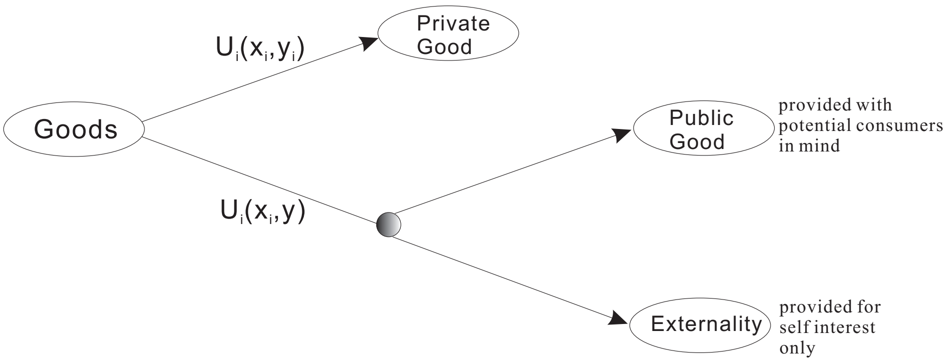
外 部 性

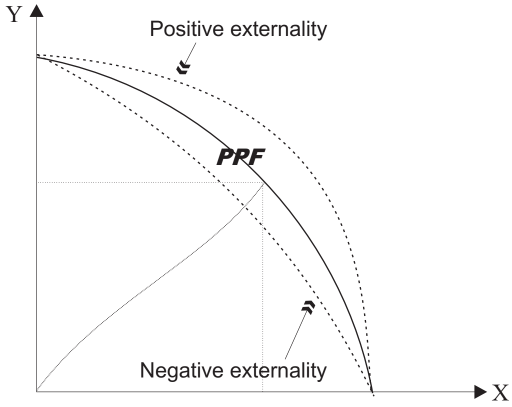
1 Physical v. Pecuniary Externality

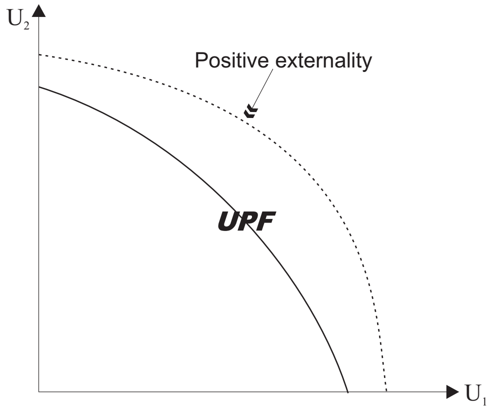
- Lacking of market (price) mechanism



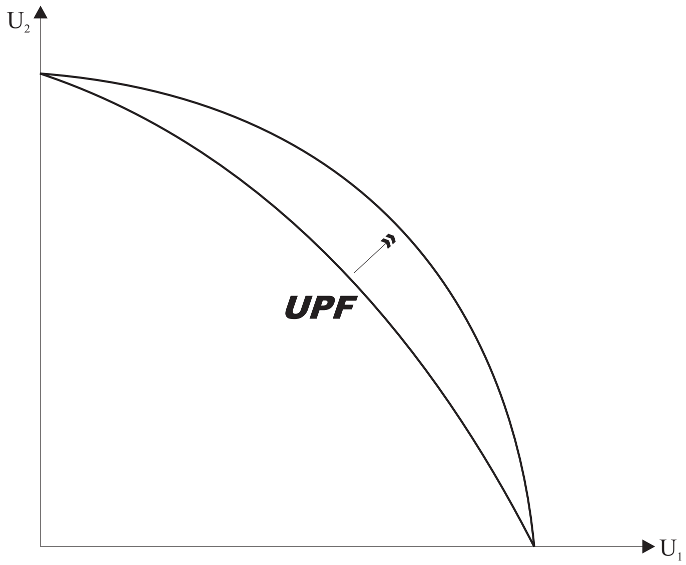
2 Four Possible Types

- "Producer-producer" externality: PPF and UPF change[1]





