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# 1. The Cornes-Sandler Framework

## 1.1. Additive Public Good

- $n$ consumers
- 2 goods:
  - $x$: private good (price = 1)
  - $Q$: public good (price = $p$)

- Private contribution to PG:

$$q_i$$

- Additive PG:

$$Q = \sum q_i, \quad \tilde{Q}_i \equiv \sum_{j \neq i} q_j$$

- Pure altruistic preference:

$$U_i(x_i, Q) = U_i(x_i, q_i + \tilde{Q}_i)$$

▷ $U$ is strictly quasi-concave

- Individual budget allocation:

$$x_i + pq_i = I_i$$

- Utility-max:

$$\max_{x,\,q} \quad U(x, q+\tilde{Q}) \quad \text{s.t.} \quad x + pq = I \tag{1}$$

or:

$$\max_{q} \quad V(q,\tilde{Q}) \equiv U(I - pq, q+\tilde{Q}) \tag{2}$$

- Iso-utility (IU) curve:

$$V(q,\tilde{Q}) \; = \; k$$

▷ Slope:

$$\frac{d\tilde{Q}}{dq} \; = \; \frac{-\partial V/\partial q}{\partial V/\partial \tilde{Q}} \; = \; \frac{pU_x - U_Q}{U_Q} \; = \; p\frac{U_x}{U_Q} - 1 \tag{3}$$

E Show that "better sets"

$$B \equiv \{(q,\tilde{Q}) \mid V(q,\tilde{Q}) \geq k;\; k \in R\}$$

of function $V(q,\tilde{Q})$ are convex. □

- Nash reaction function: Fig. 1

$$q(\tilde{Q})$$

▷ Nash reaction curve $NN$:

$$\frac{d\tilde{Q}}{dq} \; = \; p\frac{U_x}{U_Q} - 1 \; = \; 0$$

hence:

$$\frac{U_Q}{U_x} \; = \; p \tag{4}$$

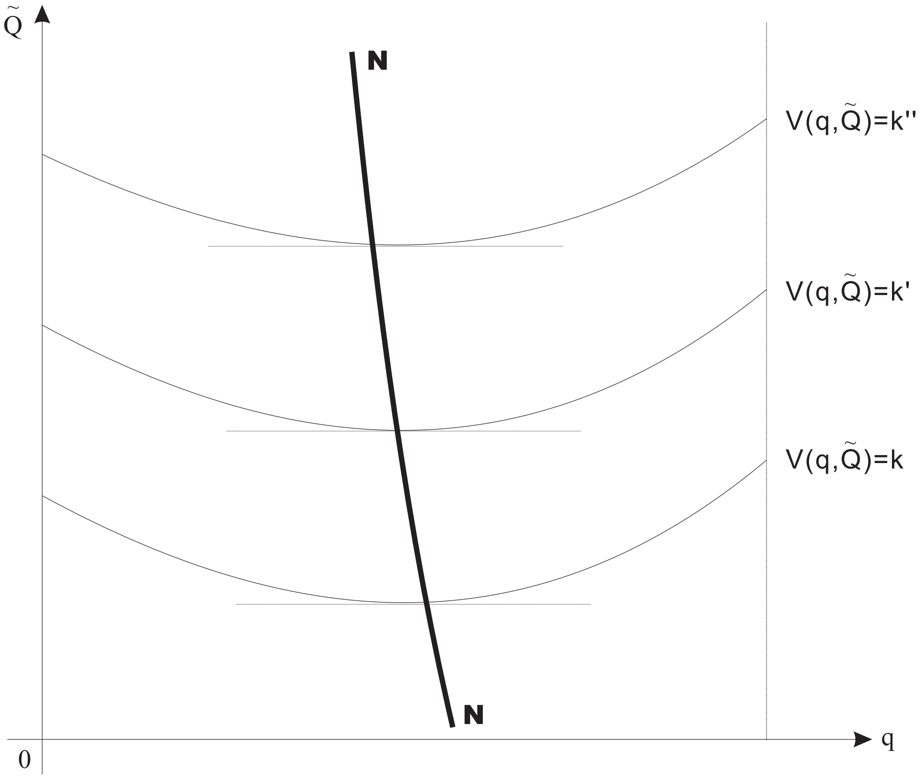
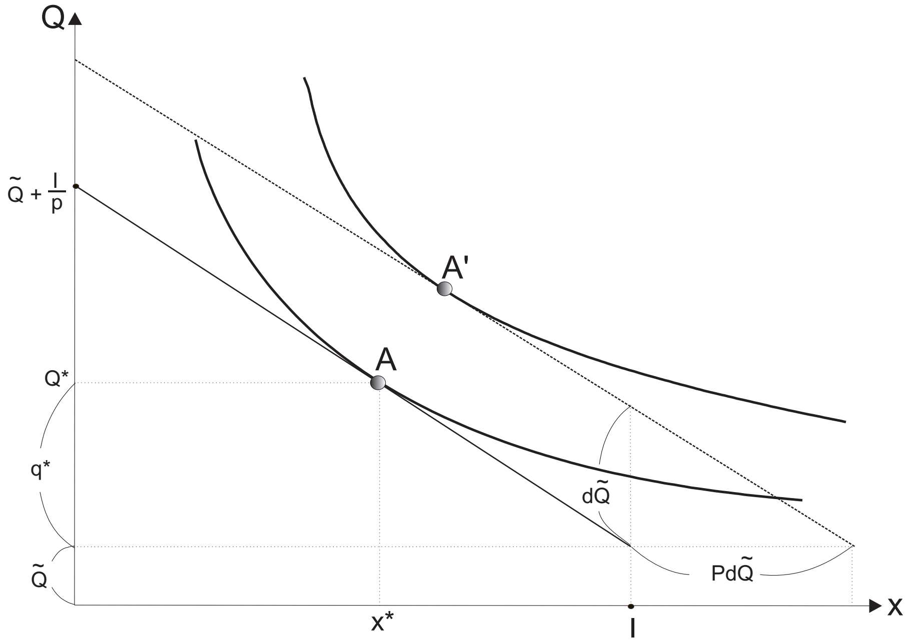



Figure 1: Iso-utility curves in $q$-$\tilde{Q}$ plane

## 1.2. Individual Nash Choice

- Choice of $(x, q)$:

$$\frac{dQ}{d\tilde{Q}} = 1 + \frac{dq}{d\tilde{Q}}$$
$$\frac{dq}{d\tilde{Q}} = \frac{dQ}{d\tilde{Q}} - 1 \tag{5}$$




- Choice of $(x, Q)$: substitute $[q = Q - \tilde{Q}]$ into (1)

$$\max_{x,Q} \quad U(x, Q) \quad \text{s.t.} \quad x + pQ = I + p\tilde{Q} \tag{6}$$

hence:

$$\frac{dQ}{d\tilde{Q}} = p\frac{dQ}{dI} \tag{7}$$

or, by substituting (7) into (5):

$$\frac{dq}{d\tilde{Q}} = p\frac{dQ}{dI} - 1 \tag{8}$$

- Slope of $NN$:
  - $x$ and $Q$ are <u>both normal</u>:
    - $\triangleright$ $A'$ to the up-right of $A$

$$1 \; > \; p\frac{dQ}{dI} \; > \; 0$$

$$0 > \frac{dq}{d\tilde{Q}} > -1, \quad \frac{d\tilde{Q}}{dq} < -1 \tag{9}$$

    - $\triangleright$ $NN$ has negative slope ($< -1$)

  - $x$ is <u>normal</u>, but $Q$ is <u>inferior</u>:
    - $\triangleright$ $A'$ to the lower-right of $A$

$$p\frac{dq}{dI} \; < \; 0$$

$$\frac{dq}{d\tilde{Q}} < -1, \quad -1 < \frac{d\tilde{Q}}{dq} < 0 \tag{10}$$

    - $\triangleright$ $NN$ has negative slope ($> -1$)

