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1 Contribution Motivation

1.1 Cornes-Sandler Anomaly

- Severe free-riding (cf. moderate in experiments)
- Neutrality/crowing-out

  ▷ Experiment: Eckel et al. [JPuE 2005/v89, pp. 1543–1560][1]
- Large-population effect

  ▷ White [1989], Steiberg [1989]

1.2 Other Possible Explanations

- *Fair share* [Margolis 1982]:

  ▷ G-utility v. S-utility
- Principle of *rational commitments* (or Kantian behavior):[2]

$$\max_{x,g} \; U(x, ng) \quad \text{s.t.} \quad x + pg = I$$

  <u>Samuelson foc</u>:

$$n \cdot \mathrm{MRS}^{G,x} \;=\; p$$

- Principle of *reciprocity* [Sugden 1984]

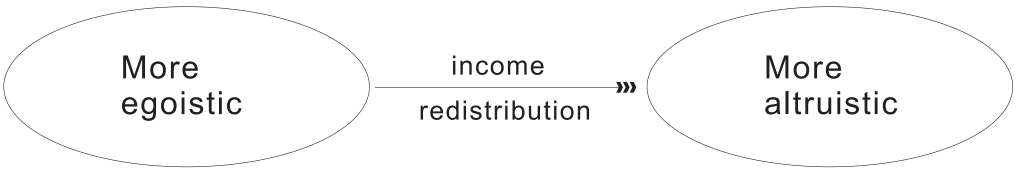
  ▷ "I should also contribute $\bar{g}$ if all other do so."
- *Sentiment* [Hollander 1990]: social approval

[1] Framing effects: crowding-out depends on player's perception about source of the funding.
[2] So called "Kantian categorical imperative". See, for example, Laffont [1975], Collard [1978], and Harsanyi [1980].

1.3 Impure Altruism: Warm Glow Theory

- Altruism v. egoism:
  - Pure egoistic:
  $$U_i(x_i, g_i)$$
  - Pure altruistic:
  $$U_i(x_i, G)$$
  - Impure altruism: Andreoni [JPE 1989, EJ 1990]
  $$U_i(x_i, g_i, G)$$

- Implications of impure altruism:
  1. Neutrality result does not hold:
     ▷ May have higher G using transfer:



  2. RKT will break down: kids will steal from head.[3]
     ▷ Parent: $U_p(X_p, X_k, t)$, more egoistic
     ▷ Kid: $U_k(X_k)$, more altruistic

[3] Ironically, now the head is more egoistic, while kids are more altruistic.

1.4 Environmental Offset

- Kotchen, M.J. (*Economic Journal*, 2009, V119, pp. 883–899)

  ▷ Contribution compensation for harmful private consumption

- Pure altruistic preference:

$$U_i(x_i, G)$$

  Consumer budget:

$$x_i + g_i = I_i$$

- Private consumption x_i diminishes PG:

$$G = G_{-i} + g_i - \beta x_i$$

  ▷ $g_i \equiv$ direct contribution by i

  ▷ $y_i = g_i - \beta x_i \equiv$ net contribution by i

- Equilibrium:

  – Mean contribution does not converge to zero as population grows large

2 Non-additive Public Goods

2.1 Social Composition Functions: Hirshleifer [PC 1983]

- Summation rule:

$$G = \sum_i g_i$$

- Best-shot:

$$G = \max\{g_1, \cdots g_n\}$$

- Weakest-link:

$$G = \min\{g_1, \cdots g_n\}$$

2.1.1 Summation Rule

- Optimality condition:

$$\sum_j \mathrm{MRS}_j^{G,x_j} = \mathrm{MC}_i(g_i), \ \ \forall i$$

- Nash interior condition:

$$\mathrm{MRS}_i^{G,x_i} = \mathrm{MC}_i(g_i), \ \ \forall i$$

2.1.2 Best-shot

- Optimality condition: with the low cost player k

$$\sum_i \mathrm{MRS}_i^{G,x_i} = \mathrm{MC}_k(G^B);\ \text{ and }\ \mathrm{TC}_k(G^B) \leq \mathrm{TC}_j(G^B),\ \forall j \neq k$$

  and

$$g_j = 0,\ \forall j \neq k$$

- Nash interior condition: with the low cost player k

$$\mathrm{MRS}_k^{G,x_k} \ = \ \mathrm{MC}_k(N^B)$$

  and

$$g_j = 0,\ \forall j \neq k$$

2.1.3 Weakest-link

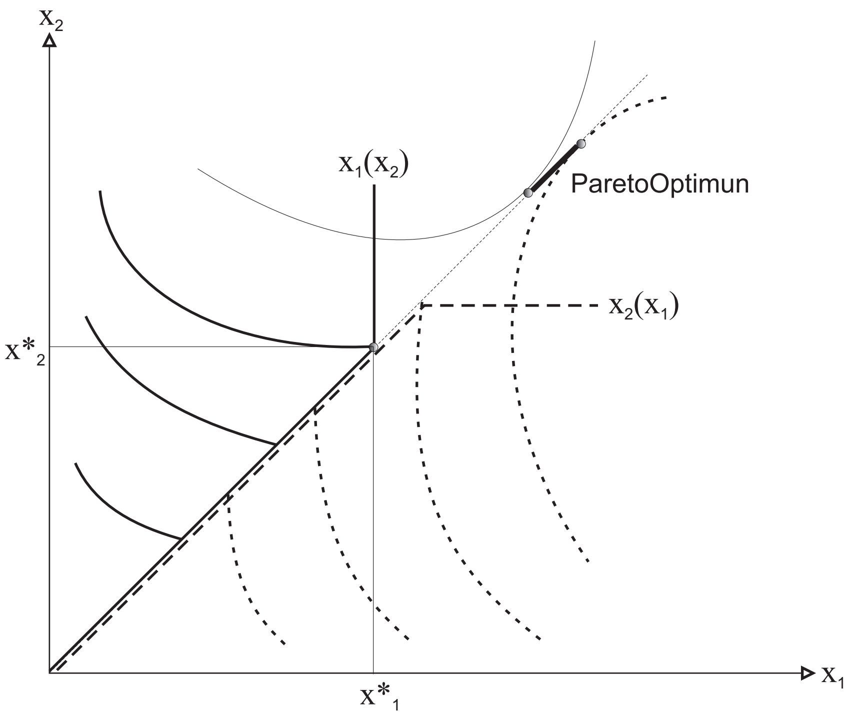
- Optimality condition:

$$\sum_i \mathrm{MRS}_i^{G,x_i} \;=\; \sum_i \mathrm{MC}_i(G^W)$$

- Nash interior condition: $\exists\, k$

$$\mathrm{MRS}_k^{G,x_k} \;=\; \mathrm{MC}_k(N^W)$$

$$\mathrm{MRS}_j^{G,x_j} \;>\; \mathrm{MC}_j(N^W),\;\; \forall j \neq k$$



2.2 Generalization

- Cornes [QJE 1993]
- Constant elasticity of substitution (CES) production function:

$$Q \;=\; \alpha \left[\frac{\sum_{i=1}^{n} q_i^{\lambda}}{n} \right]^{1/\lambda}$$

  – Summation:

$$\alpha = n, \;\; \lambda = 1$$

  – WL:

$$\alpha = 1, \;\; \lambda \to -\infty$$

  – BS:

$$\alpha = 1, \;\; \lambda \to +\infty$$

  – Average: $\alpha = \lambda = 1$, hence:

$$Q \;=\; \frac{\sum_i q_i}{n}$$

- Weaker-link: $\lambda \to +0$

$$Q = \left(\prod_{i=1}^{n} q_i \right)^{1/n}$$

▷

$$\frac{\partial Q}{\partial q_i} \;=\; \frac{Q}{n q_i} \;\; \uparrow \;\; \text{with lower} \;\; q_i$$

2.3 Group Contest/Tournament: Group-specific Public-good Prize

2.3.1 Baik [EL 1993]

- N groups: each with m_i risk-neutral members
- Effort of member k in group-i:

$$x_i^k$$

- Total group i effort:

$$X_i = \sum_{j=1}^{m_i} x_i^j$$

- Prize-winning probability:

$$p_i(X_1, \ldots, X_N)$$

with:

$$\frac{\partial p_i}{\partial X_i} \geq 0, \quad \frac{\partial^2 p_i}{\partial X_i^2} \leq 0; \quad \frac{\partial p_i}{\partial X_j} \leq 0, \quad \frac{\partial^2 p_i}{\partial X_j^2} \geq 0$$

- Member-specific prize value: v_i^k (> 0)

$$\mathrm{EU}_i^k \ = \ v_i^k p_i(X_1, \ldots, X_N) - x_i^k$$

- Assuming, for each group i:

$$v_i^1 \ \geq \ v_i^2 \ \geq \ \cdots \ \geq \ v_i^{m_i} \ (> 0)$$

- player-k-best response:

$$\tilde{X}_i^k \ \equiv \ \mathrm{argmax}_{X_i} \ v_i^k p_i(X_i | X_{-i}) - X_i \quad \text{s.t.} \quad X_i \geq 0$$

$\triangleright$

$$\tilde{X}_i^1 \ \geq \ \tilde{X}_i^2 \ \geq \cdots \geq \ \tilde{X}_i^{m_i}, \ \ \forall i$$

- Equilibrium:

$$(X_1^*, \cdots, X_N^*)$$

  – Only member 1 will put out effort: $X_i^* = x_i^1 = \tilde{X}_i^1$

$$v_i^1 \cdot \frac{\partial p_i}{\partial X_i}(x_i^1, X_{-i}^*) \ = \ 1$$

  – Other members ($j \neq 1$) will free ride ($x_i^j = 0$):

$$v_i^j \cdot \frac{\partial p_i}{\partial X_i}(x_i^1, X_{-i}^*) \ < \ 1$$

2.3.2 Plurality rule

- Baik-Shogren [1998][4]

- Winning probability:

$$p_1(X_1, X_2) \ = \ F(X_1 - X_2), \quad p_2(X_1, X_2) \ = \ 1 - F(X_1 - X_2)$$

with:

$$F(0) = 1/2, \ F(-d) = 1 - F(d)$$

$$0 \ < \ F(d) \ < \ 1, \ \ \forall\, d \in \mathcal{R}$$

$$F'(\cdot) > 0, \ F''(0) = 0, \ F''(d)d < 0$$

[4] K.H. Baik and J.F. Shogren, "A Behavioral Basis for Best-Shot Public-Good Contest," in *Advances in Applied Microeconomics (Volume 7)*, JAI Press, pp. 169–78, 1998.

