Public Choice

1. Public Choice Theory

- Social/public choice: the process of collective decision-making

- Elements:
  - Players/voters/consumers/agents: $i = 1, \cdots, N$
  - Candidates/alternatives/options: choice set A
  - Individual preference/ranking over A: R_i

- Preference aggregation mechanism:
  - Social decision rule (SDR): collective ranking R over A

E Beauty contest, sports event

  - Social choice function (SCF): a single choice $a \in A$

E Political election, travel destination, movie/restaurant

- Saari [1988] story: choice of drink in department meeting

15 voters	1st	2nd	3rd
6	Milk	Juice	Beer
5	Beer	Juice	Milk
4	Juice	Beer	Milk

  - "Milk" chosen initially as most favored:

$$M6 \ : \ B5 \ : \ J4$$

  - "Beer" served in meeting for lack of Milk
  - But people found "Juice" (10) is actually preferred to "Beer" (5)
  - Further: "Milk" least favored by pairwise comparison

$$J9 \ : \ M6$$

$$B9 \ : \ M6$$

2. Direct democracy

2.1. Unanimity rule (一致決): Wicksell [1896]

- Consistent with Pareto criterion

  ▷ Bills passed will surely make everyone better off

- Problems:

  – Theoretical:

    √ Social ranking is not "complete"

    √ Agreement is rarely reached

  – Practical:

    √ Distribution/jealousy issue not considered

      ▷ Some may prefer *non-Paretian* situation

    √ Everyone has veto power: transaction costs high

      ▷ Outcome subject to negotiation and strategic behaviors

      E 釘子戶

- Unanimity with *compensation/side-payment*

  ▷ 賄選合法化:「股東會出席通知書」(上有股東戶號, 名稱, 股數) 收購

  ? 錢多者當選?

2.2. Majority voting (多數決)

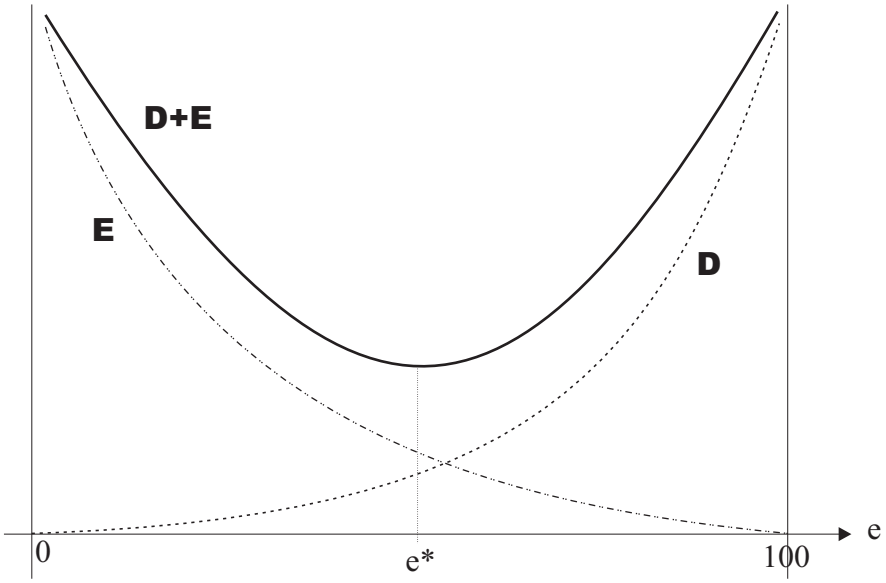
- Relative majority: $\eta\%$ ($\geq$ 50%) required

- Constitutional choice: [Buchanan-Tullock 1962][1]

$$\min_{\eta} \quad \text{ETSC} \equiv D + E$$

$\surd$ External costs (外部成本) E: damages imposed on minority

$\surd$ Decision costs (交易成本) D: costs for reaching decisions



$\triangleright$ Economic justification of the simple majority rule

[1] J.M. Buchanan and G. Tullock, Chapter 6 in *The Calculus of Consent – Logical Foundations of Constitutional Democracy*, 1962, University of Michigan Press.