- "Consumer-consumer" externality: UPF affected



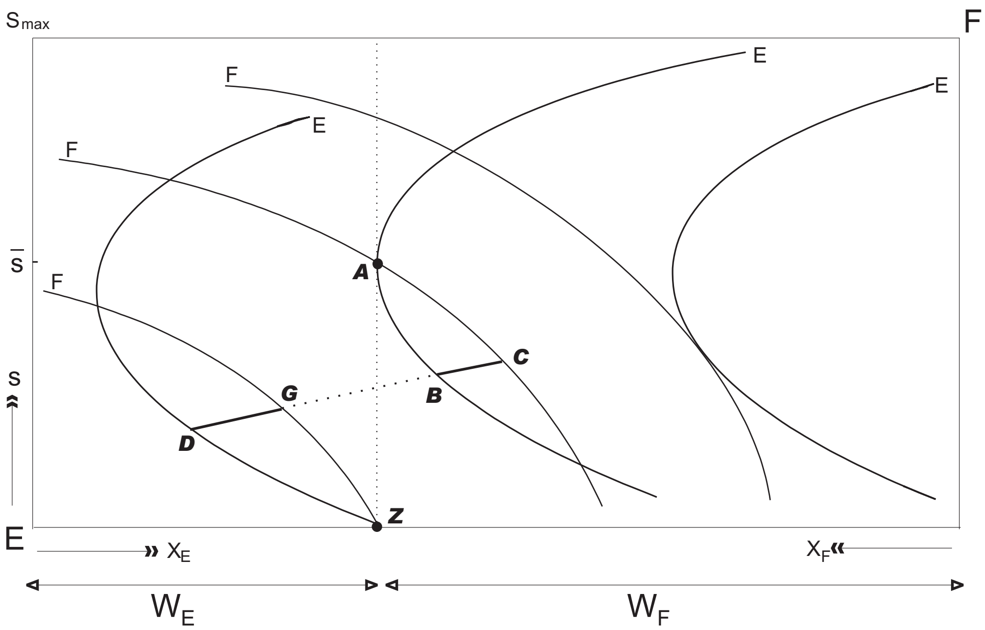
- "Producer-consumer" externality
- "Consumer-producer" externality

[1]See R.C. Griffin, "The Welfare Analytics of Transaction Costs, Externalities, and Institutional Choice," *AJAE*, 1991, 73:601–14. For example, original PPF may be $x^2 + y^2 = 1$. New PPF under positive externality may be $x^3 + y^3 = 1$, whereas PPF under negative externality may be $x + y = 1$.

3 Coasian Bargaining: Contractarian Solution

- Relevant when:
  - Few people involved
  - No transaction cost
- Property right assignment by law will not affect efficiency
- Income redistribution effect

	Smoking "allowed"	Smoking "not allowed"
No negotiation	A	Z
Negotiation possible	BC* (F pays E)	DG* (E pays F)



4 Basic Model

- 2 consumers: $(A,\ B)$

  ▷ Total income W

- 2 goods: $(1,\ 2)$

  ▷ Price (p_1, p_2)

- Mutual externality: consumption of good 1[2]

- Consumer utility:

$$U^A(x_1^A, x_2^A, x_1^B)$$

$$U^B(x_1^B, x_2^B, x_1^A)$$

- Pareto optimality:

$$\max_{\{x_1^A, x_2^A, x_1^B, x_2^B\}} \quad U^A \quad \text{s.t.} \quad \begin{cases} U^B \ \geq \ \bar{U} \\ p_1[x_1^A + x_1^B] + p_2[x_2^A + x_2^B] \ \leq \ W \end{cases}$$

foc:

$$\sum_{i=A,B} \frac{\partial U^i/\partial x_1^A}{\partial U^i/\partial x_2^i} \ = \ \frac{p_1}{p_2}$$

$$\sum_{i=A,B} \frac{\partial U^i/\partial x_1^B}{\partial U^i/\partial x_2^i} \ = \ \frac{p_1}{p_2}$$

[2]E.g., music listening.

5 Altruistic Preferences: Psychic externalities

5.1 Consumption Dependency

- Caring:

$$U_A(x_A, y_A, x_B, y_B), \quad U_B(x_B, y_B, x_A, y_A)$$

with:

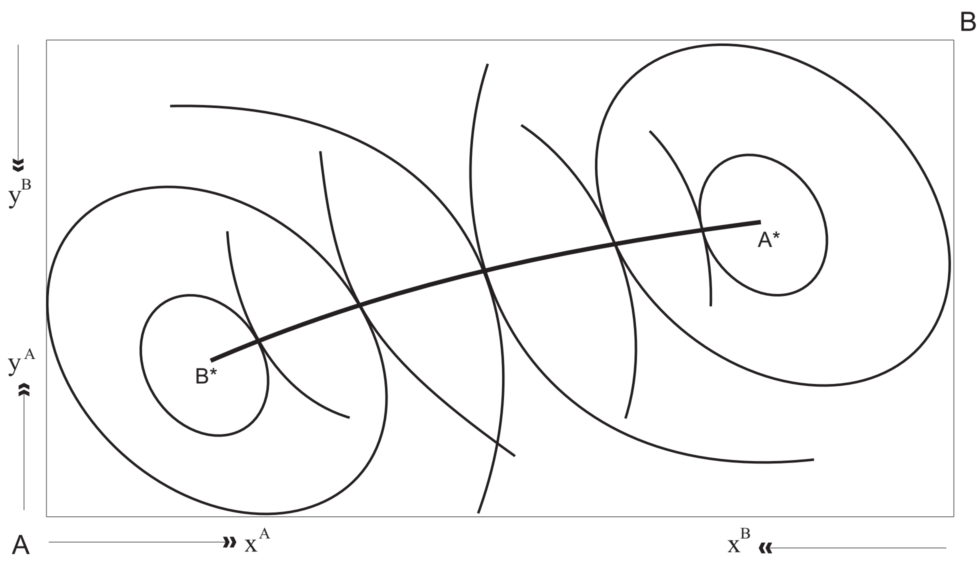
$$\frac{\partial U_A}{\partial x_B} > 0, \; \frac{\partial U_A}{\partial y_B} > 0; \quad \frac{\partial U_B}{\partial x_A} > 0, \; \frac{\partial U_B}{\partial y_A} > 0$$

or, for *jealousy*:

$$\frac{\partial U_A}{\partial x_B} < 0, \; \frac{\partial U_A}{\partial y_B} < 0; \quad \frac{\partial U_B}{\partial x_A} < 0, \; \frac{\partial U_B}{\partial y_A} < 0$$

- Pareto Optimality:
  - Individual ideal point: A^* and B^*
  - Aggregate budget:

$$x_A + x_B \; = \; w_x; \quad y_A + y_B \; = \; w_y$$



– Net MU of x_A to A:

$$\mathrm{MU}^A_{x_A} \ - \ \mathrm{MU}^A_{x_B}$$

– PO (tangency at contract curve A^*B^*):

$$\mathrm{MRS}^A_{x,y} \ = \ \frac{\mathrm{MU}^A_{x_A} - \mathrm{MU}^A_{x_B}}{\mathrm{MU}^A_{y_A} - \mathrm{MU}^A_{y_B}} \ = \ \frac{\mathrm{MU}^B_{x_B} - \mathrm{MU}^B_{x_A}}{\mathrm{MU}^B_{y_B} - \mathrm{MU}^B_{y_A}} \ = \ \mathrm{MRS}^B_{x,y} \quad (1)$$

- Competitive equilibrium (CE):

$$\mathrm{MRS}^A_{x,y} \ = \ \frac{\mathrm{MU}^A_{x_A}}{\mathrm{MU}^A_{y_A}} \ = \ \frac{P_A}{P_B} \ = \ \frac{\mathrm{MU}^B_{x_B}}{\mathrm{MU}^B_{y_B}} \ = \ \mathrm{MRS}^B_{x,y} \quad (2)$$

▷ Market outcome is ineffecient (due to externality)

- Non-paternalism: non-paternalistic preferences[3]

$$\frac{\mathrm{MU}^A_{x_B}}{\mathrm{MU}^A_{y_B}} \ = \ \frac{\mathrm{MU}^B_{x_B}}{\mathrm{MU}^B_{y_B}}$$

$$\frac{\mathrm{MU}^B_{x_A}}{\mathrm{MU}^B_{y_A}} \ = \ \frac{\mathrm{MU}^A_{x_A}}{\mathrm{MU}^A_{y_A}}$$

▷ Respecting mutual preference

▷ CE is PO: (2) implies (1)

[3]Archibald, G.C. and D. Donaldson (1976) "Non-paternalism and the Basic Theorems of Welfare Economics," *Canadian Journal of Economics*, 9(3):492-507.

