset$, then $|M(R^i, S)| = 1$. [ie, $|M(R^i, S)| = 0$ or 1] ■

[1] For finite X, `COMP` and `ACYC` are enough for $M(R, S) \neq \emptyset$. But for infinite X, `LC` is required.

Def 5 (Compactness) Individual preference R^i is:

(1) compact if contour set $R_i(x)$ is compact for all x.

(2) CCC if it is continuous, convex, and compact.

Def 6: Utility function: $u_i(\cdot)$ such that: $u_i(x) > u_i(y) \rightleftharpoons {}_xP^i{}_y$

$\triangleright$ $u_i(\cdot)$ is strictly quasi-concave iff R^i is strictly convex.

Lmm (Fan) If X is compact and convex, and R^i is LC and semi-convex, then R^i satisfies condition F on X. And hence $M(R^i, \cdot) \neq \emptyset$ [by Fan's Thm]. ■

Lmm (McKelvey 1979:Econ) For relation R^i that is CCC:

(1) $P_i(x)$ is open.

(2) $I_i(x)$ is closed without interior (ie: thin indifference sets). ■

2 Collective Preference on K-dim $X^{(K)} \subseteq \mathcal{R}^K$

Def 7 (Core) $C_f(\rho, X) \equiv M(f(\rho), X)$

$\triangleright$ If x is in core, then: $\nexists y \in X$, ${}_yP_x$.

$\triangleright$ Each $x \in C_f(\rho, X)$ is a Condorcet winner.

Def 8 (Coalition contour set) For any non-empty coalition $L \subseteq N$ and $x \in X^{(K)}$:

(1) Common better set $P_L(x) \equiv \bigcap_{i\in L} P_i(x)$: $y \in P_L(x) \Rightarrow \forall\, i \in L, {}_yP^i{}_x$

(2) Common worse set $\tilde{P}_L(x) \equiv \bigcap_{i\in L} \tilde{P}_i(x)$: $y \in \tilde{P}_L(x) \Rightarrow \forall\, i \in L, {}_xP^i{}_y$

Def 9 (Collective contour set) For a given simple rule f and $x \in X^{(K)}$:

(1) Win set $P_f(x) \equiv \{y \in X \mid {}_yP_x\} = \bigcup_{L\in\mathcal{L}(f)} P_L(x)$

(2) Lose set $\tilde{P}_f(x) \equiv \{y \in X \mid {}_xP_y\} = \bigcup_{L\in\mathcal{L}(f)} \tilde{P}_L(x)$

(3) Tie set $I_f(x) \equiv \{y \in X \mid {}_yI_x\}$

Lmm (McKelvey 1979) For preference R^i ($\forall\, i$) that is CCC and plurality rule f:

(1) $P(x)$ and $\tilde{P}(x)$ is open.

(2) $I(x)$ is closed without interior (ie: thin tie sets). ■

Lmm (Thin Tie Sets) (McKelvey 1979) Let R be st-convex and continuous. If $y \in I(x)$, then for every neighborhood δ_y around y:

$$\delta_y \cap P(x) \neq \emptyset \quad \text{and} \quad \delta_y \cap \tilde{P}(x) \neq \emptyset \quad ■$$

$\triangleright$ If ${}_yI_x$, then $[\exists\, z \in \delta_y : {}_zP_x$ and $\exists\, w \in \delta_y : {}_xP_w]$

Lmm: Let $\rho = (R^1, \ldots, R^n)$ and all R^i be `LC`, then any simple rule $f(\rho)$ is `LC`. ■ [2]

Lmm: Let domain $X^{(K)}$ be compact and convex, and f be a simple rule with $K \leq v(f) - 2$. Then if profile ρ is semi-convex, $f(\rho)$ is also semi-convex. ■

Cor (Schofield) Let domain $X^{(K)}$ be compact and convex, and profile ρ be `LC` and semi-convex. Then for any simple rule f with $K \leq v(f) - 2$, core $C_f(\rho, X) \neq \emptyset$. ■

Thm (Schofield) For any non-collegial simple rule f with $K \geq v(f) - 1$, there exists a continuously differentiable st-convex profile ρ with $C_f(\rho, X) = \emptyset$. ■

▷ For majority rule f, $v(f) = 3$, so $K = 1$ is required for $C_f(\rho, X) \neq \emptyset$.