- Voting procedure: for more than 2 candidates
  - Pairwise comparison (單挑): binary agenda
    - ▷ *Condorcet winner*: winner against any other candidate
  - Plurality rule (一起上, 打群架): simultaneous voting[2]

  ? Condorcet winner may not be plurality winner:

(9 voters)	1st	2nd	3rd
2	A	B	C
3	B	A	C
4	C	A	B

  - ▷ C is Plurality winner; A is Condorcet winner

- May's Theorem: with only 2 candidates[3]
  - ▷ *Only majority rule can satisfy the following:*
  - √ Anonymity: symmetry among all voters (treated equally)
  - √ Neutrality: symmetry among all candidates
  - √ Decisiveness: a winner will always be picked
  - √ Positive responsiveness: more votes, more likely to win ■

[2]Hindriks-Myles, 2006, MIT press, p.319.
[3]Hindriks-Myles, 2006, MIT press, p.306.

- Voting paradox [Condorcet 1785]:

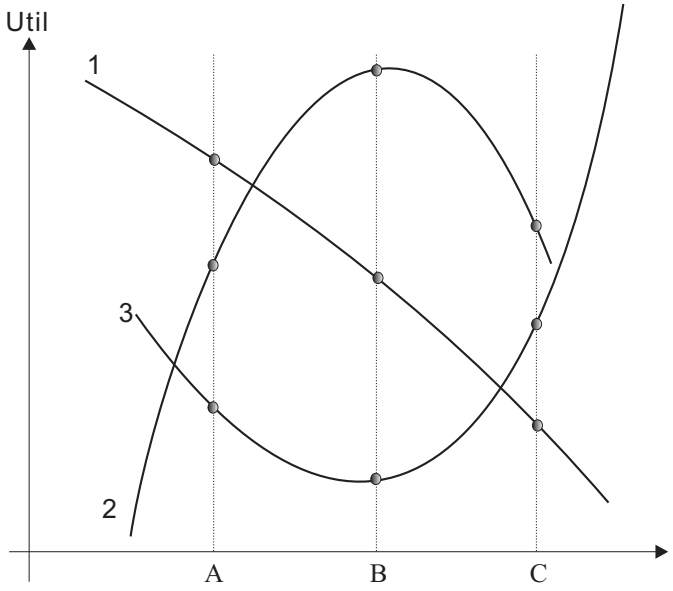
Ranking	1st	2nd	3rd
Voter 1	A	B	C
Voter 2	B	C	A
Voter 3	C	A	B

  - Voting cycles:

$$A \succ_{1,3} B \succ_{1,2} C \succ_{2,3} A$$

    - ▷ Outcome uncertain
    - ▷ Outcome subject to *agenda manipulation*

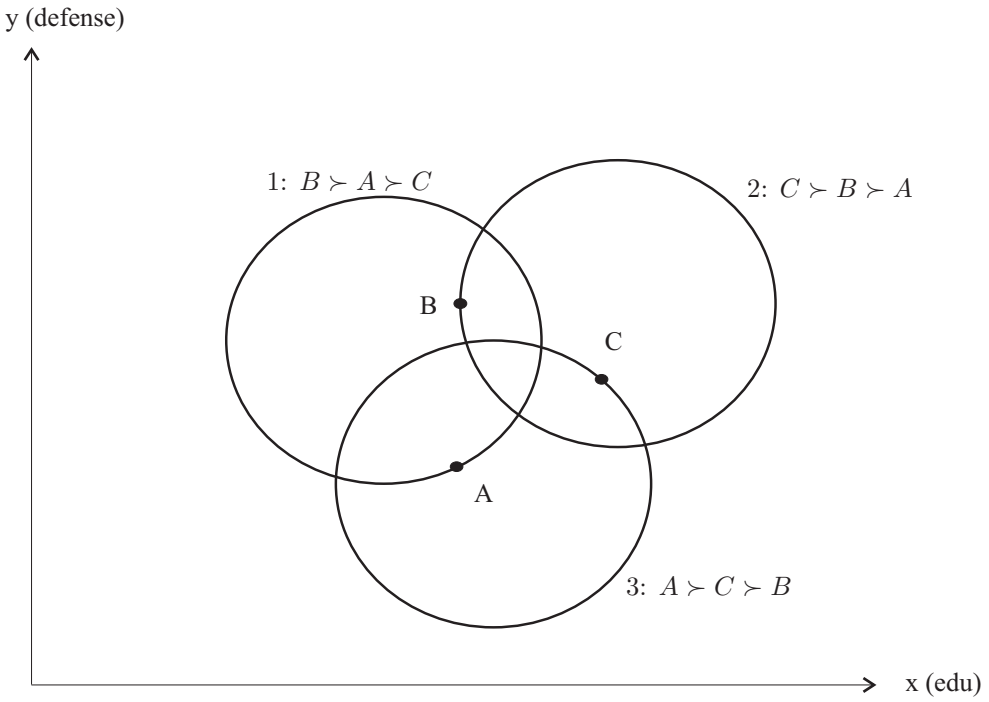
  - Single-peaked preferences (單峰偏好) [Black]:



