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1 Median Voter Approach: Demand-side Model

- Bergstrom-Goodman [AER 1973]

1.1 Assumptions

- A1 Public good y:
  – Price q for all communities (supply of y is horizontal)
- A2 Consumer i:
  – Wealth: w_i
  – Local tax rate: $\tau_i(w_i)$
  – Tax price of public good y: $\tau_i q$
- A3 Consumer utility-max:

$$\max_{x_i, y} \ U_i(x_i, y) \quad \text{s.t.} \quad x_i + [\tau_i q] y = w_i$$

or simply:

$$\max_{y} \ V_i(y) \ \equiv \ U_i(w_i - \tau_i q y, \, y)$$

▷ $U_i(x_i, y)$ is *strictly quasi-concave* in (x, y)

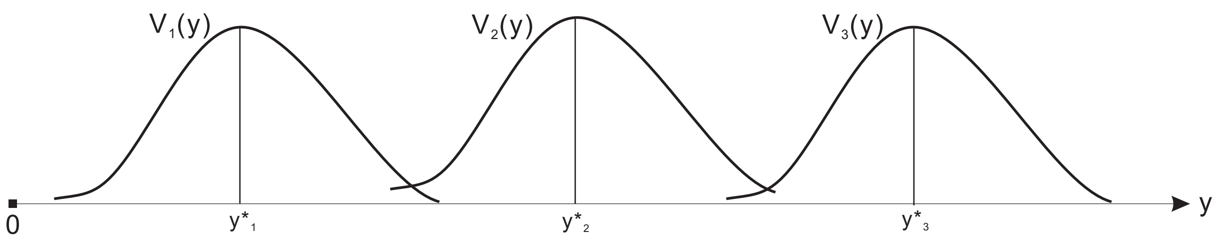
▷ $V_i(y)$ is *strictly quasi-concave* in y

▷ $y_i^* \ \equiv \ i$'s unique optimal choice (i.e., ideal amount of y)

- A4 Community public good level is determined by majority voting
- A5 Voting outcome $\hat{y}^*$ is demand of the median wealth resident

1.2 Identifying voting outcome: majority voting in [A4]

- Duncan Black Theorem: $V_i(y)$ is *single-peaked* in y
  - ▷ No voting cycle
  - ▷ There exists a *unique Condorcet winner*
- Bowen Equilibrium: winner is median $\hat{y}^*$ of all y_i^*
- Median Voter: voter m who has the median demand $y_m^* = \hat{y}^*$



1.3 Identifying the median voter: justifying [A5]

- By [A3], individual demand is:

$$x(\tau, w), \quad y(\tau, w)$$

Since τ may depend on w, we can write:

$$x(\tau(w), w), \quad y(\tau(w), w)$$

- Total differetiation:

$$\frac{dy(\tau(w), w)}{dw} = \frac{\partial y}{\partial \tau} \cdot \frac{d\tau}{dw} + \frac{\partial y}{\partial w}$$

Using elasticity:

$$\frac{dy/y}{dw/w} = \frac{\partial y/y}{\partial \tau/\tau} \cdot \frac{d\tau/\tau}{dw/w} + \frac{\partial y/y}{\partial w/w} \equiv \delta \cdot \xi + \varepsilon \tag{1}$$

where:

$\delta \equiv (\partial y/y)/(\partial \tau/\tau)$ = *price elasticity* of demand y

$\varepsilon \equiv (\partial y/y)/(\partial w/w)$ = *wealth elasticity* of demand y

$\xi \equiv (\partial \tau/\tau)/(\partial w/w)$ = *wealth elasticity* of local tax rate

- In general:

$$\frac{dy/y}{dw/w} = \delta\xi + \varepsilon \gtreqless 0$$

– For normal and ordinary y:

$$\delta < 0, \;\; \varepsilon > 0$$

– In a regular progressive local tax system:

$$\xi > 0$$

- Possible cases:

1. $(\delta\xi + \varepsilon)$ is positive for all $w > 0$: Fig. 1(top)

   Then y is monotonically increasing in w

   More y is demanded by wealthier voters

   ▷ Median ($\hat{y}$) of y is desired by voter with median w ($\hat{w}$).[1]

2. $(\delta\xi + \varepsilon)$ is negative for all $w > 0$:

   Then y is monotonically decreasing in w

[1]Public goods of this nature include security, concert, museum, and environmental quality.