▷ For $n = 3$ with different ideal points x_i^* on $X^{(2)}$, majority rules have empty core.

3 Induced Preferences

Def 10: Lines and half-lines:

$\gamma(x, y) \equiv \{z \in \mathcal{R}^K \mid \exists\, t \in \mathcal{R}, z = tx + [1-t]y\}$: line through x and y in $\mathcal{R}^K$

$\gamma_x \equiv$ a line through x in $\mathcal{R}^K$

$\Gamma_x \equiv$ set of all lines through x in $\mathcal{R}^K$

$h_y^+(\gamma_x)$, $h_y^-(\gamma_x) \equiv$ *open half lines* of γ_x divided by point y

Def 11: Let R^i be continuous and st-convex, and X be compact and convex. Then for any $x \in X$, i's induced ideal point on line γ_x is:

$$b^i(\gamma_x) \equiv \{z \in (\gamma_x \cap X) \mid {}_zR^i{}_y, \forall\, y \in (\gamma_x \cap X)\}$$

Def 12: Half-line coalitions divided by y on γ_x:

$$L_y^+(\gamma_x) \equiv \{i \in N \mid b^i(\gamma_x) \in h_y^+(\gamma_x)\}, \quad L_y^-(\gamma_x) \equiv \{i \in N \mid b^i(\gamma_x) \in h_y^-(\gamma_x)\}$$

Def 13: Induced f-median on γ_x: For simple rule f and $x \in X$,

$$\mu_f(\gamma_x) \equiv \{\, y \in (\gamma_x \cap X) \mid L_y^+(\gamma_x) \notin \mathcal{L}(f) \quad \text{and} \quad L_y^-(\gamma_x) \notin \mathcal{L}(f)\,\}$$

Thm (Cox 1987) Let f be simple and ρ be continuous and st-convex, then:

$$x \in C_f(\rho, X) \quad \rightleftharpoons \quad x \in \mu_f(\gamma_x), \ \ \forall\, \gamma_x \in \Gamma_x \quad ■$$

▷ Core requires radial symmetry among voters: *median in all directions.*

[2] Proof: $\tilde{P}_L(x)$ is finite intersection of open sets $\{\tilde{P}_i(x)\}$ over i, so $\tilde{P}_L(x)$ is open. Then $\tilde{P}_f(x)$, which is union of $\tilde{P}_L(x)$ over L, is open.

Remark 1: When the core does not exist for sure, it *rarely* exists.

– Voters' ideal points must line up symmetrically.
– Plott [1967:AER]: extension to non-Euclidean preferences.
– McKelvey/Schofield [1987:Econ]: no coalition can agree on where to move together.

Remark 2: When the core does not exist for sure, it is *fragile* even if it exits.[3]

EX: Non-empty cores: 4 voters at corners of a square.

4 Median Voter Theorem (for 1-dim Choices)

Def 14 (Single-peakedness) Let $X \subset \mathcal{R}$. A profile $\rho \in \Psi^n$ is single-peaked (SP) iff:

There exists an *order* of X on $\mathcal{R}$ such that $\forall\, i \in N$, $\exists\, x_i^* \in X$:

(1) ${}_{x_i^*}P^i{}_y$, $\forall\, y \in X$
(2) $y < z < x_i^* \;\Rightarrow\; {}_zP^i{}_y$
(3) $x_i^* < z < y \;\Rightarrow\; {}_zP^i{}_y$

Lmm: Let $X \subset \mathcal{R}$ be convex and $R^i \in \Psi$, then:

(1) if individual preference R^i is SP, then R^i is strictly convex.
(2) if R^i is strictly convex and $M(R^i, X) \neq \emptyset$, then R is SP. ■