2.3.3 Two-stage Game

- Baik-Lee [1998][5]

- Two stages:

  S1 *Inter-group contest*:

$$p_i = \frac{X_i}{\sum_j X_j}$$

  S2 *Intra-group competition*: for share α_i of the prize

2.3.4 All-pay Auction

- Baik-Kim-Na [JPuE, 2001/v82, PP. 415–429]

- Winning probability:

$$p_i(X_1, X_2) = \begin{cases} 1, & \text{if } X_i > X_j \\ 1/2, & \text{if } X_i = X_j \\ 0, & \text{if } X_i < X_j \end{cases}$$

[5] K.H. Baik and S. Lee, "Group Rent Seeking with Sharing," in *Advances in Applied Microeconomics (Volume 7)*, JAI Press, pp. 75–85, 1998.

3 Binary/Discrete/Threshold Public Goods

3.1 Continuous/variable Contributions

1. The case of Oral Roberts

2. Bagnoli-McKee [EI 1991]

   - Binary PG: price/cost C
   - N players: income w_i, WTP for PG V_i, contribution c_i
   - Assume:

$$C > w_i > V_i, \ \ \forall\, i$$

   - Game rule:
     - $\sum_i c_i > C$: PG provided, player i gets payoff:

$$\pi_i \ = \ V_i + [w_i - c_i]$$

     - $\sum_i c_i < C$: no PG, c_i is refunded, i gets payoff:

$$\pi_i \ = \ w_i$$

   - Nash equilibrium: 3 cases
     - $\sum_i c_i > C$: $(c_1, \ldots, c_N)$ cannot be Nash.[6]
     - $\sum_i c_i = C$: <u>stable Nash</u> with $c_i \leq V_i, \ \ \forall\, i$
     - $\sum_i c_i < C$: Nash (but not trembling-hand perfect) if

$$V_i + \sum_{j \neq i} c_j \ < \ C, \ \ \forall\, i$$

[6] Player i would want to lower c_i, given other players' contributions.

3.2 Binary Contributions

3.2.1 Palfrey-Rosenthal [JPuE 1984]

1. Analysis goal:
   - Two designs: NoRefund ($\aleph$) v. Refund ($\Re$)
   - Two possible reasons for not contributing: Greed v. Fear
2. The Model:
   - M players
   - Binary PG: provided if w ($\leq M$) players contribute
     - Cost c for contributors, 0 for non-contributors
     - Player gets utility 1 with PG, 0 without
   - 3 groups of players:
     (a) Contributors: $|G^1| = m^1$
     (b) Non-contributors: $|G^2| = m^2$
     (c) Randomizers (contribute with probability q): $|G^3| = m^3$
       - $\bar{m}^3 \equiv$ number of players actually contribute in G^3
       - $\bar{m}^3_{-i} \equiv$ number of contributors excluding i in G^3
       - $\bar{m} \equiv$ number of total contributors
3. NoRefund ($\aleph$)
   - Pure-strategy Nash ($m^3 = 0$):
     (a) $w = 1$: M equilibria ($m^1 = 1$, $m^2 = M - 1$)
     (b) $w \geq 2$:

i. $m^1 = 0$, $m^2 = M$: no one controbutes, no PG

ii. $m^1 = w$, $m^2 = M - w$: exactly w contributors, PG provided

- Mixed-strategy Nash ($m^3 > 0$): equilibrium conditions are

  – G^1: EU is greater with contributing

$$P(\bar{m}^3 \geq w - m^1) - c \ \geq \ P(\bar{m}^3 \geq w - m^1 + 1)$$

so $c \leq P(\bar{m}^3 = w - m^1)$, or:

$$c \ \leq \ \begin{pmatrix} M - m^1 - m^2 \\ w - m^1 \end{pmatrix} q^{w-m^1}[1-q]^{M-w-m^2} \tag{1}$$

where $P(X)$ is the probability of event X.

  – G^2: EU is greater without contributing

$$P(\bar{m}^3 \geq w - m^1) \ \geq \ P(\bar{m}^3 \geq w - m^1 - 1) - c$$

so $c \geq P(\bar{m}^3 = w - m^1 - 1)$, or:

$$c \ \geq \ \begin{pmatrix} M - m^1 - m^2 \\ w - m^1 - 1 \end{pmatrix} q^{w-m^1-1}[1-q]^{M-w-m^2+1} \tag{2}$$

  – G^3: equal EU either way

$$P(\bar{m}^3_{-i} \ \geq \ w - m^1 - 1) - c = P(\bar{m}^3_{-i} \geq w - m^1)$$

so $c = P(\bar{m}^3_{-i} = w - m^1 - 1)$, or:

$$c \ = \ \begin{pmatrix} M - m^1 - m^2 - 1 \\ w - m^1 - 1 \end{pmatrix} q^{w-m^1-1}[1-q]^{M-w-m^2} \tag{3}$$

  – For mixed strategy Nash (m^1, m^2, m^3, q):

(a) if $(m^1 = 0)$: (m^2, m^3, q) must satisfy (2, 3)

(b) if ($m^2 = 0$): (m^1, m^3, q) must satisfy (1, 3)

(c) if ($m^1 = m^2 = 0$, $m^3 = M$): q only have to satisfy (3)

(d) otherwise: all 3 eqs (1, 2, 3) must hold

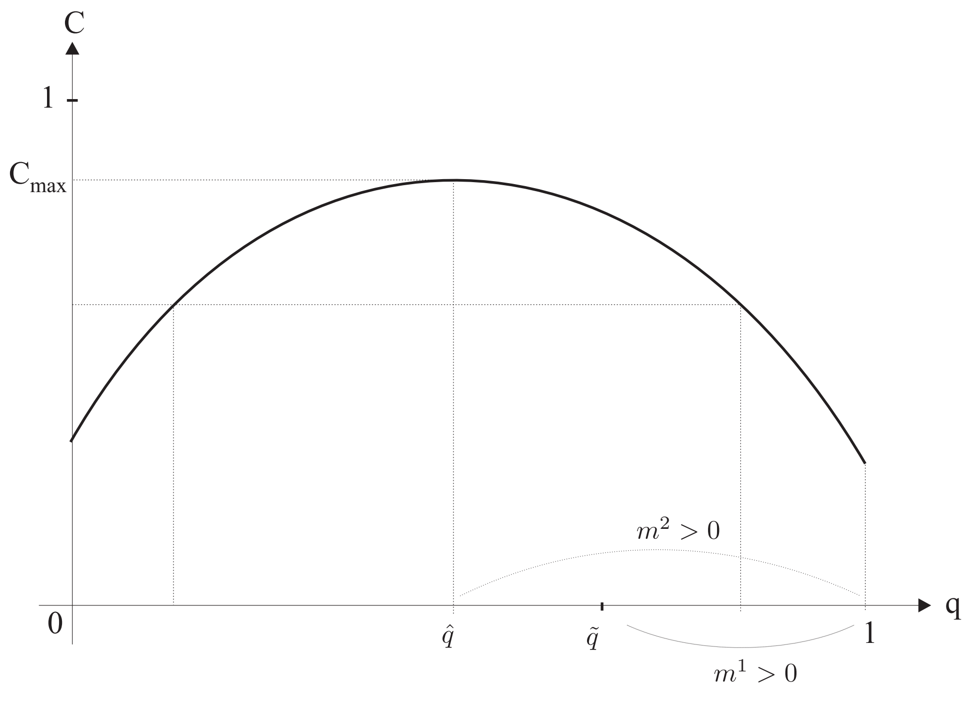
– Admissible (m^1, m^2, m^3, q), given (M, w, c), satisfies:

(1) $m^1 \leq w - 1$: or else there must be PG, hence no need to mix

(2) $m^2 \leq M - w$: or else there must be no PG, hence no need to mix

– $c(q)$, by Eq.(3), must be uni-modal, and peaks at $c_{\max} = c(\hat{q})$:

$$\hat{q} \equiv \frac{w - m^1 - 1}{M - m^1 - m^2 - 1}$$



– iff condition for existence of mixed-strategy Nash, given any admissible (m^1, m^2, w, M):

$$c \le \begin{cases} c(\hat{q}), & \text{if } m^1 = 0 \\ c(\tilde{q}), & \text{if } m^1 > 0 \end{cases}$$

* If $m^2 > 0$, then by Eqs.(2)(3): $q \ge \hat{q}$。
* If $m^1 > 0$, then by Eqs.(1)(3):

$$q \ge \tilde{q} \equiv \frac{w - m^1}{M - m^1 - m^2} > \hat{q}$$

* If $m^1 > 0$ and $m^2 > 0$: $q \ge \tilde{q}$ (as $\hat{q}$ is not binding)

– Nash (m^1, m^2, m^3, q) can hence be obtained, for any chosen (m^1, m^2).[7]

E Assume ($M = 4$, $w = 2$, $c = 0.096$):

* Pure Nash: ($m^1 = 0$, $m^2 = 4$) and ($m^1 = m^2 = 2$)
* Mixed Nash:

(1) $m^1 = m^2 = 0$: two solutions

$$q = 0.800, \quad E(\bar{m}) = 3.2 > w$$

$$q = 0.034, \quad E(\bar{m}) = 0.14 < w$$

(2) $m^1 = 0$, $m^2 = 1$: $q = 0.9494$, $E(\bar{m}) = 2.85 > w$

(3) $m^1 = 1$, $m^2 = 0$: $q = 0.69$, $E(\bar{m}) = 3.07 > w$

(4) $m^1 = m^2 = 1$: $q = 0.904$, $E(\bar{m}) = 2.81 > w$

* NB: It is possible that everyone contributes ($m^2 = 0$), and PG is over-provided ($\bar{m} > w$). ■

[7] When $m^1 > 0$ or $m^2 > 0$, we have no more than one solution. Otherwise ($m^1 = m^2 = 0$), we have at most two solutions.