Less y is demanded by wealthier voters

$\triangleright$ Still, median ($\hat{y}$) of y is desired by voter with median w.[2]

3. If $(\delta\xi + \varepsilon)$ is first negative, then positive: Fig. 1(bottom)

 ! In this case, [A5] may not hold.[3]

1.4 Data Collection

- Each community is an observation point:
  - PG quantity: community PG expenditures
  - PG price: tax price $\hat{\tau}$ of the median-wealth resident

- Individual tax price τ_i:
  - Local PG is financed by local property tax (mainly house and land taxes)
  - Property tax depends on property value:

$$\tau_i = \frac{H_i}{\sum_j H_j}$$

where: H_i is value of resident i's realty

- Congestion effect of local PG:

$$z = y \cdot N^{\gamma}, \quad \gamma \leq 0 \tag{2}$$

where: z is PG level actually enjoyed by each resident

[2] An example is mediocre local park.

[3] Now voting outcome will be $\hat{y}$ (still median of all y), but this is not demand of the median income voter. Half the voters have income in ab, with the other half in $(\underline{w}a \cup b\bar{w})$.

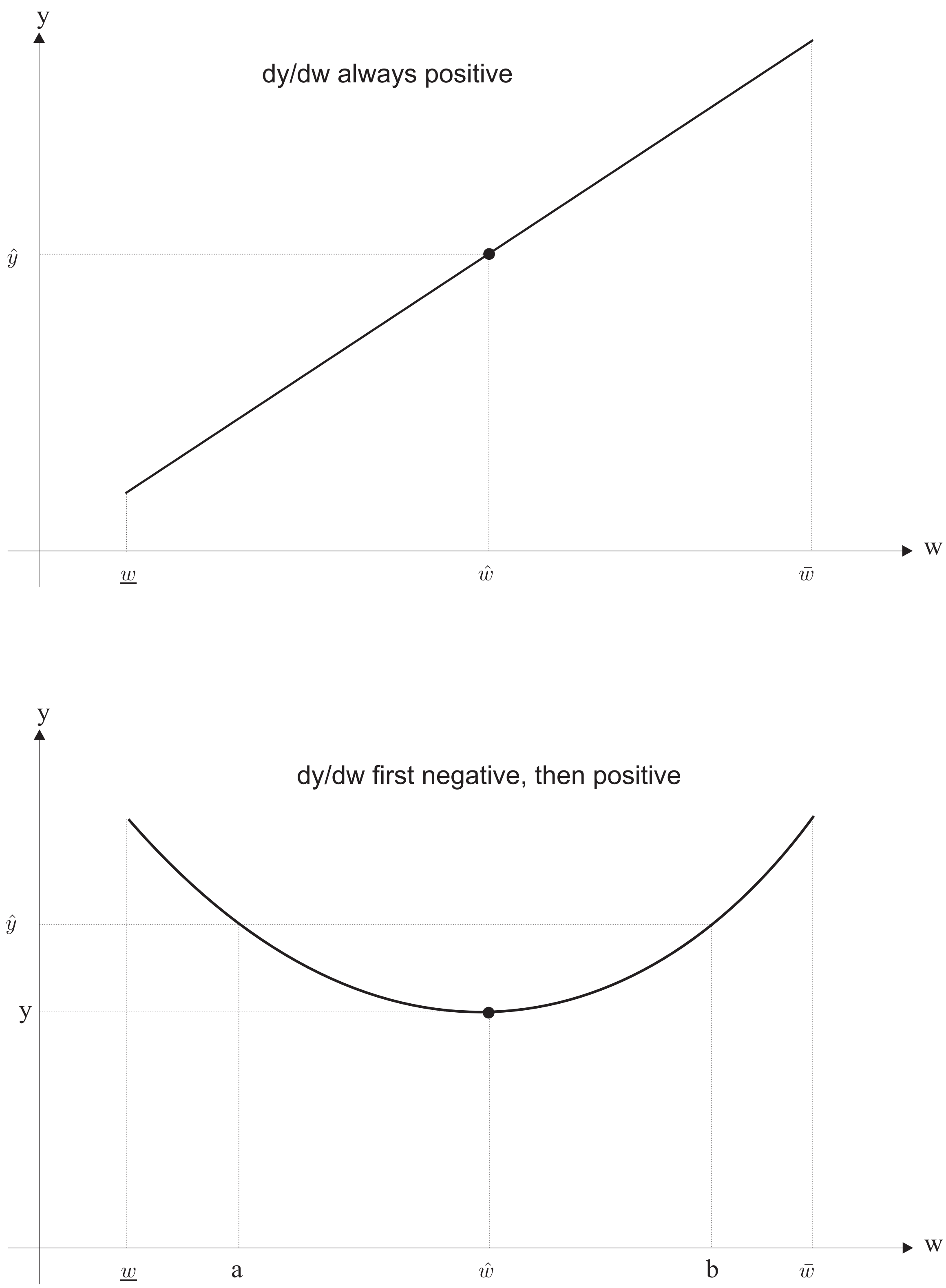


Figure 1: Income effect of public good demand.

- $\gamma = 0$: $z = y$, pure PG
- $\gamma \in (0, -1)$: impure PG with congestion
- $\gamma < -1$: serious crowding

1.5 Estimation Procedure

- Log-linear demand function:

$$z = cp^{\delta} w^{\varepsilon} e^{\beta x} \tag{3}$$

where:

z = actual level of PG enjoyed

p = price of z

w = individual wealth

x = other socio-economic variables

- Transforming unobserved (z, p) into observable (y, τ_i):
  - By consumer budget:

$$w = x + \tau y = x + [\tau N^{-\gamma}]z$$

▷ price of z is:

$$p = \tau N^{-\gamma}$$

  - Combining (2)(3):