Thm (Median Voter) (Black 1958) Let f be simple and $X \subset \mathcal{R}$. Then if ρ is SP and continuous on X, then $C_f(\rho, X) = \mu_f(\rho, X) \neq \emptyset$. ■

5 Sincere/Myopic Voting (under Centralized Agenda Setting)

Def 15a: Hyperplanes and half-spaces:

(1) Hyperplane: $H_{y,c} \equiv \{x \in \mathcal{R}^K \mid x \cdot y = c\}$ for vector $y \in \mathcal{R}^K$ and scalar $c \in \mathcal{R}$.[4]
(2) Open half space: $H^+_{y,c} \equiv \{x \in \mathcal{R}^K \mid x \cdot y > c\}$, $H^-_{y,c} \equiv \{x \in \mathcal{R}^K \mid x \cdot y < c\}$
(3) Closed half space: $\bar{H}^+_{y,c} \equiv \{x \in \mathcal{R}^K \mid x \cdot y \geq c\}$, $\bar{H}^-_{y,c} \equiv \{x \in \mathcal{R}^K \mid x \cdot y \leq c\}$

Def 15b (Median hyperplane) $H_{y,c}$ with $|\{i \mid x_i^* \in H^+_{y,c}\}| \leq \frac{n}{2}$ and $|\{i \mid x_i^* \in H^-_{y,c}\}| \leq \frac{n}{2}$

$\triangleright$ Convention: denoted H_y with $\| y \| = 1$ and minimal c.

Def 15c (Total median) x^* is a TM if $\forall\, y$, $\exists$ median hyperplane $H_{y,c}$ with $x^* \in H_{y,c}$

Def 15d: A total median x^* is strong if $H_{y,c}$ is unique for all y.

[3] That is, it will be gone with just a little perturbation.
[4] It is a plane perpendicular to vector y.

Thm (Davis/Degroot/Hinich 1972:Econ) For any majority rule f:

(1) There exists a total median iff:

$$\bigcap_y \bar{H}_y^+ \neq \emptyset$$

(2) x^* is a total median iff $x \in C_f(\rho, X)$.

(3) If x^* is strong, then social order R is transitive on X:

$${}_xR_y \iff \| x - x^* \| \leq \| y - x^* \| \quad \blacksquare$$

Remark: For majority rules and Euclidean individual preferences:

- n odd: `TM` is unique and strong, and hence R is transitive.
- n even: `TM` not strong, and R not transitive

(Eg) x_i^* on 4 corners of a square: `TM` $x^* = \frac{\sum x_i^*}{4}$ is not strong, and R not transitive.

Lmm (Helley) Let $H_1, \ldots, H_{K+m}$ $(m > 0)$ be compact and convex sets in $\mathcal{R}^K$, If intersection of every sub-family of $(K+1)$ sets is non-empty, then $H_1 \cap \cdots \cap H_{K+m} \neq \emptyset$. $\blacksquare$

Thm (Chaos) (McKelvey 1976:JET) Let n (≥ 3) be finite and X $(\subset \mathcal{R}^K)$ be compact and convex. Individual preference $u_i : X \mapsto \mathcal{R}$ is Euclidean:

$$u_i(x) = \phi_i(\| x - x^* \|) \quad \text{where} \quad \phi_i'(\cdot) < 0$$

For any majority rule f, if $C_f(\rho, X) = \emptyset$, then for any $x, y \in X$, there exists a finite sequence $z_0, \ldots, z_T \in X$ such that (i) $z_0 = x$, $z_T = y$; and (ii) ${}_{z_{t+1}}P_{z_t}$, $0 \leq t < T$. $\blacksquare$

$\triangleright$ *Global cycling*: majority rule may wander anywhere with a naive voting body!

$\triangleright$ For open decentralized agenda formation, no equilibrium exists!

$\triangleright$ Generalization of core: uncovered set (McKelvey 1986)

6 Sophisticated Voting (under Centralized Agenda Setting)

EX: Committee chair with tie-breaking power. [Farguharson 1969]

Congress voting on pay raise.