– Eventually, when M is very large, only pure Nash exists.[8]

4. Refund ($\Re$)

- Pure-strategy Nash ($m^3 = 0$):
  - $w \leq 2$: same as in ($\aleph$)
  - $w > 2$: besides those in $\aleph$, we have $(0 < m^1 \leq w - 2, m^2 = M - m^1)$
  - NB: Now we have more Nash than in $\aleph$, but only $(m^1 = w, m^2 = M - w)$ is strong.[9] In contrast, all Nash in $\aleph$ are strong.。
- Mixed-strategy Nash ($m^3 > 0$):
  - Define:

$$C(N, n, q) \equiv \begin{pmatrix} N \\ n \end{pmatrix} q^n [1-q]^{N-n}$$

  - Equilibrium condition for mixed Nash:
    * G^1: now cost c is incured only when PG is provided

$$P(\bar{m}^3 \geq w - m^1)[1-c] + P(\bar{m}^3 < w - m^1) \cdot 0$$

$$\geq \; P(\bar{m}^3 \geq w - m^1 + 1) \cdot 1 + P(\bar{m}^3 < w - m^1 + 1) \cdot 0$$

so:

$$c \leq \frac{P(\bar{m}^3 = w - m^1)}{P(\bar{m}^3 \geq w - m^1)}$$

[8] The proof goes as follows: First begin with an admissible (m^1_0, m^2_0, w_0, M_0), define a sequence:

$$\{(m^1_n, m^2_n, w_n, M_n)\}_{n=1}^{\infty}; \;\; m^1_n \equiv n m^1_0, \; m^2_n \equiv n m^2_0, \; w_n \equiv n w_0, \; M_n \equiv n M_0$$

and let $c^n_{\max}$ be the corresponding $c_{\max}$ of (m^1_n, m^2_n, w_n, M_n). Then by limiting property of binomial distribution, we know:

$$\lim_{n\to\infty} c^n_{\max} = 0$$

Therefore, $\lim_{n\to\infty} q^n = 0$, any mixed Nash is actually pure Nash.

[9] "Strong" means that a player will get get strictly lower utility if he/she deviates from Nash strategy.

$$= \frac{C(M - m^1 - m^2, w - m^1, q)}{\sum_{t=w-m^1}^{M-m^1-m^2} C(M - m^1 - m^2, t, q)} \tag{4}$$

* G^2:

$$P(\bar{m}^3 \geq w - m^1) \cdot 1 \;\geq\; P(\bar{m}^3 \geq w - m^1 - 1)[1 - c]$$

or:

$$c \;\geq\; \frac{P(\bar{m}^3 = w - m^1 - 1)}{P(\bar{m}^3 \geq w - m^1 - 1)}$$

$$= \frac{C(M - m^1 - m^2, w - m^1 - 1, q)}{\sum_{t=w-m^1-1}^{M-m^1-m^2} C(M - m^1 - m^2, t, q)} \tag{5}$$

* G^3:

$$P(\bar{m}^3_{-i} \geq w - m^1 - 1)[1 - c] \;=\; P(\bar{m}^3_{-i} \geq w - m^1) \cdot 1$$

or:

$$c \;=\; \frac{P(\bar{m}^3_{-i} = w - m^1 - 1)}{P(\bar{m}^3_{-i} \geq w - m^1 - 1)}$$

$$= \frac{C(M - m^1 - m^2 - 1, w - m^1 - 1, q)}{\sum_{t=w-m^1-1}^{M-m^1-m^2-1} C(M - m^1 - m^2 - 1, t, q)} \tag{6}$$

– Note that eqs(4)(5) are not binding. Then by eq.(6):

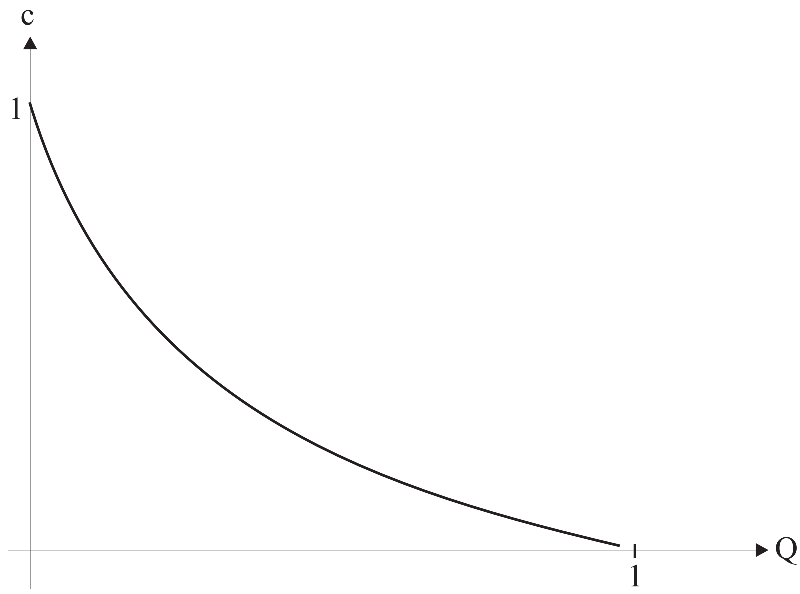
* c is continuously differentiable in $q \in (0, 1)$

$$c(0) = 1, \;\; c'(q) < 0, \;\; \lim_{q \to 1} c(q) = 0$$

* $c(q)$ has inverse function $Q(c)$:

$$Q(1) = 0, \;\; Q'(c) < 0, \;\; \lim_{c \to 0} Q(c) = 1$$

* Unique Q for any c ($\in (0, 1)$) and admissible (m^1, m^2, w, M)



– Comparision:

∗ PG more likely in $\Re$ than in $\aleph$ when c is high.[10]

∗ For any admissible (m^1, m^2, w, M) and $c \leq c_{\max}(m^1, m^2, w, M)$, by comparing eq.(3)(6), we know $Q(c)$ in $\Re$ is strictly greater than $q(c)$ in $\aleph$.[11]

E Take again ($M = 4$, $w = 2$, $c = 0.096$), now $\Re$ has same pure Nash as $\aleph$, but mixed nash are:

(a) $m^1 = m^2 = 0$: $Q = 0.802$

(b) $m^1 = 0$, $m^2 = 1$: $Q = 0.9496$

(c) $m^1 = 1$: same as in $\aleph$

$\triangleright$ $Q > q$ (but very close) ■

[10] PG is possible only when $c \leq c_{\max}$ in $\aleph$.
[11] When $m^1 = m^2 = 0$, $Q(c)$ must be greater than the two solutions in $\aleph$.

3.2.2 Palfrey-Rosenthal [JPuE 1988]

1. Perfect-info game: contribution cost $c \in (0, 1)$, PG utility 1

A \ B	Contribute	Not
Contribute	$(1 - c,\ 1 - c)$	$(1 - c,\ 1)^*$
Not	$(1,\ 1 - c)^*$	$(0,\ 0)$

- 2 pure Nash: either one contributes
- 1 mixed Nash: both contribute with probability $p = 1 - c$

2. Bayesian game:

- "Warm glow" utility $d_i \geq 0$ $(i = A, B)$: private info

  ▷ CDF $F(\cdot)$ is common knowledge

- Normal form:

A \ B	Contribute	Not
Contribute	$(1 - c + d_A,\ 1 - c + d_B)$	$(1 - c + d_A,\ 1)$
Not	$(1,\ 1 - c + d_B)$	$(0,\ 0)$

- Player i's strategy:
  - If $d_i > c$: should contribute
  - If $d_i < c - 1$: should not contribute
  - $\exists\ d^* \in [c - 1, c]$: contribute <u>iff</u> $d_i > d^*$

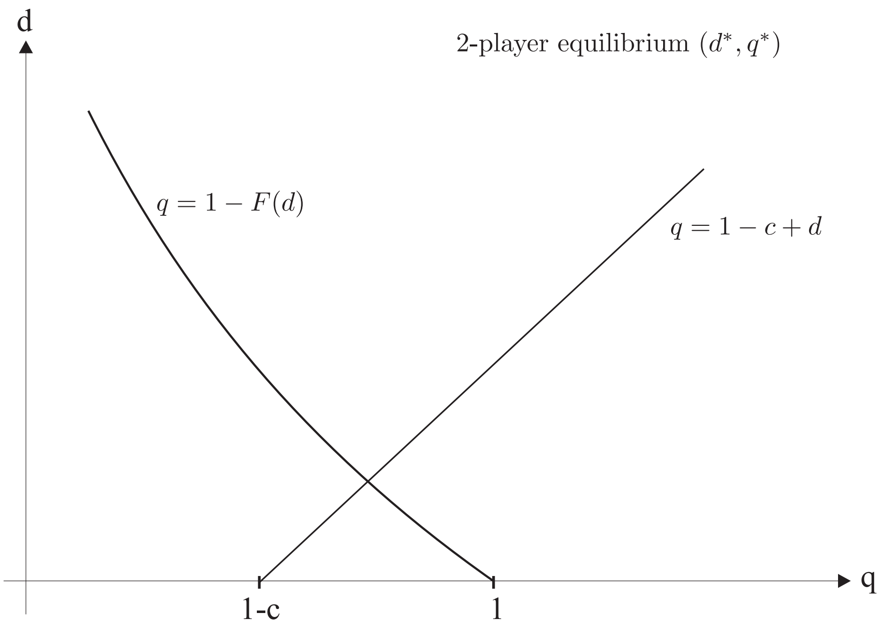
- Equilibrium:
  - For either player i, probability(j will contribute) is:

$$q^* = 1 - F(d^*)$$

  - At threshold $d_i = d^*$, same utility from contributing or not:

$$1 - c + d^* = q^*$$

  - Can solve for equilibrium (d^*, q^*) from the above two eqs.[12]



3. N-player games:

- Dummy s_i: $= 1$ for contributing, $= 0$ for not contributing
- Warm-glow EU:

$$U_i(m_i, s_i) = \begin{cases} V(m_i, 1) + d_i, & \text{if} \quad s_i = 1 \\ V(m_i, 0), & \text{if} \quad s_i = 0 \end{cases}$$

[12] (d^*, q^*) must exist if F is continuous.

where: $V(m_i, s_i) \equiv \text{EU}_i$ if there are m_i other contributors.

