

## 成本效益分析

### 1 Project Evaluation

- Welfare Economics Approach:

- ▷ Compute SW function before/after project

- Enormous information required

- Interpersonal utility comparison

- Cost-Benefit Analysis: a practical procedure

- Stream of benefits and costs:

$$(B_0, B_1, \dots, B_T), (C_0, C_1, \dots, C_T)$$

- Determine net benefit of a projects

- For Pareto/fairness: losers should be compensated

- Intangibles: life, environmental quality

## 2 Long-term Time Consideratns

- Present value (PV): discount rate  $r$

- $r$  reflects opportunity cost of funding (market interest rate)
- Time discounting: \$1 now is worth  $\$[1+r]$  in next period
- Present value of \$1 in  $t^{\text{th}}$  year:

$$\frac{1}{[1+r]^t}$$

- Time-varying discount rate:

$$PV = \frac{1}{[1+r_1][1+r_2]\cdots[1+r_t]}$$

- Stream of returns:

$$PV = R_0 + \frac{R_1}{[1+r]} + \frac{R_2}{[1+r]^2} + \cdots$$

- Inflation (通貨膨脹): rate  $\pi$

- Nominal return ( $\tilde{R}$ ) v. Real return ( $R$ ):

$$R = \frac{\tilde{R}}{[1+\pi]}$$

$$\tilde{R} = R[1+\pi]$$

- Expected market interest rate:

$$\tilde{r} \approx r + \pi$$

▷

$$PV = R_0 + \frac{R_1}{[1+\tilde{r}]} + \frac{R_2}{[1+\tilde{r}]^2} + \frac{R_3}{[1+\tilde{r}]^3} + \cdots$$

### 3 Private-sector project evaluation

#### 3.1 Single-period Decision

- Choice between 2 projects:  $X$  or  $Y$
- Benefits and costs:

$$(B^X, C^X), (B^Y, C^Y)$$

- Criteria:

- Net return (淨回收):

$$N^X = B^X - C^X$$

$$N^Y = B^Y - C^Y$$

- Admissible (可行性):

$$N^X > 0$$

$$N^Y > 0$$

- Preferable (較佳者): highest net return

## 3.2 Multiple-period Decision

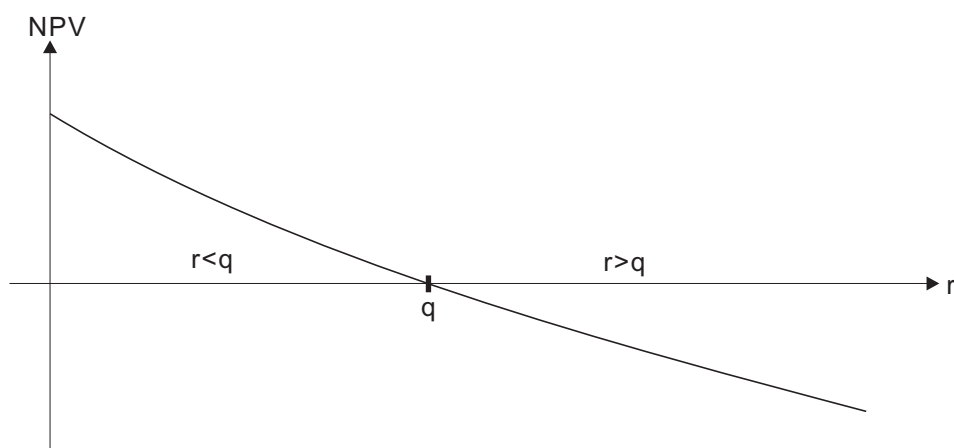
### 3.2.1 Net Present Value (NPV)

- Compare: project return v. bank return
- PV of net income stream: use *post-tax* market interest rate  $r$

$$\mathcal{N}^X = [B_0^X - C_0^X] + \frac{B_1^X - C_1^X}{[1+r]} + \frac{B_2^X - C_2^X}{[1+r]^2} + \dots$$

$$\mathcal{N}^Y = [B_0^Y - C_0^Y] + \frac{B_1^Y - C_1^Y}{[1+r]} + \frac{B_2^Y - C_2^Y}{[1+r]^2} + \dots$$

- Criteria:
  - ▷ Admissibility:  $\text{NPV} > 0$
  - ▷ Preferable: high NPV
- When  $r \uparrow$ :  $\text{NPV} \downarrow$  for all projects.
  - ▷ Early-return project is preferred.



### 3.2.2 Internal Rate of Return (IRR)

- Compare: project return v. bank return
- Def:  $\rho$  solving

$$\text{NPV}(\rho) = B_0 - C_0 + \frac{B_1 - C_1}{1 + \rho} + \frac{B_2 - C_2}{[1 + \rho]^2} + \dots = 0$$

▷ Unique  $\rho$  if  $B_t - C_t < 0$  before  $t = \tau$ , and  $B_t - C_t > 0$  after

- Criteria:
  - Admissibility:  $\rho > r$  (i.e.,  $\mathcal{N} > 0$ )
  - Preferable: high  $\rho$

- Problems:

1. Not applicable when market  $r$  varies in time !
2. Project scale not considered !

**[E]** 2 projects (X, Y) with market  $r=6\%$

	C	B	$\rho$	Nominal Net	Real Profit
X	100	110	10%	\$10	\$4
Y	1000	1080	8%	\$80	\$20

▷ Y has higher profit (preferred), but lower  $\rho$  ■

## 3. Timing not considered! (B&amp;W p.193)

	$t = 0$	$t = 1$	$t = 2$	$\rho$	$\mathcal{N}_{2\%}$	$\mathcal{N}_{5.2\%}$	$\mathcal{N}_{7\%}$
X	-1000	0	1210	0.10	163*	93*	57
Y	-1000	1150	0	0.15*	127	93*	75*

