資訊不對稱與機制設計

1 Analysis Framework

- n agents:

$$u_i(x, \theta_i)$$

  - x ($\in X$) is a social outcome
  - θ_i ($\in \Theta_i$) is agent type, private info

- Agent type profile:

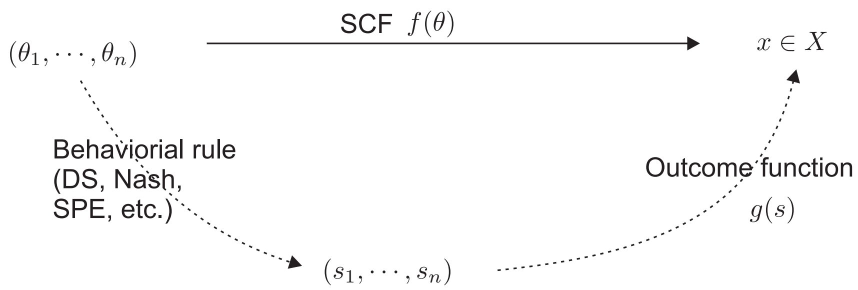
$$\theta \equiv (\theta_1, \cdots, \theta_n)$$

- Social choice/goal function:

$$f(\theta) \in X$$

- Mechanism/game design:

$$\Gamma(S_1, \cdots, S_n, g(\cdot))$$



– Player strategy:

$$s \equiv (s_1, \cdots, s_n), \;\; s_i(\theta_i) \in S_i$$

– Outcome function:

$$g(s) \in X$$

– Γ implements f if there exists am equilibrium strategy

$$(s_1^*(\theta_1), \cdots, s_n^*(\theta_n))$$

such that, for any θ:

$$g(s_1^*(\theta_1), \cdots, s_n^*(\theta_n)) = f(\theta) \quad \square$$

- Revelation principle:

  Any f that is implementable can be implemented by a "direct revelation" mechanism.

- Equilibrium concepts:

  – Dominant-strategy mechanism:
  Clarke [1971], Groves-Loeb [1975]

  – Nash mechanism:
  Groves-Ledyard [1977]

  – SPE mechanism:
  Moore-Repullo [1988]

1. Dominant-strategy Mechanism

1.1. Consumer Mechanism [Clarke, PC 1971]

- The Model
  - 2 goods:

    $x \equiv$ private good

    $y \equiv$ public good
  - Consumer i: income

$$w_i$$

  - Quasi-linear utility function:

$$U_i(x_i,\, y) \;=\; x_i + f_i(y), \quad f_i' > 0, \quad f_i'' < 0$$

  - PG production cost:

$$C(y)$$

- Govt goal: maximize Benthamite social welfare

$$\begin{aligned}\max_y \;\; W \;&\equiv\; \sum_i U_i(x_i, y) \;=\; \sum_i x_i \;+\; \sum_i f_i(y)\\ &=\; \left[\sum w_i \;-\; C(y)\right] \;+\; \sum_i f_i(y)\end{aligned}$$

▷

$$\max_y \quad \sum_i f_i(y) - C(y)$$

- Info asym: govt does not know $f_i(y)$

  ▷ Design a mechanism so consumers will truthfully reveal $f_i(y)$

- Clarke mechanism/tax: 2 stages

  S1 (PG rule) Each consumer i reports his/her $f_i(y)$ as $M_i(y)$

  Govt then takes $M_i(y)$ as real $f_i(y)$ and chooses y accordingly:

$$\max_y \sum_i M_i(y) - C(y)$$

  $\triangleright$ $y(M)$, where $M \equiv (M_1, \ldots, M_n)$

  S2 (Cost-sharing) Each consumer i pays tax:

$$T_i(M) = C(y(M)) - \sum_{j \neq i} M_j(y(M)) - R_i(M_{-i})$$

  where $M_{-i} \equiv (M_1, \ldots, M_{i-1}, M_{i+1}, \ldots, M_n)$. $\square$

- Consumer incentive:

  - i's utility after reporting $M_i(y)$:

$$\begin{aligned} U_i^* &= [w_i - T_i(M)] + f_i(y(M)) \\ &= w_i - \Big[C(y(M)) - \textstyle\sum_{j \neq i} M_j(y(M)) - R_i(M_{-i})\Big] + f_i(y(M)) \end{aligned}$$