- Can solve for equilibrium (d^*, q^*) with the two eqs:

$$q^* = 1 - F(d^*)$$

$$\sum_{j=0}^{N-1} \pi_j V(j,1) + d^* = \sum_{j=0}^{N-1} \pi_j V(j,0)$$

where

$$\pi_j \equiv \binom{N-1}{j} [q^*]^j [1-q^*]^{N-1-j}$$

is probability that j of the other $N-1$ players will contribute.

4. Possible games: w contributors required for PG provision

- Chicken: players have both "greed" and "fear"[13]

	$m_i < w-1$	$m_i = w-1$	$m_i > w-1$
$V(m_i,1)$	$-c$	$[1-c]$	$[1-c]$
$V(m_i,0)$	0	0	$[1]$

- NoFear: *cost refund* if PG is not provided

	$m_i < w-1$	$m_i = w-1$	$m_i > w-1$
$V(m_i,1)$	0	$[1-c]$	$[1-c]$
$V(m_i,0)$	0	0	$[1]$

- NoGreed: *cost sharing*, everyone must incur c if PG is provided

	$m_i < w-1$	$m_i = w-1$	$m_i > w-1$
$V(m_i,1)$	$-c$	$[1-c]$	$[1-c]$
$V(m_i,0)$	0	0	$[1-c]$

[13] Parentheses in table indicate cases that PG is provided.

- Control: with both refund and cost sharing

	$m_i < w-1$	$m_i = w-1$	$m_i > w-1$
$V(m_i, 1)$	0	$[1-c]$	$[1-c]$
$V(m_i, 0)$	0	0	$[1-c]$

- Equilibrium strategy:

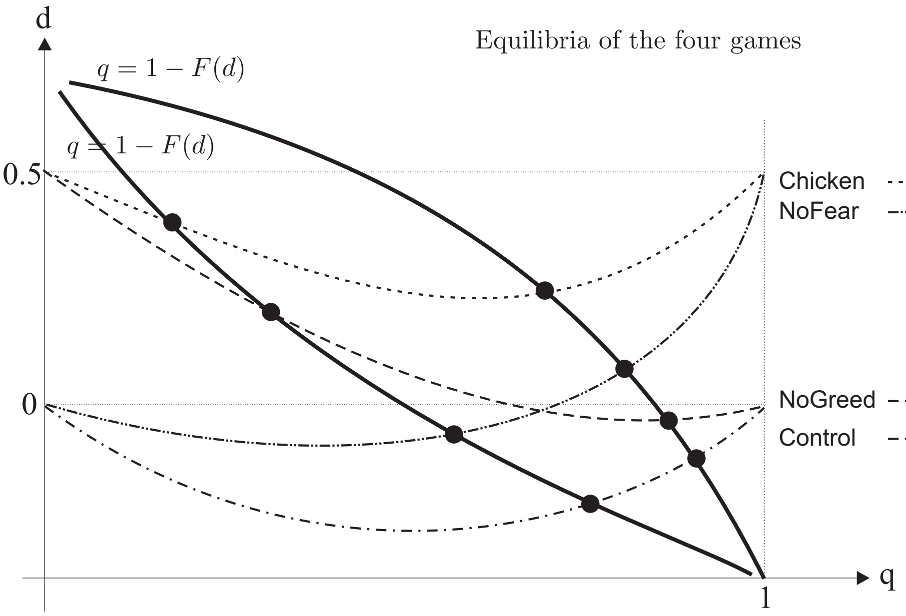
$$\begin{array}{llll} \text{(Chicken)} & d^* & = & \sum_{j=0}^{N-1} \pi_j c - \pi_{w-1} \;=\; c - \pi_{w-1} \\ \text{(NoFear)} & d^* & = & \sum_{j=w-1}^{N-1} \pi_j c - \pi_{w-1} \\ \text{(NoGreed)} & d^* & = & \sum_{j=0}^{w-1} \pi_j c - \pi_{w-1} \\ \text{(Control)} & d^* & = & \pi_{w-1}[1-c] \end{array}$$

- Comparision: given N, w, c, and $F(\cdot)$

$$q^*_{\text{Chicken}} \le q^*_{\text{NoFear}} \le q^*_{\text{Control}}$$

$$q^*_{\text{Chicken}} \le q^*_{\text{NoGreed}} \le q^*_{\text{Control}}$$

▷ People are more likely to contribute without greed or fear.



4 Contribution-based Group Formation

- Gunnthorsdottir *et al.* [JPuE, 2010/v94, pp. 987–994]
- *Within-group* public good contribution:
  - Group members: $i = 1, \cdots, n$
    - ▷ Equal endowment: w
    - ▷ Payoff for each member from \$1 contribution by any member:

$$m \;\in\; \left(\frac{1}{n},\, 1\right)$$

  - Group optimality: all members make full contribution

$$g_i \;=\; w, \;\; \forall\, i$$

  - Individual incentive:

$$g_i \;=\; 0, \;\; \forall\, i$$

- Competitive grouping: based on contributions
  - Total consumers: N
  - Fixed number of equal-sized groups: K
    - ▷ Equal group size:

$$n \;=\; \frac{N}{K}$$

  - Individuals are *ranked* according to their contributions, and then *partitioned* into K groups.[14]
- Inefficient no-contribution equilibrium E^0:

$$g_i \;=\; 0, \;\; \forall\, i \;=\; 1, \cdots, N$$

[14] Ties are broken by random draw.

- Near-efficient positive-contribution equilibrium (NEE, E^+):
  - *Class*: set of consumers with identical contribution
    - ▷ $c_i \equiv$ number of consumers in Class C_i
    - ▷ $\bar{g}_i \equiv$ individual contribution of consumers in Class C_i

$$\bar{g}_1 > \bar{g}_2 > \bar{g}_3 > \cdots$$

  - *Group*: $1,\ 2,\ 3,\ \cdots,\ K$
  - Equilibrium E^+ construction:

1. There must be at least 2 classes, C_1 and C_2.[15]
2. Group 1 contains only C_1 players.[16]
   - ▷ c_1 must be greater than (but not divisible by) n.[17]
   - ▷ Some C_1 players will be mixed with C_2 players in a group.
3. All C_1 players make full contribution:[18]

$$\bar{g}_1 = w$$

4. The mixed group (with both C_1 and C_2 players) must be the last group K.[19]
   - ▷ In any E^+, there are exactly two classes (C_1 and C_2).
   - ▷ All C_2 players are contained in last group K:

$$c_2 < n$$

[15] If all consumers have equal positive contributions (i.e., only 1 class), any one will want to deviate to $g = 0$.
[16] Otherwise any C_1 player i will want to reduce his contribution as long as $g_i > \bar{g}_2$.
[17] Otherwise any C_1 player can reduce his contribution and remain in the same group.
[18] Otherwise each C_1 player i will increase his g_i by ϵ to avoid being assigned to the mixed group.
[19] Assume that there are still groups below the mixed group. Then we have two possibilities: (1) Class C_2 extends beyond the mixed group further below: now each C_2 player i will increase his g_i by ϵ to stay for sure in the mixed group to be with the C_1 players. (2) Class C_2 does not extend beyond the mixed group: now each C_2 player i can slightly decrease his g_i by ϵ without being kicked out of the mixed group. Neither case is possible in equilibrium.

5. All C_2 players contribute nothing:

$$\bar{g}_2 = 0$$

- Existence and uniqueness of E^+:
  - Only equilibrium E^0 exist if

$$m < \frac{N-n+1}{Nn-n^2+1}$$

  - Both E^0 and E^+ exist if

$$m > \frac{N-n+1}{Nn-n^2+1}$$

    and, for E^+, c_2 is an integer between b and $b+1$, with:

$$b = \frac{N-mN}{mN-mn+1-m}$$

    ▷ Equal expected payoffs for both C_1 and C_2 players.

- Experimental test:
  - In general, experimental results support NEE.

5 War of Attrition: Time Dynamics

5.1 Incomplete Info [Bliss-Nalebuff, JPuE 1984]

- The problem:
  - Only 1 contributor is needed for an indivisible PG
  - Who and when will someone contribute?
- Examples:
  - √ *Dragon slaying and Ballroom dancing* [paper title]
  - √ Who will clean up the house/toilet?
  - √ Who will get up to feed the crying baby at night?
  - √ Who will turn in the exam first?
  - √ The mice v. cat story
- The Model:
  - $n+1$ players, everyone gets one-time utility 1 when PG is provided
  - Individual contribution cost: private info

$$c \in [0,1]$$

    ▷ c follows distribution pdf $f(\cdot)$ and cdf $F(\cdot)$

- Asymmetric equilibrium:
  - $(n+1)$ equilibria: any one contributes at time 0, others free ride
  - Not justifiable

- Symmetric-strategy equilibrium:
  - Identical strategy: waitig time limit

$$T(n, c)$$

  - Can I benefit from deviation (pretending I am $c^* > c$)?[20]

$$E[U(n, c, c^*)] = [1 - c]e^{-T(n,c^*)}[1 - F(c^*)]^n + \int_0^{c^*} e^{-T(n,x)} nf(x)[1 - F(x)]^{n-1} dx$$

  - FOC for optimal deviation c^*:

$$\frac{\partial E[U(n, c, c^*)]}{\partial c^*} = 0$$

  or:

$$\frac{\partial T(n, c^*)}{\partial c^*} = \frac{ncf(c^*)}{[1 - c][1 - F(c^*)]}$$

  - However, by definition, $T(n, c)$ implies $c^* = c$:

$$\frac{\partial T(n, c)}{\partial c} = \frac{ncf(c)}{[1 - c][1 - F(c)]} > 0 \quad (7)$$

    ▷ First-order P.D.E. with border condition:

$$T(n, 0) = 0, \quad \forall n$$

- Properties of $T(n, c)$:
  - People with higher costs wait longer:

$$\frac{\partial T}{\partial c} > 0$$

[20] With all others following $T(n, c)$.