5.2 Utility Interdependency

- Consumer utility:

$$U_A(x_A, y_A, u_A(x_B, y_B))$$

$$U_B(x_B, y_B, u_B(x_A, y_A))$$

- *Non-paternalistic preferences*:

$$\frac{\partial u_A/\partial x_B}{\partial u_A/\partial y_B} = \frac{\partial U_B/\partial x_B}{\partial U_B/\partial y_B}$$

$$\frac{\partial u_B/\partial x_A}{\partial u_B/\partial y_A} = \frac{\partial U_A/\partial x_A}{\partial U_A/\partial y_A}$$

- Example:[4]

$$U_A(x_A, y_A, U_B), \quad U_B(x_B, y_B, U_A)$$

[4] This is a case of the "Pareto-irrelevant externality". See R.P Parks, "Pareto Irrelevant Externality," *JET*, 54:165–79。

5.3 The Dilemma of Romeo and Juliet [Bergstrom, JEP 1988]

- Altruistic preferences:

$$U_R(S_R, U_J); \quad U_J(S_J, U_R)$$

with:

$$(R) \quad \frac{\partial U_R}{\partial S_R} > 0, \ \frac{\partial U_R}{\partial U_J} > 0; \ (J) \quad \frac{\partial U_J}{\partial S_J} > 0, \ \frac{\partial U_J}{\partial U_R} > 0$$

- Assume:

$$\begin{cases} U_R = \sqrt{S_R} + a \cdot U_J; & a > 0 \\ U_J = \sqrt{S_J} + b \cdot U_R; & b > 0 \end{cases} \tag{3}$$

▷ Diminishing marginal utility of private consumption S:

$$\frac{\partial U_i}{\partial S_i} > 0, \ \frac{\partial^2 U_i}{\partial S_i^2} < 0$$

- Reduced form:

$$\begin{aligned} V_R &= \frac{1}{1-ab}\sqrt{S_R} + \frac{a}{1-ab}\sqrt{S_J} \equiv \alpha_R\sqrt{S_R} + \alpha_J\sqrt{S_J} \\ V_J &= \frac{b}{1-ab}\sqrt{S_R} + \frac{1}{1-ab}\sqrt{S_J} \equiv \beta_R\sqrt{S_R} + \beta_J\sqrt{S_J} \end{aligned}$$

▷

$$\alpha, \beta > 0 \ \rightleftharpoons \ ab < 1$$

- Individual goal of consumption distribution:

$$R: \quad \max_{S_R, S_J} V_R(S_R, S_J) \quad \text{s.t.} \quad S_R + S_J = W$$

$$J: \quad \max_{S_R, S_J} V_J(S_R, S_J) \quad \text{s.t.} \quad S_R + S_J = W$$

– Ideal (S_R, S_J) for Romeo:

$$\text{MRS}_R^{S_R,S_J} = \frac{\sqrt{S_J}}{a\sqrt{S_R}} = 1$$

▷

$$S_R = \frac{S_J}{a^2}$$

– Ideal (S_R, S_J) for Juliet:

$$\text{MRS}_J^{S_R,S_J} = \frac{b\sqrt{S_J}}{\sqrt{S_R}} = 1$$

▷

$$S_R = b^2 \cdot S_J$$

- Dilemma:

– If $ab < 1$:

$$\frac{1}{a^2} > b^2$$

▷ Romeo wants to eat more spagette than Juliet allows him

▷ They fight for food!

– If $ab > 1$:

$$\alpha < 0, \ \ \beta < 0$$

▷ Both hate spagette, and dump spagette onto the other

- What is the problem with (3)?

- Fixing the problem:

$$\begin{cases} U_R &= -\sqrt{S_R} + a \cdot U_J, \quad a > 0 \\ U_J &= -\sqrt{S_J} + b \cdot U_R, \quad b > 0 \end{cases}$$

▷ Reduced form:

$$V_R = \frac{-1}{1-ab}\sqrt{S_R} + \frac{-a}{1-ab}\sqrt{S_J}$$

$$V_J = \frac{-b}{1-ab}\sqrt{S_R} + \frac{-1}{1-ab}\sqrt{S_J}$$

▷ Preferred consumption combination: same as before

- Now with $ab > 1$:
  - Both like spaghetti
  - Both want the other to eat more of the good stuff

5.4 Additive Altruistic Utility Function

- Additive benevolent system:[5]

$$V_i \;=\; \beta_{ii}U_i(x_i) \;+\; \sum_{j\neq i}\beta_{ij}U_j(x_j), \quad \text{with} \quad \beta_{ii}=1,\ \beta_{ij}\geq 0\ \ (\forall\, i,j)$$