$$\log z = \log y + \gamma \log N = c' + \delta \log p + \varepsilon \log w + \beta x; \quad c' \equiv \log c$$

or:

$$\begin{aligned} \log y &= c' - \gamma \log N + \delta \log(\tau N^{-\gamma}) + \varepsilon \log w + \beta x \\ &= c' - \gamma[1 + \delta] \log N + \delta \log \tau + \varepsilon \log w + \beta x \end{aligned}$$

where:

$\delta = \partial \log y / \partial \log \tau$ is price elasticity of demand y

$\varepsilon = \partial \log y / \partial \log w$ is income elasticity of demand y

  - Can obtain estimates of $(\delta, \varepsilon, \gamma)$

1.6 Estimation Results

- Data: 826 US cities with population between (10K, 150K) in 1960
- With $\hat{\xi} \approx$ 1–1.3:

$$\hat{\delta}\hat{\xi} + \hat{\varepsilon} > 0$$

	Total PG	Policing	Parks
income elasticity $\hat{\varepsilon}$	0.64	0.71	1.32
price elasticity $\hat{\delta}$	-0.23	-0.25	-0.19
congestion $\hat{\gamma}$	-1.09	-1.07	-1.44

2 Median Voter Approach: Supply-side Model

- Borcherding-Deacon [AER 1972]
- Assumptions:

  [A1] Local government are chosen by residents using majority rule. As such, government policies will reflect preferences of the median voter.[4]

  [A2] Median voters in all communities have similar perferences.

  [A3] Local public goods/services are supplied with minimal costs.[5]

  [A4] Local PG is shared by all local residents. Its production costs also fall equally on all residents. So all residents have same PG tax price.

- The Model
  - Cobb-Douglas production technology:

$$X = aL^{\beta}K^{1-\beta}, \quad 0 < \beta < 1 \tag{4}$$

where:

$X \equiv$ local PG level

$L \equiv$ labor input in PG production

$K \equiv$ capital input

[4] Namely, local government seeks to maximize median voter's welfare.
[5] Or, local PG is produced with efficiency.