  - Consumer reporting strategy:

$$\max_{M_i(y)} \; f_i(y(M)) + \sum_{j \neq i} M_j(y(M)) - C(y(M))$$

  - Dominant strategy: truthful revelation

$$M_i(y) = f_i(y)$$

- Government budget
  - Cannot guarantee govt budget balance
  - To insure surplus: let θ_i be i's cost share[1]

$$R_i(M_{-i}) \equiv \min_y \; [1-\theta_i]C(y) - \sum_{j\neq i} M_j(y)$$

then:

$$T_i(M) \geq \theta_i C(y(M)), \;\; \forall i$$

hence;

$$T(M) = \sum_i T_i(M) \geq \sum_i [\theta_i C(y(M))] = C(y(M))$$

  - Surplus cannot be returned to consumers

    ▷ Must be discarded, or given to another economy

[1] Therefore, $\sum_i \theta_i = 1$. If PG cost is to be shared equally, then $\theta_i = 1/n$.

1.2. Producer Mechanism [Groves-Loeb, JPuE 1975]

- Firm i: profit-maximizing

$$\pi_i(K) \equiv \max_{L_i} H_i(L_i, K)$$

where:

H_i = profit function

L_i = labor input

K = public service

- Assume: $\pi_i(K)$ is
  - differentiable
  - mono-increasing
  - strictly concave

- Government:
  - provide K at unit price p
  - not knowing $\pi_i(K)$

- Groves-Loeb 2-stage mechanism:

S1 (PG rule) Firm i reports its $\pi_i(K)$ as $M_i(K)$

▷ Govt then treats $M_i(K)$ as real $\pi_i(K)$ and chooses K:

$$\max_K \sum_i M_i(K) - pK$$

⇒ $K(M)$, with

$$M \equiv (M_1, \ldots, M_n)$$

S2 (Cost-sharing) Each firm pays tax:

$$T_i(M) \equiv pK(M) - \sum_{j \neq i} M_j(K(M)) + R_i(M_{-i})$$

with:

$$M_{-i} \equiv (M_1, \ldots, M_{i-1}, M_{i+1}, \ldots, M_n) \quad \square$$

- Firm's goal: choose $M_i(y)$ to max:

$$\pi_i(K(M)) - T_i(M)$$

$$= \pi_i(K(M)) + \sum_{j \neq i} M_j(K(M)) - pK(M) - R_i(M_{-i})$$

▷ Dominant strategy: truthful revelation

$$M_i(K) = \pi_i(K)$$

- Government budget surplus:[2]

$$\sum_i T_i(M) - pK(M)$$

$$= \sum_i R_i(M_{-i}) - (n-1)\left[\sum_i M_i(K(M)) - pK(M)\right]$$

[2]In some special cases, govt budget balance can be obtained. For example, firms may have quadratic $\pi_i(K)$:

$$\pi_i(K) = \alpha_i K - \frac{K^2}{2}$$

and govt does not know α_i. After firms report their α_i as a_i, govt will maximize total profit $[\sum \pi_i(K) - pK]$ and choose

$$K(a) = \frac{\sum a_i - p}{n}; \quad p < \sum_i a_i$$

With firm tax set as

$$T_i(a) = pK(a) - \sum_{j \neq i}\left[a_j K - \frac{K^2}{2}\right] + R_i(a_{-i}), \quad R_i(a_{-i}) \equiv \frac{[\sum_{j \neq i} a_j - p]^2}{2n} + \frac{\sum_{j \neq k;\, j,k \neq i} a_j a_k}{2n[n-2]} - \frac{p^2}{2n^2}$$

firms will tell the truth, and govt will have balanced budget.