  * $*$ Lowest-cost player will contribute at time limit $T(n, c)$
  * $*$ Efficiency loss: delayed provision of PG

– People wait longer with larger population:

$$\frac{\partial T}{\partial n} > 0$$

▷ Since, by eq. (7):

$$T(n, c) = nS(c)$$

we know:

$$T(1, c) = S(c)$$

and

$$T(n, c) = n \cdot T(1, c)$$

E Let:

$$T(1, c) = S(c) = 10$$

then:

$$T(2, c) = 2 \cdot T(1, c) = 20$$

$$T(3, c) = 3 \cdot T(1, c) = 30$$

$$\ldots \quad \square$$

- It is still desirable to have more people (higher n):

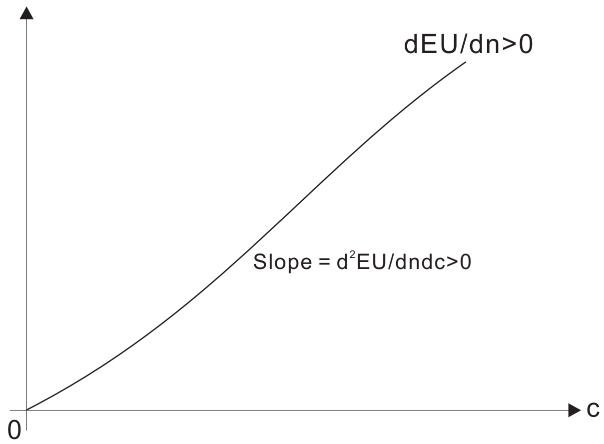
$$\frac{\partial E[U(n, c, c)]}{\partial n} > 0, \quad \forall n, \ \forall c$$

Pf We know

$$\frac{\partial E[U(n, 0, 0)]}{\partial n} = 0, \quad \forall n$$

and

$$\left.\frac{\partial^2 E[U(n,c,c^*)]}{\partial n\,\partial c}\right|_{c^*=c} = \frac{\partial(-e^{-nT(2,c)}[1-F(c)]^n)}{\partial n} \geq 0,\ \forall c \quad \square$$



- Large population: let $\hat{c} \equiv \sup\{x : F(x) = 0\}$

$$\lim_{n\to\infty} E[e^{-T_{\min}}] = 1 - \hat{c}$$

$$\lim_{n\to\infty} T(C_{\min}) = \frac{\hat{c}}{1-\hat{c}}$$

▷ No efficiency loss: a zero-cost player will come out at time 0.

5.2 Perfect Info [Bilodeau-Slivinski JPuE 1996]

- *Department chairing and toilet cleaning* [paper title]
- Complete information
- $n + 1$ players: one contributor needed
- Utility without PG:

$$v_i$$

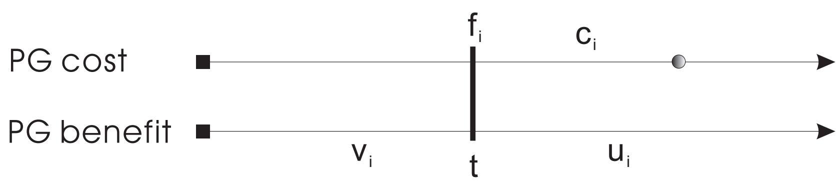
- PG benefit for i: higher utility after PG is provided

$$u_i \ (> v_i)$$

- Contributor cost:
  - One-time cost:

$$f_i$$

  - Prolonged cost c_i for Δ periods.



5.2.1 Infinite Time Horizon

- All players live forever to $t = \infty$.
- i's lifetime utility (PV at $t = 0$) with PG provided at t:[21]

$$F_i(t) \;=\; \frac{v_i}{r_i}\left[1 - e^{-r_i t}\right] + \frac{u_i}{r_i} e^{-r_i t}$$

- i's cost (PV at $t = 0$) if she provides PG at t:

$$C_i(t) \;=\; f_i e^{-r_i t} + \frac{c_i}{r_i}\left[e^{-r_i t} - e^{-r_i[t+\Delta]}\right]$$

- i's lifetime utility if she provides PG at t:

$$L_i(t) \;=\; F_i(t) \;-\; C_i(t)$$

- i's lifetime utility if PG never occurs:

$$S_i = \frac{v_i}{r_i}$$

▷ Ignore people with $L_i(0) < S_i$: they cannot be the contributor!

- SPE: anyone contributes at $t = 0$, with others free riding!

  [!] Multiple equilibria

  [!] Not justifiable!

[21] Note that

$$\int_a^\infty x e^{-rt} dt \;=\; \frac{x e^{-ra}}{r}$$

and hence:

$$\int_a^b x e^{-rt} dt \;=\; \frac{x[e^{-ra} - e^{-rb}]}{r}$$

5.2.2 Finite Time Horizon

- Player i dies at T_i $(< \infty)$

- i's lifetime utility with PG provided at t:

$$F_i(t) \;=\; \frac{v_i}{r_i}\left[1 - e^{-r_i t}\right] + \frac{u_i}{r_i}\left[e^{-r_i t} - e^{-r_i T_i}\right]$$

- i's cost if she provides PG at t:

$$C_i(t) \;=\; f_i e^{-r_i t} + \frac{c_i}{r_i}\left[e^{-r_i t} - e^{-r_i \tau_i(t)}\right]$$

with:

$$\tau_i(t) \;\equiv\; \min\{T_i, t + \Delta\}$$

- i's lifetime utility if she provides PG at t:

$$L_i(t) \;=\; F_i(t) \;-\; C_i(t)$$

- i's lifetime utility if PG never occurs:

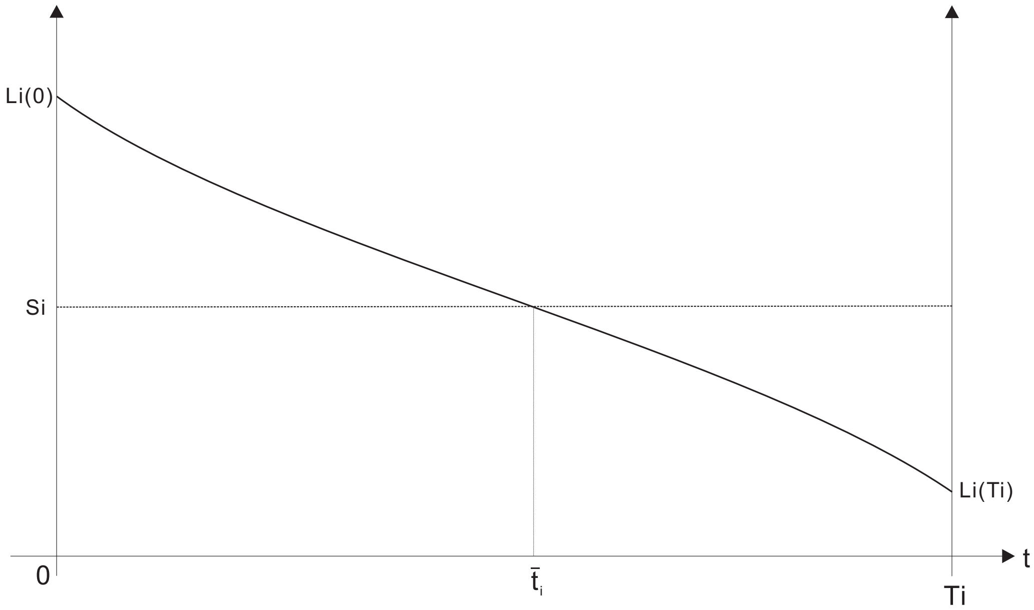
$$S_i \;=\; \frac{v_i}{r_i}\left[1 - e^{-r_i T_i}\right]$$

- The latest time $\bar{t}_i$ that i may provide PG: Figure

$$L_i(\bar{t}_i) = S_i$$

and hence:

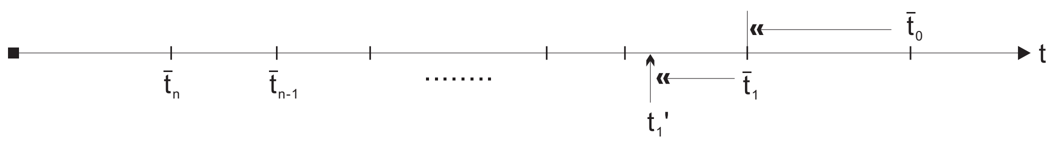
$$\bar{t}_i \;=\; T_i \;-\; \frac{1}{r_i}\ln\frac{B_i}{B_i - 1}, \quad B_i \;\equiv\; \frac{u_i - v_i - c_i}{r_i f_i}$$



- SPE (using backward induction):
  - Re-arranging players:

$$\bar{t}_n < \bar{t}_{n-1} < \cdots < \bar{t}_1 < \bar{t}_0$$

  - Efficiency: player 0 will contribute at $t = 0$ (no delay)[22]