– Output-max with fixed costs E:[6]

$$\max_{L,K} \quad X = aL^{\beta}K^{1-\beta} \quad \text{s.t.} \quad rK + wL \leq E \tag{5}$$

<u>where</u>:

$r \equiv$ unit price of capital K

$w \equiv$ unit price of labor L

– Solution (L^*, K^*) to (5):

$$L^* \;=\; \frac{\beta E}{w}; \quad K^* = \frac{[1-\beta]E}{r}$$

– Substitute (L^*, K^*) into (4):

$$X = a\left[\frac{\beta}{w}\right]^{\beta}\left[\frac{1-\beta}{r}\right]^{1-\beta} E \tag{6}$$

▷ By CRTS of C-D technology: X doubles when E doubles.[7]

– Inverting (6), we have:

$$E \;=\; \frac{1}{a}\left[\frac{w}{\beta}\right]^{\beta}\left[\frac{r}{1-\beta}\right]^{1-\beta} X$$

▷ X has constant marginal production cost:

$$c \;=\; \frac{1}{a}\left[\frac{w}{\beta}\right]^{\beta}\left[\frac{r}{1-\beta}\right]^{1-\beta} \tag{7}$$

– Assuming: r is constant across communities, but w may vary.

– Can simplify (7):

$$c \;\equiv\; a'w^{\beta}; \;\; a' \equiv \frac{1}{a\beta^{\beta}}\left[\frac{r}{1-\beta}\right]^{1-\beta} \tag{8}$$

[6] Alternatively, we can consider the following cost-min problem:

$$\min_{L,K} \;\; rK + wL \quad \text{s.t.} \quad aL^{\beta}K^{1-\beta} \geq \bar{X}$$

[7] That is, $X(E)$ is a homogeneous function of degree 1.

- Congestion consideration:

$$q = \frac{X}{N^{\alpha}} \tag{9}$$

where:

$q \equiv$ local PG level actually consumed

$N \equiv$ community population

$\alpha \equiv$ congestion parameter

- PG nature:

(1) $\alpha = 0$: X is pure PG

(2) $\alpha = 1$: X is pure private good

(3) $\alpha \in (0, 1)$: impure PG

- Tax price calculation:
  - PG level X
  - Production cost cX:

    ▷ Individual share is:

$$\frac{cX}{N}$$

  - Actual enjoyment level: q
  - Price t of q:

$$t = \frac{cX}{Nq} = cN^{\alpha-1} \tag{10}$$

- Estimation procedure:

– Log-linear demand:

$$q = At^{\eta}y^{\delta} \tag{11}$$

where: $y \equiv$ individual income

– By (11), we know:

$$\eta = \frac{\partial \log q}{\partial \log t}; \quad \delta = \frac{\partial \log q}{\partial \log y}$$

where:

$\eta =$ price elasticity of PG demand

$\delta =$ income elasticity of PG demand

– Substituting (9)(10) into (11), and using (8):

$$\begin{aligned} X &= N^{\alpha}A[cN^{\alpha-1}]^{\eta}y^{\delta} \\ &= N^{\alpha}A[a'w^{\beta}N^{\alpha-1}]^{\eta}y^{\delta} \\ &= A'w^{\beta\eta}N^{\eta(\alpha-1)+\alpha}y^{\eta}; \quad A' \equiv Aa'^{\eta} \end{aligned} \tag{12}$$

– Let $e \equiv E/N = cX/N$:[8]

$$e = \frac{cX}{N} = A''w^{\beta(\eta+1)}N^{(\alpha-1)(\eta+1)}y^{\eta}; \quad A'' \equiv a'A' \tag{13}$$

– Put (13) in log form:

$$\ln e = A''' + [\eta+1]\ln(w^{\beta}) + [(\alpha-1)(\eta+1)]\ln N + \delta \ln y; \quad A''' \equiv \ln A'' \tag{14}$$

– Data collection: for each community

  * Calculate e from community E and N
  * Calculate (w^{β}) from community wage rate[9]

[8] Public expenditure per capita.
[9] β is obtained from other research.

- Estimation results:
  - Data: 44 US states in 1962
  - 8 PGs
  - Can obtain η and δ from coefficient estimates of $\ln(w^{\beta})$ and $\ln y$. Then we can have α from estimates of $[(\alpha-1)(\eta-1)]$ (coefficient of $\ln N$).

3 Median Voter Method in Tiebout Equilibrium

- Tiebout equilibrium:
  - Homogeneous residents in all communities
  - Estimation procedure: random selection in each community

- The problem:
  - Communities are in Tiebout equilibrium
  - PG demand estimated using median voter method
  - Are the estimates unbiased?

