# 配對理論

## 1 Applications

- $\bullet$  Job search
- School application
- Internship of medical students
- Master/angel game
- Marriage

# 2 Social Welfare Maximization

• 3-pair example: male (A, B, C), female (a, b, c)

Utility (M, F)	a	b	с
А	(1, 9)	5, 2	$3, 6^{*}$
В	4, 4	0, 0	(4, 5)
С	3, 2	(6, 9)	1,  1

#### • SW-max mating:

$$(A, a), (B, c), (C, b)$$

- Not stable: (A, c) will have an affair!
  - Male 'A': prefers c (util 3) to his wife a (util 1)
  - Female 'c': prefers A (util 6) to her husband B (util 5)

#### 3 Stable Matching

- Gale-Shapley algorithm<sup>1</sup>
- Proposer advantage:

Ranking	a	b
А	(1, 2)	(2, 1)
В	(2, 1)	(1, 2)

- Male proposing: ((A, a), (B, b)), men get 1st choice
- Female proposing: ((A, b), (B, a)), women get 1st choice
- Strategic behaviors:

Ranking	a	b	
А	(1, 2)	(2, 1)	•••
В	(2, 1)	(1, 2)	•••
С	(1, n)	(2, n)	••••
D	(2, n)	(1, n)	
•••			•••

- Round 1: 'A' and 'C' propose to 'a'; 'B' and 'D' propose to 'b'
  ▷ 'a' and 'b' collude and take 'C' and 'D' respectively
- Round 2: male 'A' and 'B' turn to their 2nd choice 'b' and 'a' respectively

 $\triangleright$  Now 'a' and 'b' get their top choice

### 4 Strategy-proof Matching?

<sup>&</sup>lt;sup>1</sup>Gale, D. and L. Shapley, "College Admissions and the Stability of Marriage," American Math. Monthly, 69:9–15.

### 5 Rich Old Men v. Pretty Young Women

- Bergstrom/Bagnoli:「男人選美麗,女人選財力」<sup>2</sup>
- Information asymmetry:

	Value	Info	Timing
Male	financial	private	wait or not?
Female	reproduction	public	don't wait

• Separating Equilibrium Strategy:

	Strategy	Mate
LQ male	Get married young	Young LQ woman (random)
HQ male	Wait till old	Young HQ woman (sorted)
LQ female	Get married young	Young LQ man (random)
HQ female	Get married young	Old HQ man (sorted)



<sup>&</sup>lt;sup>2</sup>Bergstrom, T.C. and M. Bagnoli, "Courtship as a Waiting Game," 1993, JPE, 101(1):185–202.