2. Nash Mechanism

1. Optimal government: Groves-Ledyard [Econometrica 1977]

	Clarke mechanism	G/L mechanism
1. Reporting	Utility function	Individual demand
2. Truth-telling	Dominant strategy	Nash equilibrium
3. Govt budget	Surplus	Balance
4. Preference	Quasi-linear	Any convex preference
5. Equilibrium	Partial equilibrium (demand or supply)	General equilibrium (both D and S)

2. Original G/L Mechanism:

- 2-stage design:

  S1 After receiving individual $m_i(y)$, govt chooses PG by:

$$\max_y \sum_i m_i(y) - Py \tag{1}$$

  ▷ $y = y(M)$, where

$$M = (m_1, m_2, \cdots, m_n)$$

  S2 Consumer i has to pay tax:

$$T_i(M) = \alpha_i Py - \sum_{j \neq i} [m_j(y) - \alpha_j Py] + R_i(M_{-i})$$

  where

$$M_{-i} = (m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n)$$

  and

$$\sum_i \alpha_i = 1 \quad \square$$

- Interpretation of $T_i(M)$:
  - $\alpha_i P y$: i's fixed proportional cost share
  - $[m_j(y) - \alpha_j P y]$: j's net consumer surplus
  - $R_i(M_{-i})$: i's lump-sum income transfer (indep. of $m_i(y)$)
- Unique Nash equilibrium:
  - Honest reporting of WTP function, Pareto optimality

  [?] cannot achieve balanced budget

3. Improved G/L Mechanism:

- 3-stage design:

  [S1] Each consumer reports a real number:[3]

$$m_i \ (\in \mathcal{R}) \ \gtreqless \ 0$$

  [S2] PG level will be

$$y(M) \ = \ \sum_i m_i$$

  [S3] Given $M = (m_1, \cdots, m_n)$ and $y^*(= y(M))$, consumer i pays:

$$T_i(M) \ = \ \alpha_i P y^* + \frac{r}{2}\left[\frac{n-1}{n}(m_i - \mu_{-i})^2 - \sigma_{-i}^2\right]$$

  where:

$$\mu_{-i} \ \equiv \ \frac{\sum_{j \neq i} m_j}{n-1}; \quad \sigma_{-i}^2 \ \equiv \ \frac{\sum_{j \neq i}\sum_{k \neq i}(m_j - m_k)^2}{2[n-1][n-2]}$$

  and α_i and r can be any number such that $\sum_i \alpha_i = 1$ □

[3] It can be interpreted as i's extra demand (increment/decrement) for PG, given others' total demand.

- Interpretation of $T_i(M)$:
  - T_i goes up if his/her m_i diverges from others' average μ_{-i}
  - T_i smaller when others' m_j quite different (variance $\sigma^2_{-i} \uparrow$)
- Nash:
  - Consumer goal: given $(m_1, \ldots, m_{i-1}, m_{i+1}, \ldots, m_n)$

$$\max_{m_i} \; U_i \;=\; x_i + f_i(y^*) \;=\; [w_i - T_i(M)] + f_i(\sum_j m_j)$$

<u>foc</u>:

$$f_i'(\sum_j m_j) \;=\; T_i'(M) \;=\; \alpha_i p + r\left[m_i - \frac{\sum_j m_j}{n}\right]$$

▷ Summing up, we have the Samuelson foc:

$$\sum_i f_i'(y^*) \;=\; \sum_i [\alpha_i p] \;=\; p$$

  - Uniqueness of Nash:
    - ∗ As $f(\cdot)$ is strictly concave, y^* is unique
    - ∗ Unique individual m_i:

$$m_i \;=\; \frac{f_i'(y^*) - \alpha_i p}{r} \;+\; \frac{y^*}{n}$$

- Balanced budget:

$$\sum_i T_i(M) \;=\; py^*$$

4. Other Nash mechanism: Hurwicz [1979], Walker [1981]

3. SPE Mechanism

3.1. Monotoncity

- Monotoncity: necessary condition for Nash implementability[4]
- Monotoncity (Maskin 1997):[5] Consider a social choice function $f(\cdot)$. Let $L_i^\theta(x)$ be lower contour (worse) set of x for consumer i under any profile θ. f is monotonic if, for any other possible θ',

$$L_i^\theta(f(\theta)) \subseteq L_i^{\theta'}(f(\theta)), \forall i \quad \Rightarrow \quad f(\theta') = f(\theta) \ \square$$

3.2. Example

- Moore-Repullo [Econometrica 1988]
- 2 consumers: Fig. 1
  - Same utility type: Cobb-Douglas (C)[6] v. Leontiff (L)[7]
  - Social goal: choose $f(C)$ or $f(L)$ accordingly
  - Conflict: 1 prefers $f(C)$, 2 prefers $f(L)$
  - Type (C or L) unknown to govt:
    - ▷ 1 will claim C, 2 will claim L

[4] As for the sufficient condition, we further need "no veto power".