    - ▷ Single-peakedness insures no cycle
    - ▷ Applicable only to 1-dim voting

E 2-dim voting cycle:

$$A \succ_{1,3} C \succ_{2,3} B \succ_{1,2} A$$



– Single-crossing preferences (SC):[4]

D On a 1-dim line, for 2 voters $a < b$, and 2 options $x < y$:

$$U^a(y) > U^a(x) \;\Rightarrow\; U^b(y) > U^b(x)$$

[4]Hindriks-Myles, 2006, MIT, pp.310.

and

$$U^b(x) > U^b(y) \;\Rightarrow\; U^a(x) > U^a(y) \quad \square$$

∗ If voter preferences satisfy SC, then there is no cycle.

∗ Condorcet winner is preferred option of the median voter M.[5]

– Cycle probability: 1–2%

▷ Not detectable when it arises!

E 3 people dividing \$1: no Condorcet winner!

Round	A	B	C
1	1/3	1/3	1/3
2	1/2	1/2	0
3	2/3	0	1/3
4	0	1/2	1/2
...	...	...	...

E Bundled voting: no Condorcet winner!

Voter value	A	B	C
1	500	−100	−100
2	−100	500	−100
3	−100	−100	500

▷ Cycle:[6]

$$(n,n,n) \rightarrow_{1,2,3} (y,y,y) \rightarrow_{1,2} (y,y,n) \rightarrow_{2,3} (n,y,n) \rightarrow_{1,3} (n,n,n)$$

[5] Because, for any 2 options $x < y$, if M prefers x, then all voters to his left will also prefer x. If M prefers y, then all voters to his right must also prefer y. □

[6] Any proposal changing a "y" to "n" will pass with two votes. But then (n,n,n) will be defeated by a proposal replacing any two "n" with two "y".

- Independence from Irrelevant Alternatives (IIA) may be violated

E Example:

#voters / ranking	1st	2nd	3rd
9	A	B	C
4	B	C	A
6	C	B	A

  - With all 3 candidates: (A9 : B4 : C6) $\Rightarrow$ A elected
  - If C drops out: (A9 : B10) $\Rightarrow$ B elected
  - Need IIA to avoid sabotage (攪局)[7] and strategic voting (棄保策略)[8]

- Pareto principle may be violated:

Ranking	1st	2nd	3rd	4th	5th	6th	7th
Voter 1	A	B	C	D	E	F	G
Voter 2	C	D	A	F	G	B	E
Voter 3	D	A	G	B	C	E	F

  $\triangleright$ Possible agenda/outcome:

$$A \to D \to C \to B \to G \to F \to E$$

  $\triangleright$ E is Pareto inferior to (A, B, C, D) for all voters:

$$A \succ_i E, \forall i, \quad \text{but} \quad E \succ A$$

[7] 真實例子: 1994 台北市長選舉 (陳水扁 v. 趙少康 v. 黃大洲), 2000 總統選舉 (陳水扁 v. 連戰 v. 宋楚瑜), 及 2012 總統選舉 (蔡英文 v. 馬英九 v. 宋楚瑜)。

[8] People may vote for 2nd choice, if they feel their top choice has no chance to win.