  - $Q$ is <u>normal</u>, but $x$ is <u>inferior</u>:
    - $\triangleright$ $A'$ to the up-left of $A$

$$p\frac{dQ}{dI} \; > \; 1$$

$$\frac{dq}{d\tilde{Q}} > 0, \quad \frac{d\tilde{Q}}{dq} > 0 \tag{11}$$

    - $\triangleright$ $NN$ has positive slope

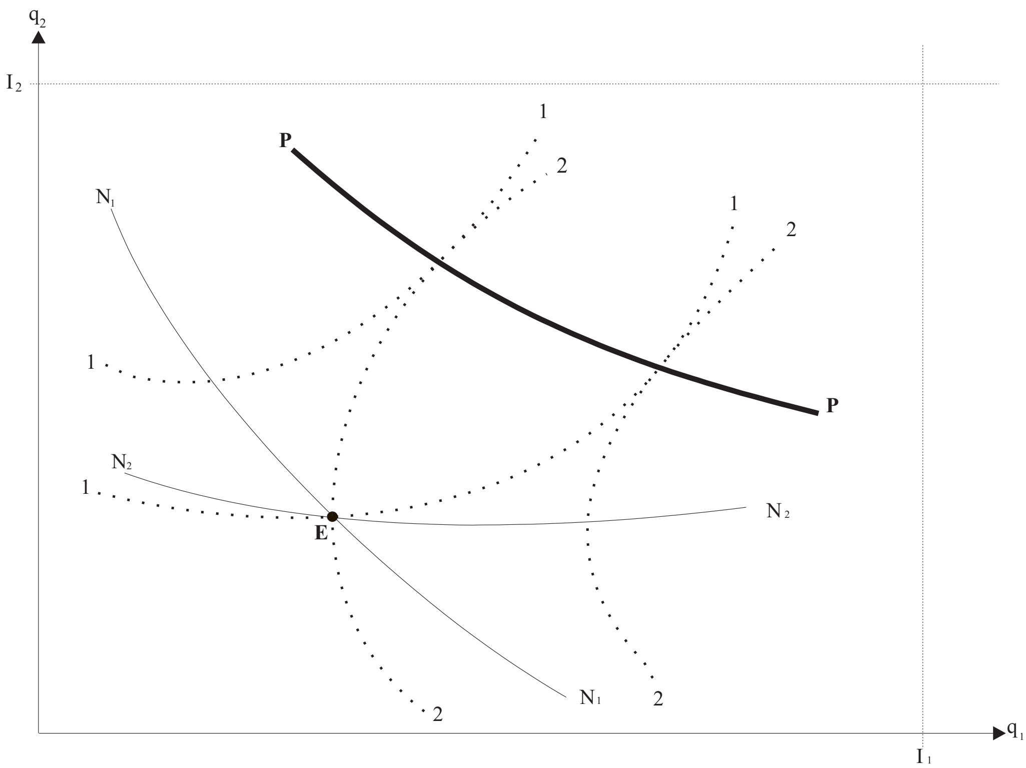
## 2. 2-player Normal-good Equilibrium

- Individual util-max:

$$V^1(\bar{q}_2 \mid I_1) \equiv \max_{x_1,q_1} U(x_1, q_1 + \bar{q}_2) \quad \text{s.t.} \quad x_1 + pq_1 = I_1$$
$$V^2(\bar{q}_1 \mid I_2) \equiv \max_{x_2,q_2} U(x_2, \bar{q}_1 + q_2) \quad \text{s.t.} \quad x_2 + pq_2 = I_2$$