- Likely contributor:
  - T_i large: live longer

[22]Note that t_1' is where:

$$L_1(t_1') = F_1(\bar{t}_1).$$

– r_i large: impatient to wait

– B_i large: benefit-cost ratio higher

6 Sequential-move Games

6.1 Stackelberg v. Nash Games [Varian, JPuE 1994]

- Additive PG: 2 players ($i = 1, 2$)
  - Individual budget:

$$w_i \;=\; x_i \;+\; g_i$$

  - PG:

$$G \;=\; g_1 \;+\; g_2$$

  - Quasilinear utility:

$$U_i(x_i, G) \;=\; u_i(G) + x_i \;=\; u_i(g_1 + g_2) + [w_i - g_i]$$

- Stand-alone contribution: i's contribution when $g_j = 0$

$$\bar{g}_1 \;=\; \text{argmax}_{g_1}\; u_1(g_1) + [w_1 - g_1]$$

$$\bar{g}_2 \;=\; \text{argmax}_{g_2}\; u_2(g_2) + [w_2 - g_2]$$

  ▷ Assume player 1 likes PG more:

$$\bar{g}_1 \;>\; \bar{g}_2$$

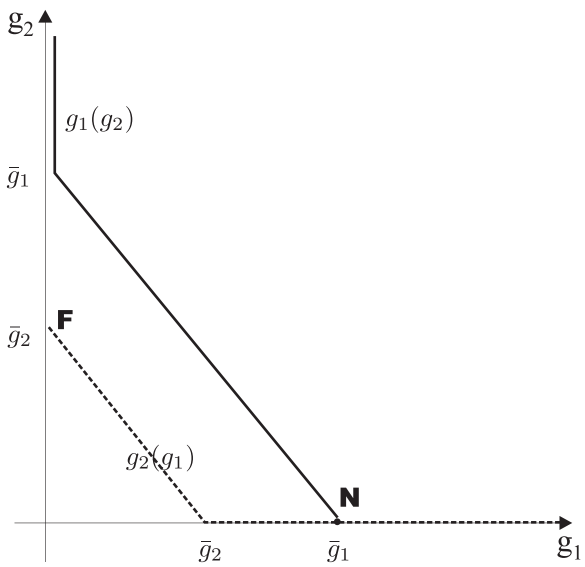
- Nash reaction function: [Figure below]

$$g_1(g_2) \;=\; \max\{\bar{g}_1 - g_2, 0\}$$

$$g_2(g_1) \;=\; \max\{\bar{g}_2 - g_1, 0\}$$

- Simultaneous-move equilibrium: intersection N of Nash reaction curves

$$G^N \;=\; \bar{g}_1$$



- Sequential-move equilibrium:

1. Player 2 as leader: since

$$u_2(\bar{g}_1) \;>\; u_2(\bar{g}_2) \;>\; u_2(\bar{g}_2) - \bar{g}_2$$

▷ Player 2 will free ride:

$$g_2 = 0, \quad g_1 = \bar{g}_1$$

▷ PG level same as in Nash: $G^S = \bar{g}_1$

2. Player 1 as leader:

$$V_1(g_1) \;=\; u_1(g_1 + g_2(g_1)) + [w_1 - g_1]$$

$$=\; u_1(g_1 + \max\{\bar{g}_2 - g_1, 0\}) + [w_1 - g_1]$$

or:

$$V_1(g_1) \;=\; \begin{cases} u_1(\bar{g}_2) + [w_1 - g_1], & g_1 \le \bar{g}_2 \\ u_1(g_1) + [w_1 - g_1], & g_1 \ge \bar{g}_2 \end{cases}$$

– Choice 1: $g_1 = 0$ (free ride), let $G^S = \bar{g}_2$, and get utility

$$V_1^F = u_1(\bar{g}_2) + w_1$$

▷ PG level lower than Nash:

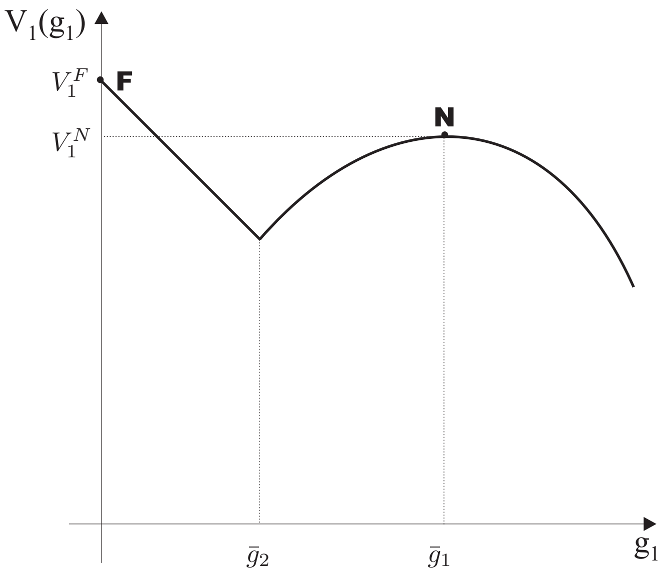
$$G^S = \bar{g}_2$$

– Choice 2: $g_1 = \bar{g}_1$, let 2 free ride, and get utility

$$V_1^N = u_1(\bar{g}_1) + w_1 - \bar{g}_1$$

▷ PG level same as in Nash:

$$G^S = \bar{g}_1$$



3. *PG level in Stackelberg may be lower than Nash*:

– when PG-lover is leader

– when PG-lover chooses to free ride.[23]

[23] This is not possible in Nash. Threat is not credible.

- Stackelberg leadership bidding:
  - Value of leadership for players:

$$b_1 = V_1^F - V_1^N = u_1(\bar{g}_2) - [u_1(\bar{g}_1) - \bar{g}_1]$$

$$b_2 = V_2^N - V_2^F = u_2(\bar{g}_1) - [u_2(\bar{g}_2) - \bar{g}_2]$$

  Therefore:

$$\begin{aligned} b_2 - b_1 &= [u_1(\bar{g}_1) - u_1(\bar{g}_2)] + [u_2(\bar{g}_1) - u_2(\bar{g}_2)] - [\bar{g}_1 - \bar{g}_2] \\ &\geq [u_1'(\bar{g}_1) + u_2'(\bar{g}_1) - 1][\bar{g}_1 - \bar{g}_2] \quad \text{(concavity)} \\ &= u_2'(\bar{g}_1)[\bar{g}_1 - \bar{g}_2] \quad \text{(foc2)} \\ &> 0 \end{aligned}$$

  - Player 2 has a higher bid than player 1.
- <u>Generalization</u>: results hold for any convex preferences.

6.2 Donation Announcement by Charities

- Romano and Yildirim [JPuE, 2001/v81, pp. 423–447]
- Charities often announce donor contributions as they accrue.

  ▷ Contributions become sequential, instead of simultaneous.

  [E] Telethon, United Ways, university fund-raising

6.3 Seed Donation for Fixed-cost PG Campaign

- Leadership giving: Andreoni [JPE, 1998]
  - Discrete PG: *fixed production costs*
    - ▷ Both <u>positive</u> and <u>zero provision</u> equilibria exist in Nash games
  - Donors may get stuck in no-provision outcome
    - ▷ Due to lack of coordination
  - *Sequential fund-raising strategy* is preferable
    - ▷ People are induced to contribute by large *initial donations*
  - <u>Lab experiment</u>: Bracha, Menietti,and Vesterlund [JPuE, 2011v95]
    - ▷ Theory confirmed for *high* (not for *low*) fixed costs

- List and Lucking-Reiley [JPE, 2001/v110(1)]
  - Field experiment
  - Both likelihood and average amount of contributing are higher with larger initial seed amounts

- Andreoni [JET 2006]
  - ?

6.4 Alternating-move Game [Admati-Perry, REStud 1991]

- Alternating contributions:
  - Player 1 makes contribution in period 1.
  - Player 2 then makes contribution in period 2.
  - Player 1 again makes contribution in period 3.
  - And so on ...
  - ▷ Game terminates when total contribution reaches PG cost.
- 2 contribution setups:
  - Contribution game: pay immediately when making commitment
  - Subscription game: pay only when PG is provided
- The 2-player model:
  - Same value of PG: V for both players
  - Cost of PG: K

6.4.1 Subscription Game

- Let $T \equiv$ time when the game terminates (when PG is provided)

  ▷ $C_i^T \equiv$ total pledged contribution by i up to period T

  ▷ Utility at period T:

$$U_i^T(C_1, C_2) = \delta^T \left[V - C_i^T\right], \quad i = 1, 2$$

- SPE, given (V, K), is:

  – $K > 2V$: PG is never provided

  – $K \in ([1-\delta]V, 2V)$: Player 1 contributes at $t = 1$:

$$C_1^* = \frac{K - [1-\delta]V}{1+\delta}$$

  then player 2 contributes at $t = 2$:

$$C_2^* \;=\; K - C_1^* \;=\; \frac{\delta K + [1-\delta]V}{1+\delta}$$

  ▷ PG provided at $t = 2$ (no delay)

  – $K = [1-\delta]V$: 2 possible SPEs.

  Player 1 may contribute all cost $C_1^* = K$ at $t = 1$.

  Or 1 makes no contribution at $t = 1$, and let 2 makes $C_2^* = K$.

  Player 1 will get same discounted utility:

$$V - K \;=\; \delta V$$

  – $K < [1-\delta]V$: player 1 takes full responsibility ($C_1^* = K$) at $t = 1$, and let 2 free ride ($C_2^* = 0$). ■

- Efficiency: PG is provided when $K < 2V$ in no more than 2 periods.

6.4.2 Contribution Game

- Let $c_i^t \equiv$ contribution by i at t
- Payment sequence: $(c_1^1, c_2^1 = 0),\ (c_2^2, c_1^2 = 0), \cdots$
- Game terminates at time T if:

$$\sum_{t=1}^{T}[c_1^t + c_2^t] \ \geq \ K$$

  – Outcome:

$$(T,\ \{c_1^t\}_{t=1}^T,\ \{c_2^t\}_{t=1}^T)$$

  – Player i utility:

$$U_i^T\left(\{c_1^t\}_{t=1}^T, \{c_2^t\}_{t=1}^T\right) \ = \ \delta^{T-1}V - \sum_{t=1}^{T}\delta^{t-1}W(c_i^t)$$

  where: $W(c_i) \equiv i$'s cost of making contribution c_i

- *SPE may be inefficienct*:
  – $W(c_i)$ is linear, say $W(c_i) = c_i$:
    ▷ PG will occur iff $V > K$, with 1 being the sole contributor $(c_1^1 = K)$
    ▷ PG will not occur (as it should) when $K \in [V, 2V]$
  – $W(c_i)$ is strictly convex: conditions for PG to be provided:
    if: $V > W(K)$
    only if: $V > W'(0)K$
    E When $K = V = 1$, $\delta = 1$, and $W(c) = c + c^2$: no PG.[24] □

[24] The efficient solution would be equal sharing ($c_1 = c_2 = 1/2$).