- Voter preference intensity not considered:

  ▷ Logrolling (選票互換): vote trading/exchange

  – (Yes) Voter intensity revealed: compromise means efficiency!

(Project)	1	2	3	NetValue	M.V.	logrolling
Hospital	200	-50	-55	95	n	y (1,2), (1,3)
Library	-40	150	-30	80	n	y (1,2), (2,3)
Park	-120	-60	400	220	n	y (2,3), (1,3)

  – (No) Special-interest gains may outweigh general losses!

(Project)	1	2	3	NetValue	M.V.	logrolling
Hospital	200	-110	-105	-15	n	y (1,2), (1,3)
Library	-40	150	-120	-10	n	y (1,2), (2,3)
Park	-180	-140	250	-70	n	y (2,3), (1,3)

- 64% majority rule [Caplin-Nalibuff, *Econometrica* 1988]

  – In k-dim elections, incumbent can guarantee only: Figure 1

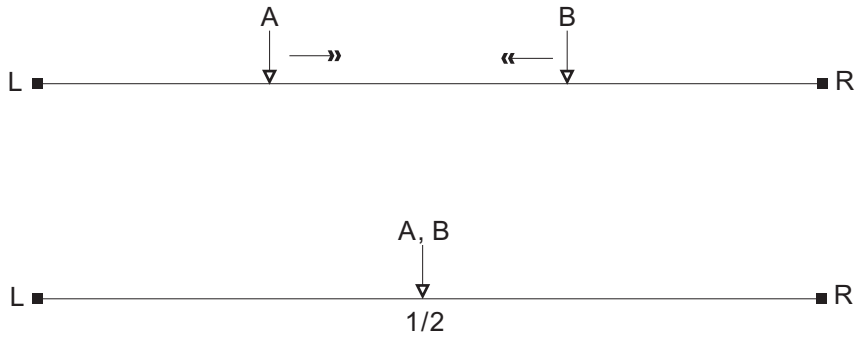
$$\sigma_k \;=\; \left(\frac{k}{k+1}\right)^k$$

  E $\sigma_1 = 1/2$, $\sigma_2 = 4/9$

  – In real-life elections, a challenger will get at least:

$$\sigma_\infty \;=\; \lim_{k\to\infty}\left[1-\left(\frac{k}{k+1}\right)^k\right] \;=\; 1-\frac{1}{e} \;\approx\; 64\% \quad \square$$