[5] Sketch of proof: Suppose ∃ Nash implementation with $s^* = s(\theta)$ as a Nash strategy:

$$g(s_1^*, \cdots, s_n^*) = f(\theta), \quad \forall \theta$$

Then by "revealed preference", s_i^* is also i's equilibrium strategy for θ', because better sets

$$B_i^\theta(f(\theta)) \supset B_i^{\theta'}(f(\theta)), \forall i$$

Hence must also let $g(s_1^*, \cdots, s_n^*) = f(\theta')$ □

[6] The goods are complements.

[7] The goods are substitutes.

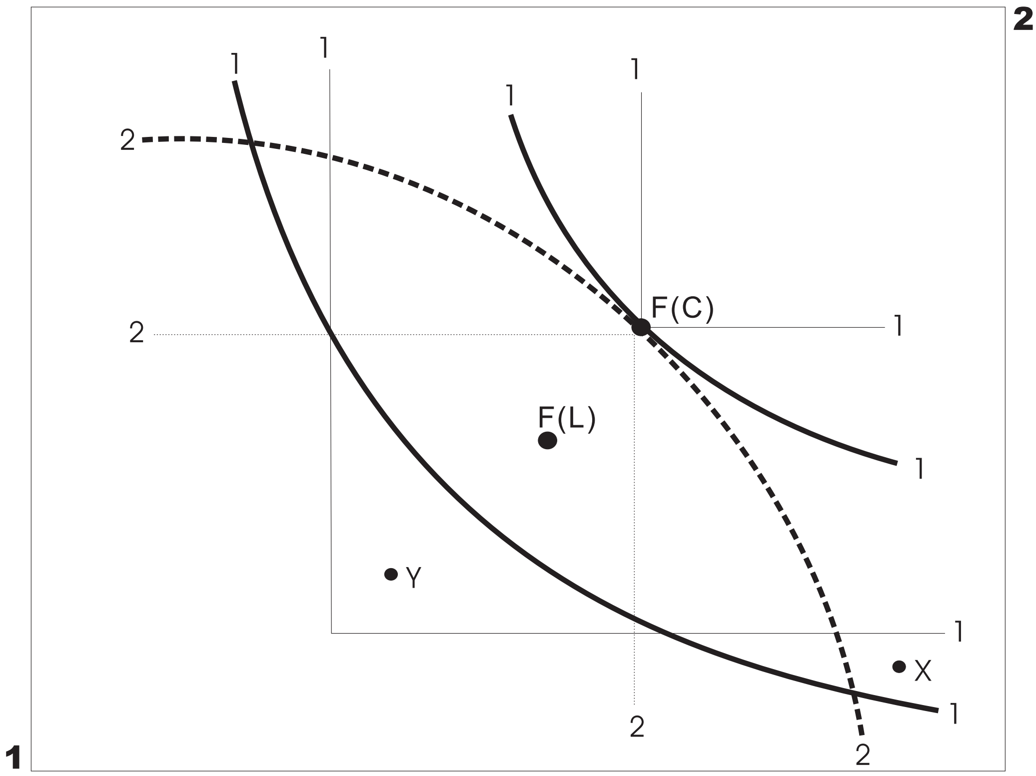


Figure 1: SPE implementation

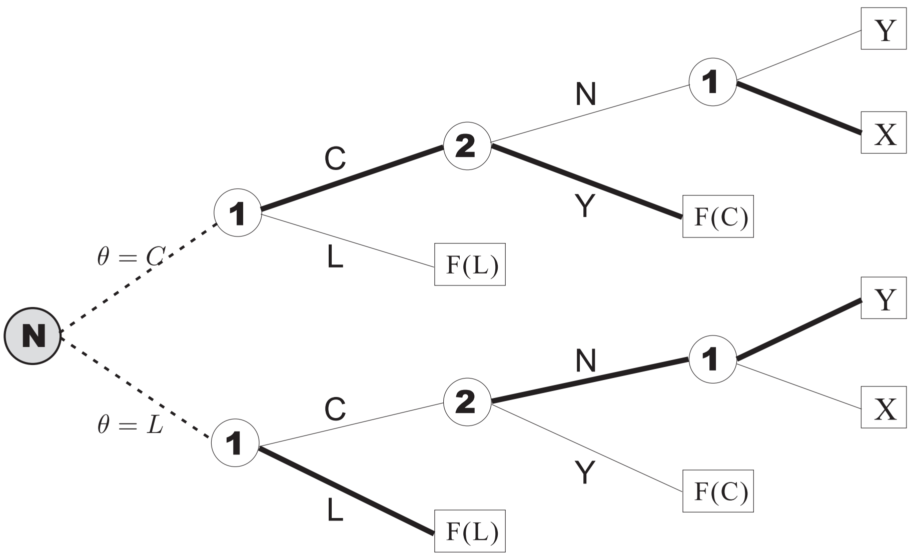


Figure 2: Game tree [Moore-Repullo 1988]

- f is not monotonic, because:

$$L_1^C(f(C)) \subset L_1^L(f(C)),\ L_2^C(f(C)) \subset L_2^L(f(C))$$

but:

$$f(C) \neq f(L) \quad \square$$

$\triangleright$ f is not Nash-implementable

- SPE-implementation: 3-stage design

[S1] First 1 declares their common type θ $(= C$ or $L)$.

If 1 admits $\theta = L$, we go for $f(L)$. [EoG]

Otherwise, we go to Stage 2.

[S2] Now 2 will confirm/reject 1's announced θ.

If 2 agrees with 1 on type $\theta = C$, we go for $f(C)$. [EoG]

Otherwise, there is conflict. We enter Stage 3.

[S3] Finally, it is up to 1 to choose between x and y. ■

- Unique SPE: [Fig. 2]
  - 1 will tell truth in Stage 1
  - 2 will confirm in Stage 2 if necessary

3.3. The PG Problem

- 2 consumers ($i = 1, 2$):