- The model: [Goldstein-Pauly, JPuE 1981]
  - A metropolitan area consisting of many communities
  - Income distribution of all residents is unimodal, with mean y_M
  - PG demand of resident i:

$$x_i = \alpha + \beta y_i + \varepsilon_i$$

    where: x_i is PG demand, y_i is income
  - Random term:

$$\varepsilon_i \sim \text{ i.i.d. } N(0, \sigma^2)$$

  - Distribution of PG demand by all residents with same income y:

$$x \sim \text{ i.i.d. } N(\alpha + \beta y, \sigma^2)$$

▷ Resident distribution of 3 income levels: $y_2, \hat{y}, y_1$ Fig. 2

$$y_2 = \hat{y} - \delta, \quad y_1 = \hat{y} + \delta, \quad y_2 < \hat{y} < y_1 < y_M$$

- Tiebout equilibrium in the metropolis:
  - All residents in a community have same PG demand
  - Consider a community with some PG level x:

$$\hat{x} \;=\; E[\alpha + \beta\hat{y} + \varepsilon] = \alpha + \beta\hat{y}$$

  - Community income distribution: Fig. 3
    - There are f_0 residents with income $\hat{y}$
    - There are f_1 residents with income $y_1 > \hat{y}$
    - There are f_2 residents with income $y_2 < \hat{y}$

    ▷ Median community income $y' > \hat{y}$ (because $f_1 > f_2$)
  - Similarly, for income $\bar{y} > y_M$ and community supplying PG

$$\bar{x} \;=\; \alpha + \beta\bar{y}$$

    ▷ Median community income

$$y'' \;<\; \bar{y}$$

- Estimation bias: Fig. 4
  - Actual demand curve: thick line L

    ▷ Slope β, with 2 points on L:

$$(\hat{x}, \hat{y}), \quad (\bar{x}, \bar{y})$$

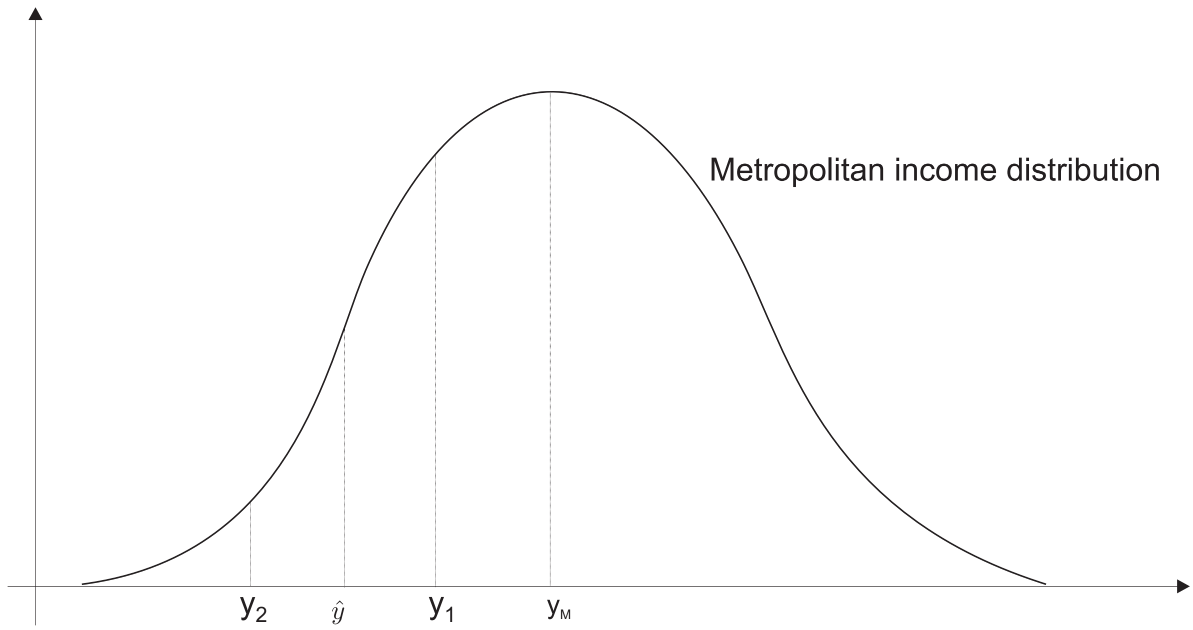


Figure 2: Metropolitan income distribution

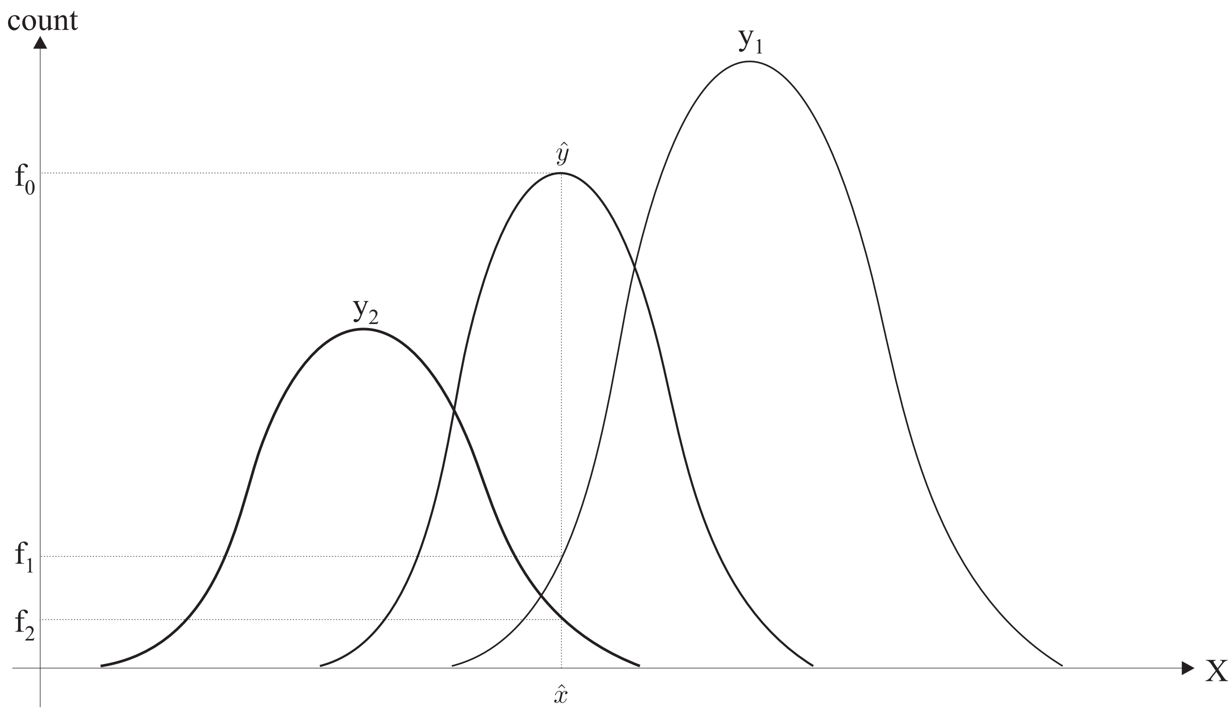


Figure 3: Community PG demand distribution

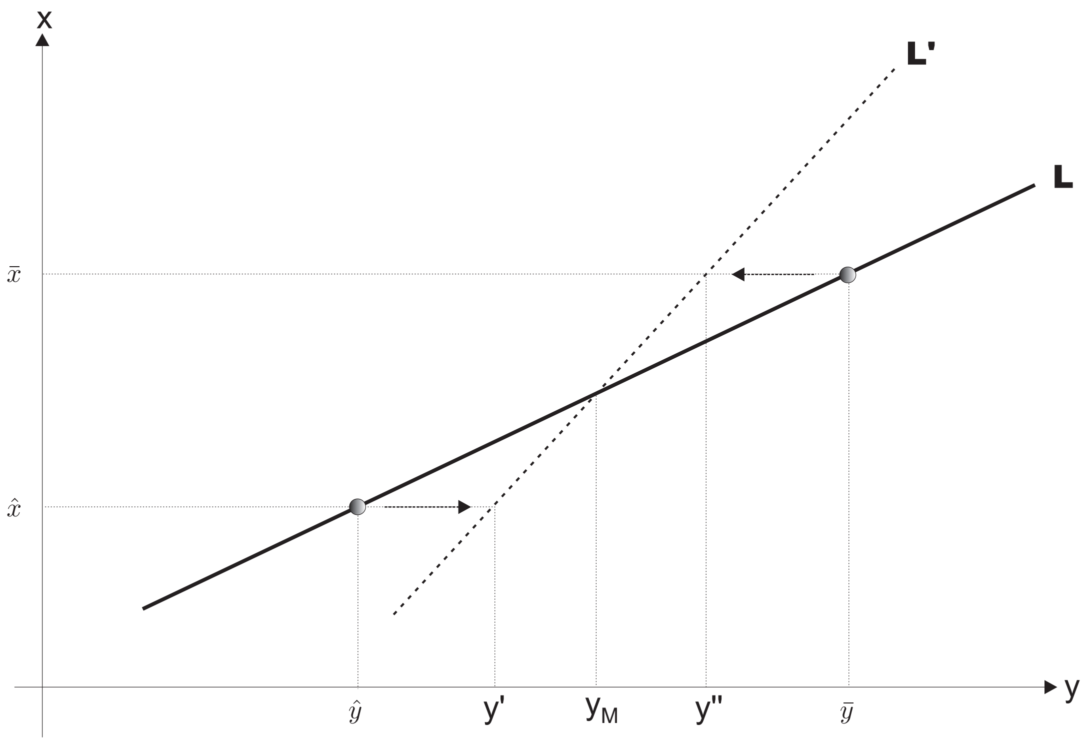


Figure 4: Demand estimation bias

– Estimation using MVM:

▷ Median income is y' in community $\hat{x}$

▷ Median income is y'' in community $\bar{x}$

▷ Estimated demand is thin dashed line L' with 2 points:

$$(\hat{x}, y'), \quad (\bar{x}, y'')$$

$\Rightarrow$ Slope greater than β

4 Survey Approach

- Bergstrom et al. [Econometrica 1982]
- Data: 2001 questionaires after 1978 election
  - Asking: people's opinion on government public school spending
  - Answer: "more/less/same"
- Preliminary design:
  - PG demand function:

$$g_i = D(x_i)/\epsilon_i \tag{15}$$

    where:

    $g_i \equiv i$'s ideal PG expenditures

    $x_i \equiv i$'s socio-economic variables

    $D(x_i) \equiv$ deterministic part of demand

    $\epsilon_i \equiv$ random term

  - Let $a_i \equiv$ PG expenditure level where i resides.
    * If $g_i > a_i$, then i will answer "more"
    * If $g_i = a_i$, then i will answer "same"