Hotelling Spatial Model: 1-dimensional Voting



2-dimensional Voting

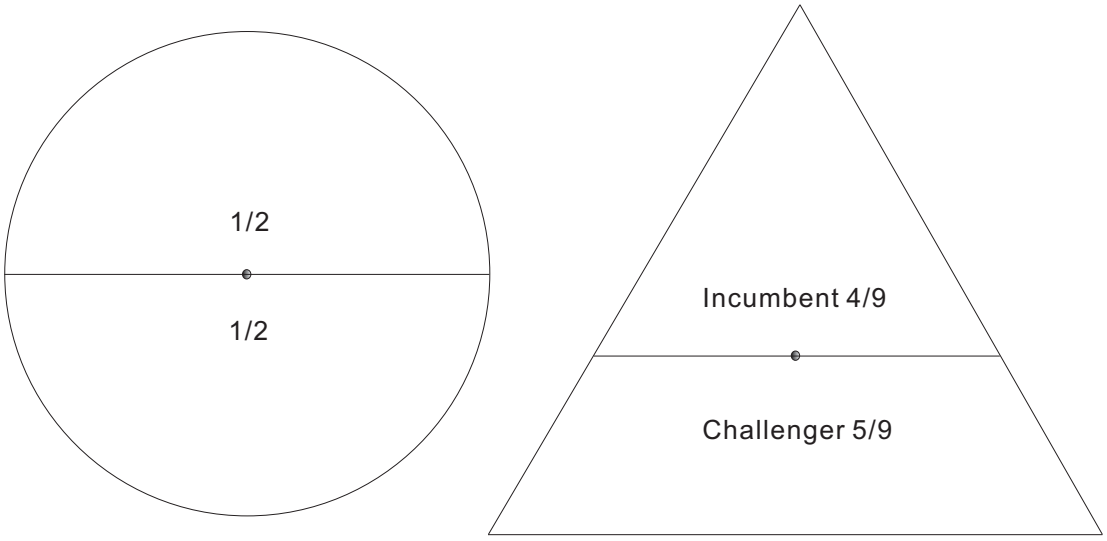
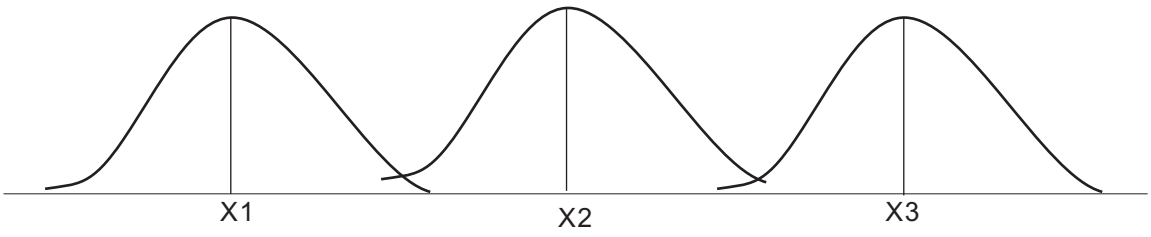


Figure 1: Justification for 2/3 majority rule

- Median Voter Theorem (中值選民定理)[9]



  - X_2 is Condorcet winner (by pairwise comparison)
  - Voting outcome is the demand of the medium voter
  - Democracy reflects preference of medium-wealth citizens
  - Voting outcome usually inefficient

[9]Holcombe pp.175–76; Hyman p.165.

2.3. Borda count (包達計數法)

- Counting procedure: choose one with lowest count

#voters	Keynes	Becker	Chair
10 Macro	1	2	3
10 Micro	2	1	3
1 Chair	2	3	1
Rank/Score	1(32)*	2(33)	3(61)

  ▷ No cycles

  ▷ May set rank values to reflect relative weights (eg, 1,2,3,10,...)

- Strategic manipulation:

  E 10 Micros now claim "Chair as 2nd, Keynes as 3rd"

#voters	Keynes	Becker	Chair
10 Macro	1	2	3
10 Micro	3	1	2
1 Chair	2	3	1
Rank/Score	2(42)	1(33)*	3(51)

- IIA violated:

#voters	Keynes	Becker
10 Macro	1	2
10 Micro	2	1
1 Chair	1	2
Rank/Score	1(31)*	2(32)

2.4. Approval voting (同意決)

- Can vote for any number of alternatives, each vote counts as 1.[10]
- Voter flexibility.
- Outcome indeterminacy:

#voters / ranking	1st	2nd	3rd
6	x	z	y
5	y	z	x
4	z	y	x

  - x wins: if everyone votes only for 1st choice (x6 : y5 : z4)
  - y wins: if group 3 votes for top 2 choices (x6 : y9 : z4)
  - z wins: if everyone votes for top 2 choices (x6 : y9 : z15)

  ▷ Condorcet winner may not be picked.

2.5. Runoff voting (兩階段決選)

- Top 2 winners in Round 1 will enter Round 2.[11]
- Condorcet winner may not win.
- Positive Responsiveness may be violated.

[10] Hindriks-Myles, 2006, MIT press, p.320.
[11] Hindriks-Myles, 2006, MIT press, p.321.

Count	1st	2nd	3rd
6	a	b	c
5	c	a	b
4	b	c	a
2	b	a	c

2.6. Elimination (刪除法)

- Everyone votes for the candidate you dislike most.

  ▷ The candidate who receives least votes get elected.

- May have cycle.

- IIA violated.