  - Prefernce/type: private info θ_i
  - Utility from PG d:

$$u_i(d, \theta_i)$$

- Government goal: with true (θ_1, θ_2)
  - Desired PG level:

$$d(\theta_1, \theta_2)$$

  - Tax on 1:

$$t_1(\theta_1, \theta_2) > 0$$

  - Subsidy for 2:[8]

$$t_2(\theta_1, \theta_2) \; > \; 0$$

- Social choice function f:

$$f(\theta_1, \theta_2) \; = \; (d(\theta_1, \theta_2),\, t_1(\theta_1, \theta_2),\, t_2(\theta_1, \theta_2))$$

  and hence:

$$U_1 \; = \; u_1(d, \theta_1) - t_1$$

$$U_2 \; = \; u_2(d, \theta_2) + t_2$$

[8] We can set $t_1 = t_2$ for balanced govt budget. But in general, govt may allow any positive (t_1, t_2) combination.

- Moore-Repullo 3-stage design: *inquire* θ_i *one-by-one*

  [S1] Consumer 1 announces his type θ_1

  [S2] We next check with consumer 2.

  – If she agrees, we accept θ_1. [EoG]

  – Otherwise, we ask 2 to disclose 1's true type. Let it be ϕ_1.

  [S3] Again it is up to 1 to choose between X and Y:

$$\begin{aligned} X &= (x,\, t_x + \Delta,\, t_x - \Delta) \\ Y &= (y,\, t_y + \Delta,\, t_y + \Delta) \end{aligned}$$

  where: Δ is a very large positive number, and (x, y, t_x, t_y) satisfy:

$$\begin{aligned} u_1(x, \theta_1) - t_x &> u_1(y, \theta_1) - t_y \\ u_1(x, \phi_1) - t_x &< u_1(y, \phi_1) - t_y \end{aligned}$$

■

- SPE: use *backward induction*

  S3 This stage is reached only when 1 says θ_1 and 2 says ϕ_1.