- Slope of Nash reaction curves:

$$\frac{dq_1(q_2)}{dq_2} < -1, \quad \frac{dq_2(q_1)}{dq_1} < -1$$




- Nash equilibrium: intersection of Nash curves $NN_1$ and $NN_2$

- Pareto efficiency: tangency of iso-utility curves (by slope (3))

$$\frac{dq_2}{dq_1} = p\frac{U_x^1}{U_Q^1} - 1$$

$$\frac{dq_1}{dq_2} = p\frac{U_x^2}{U_Q^2} - 1$$

▷ Tangency on ($q_1$-$q_2$) plane:

$$p\frac{U_x^1}{U_Q^1} - 1 = \left[p\frac{U_x^2}{U_Q^2} - 1\right]^{-1} \tag{12}$$

▷ Hence the Samuelson FOC:

$$\text{MRS}_1 + \text{MRS}_2 \equiv \frac{U_Q^1}{U_x^1} + \frac{U_Q^2}{U_x^2} = p$$

- Under-provision of Nash:

$$Q^E = q_1 + q_2 < PP$$

- Interpretation: Prisoners' Dilemma

- Nash must exist, but may not be unique!

## 3. Free-riding

- Definition:

  1. Micro-level:

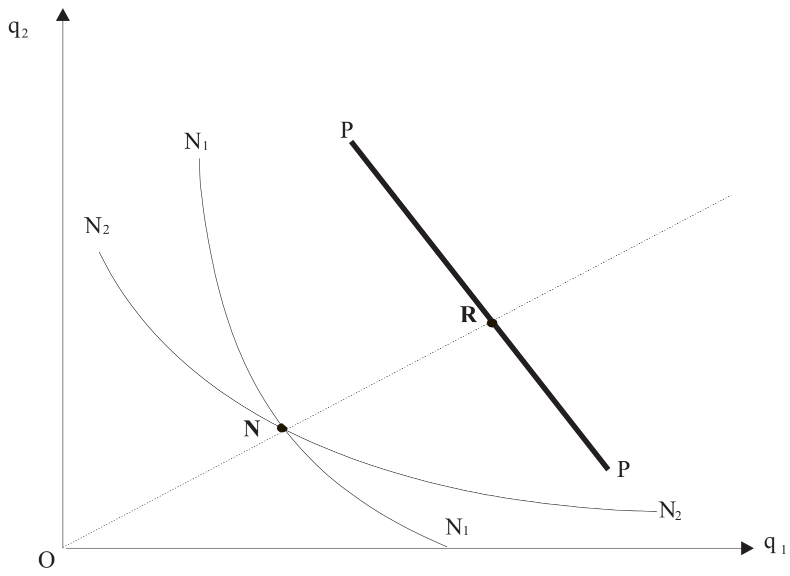
$$q'(\tilde{Q}) \; < \; 0$$

  2. Systemic level:

$$Q^E \; = \; q_1 + q_2 \; < \; PP$$

- Index/measurement:

$$\delta \; \equiv \; \frac{ON}{OR} \; \in \; (0, 1)$$




- $n$-player case: on $q$-$\tilde{Q}$ space

  – Symmetric solution: both Nash and PO have slope $(n-1)$:

$$q \; = \; \frac{Q}{n} \; = \; \frac{\tilde{Q}}{n-1}$$

  – Symmetric Nash: intersection of NN and ray of slope $(n-1)$

– Symmetric Pareto: tangency of EU and ray of slope $(n-1)$

[P] Since

$$p\frac{U_x}{U_Q} - 1 \;=\; n-1$$

we have the Samuelson foc:

$$p \;=\; n \cdot \text{MRS}^{Q,x} \quad \square$$

- Population effect: as $n \uparrow$

– Quasi-linear:

$$U(x,Q) \;=\; x + f(Q), \quad f' > 0, \quad f'' < 0$$

▷ $\delta$ goes down as $n \uparrow$

[P] Since $dQ/dI = 0$,

$$\frac{dq}{d\tilde{Q}} \;=\; p\frac{dQ}{dI} - 1 \;=\; -1$$

Nash $\hat{Q} = q + \tilde{Q}$ is fixed. However, Pareto $Q^*$ must satisfy

$$nf'(Q^*) \;=\; p$$

and hence will go up with $n$. ■

– Cobb-Douglas (or log-linear):

$$U(x,Q) \;=\; \log x \;+\; r\log Q$$

▷ $\delta$ goes down as $n \uparrow$, eventually approaching 0.