Count	1st	2nd	3rd	4th
9	A	B	C	D
4	B	C	D	A
6	C	D	A	B
5	D	A	B	C

  – 4 candidates: (A4 : B6 : C5 : D9) $\Rightarrow$ A elected.

  – If B withdraws: (A10 : C5 : D9) $\Rightarrow$ C elected.

2.7. Indeterminacy of Collective Choice

Collective choice depends on voting mechanism:

E 7 voters, 4 alternatives:

V1	V2	V3	V4	V5	V6	V7
A	A	A	B	B	C	C
B	B	B	C	C	D	D
C	C	C	D	D	A	A
D	D	D	A	A	B	B

- Plurality rule: A*(3) : B(2) : C(2) : D(0)
- Borda count: A(17) : B(16) : C*(15) : D(22)
- Approval (2 votes): A(3) : B*(5) : C(4) : D(2)
- Pairwise comparison: cycle, no Condorcet winner

$$A \succ_{5:2} B \succ_{5:2} C \succ_{7:0} D \succ_{4:3} A$$

2.8. Arrow's Impossibility Theorem [1951] (不可能定理)

1. Axiomatic approach

2. No social decision rule can satisfy the following:

   - Universality (全域性): no restriction on voter preferences
   - Consistency (一致性): social ranking is *transitive* (i.e., *no cycle*)
   - Pareto axiom: social ranking obeys *unanimous* preference
   - IIA (independence of irrelevant alternatives)
   - Non-dictatorship ■

3. Use of cardinal social welfare functions: measurement problem

4. Satherswaite Theorem: strategy-proofness (instead of IIA) is required

2.9. About IIA

- Example: consumer ice cream choice

  (vanilla, choco, strawberry) v. (vanilla, strawberry)

  ▷ Not reasonable; IIA seems desirable

- Minimax strategy: minimize maximal possible regret [Savage 1951]
  - Regret: loss/damage of choosing a wrong action/choice
  - Applicable cases:
    - *Should I bring umbrella?* ["Yes", if being wet is disaster]
    - *Should we believe in God?* [Pascal: "Yes"]
    - *Should we try to contact aliens?* [Hawking: "No"]
    - *Nuclear power plant, cancer insurance, committing a crime*
  - Minimax strategy may violate IIA

    E 3 possible states (l, m, r), 3 options (A, B, C)

Payoff	A	B	C
l	1	2	3
m	2	3	1
r	3	1	2

Regret	A	B	C
l	2	1	0
m	1	0	2
r	0	2	1

Regret	A	B
l	1	0
m	1	0
r	0	2

    ▷ Given choice set (A, B, C): $A \sim B$
    ▷ Given choice set (A, B): $A \succ B$

- Is IIA essential? — Consumer choice re-visited
  - Consistent underlying consumer food preference:

    $$\text{beef} \succ \text{chicken}$$

  - Observed/explicit consumer choice in restaurants:

    (chicken, beef) v. (chicken, beef, seafood)