  Here 1 will choose X if he was honest (with true type θ_1).

  Otherwise 1 will prefer Y (with true type ϕ_1).

  S2 Player 2 will act as she should:

  – Confirm if 1 told the truth in S1.[9]

  – Challenge if 1 lied in S1.[10]

  S1 1 should tell truth, instead of lying, since:

$$u_1(d, \theta_1) - t_1 \ > \ u_1(y, \theta_1) - t_y - \Delta$$

■

▷ Equilibrium path: 1 tells truth, then 2 confirms.

[9] Player 2 is better off confirming (than to challenge and get X in S3) if θ_1 is true:

$$u_2(d, \theta_2) + t_2 \ > \ u_2(x, \theta_2) + t_x - \Delta$$

[10] By challenging, she will get Y in S3, much better than being silent:

$$u_2(d, \theta_2) + t_2 \ < \ u_2(y, \theta_2) + t_y + \Delta$$

4. Binary Choice

4.1. Pivot Mechanism: Tideman-Tullock [JPE 1976]

1. Choice between two options:

   - Indivisible PG: "yes" or "no"
   - Project choice:

   $$\alpha \text{ v. } \beta$$

2. The context: choice between two options (α v. β)

   - Preference intensity considered (cf. majority voting)
   - Value/WTP of α (against β) for agent i:

   $$v_i \gtreqless 0$$

   ▷ private info

   - Social goal: choose α iff

   $$\sum_i v_i \geq 0$$

   - Self-reporting incentives: mis-representation of v_i

3. T/T mechanism:

   - Each agent i reports his/her valuation of α (against β) as $\hat{v}_i$.
   - Let $\hat{V}$ be the sum of individual $\hat{v}_i$:

   $$\hat{V} \equiv \sum_i \hat{v}_i$$

- We choose:
  - α: if $\hat{V} \geq 0$
  - β: if $\hat{V} < 0$
- Payment rule: only pivotal consumers have to pay[11]
  - If $\hat{v}_i > \hat{V} > 0$, then i must pay

$$\hat{v}_i - \hat{V} \quad (> 0)$$

  - If $0 > \hat{V} > \hat{v}_i$, then i must pay

$$\hat{V} - \hat{v}_i \quad (> 0) \quad \square$$

E Consider 5 consumers, who claim their valuation for α as:

$$v_1 = 12,\ v_2 = 11,\ v_3 = 8,\ v_4 = 1,\ v_5 = -22$$

So the reported total is $\hat{V} = 10$, and the choice would be α.

Now, only 1 and 2 are required to pay (\$2 and \$1, respectively). $\square$

4. Dominant-strategy equilibrium: honest reporting ($\hat{v}_i = v_i$)

(a) Suppose an agent i prefers α (that is, $v_i > 0$), and let

$$\hat{V}_{-i} \equiv \sum_{j \neq i} \hat{v}_j$$

(b) Consider her best reporting strategy in 4 cases below:

- $\hat{V}_{-i} > 0$
- $\hat{V}_{-i} = 0$

[11] A person is called pivotal because, without his/her reported value $\hat{v}_i$, the outcome will be reversed. I.e., $\sum_{j \neq i} \hat{v}_j$ and $\hat{V}$ have opposite sign. Therefore, a pivotal person should pay for the loss he/she imposes on other people.

- $\hat{V}_{-i} \in (-v_i, 0)$
- $\hat{V}_{-i} < -v_i < 0$

▷ Lying will not do you any good, and can only hurt you.

5. Problems with T/T mechanism:

- People may collude and get whatever they like, without paying anything.[12]
- Govt budget deficit: with large population, no one has to pay.
- Waste of resources: payment collected must be trashed (or donated to charity).

[12] This surely will not happen when there are two agents. So T/T design is quite suitable for couples to make movie or restaurant decisions.