Def 16: Binary agenda under amendment process: $\mathcal{B} = (x_1, \ldots, x_t)$ with $x_1 \equiv$ status quo

(1) Forward agenda: $(x_1, \ldots, x_t) = (((((x_1, x_2), x_3), \cdots), x_t)$

(2) Backward agenda: $(x_1, \ldots, x_t) = (x_1, \cdots, (x_4, (x_5, x_6)))))$

Def 17: Sophisticated voting [Farguharson 1969]: iterated elimination of dominated strategies

Multi-stage sophisticated voting [McKelvey/Niemi 1978:JET]

Def 18: For a forward binary agenda $\mathcal{B} = (x_1 \ldots, x_t)$, its sophisticated equivalent (SEQ) $\mathcal{Z} = (z_1 \ldots, z_t)$ is constructed as:

$$(i)\;\; z_t = x_t; \qquad (ii)\;\; \text{for } i < t,\; z_i = \begin{cases} x_i, & \text{if } {}_{x_i}P_{z_j},\ \forall\, j > i \\ z_{t+1}, & \text{otherwise} \end{cases}$$

Thm (Equivalence) (Shepsle/Weingast 1984:AJPS) For a forward binary agenda $\mathcal{B}$, the first element z_1 of its SEQ $\mathcal{Z}$ identifies the sophisticated outcome. And z_1 is called the sophisticated voting equilibrium (SVE). ■

Thm (Intersecting Win-sets) (S/W 1984:AJPS Thm 2) For agenda $\mathcal{B}$ of length t, its SVE z_1 satisfies:

$$z_1 \;\in\; \bigcap_{j=2}^{t} P(z_j) \quad ■$$

Def 19: x dominates y $[{}_xD_y]$ iff $P(x) \subseteq P(y)$ and $R(x) \subseteq R(y)$

Undominated set $\mathrm{UD}(X) \equiv \{x \in X \mid \nexists\, y \in X : {}_yD_x\} = \{x \mid \forall\, y \in X : \sim {}_yD_x\}$

$D(x) \equiv \{y \in X \mid {}_yD_x\}$

$\tilde{D}(x) \equiv \{y \in X \mid {}_xD_y\}$

Def 20: x covers y $[{}_xC_y]$ iff ${}_xP_y$ and $P(x) \subset P(y)$ [S/W 1984]

Uncovered set $\mathrm{UC}(x) \equiv \{y \in X \mid \sim {}_xC_y\}$

Uncovered set $\mathrm{UC}(X) \equiv \{x \in X \mid \nexists\, y \in X : {}_yC_x\} = \{x \mid \forall\, y \in X : \sim {}_yC_x\}$

▷ ${}_xC_y \;\rightleftharpoons\; {}_xP_y$ and ${}_xD_y$ [S/W 1984 footnote 8]

▷ $C_f(\rho, X) \;\subseteq \mathrm{UC}(X)$ [$\because\; {}_xR_y \;\Rightarrow \sim {}_yC_x$]

▷ $\mathrm{UD}(X) \subseteq \mathrm{UC}(X)$ [$\because \sim {}_yD_x \;\Rightarrow \sim {}_yC_x$]

▷ $C_f(\rho, X) \nsubseteq \mathrm{UD}(X)$, $\mathrm{UD}(X) \nsubseteq C_f(\rho, X)$

Lmm (McKelvey 1986:AJPS Prop 3) For continuous and convex R^i:

(1) Relation C and D are `SYM`, `IRR`, `TRAN`, and `ACYC`.