  - Possible explanation: information

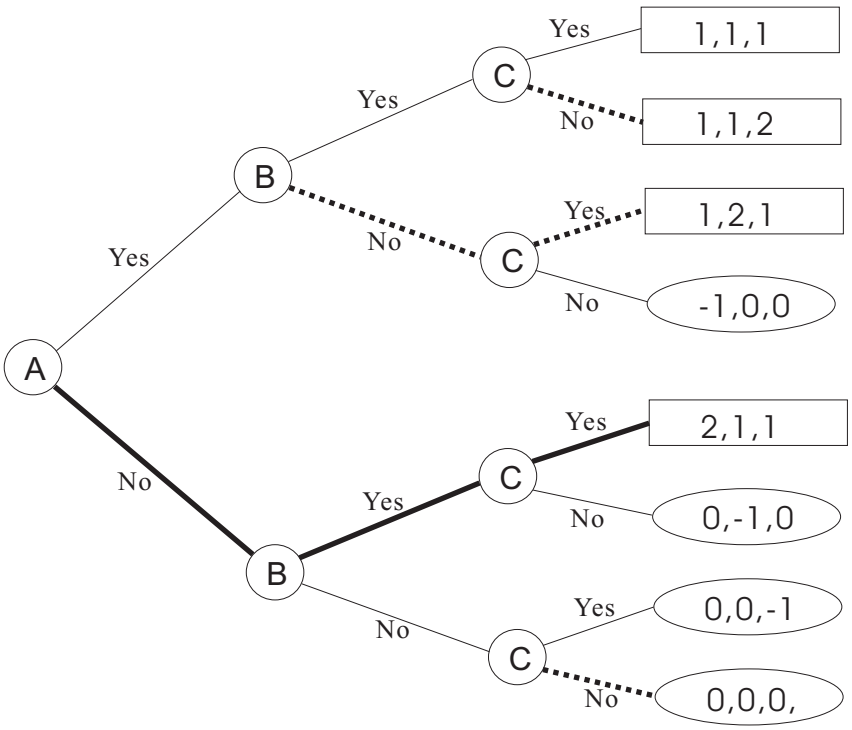
    Available "seafood" option signals good quality of the restaurant

  - Rational consumer choices/behaviors may actually violate IIA

2.10. (Application) Congress Voting on Own Pay Raise

Payoff	Bill "pass"	Bill "fail"
Vote "yes"	1	-1
Vote "no"	2	0

Congress pay-raise voting:



2.11. (Application) Tie-breaking Power

[Farquharson 1969, p.50]

▷ Vote by majority rule, voter 1 can break tie.

Voter	1st	2nd	3rd
1	A	C	B
2	B	A	C
3	C	B	A

Figure 2

Voting Outcome:

(3=A)

1/2	A	B	C
A	A	A	A
B	A	B	B*
C	A	C*	C

(3=B)

1/2	A	B	C
A	A	B	A*
B	B	B	B
C	C*	B	C

(3=C)

1/2	A	B	C
A	A	A*	C
B	B*	B	C
C	C	C	C

Elimination of dominated strategies (Round 1):

(3=A)

1/2	A	B	C
A	A	A	A
B	A	B	B*
C	A	C*	C

(3=B)

1/2	A	B	C
A	A	B	A*
B	B	B	B
C	C*	B	C

(3=C)

1/2	A	B	C
A	A	A*	C
B	B*	B	C
C	C	C	C

Elimination of dominated strategies (Round 2):

(3=A)

1/2	A	B
A	A	A

(3=B)

1/2	A	B
A	A	(B)

(3=C)

1/2	A	B
A	A	A*

Equilibrium outcome: B (1 for A, 2 for B, 3 for B), 1 gets worst!

Figure 2: Tie-breaking power may hurt you!

3. Representative Democracy

1. Rational:

   √ Transaction costs low (fewer people)

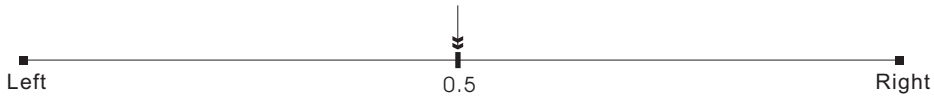
   √ Gains from specialization

2. Iron triangle (鐵三角)

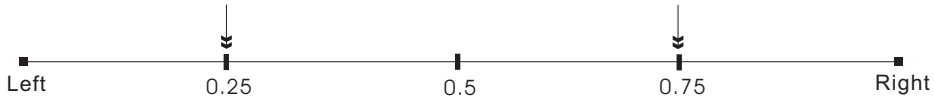
   • Elected politicians (民選政客):

   – Hotelling's spatial model (EJ 1929):

   ▷ 2 candidates:



   ▷ 4 candidates:



   ▷ No equilibrium for 3-candidate election

   – Voting paradox

   – Government by jury [Varian-Bergstrom]