7 Mechanism Design for Optimality

7.1 Matching Game

- Indogenous linear matching: Guttman [AER 1978, 68:251–255]
  - Two-stage game:
    - 1 Each player i announces his/her *matching rate* b_i
    - 2 Each player decides his/her *flat contribution* a_i
  - PG contribution:

$$x_i \;=\; a_i \;+\; b_i \sum_{j \neq i} a_j, \;\; X \;=\; \sum_i x_i$$

  - Quasi-linear player payoff:

$$\pi_i \;\equiv\; f_i(X) - x_i \;=\; f_i\left(\sum_i [a_i + b_i \sum_{j \neq i} a_j]\right) - \left[a_i + b_i \sum_{j \neq i} a_j\right]$$

  - Efficiency: SPE satisfies Samuelson condition.

- Exogenous matching rates μ_{ij}:
  - Buchholz-Cornes-Rubbelke [JPuE 2011, 95:639–645]
  - For a matching game equilibrium to be Pareto efficient, all players must be *contributors* (i.e., *interoir*)
  - Income distrubution is cricual for interior solution
  - Warr neutrality no longer holds

E 2 players: $U_i(x_i, G) = x_i G$, $W = 2$, $\mu_{ij} = 1 \Rightarrow\; W_1 = W_2 = 1$ □

7.2 Contribution Deposit

7.2.1 Introduction

- Gerber and Wichardt [JPuE 2009, 93:429–439]
- To implement any social goal $\wp$ that is P-superior to Nash outcome $\aleph$
  - Lack of centralized sanctioning intitutions
  - Voluntary participation
- Applications:
  - International environmental agreement: Kyoto Protocal
  - Private contribution to public good
  - n-person Prisoners' Dilemma
- 2-stage mechanism:

  [1] Deposit (押金) stage

  [2] Contribution stage
- Subgame-perfect Nash equilibrium:
  - Unanimous deposit payment
  - Full ex-post contributions

7.2.2 2-player Example: Symmetric Linear Public Good

- The Nash contribution game Γ^0:
  - 2 players ($i = 1, 2$) with equal endowment e
  - Voluntary individual PG contribution:
  $$c_i \in [0, e]$$
  - Additive PG:
  $$C = c_1 + c_2$$
  - Linear Utility:
  $$U_i(c_1, c_2) = [e - c_i] + \alpha[c_1 + c_2], \quad \frac{1}{2} < \alpha < 1$$

  $\boxed{\text{Q}}$ Why do we need $\alpha \in (\frac{1}{2}, 1)$?

- Unique Nash equilibrium $\aleph$:
$$c_1 = c_2 = 0$$
- Full-contribution optimum $\wp$:
$$c_1 = c_2 = e = \bar{c}$$
  ▷ $\aleph$ is Pareto dominated by $\wp$:
  $$U_i(\bar{c}, \bar{c}) > U_i(0, 0), \quad i = 1, 2$$
- How can we implement $\wp$?

- 2-stage game design to implement $\wp$:

  S1 Both players decide simultaneously whether to pay deposit $\bar{d}$.

  – Payment d_i is hence either 0 or $\bar{d}$.

  – Payment decision (d_1, d_2) is public info. □

  S2 If either $d_i = 0$: all deposits are refunded, game Γ^0 is played.

  If $d_1 = d_2 = \bar{d}$: players contributing full $\bar{c}$ get refund $\bar{d}$. ■

- SPE:

  S2 Stage game Nash equilibrium:

  ∗ If either $d_i = 0$: player dominant strategy is

$$c_1 = c_2 = 0$$

  and utility is:

$$U_1^0 = U_2^0 = e \quad \square$$

  ∗ If $d_1 = d_2 = \bar{d}$: player payoffs are:

$$\pi_i(c_i, c_j) = \begin{cases} e - \bar{d} - c_i + \alpha[c_i + c_j], & \text{if } c_i \neq \bar{c} \\ e - \bar{c} + \alpha[\bar{c} + c_j], & \text{if } c_i = \bar{c} \end{cases}$$

  or:

$$\pi_i(c_i, c_j) = \begin{cases} e - \bar{d} - [1 - \alpha]c_i + \alpha c_j, & \text{if } c_i \neq \bar{c} \\ e - [1 - \alpha]\bar{c} + \alpha c_j, & \text{if } c_i = \bar{c} \end{cases}$$

  ▷ Dominant strategy is $c_i = \bar{c}$ $(i = 1, 2)$ if[25]

$$e \geq \bar{d} > [1 - \alpha]\bar{c} = [1 - \alpha]e$$

[25] Note that if $c_i \neq \bar{c}$, i should choose $c_i = 0$. Hence we are comparing:

$$\pi_i(c_i, c_j) = \begin{cases} e - \bar{d} + \alpha c_j, & \text{if } c_i = 0 \\ e - [1 - \alpha]\bar{c} + \alpha c_j, & \text{if } c_i = \bar{c} \end{cases}$$

and utility is:

$$U_1^* \;=\; U_2^* \;=\; e + [2\alpha - 1]\bar{c} \quad \square$$

S1 Payment $d_i = \bar{d}$ is a weakly dominant strategy for either player.

* If $d_j = \bar{d}$: then $d_i = \bar{d}$ is strictly better than $d_i = 0$ for i

$$U_i^*(\bar{c}, \bar{c}) \;>\; U_i^0(0, 0)$$

* If $d_j = 0$: then both $d_i = \bar{d}$ and $d_i = 0$ yield same utility

$$U_i^0(0, 0) \;=\; e \quad \square$$

- Intuition:
  - Players are now forced to choose between Pareto-superior $\wp$ and Nash $\aleph$.
  - Threats: either commit to $\wp$, or revert to the Nash outcome $\aleph$.[26]
  - Players cannot choose individual c_i (hence cannot free ride).

[26] For PD game: if you betray me, you won't get deposit back.

7.2.3 General n-player Model

1. Assumptions

- Utility function: x private, y public

$$U^i(x_i, y);\;\; U^i_x > 0,\;\; U^i_y > 0$$

- Endowment: e_i (units of x)
- Individual PG contribution:

$$c_i \;\in\; [0, e_i]$$

▷ budget constraint:

$$x_i \;+\; c_i \;=\; e_i$$

- Aggregate PG production:

$$y \;=\; F\left(\sum_i f_i(c_i)\right),\;\; f_i' > 0,\; f_i'' < 0,\; F' > 0$$

- For any $c = (c_1, \cdots, c_n) = (c_i, c_{-i})$: utility

$$\pi_i(c_i, c_{-i}) \;=\; U_i\left(e_i - c_i,\, F\left(\sum_i f_i(c_i)\right)\right)$$

- A1 Spending on x yields higher marginal return than y: $\forall\, i$

$$U^i_x \;>\; U^i_y F' f_i'$$

▷ Contribution lowers utility:

$$\frac{\partial \pi_i(c_i, c_{-i})}{\partial c_i} \;<\; 0,\;\; \forall\, i,\; \forall\, c_i \geq 0,\; \forall\, c_{-i}$$

! Very strict restriction on utility function

- Equilibrium $\aleph$ of the Nash game Γ^0: strictly dominant strategy

$$c_i^0 \;=\; 0, \;\; \forall\, i$$

$\boxed{\text{E}}$ Linear utility function:

$$U^i(x_i, y) = x_i + ay;\; a < 1$$

$$f_i(c_i) = c_i, \;\; F(x) = x \quad \square$$

2. Design

- Can implement <u>any</u> $c^* = (\bar{c}_1, \cdots, \bar{c}_n)$ that <u>Pareto-dominates</u> $\aleph$:

$$\pi_i(\bar{c}_1, \cdots, \bar{c}_n) \; > \; \pi_i(0, \cdots, 0), \;\; \forall\, i$$

<u>Example</u>: The PD game

- 2-stage game:

S1 Everyone pays deposit: $d_i \; \in \; \{0, \bar{d}_i\}$

$\triangleright$ $d \equiv (d_1, \cdots, d_n)$ is public info at end of S1.

S2 Depending on $d = (d_1, \cdots, d_n)$ in S1:

- If any $d_i = 0$:
  - All deposits d_i are refunded.
  - Nash game Γ^0 is played, all players get utility

$$\pi_i(0, \cdots, 0)$$

- If all $d_i = \bar{d}_i$: game Γ^* below is played:
  - Players contributing full $c_i = \bar{c}_i$ get refund $\bar{d}_i$.

    Others (with $c_i < \bar{c}_i$) receive no refund.
  - Player i gets payoff:

$$\pi_i(c_i, c_{-i}) \; = \; \begin{cases} U^i(e_i - c_i - \bar{d}_i, F(\sum_j f_j(c_j))), & \text{if} \;\; c_i < \bar{c}_i \\ U^i(e_i - \bar{c}_i, F(\sum_j f_j(c_j))), & \text{if} \;\; c_i = \bar{c}_i \end{cases} \qquad \square$$

- **A2** Deposit $(\bar{d}_1, \cdots, \bar{d}_n)$: $\bar{d}_i$ $(\leq e_i)$ is chosen such that: $\forall\, c_{j\,(\neq i)} \leq e_j$

$$U^i\left(e_i - \bar{c}_i, F(f_i(\bar{c}_i + \sum_{j \neq i} f_j(c_j))\right) \; > \; U^i\left(e_i - \bar{d}_i, F(\sum_{j \neq i} f_j(c_j))\right)$$

- SPE of the 2-stage game:

  S1 Weakly dominant strategy for all i:

$$d_i = \bar{d}_i$$

  S2 Strictly dominant strategy for all i:

$$c_i(d) = \begin{cases} \bar{c}_i, & \text{if } d_j = \bar{d}_j, \ \forall j \\ 0, & \text{if } d_j = 0 \text{ for some } j \end{cases} \qquad \square$$

- Sketch of Proof:

  S2 By A1, subgame Γ^0 has unique DSE $c_i = 0$ (all i).