<u>where</u>:

$x_i \;\equiv i$'s consumption vector

$U_i(x_i) \;\equiv i$'s private consumption utility[6]

▷ The matrix form:

$$V \;=\; BU \tag{4}$$

- Alternatively, with recursion:[7]

$$\begin{aligned} V_i \;&=\; \gamma_i U_i(x_i) \;+\; \textstyle\sum_{j\neq i}\delta_{ij}V_j \\ &=\; U_i(x_i) \;+\; \textstyle\sum_j \alpha_{ij}V_j,\;\; \alpha_{ii}\equiv \frac{\gamma_i-1}{\gamma_i},\;\alpha_{ij}\equiv\frac{\delta_{ij}}{\gamma_i} \end{aligned} \tag{5}$$

▷ The matrix form:[8]

$$V \;=\; U \;+\; AV, \quad \text{or} \quad [I-A]V = U \tag{6}$$

- Conversion between (4) and (6):[9]

$$A \;=\; I \;-\; B^{-1},\;\; B \;=\; [I-A]^{-1}$$

[5] See, for example, Becker [JPE 1974] and Abel-Bernheim [Econometrica 1991].

[6] $\beta_{ii}U_i(x_i)$, the 1st term of RHS, is i's "ego" utility, while the second term is called the "alter" utility.

[7] For example: Barro [JPE 1974], Bergstrom [JEP 1989], and Bernheim-Stark [AER 1988].

[8] Note that, after rearranging terms, (5) becomes:

$$V_i \;=\; \frac{U_i(x_i)}{1-\alpha_{ii}} \;+\; \sum_{j\neq i}\frac{\alpha_{ij}}{1-\alpha_{ii}}V_j$$

Therefore:

$$\gamma_i \;=\; \frac{1}{1-\alpha_{ii}},\;\; \delta_{ij} \;=\; \frac{\alpha_{ij}}{1-\alpha_{ii}}$$

And hence:

$$\alpha_{ii} \;=\; \frac{\gamma_i-1}{\gamma_i},\;\; \alpha_{ij} \;=\; \frac{\delta_{ij}}{\gamma_i}$$

[9] Assuming inverse matrix B^{-1} exists. Then by (4), we have

$$U = B^{-1}V$$

- The consistency issue:
  - β_{ij} (in 4) and δ_{ij} (in 6) should both be positive
  - Felicitous well-behaved system:
  - Necessary conditions: Bergstrom [1990]

  E 3-consumer example: Ley [EL 1997]

$$V_i \;=\; U_i(x_i) \;+\; \sum_{j \neq i} V_j$$

  ▷ Direct utility after conversion:

$$V_i \;=\; \frac{-1}{2} \sum_{j \neq i} U_j(x_j)$$

  ▷ Consumers don't care about own x_i, and hate others' x_j.

  ▷ For max SW, better to destroy all goods in economy! □

- Samuelson FOC must still hold: Ley [EL 1997]
  - Model:

    $U_i(x_i, y)$: y is PG

    $I = \sum_i I_i$ is total wealth

    $X = \sum_i x_i =$ is aggregate consumption

    $y = I - X$ is PG level

Substitute into (6), we get:

$$V \;=\; B^{-1}V \;+\; AV$$

Hence:

$$A \;=\; I \;-\; B^{-1}$$

To go the other way, assume inverse matrix $[I - A]^{-1}$. We then know, by (6):

$$V \;=\; [I - A]^{-1}U$$

Thus, B in (6) is simply:

$$B \;=\; [I - A]^{-1}$$

– Proof:

(i) From benevolent $V = BU$, we can define egoistic $V = \hat{B}U$:

$$\beta_{ii} = 1 \ (\forall\, i), \quad \beta_{ij} = 0 \ (\forall\, j \neq i)$$

(ii) For PO of $V = BU$, we can solve (for all $\lambda \equiv (\lambda_1, \cdots, \lambda_n) > 0$):

$$\begin{array}{rl} \max & \sum_i \lambda_i V_i \\ = & \sum_i \left[\lambda_i \sum_j \beta_{ij} U_j(x_j, I - X)\right] \\ = & \sum_j \left[U_j(x_j, I - X) \sum_i \lambda_i \beta_{ij}\right] \\ = & \sum_j \mu_j U_j(x_j, I - X) \end{array}$$