4.2. Cost-sharing Mechanism: Jackson-Moulin [JET 1992]

1. The context:
   - Indivisible PG: provision cost C
   - n consumers: PG benefit b_i is private info

2. The implementation problem:
   - Choice efficiency: providing PG if

   $$C \;\leq\; B \;\equiv\; \sum_i b_i$$

   - Proper cost sharing: c_i $(i = 1, \cdots, n)$ with

   $$\sum_i c_i \;\geq\; C$$

   - Individual rationality (participation):

   $$b_i \;\geq\; c_i, \;\; \forall\, i$$

3. The 2-stage 2-agent mechanism: $i = 1, 2$

   [S1] Both declare their estimated total PG value

   $$b_1 \;+\; b_2$$

   as V_1 and V_2, respectively. Assume $V_1 \geq V_2$.
   - If $V_1 \geq C$, we proceed to S2.[13]
   - Otherwise $C > V_1 \geq V_2$, we stop. [EoG][14]

[13] Now at least one of them considers the PG worthwhile.
[14] They both think the PG has too little value.

S2 Now each reports her individual value b_i as β_i.

- If $\beta_1 + \beta_2 > V_1$: PG provided, and they each have to pay:

$$c_1 = \frac{\beta_1 C}{V_1}, \quad c_2 = \frac{[V_1 - \beta_1]C}{V_1}$$

- If $\beta_1 + \beta_2 < V_1$: no PG, and 1 has to compensate 2:

$$[V_1 - \beta_1] - \frac{[V_1 - \beta_1]C}{V_1}$$

- If $\beta_1 + \beta_2 = V_1$: 1 may choose one of the above. ■

4. Unique undominated Nash: honest reporting

$$(S1) \quad V_1 = V_2 = b_1 + b_2$$

$$(S2) \quad \beta_1 = b_1, \beta_2 = b_2 \quad \square$$

Proof: using backward induction:

S2 Given V_1 (in S1) and β_1 (in S2), player 2 faces three possible cases:

- $b_2 > V_1 - \beta_1$:

  If 2 tells truth ($\beta_2 = b_2$), she gets PG and utility:

$$b_2 - \frac{[V_1 - \beta_1]C}{V_1}$$

  If instead she lies (under-reports) β_2, she may lose PG and get lower utility:

$$[V_1 - \beta_1] - \frac{[V_1 - \beta_1]C}{V_1}$$

- $b_2 < V_1 - \beta_1$:

  Now 2 is better telling truth (hence no PG) than exaggerating

β_2 (to obtain PG):

$$b_2 - \frac{[V_1 - \beta_1]C}{V_1} \; < \; [V_1 - \beta_1] - \frac{[V_1 - \beta_1]C}{V_1}$$

- $b_2 = V_1 - \beta_1$:

  Now telling truth or not yields same utility for 2:

$$b_2 - \frac{[V_1 - \beta_1]C}{V_1} \; = \; [V_1 - \beta_1] - \frac{[V_1 - \beta_1]C}{V_1}$$

Therefore, being honest ($\beta_2 = b_2$) is her weakly dominant strategy.

Furthermore, if $b_1 + b_2 \geq C$, then given V_1 and $\beta_2 = b_2$, 1 should choose:

$$\beta_1 \; = \; \max\{V_1 - b_2, 0\}$$

to minimize his cost.[15]

S1 Back to stage 1, knowing that $\beta_1 = b_1$ and $\beta_2 = b_2$ in S2, player 1's goal is to set maximal V_1 (te reduce costs in S2), subject to $V_1 \leq b_1 + b_2$ (for having PG). Hence it must be:

$$V_1 \; = \; b_1 \; + \; b_2 \quad \square$$

[15] Because PG will only exist when $\beta_1 + \beta_2 > V_1$, β_1 should be at least $V_1 - b_2$. Meanwhile, to minimize his cost

$$c_1 \; = \; \frac{\beta_1 C}{V_1}$$

1's optimal choice is:

$$\beta_1 \; = \; V_1 - b_2$$

As such, the players' utility levels are:

$$U_1 \; = \; b_1 \; - \; \frac{[V_1 - b_2]C}{V_1} \; = \; b_1 + \frac{C}{V_1} b_2 - C \;\; (\geq 0 \;\; \text{if} \;\; V_1 \leq C)$$

$$U_2 \; = \; b_2 \; - \; \frac{b_2 C}{V_1} (\geq 0 \;\; \text{if} \;\; V_1 \geq C)$$