   ▷ Congressman/judge efforts are PG, no production incentive

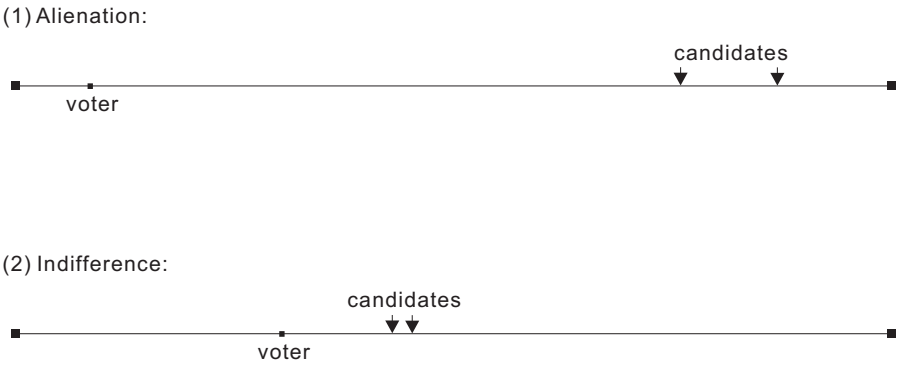
   ▷ Rational ignorance of voters: votes not intelligent

– Non-voting:

√ Abstention due to high costs

√ Abstention from alienation (疏離)

√ Abstention from indifference (無差異)



- Bureaucrats (事務官僚): [Niskanen 1971]
  - Bureaucrats: maximize own budget/power, not SW
    - SW-max:

$$Q^*: \quad \max_Q \ \text{SW} \ \equiv \ \text{TB}(Q) \ - \ \text{TC}(Q)$$

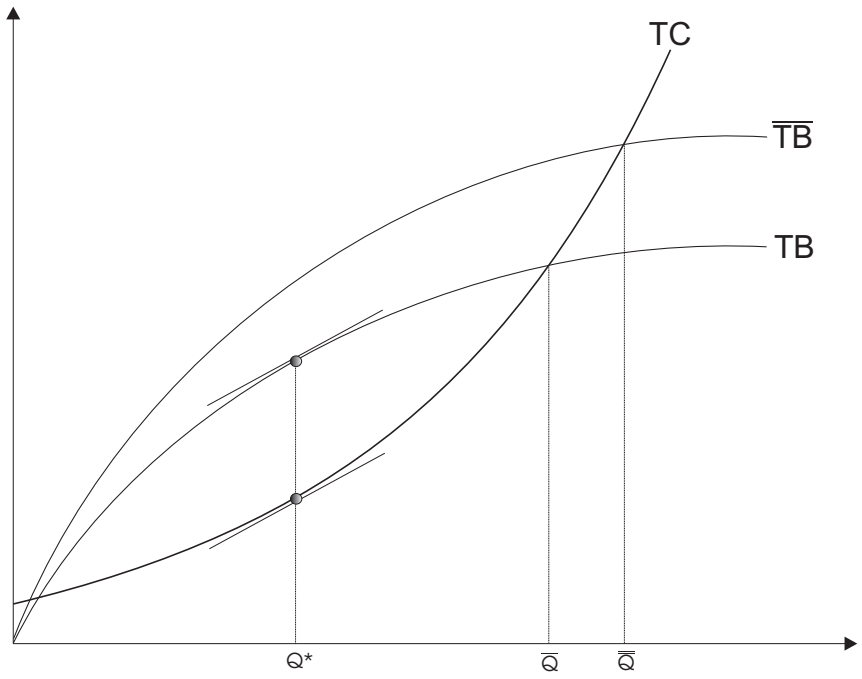
    - Bureaucrat:

$$\bar{Q}: \quad \max_Q \ Q \quad \text{s.t.} \quad \text{TB}(Q) \geq \text{TC}(Q)$$

      ▷ Bureaucrats tend to exaggerate TB to get higher Q
  - Justification:

    √ Legislature has no detailed expertise/knowledge

    √ Bureaucrat office tenure exceeds elected officials



- Special interests (利益團體): formed based on:

  √ Wealth: rich v. poor

  √ Income source: capitalist v. worker; producer v. consumer

  √ Region: industry v. agriculture v. tourism areas

  √ Demographics: sex, race, religion, age