<u>where</u>:

$$\mu_j \equiv \sum_i \lambda_i \beta_{ij}$$

(iii) Any PO of $V = BU$ must also be PO of $V = \hat{B}U$, and satisfies Samuelson:

$$\sum_i \mathrm{MRS}_i^{y,x} \ = \ \sum_i \frac{\partial U_i / \partial y}{\partial U_i / \partial x} \ = \ 1 \ \ \Box$$

– For Gorman utility:

$$U_i(x_i, y) \ = \ f(y) x_i + g_i(y)$$

▷ $\sum_i \mathrm{MRS}_i^{y,x}$ depends only on X, not on individual x_i

▷ y^* does not depend on x_i or β_{ij}

6 Information Asymmetry

6.1 Varian [AER 1994]

- 2-firm externality:
  - Two firms: (1, 2)
  - Externality: damage $e(x)$ to firm 2 by firm 1's output x
  - Both firms know $e(x)$, but govt does not

$$e'(x) \ > \ 0$$

  - Firm payoffs:

$$\pi_1(x) \ = \ rx \ - \ c(x)$$

$$\pi_2(x) \ = \ -e(x)$$

- Optimality:

$$\max_x \quad rx - c(x) - e(x)$$

<u>foc</u>:

$$r \ = \ c'(x^*) + e'(x^*)$$

- Pigouvian tax:

$$t^* \ = \ e'(x^*)$$

- 2-stage compensation scheme:

S1 (Announcement) Both firms declare desired compensation rate of firm 1 to firm 2:

$$t_1,\ t_2$$

S2 (Compensation) Given (t_1, t_2), firm 1 decides x. Firms get profits:

$$\begin{cases} \pi_1 = rx - c(x) - t_2 x - |t_1 - t_2| \\ \pi_2 = -e(x) + t_1 x \end{cases}$$

- SPE: solving backwards

S2 Given (t_1, t_2) of stage 1:

Firm 1's choice of x^* will satisfy foc 1:

$$r = c'(x^*) + t_2$$

▷ $x^* = x(t_2)$

$$\frac{dx}{dt_2} = \frac{-1}{c''(x)} < 0$$

S1 Firm 1 will set $t_1 = t_2$ to minimize cost

For firm 2:

$$\max_{t_2} \ t_1 x(t_2) - e(x(t_2))$$

foc 2:

$$[t_1 - e'(x)]x'(t_2) = 0$$

SPE Put together:

$$\begin{cases} t_1 = t_2 = e'(x) \\ r = c'(x) + e'(x) \end{cases}$$

▷ $t_1 = t_2$ is the efficient Pigouvian tax rate.[10]

[10] Note that we use $[t_1 - t_2]^2$ simply to ensure $t_1 = t_2$ in SPE. Therefore, $|t_1 - t_2|$ can also be used instead.

- 3-firm generalization: firm 1 causing different damages to 2 and 3
  - External effect on firms 2 and 3 by firm 1:
$$e_2(x), \;\; e_3(x)$$
  - Compensation rate announced by k $(i = 1, 2, 3)$ for i $(= 2, 3)$:
$$t_i^k$$
  - Compensation scheme:
$$\begin{cases} \pi_1 &= rx - c(x) - [t_2^2 + t_3^3]x - |t_2^1 - t_2^2| - |t_3^1 - t_3^3| \\ \pi_2 &= t_2^1 x - e_2(x) \\ \pi_3 &= t_3^1 x - e_3(x) \end{cases}$$
  - SPE: solved backwards

S2 Firm 1's FOC: $r = c'(x) + [t_2^2 + t_3^3] \;\; \Rightarrow \;\; x(t_2, t_3)$

S1 Firm 1 will set:
$$t_2^1 = t_2^2, \;\; t_3^1 = t_3^3$$

▷ Firm 2's foc:
$$[t_2^1 - e_2'(x)]x_1(t_2, t_3) \; = \; 0$$

▷ Firm 3's foc:
$$[t_3^1 - e_3'(x)]x_2(t_2, t_3) \; = \; 0$$

⇒
$$t_2^1 = t_2^2 = e_2'(x), \;\; t_3^1 = t_3^3 = e_3'(x)$$
$$r = c'(x) + e_2'(x) + e_3'(x)$$

7 Vaccination [Brito et al., JPuE 1991]

7.1 The Model

- Population: normalized to 1

- Individual vaccination cost: c[11]