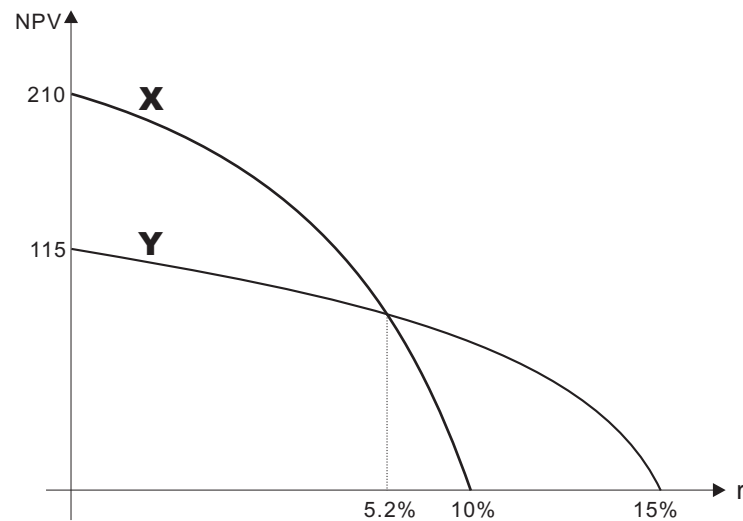
▷ Can always do better than  $Y$  alone with capital market:

( $t = 0$ ) Take  $X$ . Cost = -1000.

( $t = 1$ ) Borrow \$1150 from bank.

( $t = 2$ ) Gain \$1210 from  $X$ . Pay back bank  $\$1150 \times 1.02 = 1173$

⇒ Net consumption stream is  $(-1000, 1150, 37)$  ■



### 3.2.3 Benefit-Cost Ratio (BCR)

- B/C ratio:

$$\pi \equiv \frac{PV(B)}{PV(C)}$$

- Admissibility: same as NPV

$$\pi > 1 \Leftrightarrow PV(B) > PV(C)$$

- Problems:

1. Project scale ignored:

- Project X:  $C = 100$ ,  $B = 200$ ,  $\pi = 2$ , Net=100
- Project Y:  $C = 80$ ,  $B = 170$ ,  $\pi = 2.1$ , Net=90 ■

2. Ambiguity in B/C accounting:

	C	B	$\pi$
<b>E</b>	X 100	250	2.5
	Y 100	200	2

▷ New damage \$40 with X as  $B_X \downarrow$ :  $\pi'_X = 210/100 = 2.1 > \pi_Y$

▷ New damage \$40 with X as  $C_X \uparrow$ :  $\pi'_X = 250/140 = 1.8 < \pi_Y$  ■

### 3.3 Optimal project Scale Depends on CBA Criterion

- NPV:

$$\max_C B(C) - C$$

foc:

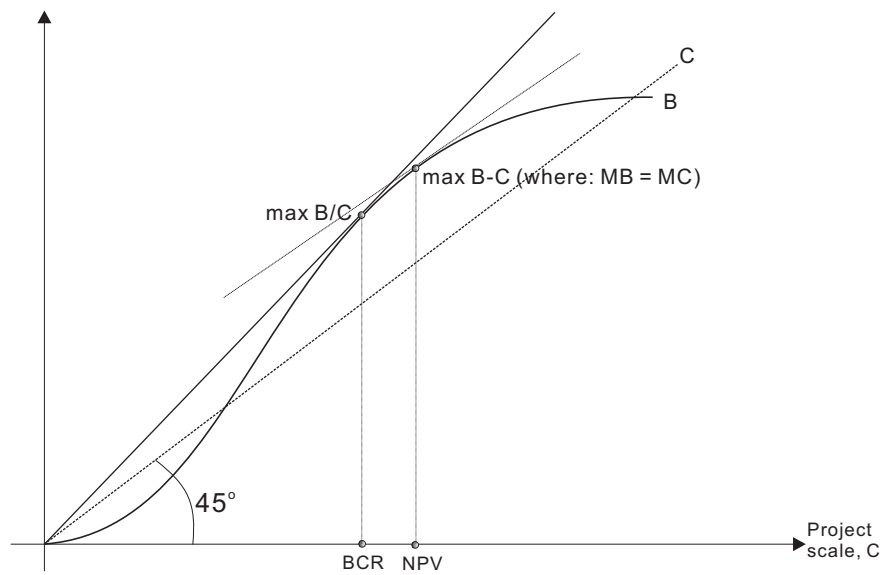
$$MB(C) = B'(C) = 1$$

- BCR:

$$\max_C \frac{B(C)}{C}$$

foc:

$$B'(C) = \frac{B(C)}{C}$$





### 3.4 Public-sector Discount Rate: $r_g$

- Measure what society places on sacrificed present consumption
  
- Maybe higher than market  $r$ :
  - Government  $r_g$ : *pre-tax* return
  - Firm  $r$ : *post-tax* return
  
- Maybe lower than market  $r$ : future benefits weight more
  - Paternalism: govt has more concern for future generation.<sup>1</sup>
  - Positive investment externality: to induce more investment

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<sup>1</sup>Private sector has “defective telescopic faculty”.

#### 4 Valuation of Life

#### 5 Valuation of Environmental Quality

- CVM/Survey: open v. closed format
  - ▷ Bias: hypothetical, starting-point, etc.
- Inferences from revealed consumer behaviors: travel cost method
  - ☐ Michigan Ludington power plant
  - ☐ Alaska Exxon Valdez accident
- ☐ Psychological value not included: existence value, option value