(2) Set $D(x)$ is closed for any $x \in X$. ■

Lmm (S/W 1984 Lmm1) $P(y) \;\subseteq\; P(x) \;\Longrightarrow\; \tilde{P}(x) \;\subseteq\; \tilde{P}(y)$ ■

Lmm (S/W 1984 Lmm2) $y \in P(x)$ but $\sim {}_yC_x \;\Longrightarrow\; P(y) \cap \tilde{P}(x) \neq \emptyset$ ■

▷ $P(y) \;\not\subset\; P(x) \;\Longrightarrow\; P(y) \cap \tilde{P}(x) \neq \emptyset$

▷ $P(y) \;\not\subset\; P(x) \;\Longrightarrow\; \exists\, z : {}_xP_zP_y$

Thm (S/W 1984:AJPS Thm 3) For any $x, y \in X$, there exists a finite agenda $\mathcal{B}$ with y being the first element and x being its SVE, iff $\sim {}_yC_x$. ■

▷ Can reach any point uncovered by y through a binary agenda.

▷ Any point in $\mathrm{UC}(y)$ can be reached as an SVE.

▷ For *open agenda processes*, the core is $\mathrm{UC}(X)$.

Thm (2-step Principle) (S/W 1984:AJPS Cor 3.1) Starting with y, for any point x that is the SVE of some finite agenda, there is an agenda that can produce x in at most two steps. ■

Def 21: Relation Q is a chain (or total order) on X iff Q is `COMP`, `IRR`, and `TRAN`.
▷ x is a maximal element with regards to relation Q iff $\forall y \in X, \sim {}_yQ_x$.
▷ Set S ($\subseteq X$) has an *upper bound* iff $\exists y \in X, \forall x \in S, {}_yQ_x$.

Lmm (Zorn) For any relation Q on X, if all chains on X are upper-bounded, then X has a maximal element with regards to Q. ■

Thm (McKelvey 1986:AJPS Thm 1) If X is compact, and individual preferences are continuous and convex, then $\mathrm{UD}(X) \neq \emptyset$ and $\mathrm{UC}(X) \neq \emptyset$. ■

Remark: S/W results not applicable to non-binary agendas.
EX: 3 voters, 7 alternatives, majority rule:
R^1: $a \succ c \succ z \succ b \succ y \succ x \succ q$
R^2: $b \succ y \succ c \succ x \succ a \succ z \succ q$
R^3: $x \succ z \succ a \succ y \succ c \succ b \succ q$
$\Longrightarrow$ z is SVE, but covered by a.[5]

Remark: For two-candidate Downsian competition, candidates are located in core.[6]

7 Structure-induced Equilibrium (SIE)

Assumptions: Policy space X ($\subseteq \mathcal{R}^K$) is compact and strictly convex. Individual preferences are continuous and strictly convex.

Def 22: $V_j(x) \equiv \{y \in X \,|\, y = x + \lambda e_j, \lambda \in \mathcal{R}\}$, where e_j ($\in \mathcal{R}^K$) is the dim-j basis vector.
$S_j(x) \equiv \{y \in V_j(x) \,|\, \nexists z \in V_j(x),\ {}_zP_y\}$: collective choice on $V_j(x)$.
▷ (Kramer 1972:JMS Lmm 3) $S_j(x)$ is non-empty, compact and convex for simple f.

Def 23 (Issue-by-issue core) $C^I_f(\rho, X) \equiv \{x \in X \,|\, x \in S_j(x),\ \forall j = 1, \ldots, K\}$

Thm (Kramer 1972:JMS Thm 1′) For issue-by-issue voting, $C^I_f(\rho, X) \neq \emptyset$. That is,

$$\exists x^* \in X : \quad x^* \in \bigcap_{j=1,\ldots,K} S_j(x^*) \quad ■$$

▷ $C^I_f(\rho, X)$ may not be SVE. (Order of issues matters.)
▷ [EX] Congress is a committee system: a decisive coalition for each issue.

[5] Since ${}_aP_z$ and $\{x\} = P(a) \subset P(z) = \{x, a, c\}$, we have ${}_aC_z$.
[6] Both candidates will choose the median voter's ideal point x^* as their platform, so outcome is in core.

Remark: (Shepsle 1979:AJPS Ex 3.1)

- TM may not be in core $C_f^I(\rho, X)$.
- The core may be outside the Pareto set.

Def 24: Individual preferences are additively separable on $X^{(K)}$ if:

$$u(x_1, \cdots, x_K) \equiv u_1(x_1) + \cdots + u_K(x_K)$$

$\triangleright$ Optimal x_i is independent of $x_j, \forall\, j \neq i$.