  ▷ Continuously distributed in $[0, \bar{c}]$

  ▷ Distribution: pdf $f(\cdot)$, cdf $F(\cdot)$

- $x \equiv$ %population not vaccinated

  $p(x) \equiv$ probability an un-vaccinated person may get sick

$$p'(\cdot) \;>\; 0$$

- Consumer certain utility: with income y

  (healthy) $U(y)$

  (sick) $\underline{U}(y)$ $(< U(y), \forall\, y)$: $\underline{U}' > 0$, $\underline{U}'' < 0$

- Consumer utility:

  (Vac) $U(y) - c$ for sure

  (NoVac) EU:

$$V(y, x) \;=\; [1 - p(x)]U(y) + p(x)\underline{U}(y)$$

  ▷

$$V_y(y, x) \geq 0, \quad V_x(y, x) \leq 0$$

- Externality: your vaccination has positive value for others

[11] Including transportation costs, time costs, and physical pains of vaccination.

7.2 Individual Vac decision: given c

- Excess utility of Vac:

$$\mathcal{E}(c\,|\,x) \;\equiv\; [U(y) - c] - V(y,x)$$

- Vac only if $\mathcal{E}(c\,|\,x) > 0$:

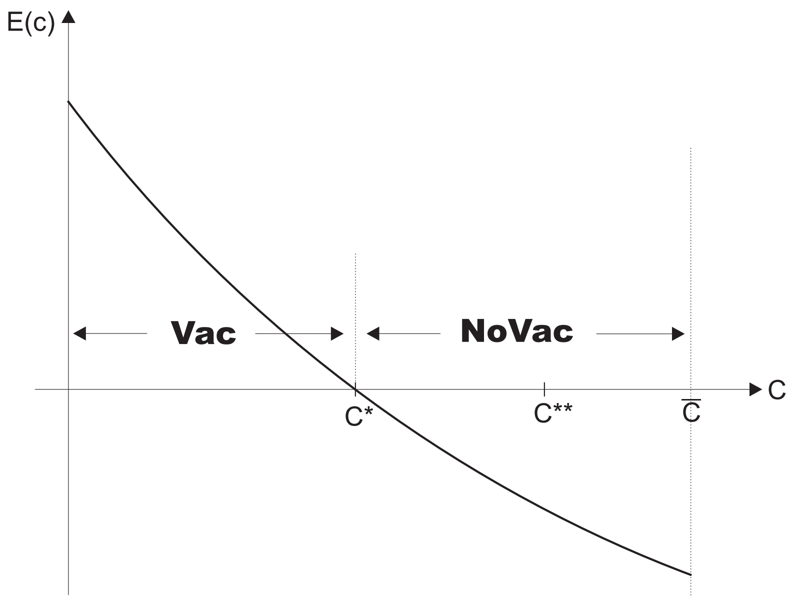
$$U(y) - c \;>\; V(y,x)$$

- Define Vac/NoVac threshold c^*:

$$\mathcal{E}(c^*) \;=\; [U(y) - c^*] - V(y, x(c^*)) \;=\; 0$$

- %population not vaccinated:

$$x(c^*) \;=\; 1 - F(c^*)$$



7.3 Interior Vac equilibrium

- Threshold $x(c^*) \in (0,1)$: some Vac, some NoVac

  $\surd$ Someone will Vac when $x = 1$:

  $$c^* > 0 : \quad \mathcal{E}(0) \;=\; [U(y) - 0] - V(y,1) \;>\; 0 \quad \text{at} \quad x = 1$$

  $\surd$ Someone will not Vac when $x = 0$:

  $$c^* < \bar{c} : \quad \mathcal{E}(\bar{c}) \;=\; [U(y) - \bar{c}] - V(y,0) \;<\; 0 \quad \text{at} \quad x = 0$$

- Separating Nash: $c^* \in (0, \bar{c})$

  $$\mathcal{E}(c^*) \;=\; [U(y) - c^*] - V(y, x(c^*)) \;=\; 0$$

  $\triangleright$ Low-cost people (with $c \le c^*$) will Vac, high-cost people will not