    * If $g_i < a_i$, then i will answer "less"
  - Estimate $D(x_i)$ with logit/probit
  - Problem:
    - Prob$(g_i = a_i) = 0$ if ϵ_i is continuously distributed
    - In dataset: 58% "same", 25% "more", 17% "less"

- Modification:
  - Assume: people do not care about minor difference[10]
    - * "more" if $g_i > \delta a_i$
    - * "less" if $g_i < a_i/\delta$
    - * "same" if

$$g_i \ \in \ \left[\frac{a_i}{\delta},\ \delta a_i\right]$$

  - Substitute into (15) and take log:
    - * "more" if:

$$\ln \epsilon_i \ < \ \ln D(x_i) - \ln a_i - \ln \delta$$

    - * "less" if:

$$\ln \epsilon_i \ > \ \ln D(x_i) - \ln a_i + \ln \delta$$

    - * "same" if:

$$\ln \epsilon_i \ \in \ [\ln D(x_i) - \ln a_i - \ln \delta,\ \ln D(x_i) - \ln a_i + \ln \delta]$$

  - Assume:
    - * ϵ_i follows logistic (with mean 1, SD σ)

      ▷

$$\varepsilon_i \ \equiv \ \frac{\ln \epsilon_i}{\sigma}$$

      follows standard logistic (mean 1, SD 1)
    - * $F(\cdot) \equiv$ CDF of ε_i
    - * $\ln D(x_i)$ is linear:

$$\ln D(x_i) \ = \ \beta_0 + \beta_1 x_{i1} + \cdots + \beta_K x_{iK} \tag{16}$$

[10] I.e., indifference relation is not transitive.