Thm (Kramer 1972:JMS Thm 2) For separable preferences, if $x^* \in C_f^I(\rho, X)$, then x^* is an issue-by-issue SVE. ■

$\triangleright$ Order of issues does not matter.
$\triangleright$ Reconsideration of issues does not matter.
$\triangleright$ Simultaneous or sequential consideration does not matter.

8 Constitutional Design

Def 25: $C_f(\rho, X, \Upsilon, \beta)$: core of the constitutional design game

State-dependent preference ρ over outcomes
Outcome function Υ: $s_1 \times \cdots \times s_n \mapsto X$, with $s_i \equiv$ strategy set of i
Behavioral model β: DSE, Nash, admissible Nash, Baysian Nash, SPE, etc.
Game design $\varphi(\rho): \Psi^n \mapsto X$
$\triangleright$ Assume $|C_f(\rho, X, \Upsilon, \beta)| = 1$.

Ex (Solomon Game) (Moore 1992) 2 women A, B fighting for a baby:

- Possible outcomes:
  - a: A gets the baby
  - b: B gets the baby
  - c: Baby cut in halves
  - d: A and B both cut in halves
- Preferences:
  - State 0 (A is mother): A: $a \succ b \succ c \succ d$; B: $b \succ c \succ a \succ d$
  - State 1 (B is mother): A: $a \succ c \succ b \succ d$; B: $b \succ a \succ c \succ d$
- Game design of Solomon: $\varphi(0) = a$, $\varphi(1) = b$

$\Longrightarrow$ Game not implementable in Nash!

Thm (Gibbard/Satterthwaite) Let X be finite with $|X| \geq 3$. Then φ is implementable in *dominant strategy* (DS) iff φ is dictatorial. ■

Thm (Zhou) Let $X \subseteq \mathcal{R}^K$ $(K \geq 2)$ be compact and convex, and preferences ρ be continuous and st-convex. Then φ is implementable in DS only if φ is dictatorial. ■

Condition NV (NoVeto) $\forall\, x, y \in X, \forall\, \rho \in R^n$: $|R(x, y; \rho)| = n - 1 \;\Rightarrow\; x \in \varphi(\rho)$.

Condition MM (Maskin Monotonicity) $\forall\, x, y \in X, \forall\, \rho, \rho' \in R^n$:

$$x \in \varphi(\rho) \quad \text{and} \quad R(x, y; \rho) \subseteq R(x, y; \rho') \implies x \in \varphi(\rho')$$

Def 26: φ is constant if $\forall\, \rho, \rho' \in R^n,\ \varphi(\rho) = \varphi(\rho')$

Lmm: φ is constant iff it is MM. ■

Thm (Maskin) Suppose φ satisfies NV, and $n \geq 3$. Then φ is `Nash-implementable` iff it satisfies MM. ■
▷ The Solomon game violates condition MM.

Thm (Palfrey/Srivastava) Let $|X| \geq 3$, and no player is completely indifferent over all alternatives. Then any φ that satisfies NV is implementable in *admissible Nash*. ■

Remark: The Solomon game is implementable in admissible Nash.
<Strategy> A and B simultaneously announce state and an integer.
<Rule> If announced states disagree, outcome is d. Otherwise, ...
<Equilibrium> Both announce true state and say 1.

9 Related Research

- Shepsle (1979 AJPS): Simple institutional arrangement (SIA)
  — gate-keeping committee
  — floor amendment via majority under germaneness rule

- Laver/Shepsle, Austin-Smith/Banks (1990 APSR): Portfolio allocation

- Romer/Rothenthal (1979): Propose-pivot paradigm
  – Proposer has proposal power: will propose own ideal point
  – Outcome distorted in favor of proposer
  – Maybe somewhat balanced by veto power of other members

- Baron/Ferejohn (1989): Pork barrel

- Groseclose/Snyder (1995), Diermeier/Myerson (1995): Vote-buying