7.4 SW calculation

- SW: with any c as separating threshold

  using $x(c) = 1 - F(c)$

  $$\begin{aligned} W(c) &= \textstyle\int_0^c [U(y) - z] f(z) dz \;+\; [1 - F(c)] V(y, x(c)) \\ &= U(y)F(c) \;-\; \textstyle\int_0^c z f(z) dz \;+\; x(c) V(y, x(c)) \end{aligned}$$

  with $x'(c) = -f(c)$:

  $$W'(c) \;=\; \{U(y) - c - V(y, x(c)) - V_x(y, x(c)) x(c)\} \, f(c)$$

  Further, assuming *global concavity*:

  $$W''(c) \;<\; 0$$

  $\triangleright$ $W(c)$ is concave

- Optimal c^{**}:

$$W'(c^{**}) = 0$$

- Nash c^* is inefficient:

$$W'(c^*) > 0, \quad c^* < c^{**}$$

- Mandatory $\bar{c}$:

$$W(\bar{c}) = \int_0^{\bar{c}} [U(y) - z] f(z) dz$$

▷

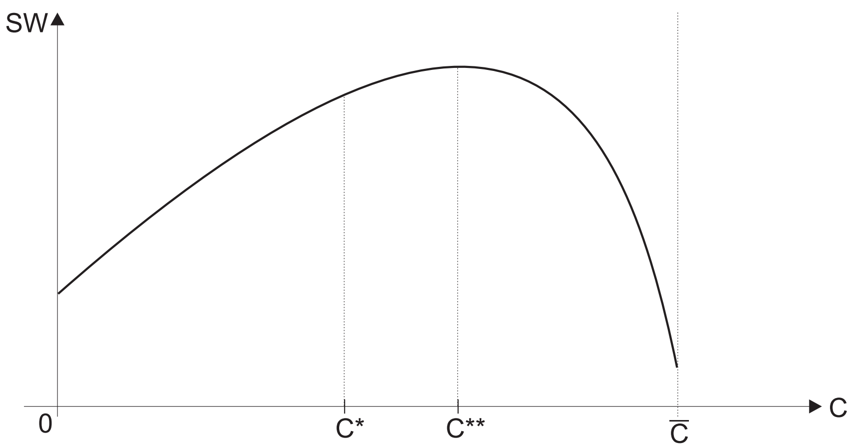
$$W(c^*) - W(\bar{c}) = \int_{c^*}^{\bar{c}} \{V(y, x(c^*)) - [U(y) - z]\} f(z) dz > 0$$

- Comparision:

$$W(\bar{c}) < W(c^*) < W(c^{**})$$

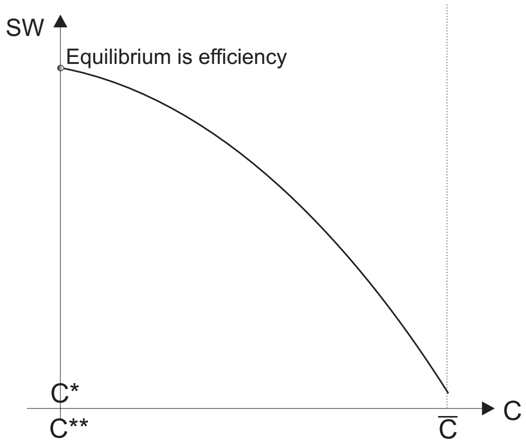
$$\bar{c} > c^{**} > c^* > 0$$

▷ Mandatory Vac is worst, better to have market outcome.

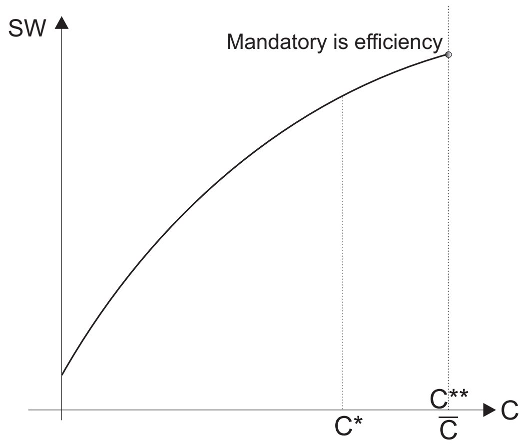


7.5 Corner cases

- Trivial common flu:



- Serious SARS threat:



7.6 Tax/subsidy for optimality

- Vac subsidy S:

$$S = -\mathcal{E}(c^{**}) > 0$$

  ▷ Excess utility curve shifts up, Nash is now at c^{**}

- Can also use NoVac tax:

$$T = S$$

  ▷ Same effect