  By A2, subgame Γ^* has unique DSE $c_i = \bar{c}_i$ (all i).

  S1 For i:

  If any $d_j = 0$: outcome is $\pi_i(0, \cdots, 0)$, independent of d_i.

  If $d_j = \bar{d}_j$, $\forall j (\neq i)$: outcome is $\pi_i(\bar{c}_1, \cdots, \bar{c}_n)$ if $d_i = \bar{d}_i$.

  $\triangleright$ $d_i = \bar{d}_i$ is weakly dominant strategy for all i. ■

3. Extensions

- Costly deposit collection/payment:
  - Payoff modification: fraction δ is payer cost

    $\triangleright$ Γ^0 game:

    $$\pi_i(c_i, c_{-i}) = \begin{cases} U^i(e_i - c_i - \delta\bar{d}_i, F(\sum_j f_j(c_j))), & \text{if } d_i = \bar{d}_i \\ U^i(e_i - \bar{c}_i, F(\sum_j f_j(c_j))), & \text{if } d_i = 0 \end{cases}$$

    $\triangleright$ Γ^* game:

    $$\pi_i(c_i, c_{-i}) = \begin{cases} U^i(e_i - c_i - \bar{d}_i, F(\sum_j f_j(c_j))), & \text{if } c_i < \bar{c}_i \\ U^i(e_i - \bar{c}_i - \delta\bar{d}_i, F(\sum_j f_j(c_j))), & \text{if } c_i = \bar{c}_i \end{cases}$$

  - Results still hold.
- Possible uses of forfeited deposits: off-equilibrium cases
  - Throw away (gift to other economies)
  - Extra/bonus refund to full contributors
  - Other inrelevent use
- Repeated games:
  - Collect a big deposit first (as long-run commitment)
  - Refund a share in each subsequent period

7.3 Category Reporting: Harbough [JPuE 1998]

7.3.1 Stylized Facts

- Many charities use category reporting for fundraising
- Donors tend to give minimum necessary to get into a category
- Donors enjoy having their donations publicized

7.3.2 Pure Egoism

- Warm glow: pure internal satisfaction from act of giving
  - Proportional to donation amount
- Prestige: utility from having their donations publicly known
  - Affected by charity reporting plans
  - Due to social recognition, business opportunities, etc.

7.3.3 The Model

- Donor choice:
  - Utility:

$$U(x, p, d), \;\; U_x > 0, \;\; U_p > 0, \;\; U_d > 0$$

    $x \equiv$ private consumption

    $p \equiv$ prestige

    $d \equiv$ warm glow (= donation)

  - Budget:

$$x + d \; = \; w$$

  - Utility max:

$$\max_{x,\,d} \;\; U(x, p, d) \quad \text{s.t.} \quad x + d = w$$

    ▷

$$\max_{d} \;\; U(w - d, p, d)$$

  - Level curves: $\boxed{\text{Fig ?}}$

$$I_w(k) \; = \; \{\, (p, d) \mid U(w - d, p, d) = k \,\}$$

    ▷ U-shaped on p-d space, with slope: 先負後正

$$\frac{dp}{dd} \; = \; \frac{U_x - U_d}{U_p}$$

    ▷ As $w \uparrow$, infection points (反轉點) shift right. ($\because \; d \uparrow$ with w)

- Prestige effect:

– Charity reports r for donation d:

$$r(d)$$

– Donor then gets prestige p from publicly known r:

$$p(r) \;=\; p(r(d))$$

– Can let $p = r$: prestige fn is absorbed into util fn

- 3 possible charity report plans $r(d)$: restriction $r(d) \leq d$

– No reporting:

$$r(d) = 0, \;\; p(d) = 0$$

– Exact reporting:

$$r(d) = d, \;\; p(d) = d$$

– Category reporting:

$$p(d) \;=\; r(d) = \begin{cases} \alpha, & \text{if } \; d \geq \alpha \\ 0, & \text{otherwise} \end{cases}$$

- Donors choose optimal (p, d) subject to report constraint $p(d)$.

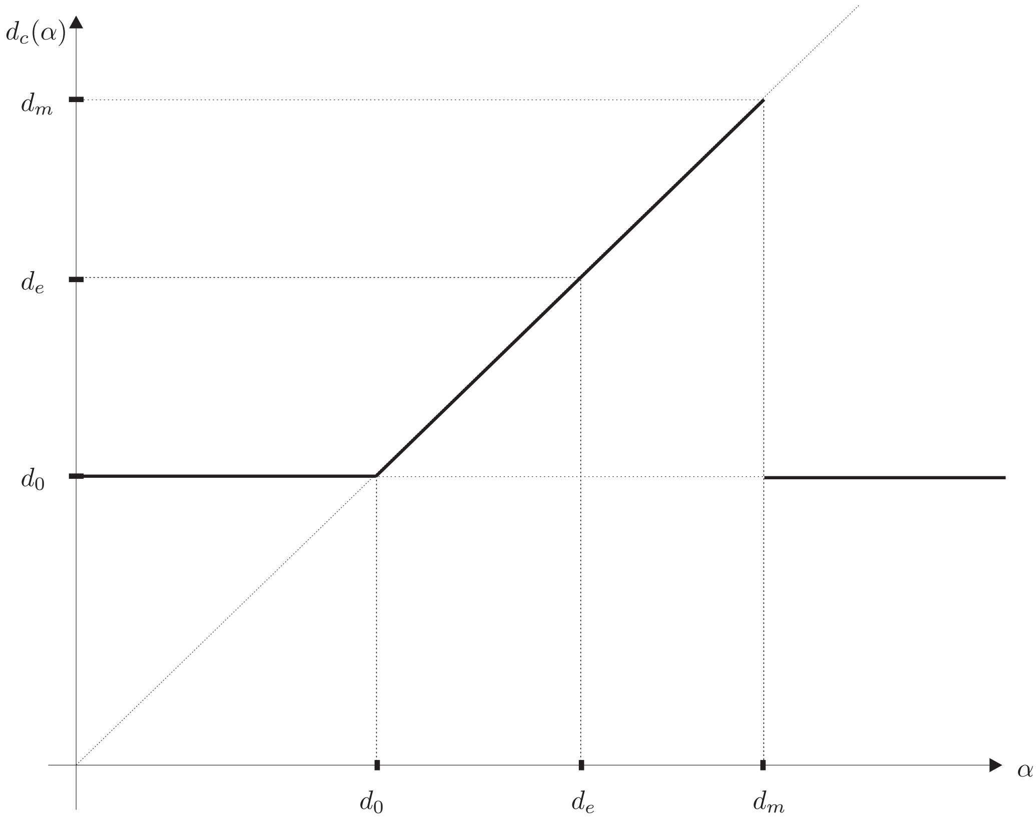
7.3.4 Effects of Reporting Plans on Donations

- No reporting d_0: $U_x = U_d$ (zero slope) on $p = 0$ line
- Exact reporting d_e: $U_x = U_d + U_p$ (slope = 1) on $p = d$ line
- Category reporting:

$$d_c = \begin{cases} d_0, & \text{if} \quad \alpha < d_0 \\ \alpha, & \text{if} \quad \alpha \in [d_0, d_e) \\ \alpha, & \text{if} \quad \alpha \in [d_e, d_m) \\ d_0, & \text{if} \quad \alpha \geq d_m \end{cases}$$

- One-donor case:

$$d_e > d_0, \text{ but } d_c \gtreqless d_e$$



- Charity strategy: to max donation, choose bracket

$$\alpha = d_m$$

- Donor bunching: donors of different incomes bunching up at bracket Fig ?

7.3.5 Optimal Solicitation Strategy of Charities

- To show: can always increase total donations by using categories.
- Assume: n types of donors with

$$d_e^1 < d_e^2 < \cdots < d_e^n$$

- Low-end category: can raise 1's donation w/o affecting others' choice.

  (C1) $d_e^1 < d_m^1 < d_e^2$: can fully exploit 1 $\boxed{\text{Fig ?}}$

$$\tilde{r}(d) = \begin{cases} d, & \text{if } d \geq d_m^1 \\ 0, & \text{otherwise} \end{cases}$$

  ▷ Exact reporting for donations above d_m^1 only

  ▷ Donor 1 change from d_e^1 to d_m^1; donor 2 remains same

  (C2) $d_e^1 < d_e^2 < d_m^1$: cannot fully exploit 1 $\boxed{\text{Fig ?}}$

$$\tilde{r}(d) = \begin{cases} d, & \text{if } d \geq d_e^2 \\ 0, & \text{otherwise} \end{cases}$$

  ▷ Exact reporting for donations above d_e^2 only

  ▷ Donor 1 change from d_e^1 to d_e^2; all others still same

- High-end category: can raise n's contribution $\boxed{\text{Fig ?}}$

$$\hat{r}(d) = \begin{cases} d, & \text{if } d \leq d_e^{n-1} \\ d_e^{n-1}, & \text{if } d \in [d_e^{n-1}, d_m^n) \\ d_m^n, & \text{if } d \geq d_m^n \end{cases}$$

  ▷ Donor n change from d_e^n to d_m^n; all others unchanged.

- <u>Note</u>: $\tilde{r}$ and $\hat{r}$ not necessarily optimal: may still raise donations further
- Similar devices: unique souvenir, building naming, trophy, etc.

7.3.6 Charity Classification and Theory Testing

- Educational institutions:
  - Monopoly on alumni donations: without substitute
  - Can fully exploit consumers ⊳ categories far apart
  - Donations publicized to a limited circle
- National organizations: E Sierra Club, RFF
  - Strong competition among charities
  - Unable to exploit consumers fully ⇒ Categories closer together
  - Aim at small donations from large population
- United Way (聯合勸募):
  - Formed to effectively use categories
  - Facilitate distribution of donation reports

7.3.7 Problems

- Not an equilibrium analysis of public-good model
- No PG in model: donors do not care about total PG level
- No consumer interaction: donors do not care about how much others donate