- Response condition:
  * "more" if:

$$\varepsilon_i \;<\; \left[\sum_{j=1}^{K} \frac{\beta_j x_{ij}}{\sigma} \;-\; \frac{\ln a_i}{\sigma} \;+\; \frac{\beta_0 - \ln\delta}{\sigma}\right]$$

  * "less" if:

$$\varepsilon_i \;>\; \left[\sum_{j=1}^{K} \frac{\beta_j x_{ij}}{\sigma} \;-\; \frac{\ln a_i}{\sigma} \;+\; \frac{\beta_0 + \ln\delta}{\sigma}\right]$$

  * "same" otherwise

- Estimation Model:
  - Response probability:
    * "more":

$$\pi_m^i \;=\; F\left(\sum_{j=1}^{K}\left[\frac{\beta_j}{\sigma} x_{ij}\right] - \frac{\ln a_i}{\sigma} + \frac{\beta_0 - \ln\delta}{\sigma}\right)$$

    * "less":

$$\pi_e^i \;=\; 1 - F\left(\sum_{j=1}^{K}\left[\frac{\beta_j}{\sigma} x_{ij}\right] - \frac{\ln a_i}{\sigma} + \frac{\beta_0 + \ln\delta}{\sigma}\right)$$

    * "same":

$$\pi_0^i \;\equiv\; 1 - \pi_m^i - \pi_e^i$$

  - MLE: maximize likelihood function
    1. Obtain $\hat{\sigma}$ from $\ln a_i$'s coefficient $1/\hat{\sigma}$.
    2. Recover $\hat{\beta}_j$ from x_{ij}'s coefficient $\hat{\beta}_j/\hat{\sigma}$.
    3. Solve for $\hat{\beta}_0$ and $\hat{\delta}$ using intercepts $(\beta_0 - \ln\delta)/\sigma$ and $(\beta_0 + \ln\delta)/\sigma)$ of π_m^i and π_e^i.

– Estimation procedure:

∗ Variables x_{ij}: tax price and post-tax income y_i of i

∗ Consumer demand for PG:

$$\ln g_i \;=\; \beta_0 + \beta_1 \ln t_i + \beta_2 \ln y_i + \sum_{j=3}^{K} \beta_j x_{ij} - \varepsilon_i \qquad (17)$$

where:

g_i = public expenditure level desired by i

y_i = post-tax income of i

t_i = MC of \$1 extra PG spending to i

∗ Equation:

$$\ln \frac{g_i}{P_e} \;=\; \beta_0 + \beta_1 \ln \frac{t_i P_e}{P_0} + \beta_2 \ln \frac{y_i}{P_0} + \sum_{j=3}^{K} \beta_j x_{ij} - \varepsilon_i \qquad (18)$$

where:

P_e = local PG price where i resides

P_0 = average price level where i resides

g_i/P_e = local PG level

y_i/P_0 = normalized post-tax income of i

$t_i P_e$ = normalized marginal tax burden on i of local PG

∗ Simplifying:

$$\ln g_i \;=\; \beta_0 + \beta_1 \ln t_i + \beta_2 \ln y_i + [1+\beta_1] \ln P_e - [\beta_1+\beta_2] \ln P_0 + \sum_{j=3}^{K} \beta_j x_{ij} - \varepsilon_i \qquad (19)$$

5 Binary PG: Bohm [JPuE 1984]

- Interval method: 2 subject groups
  - Group 1: under-report WTP (average α)
  - Group 2: over-report WTP (average β)
  - True WTP interval:

$$[\alpha, \beta]$$

  - PG provision rule: PG cost C
    · $C < \alpha$: PG provided
    · $C > \beta$: no PG
    · $\alpha < C < \beta$: indeterminate
  - Good design: interval $[\alpha, \beta]$ is small
- Bohm (1969) experiment:
  - Survey: 200 Stockholm residents in 4 groups
  - PG in question: Cable TV service
  - Payment scheme:
    * Group 1: pay full declared WTP
    * Group 2: pay fixed % of declared WTP
    * Group 3: pay a flat rate (independent of declared WTP)
    * Group 4: no payment required
  - Reporting incentives:
    * Groups 1 & 2: under-report (lower bound)

    - Groups 3 & 4: over-report (upper bound)
  - Results: not much difference between groups

- Real-world application: Bohm (1982)
  - 279 local governments of Sweden
  - WTP for some future public service
  - Payment scheme:
    - Group 1: pay fixed % of declared WTP
    - Group 2: payment depends on declared WTP
      - US$100 if WTP > 100
      - No service and no payment otherwise
  - Results:
    - Interval size is only 7.5% of LB α
    - PG provided as a result