[P] Now Nash is:

$$\hat{Q} \;=\; \frac{r}{n+r}\sum_i I_i$$

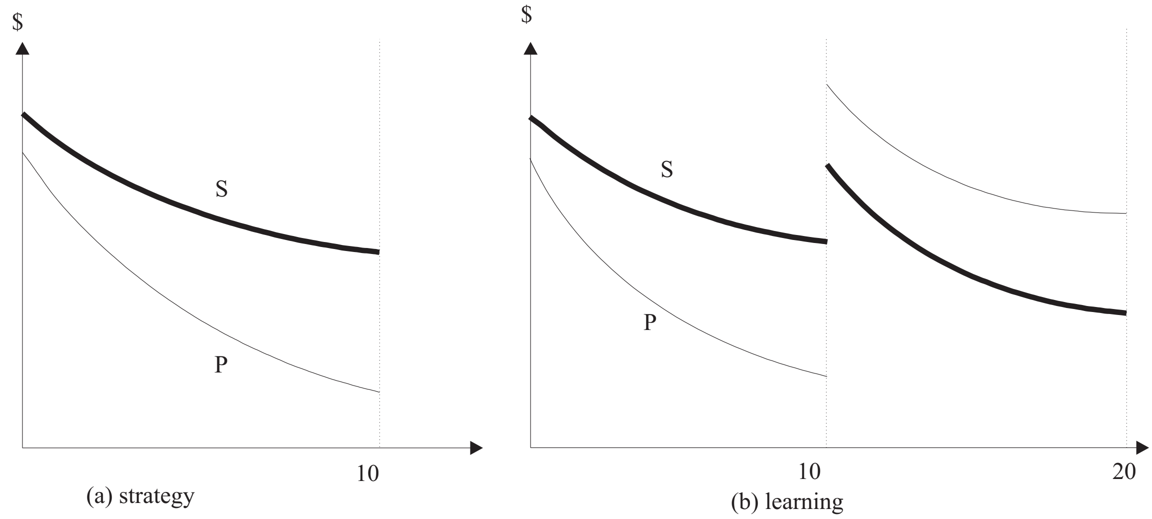
while Pareto is:

$$Q^* \;=\; \frac{r}{1+r}\sum_i I_i$$

So $\delta$ goes to 0 as $n \uparrow$. See Laffont, pp.39–41. ■

– Data testing: Haan-Kooreman [JPuE 2002/v83, pp. 277–91]
  ▷ Weak evidence that free riding increases with group size.

- Experiment design:
  – Andreoni [JPuE, 1988, 37:291–304]
  – Learning v. Strategy




  – Restart effect: Ambrus and Pathak [JPuE, 2011/v95, pp. 500–512]
    * Two player types: *selfish* v. *reciprocal*

## 4. The Neutrality/Invariance of Income Redistribution

- Bergstrom et al. [JPuE 1986]

$$U_i(x_i, G), \quad G = \sum_i g_i$$

- Original (unique) Nash equilibrium:

$$(x_1^*, \cdots, x_n^*, g_1^*, \cdots, g_n^*, G^*)$$

- Income redistribution among <u>contributors</u> ($g_i > 0$):

$$w_i' \;=\; w_i + \Delta w_i, \quad \sum_i \Delta w_i = 0$$

▷ Amount distributed not too big:

$$|\Delta w_i| \;\leq\; g_i^*, \;\text{ if }\; \Delta w_i < 0$$

$$\Delta w_i \;\leq\; G_{-i}^*, \;\text{ if }\; \Delta w_i > 0$$

- New (unique) Nash equilibrium: Fig. 2

$$(x_1', \ldots, x_n', g_1', \ldots, g_n', G')$$

with:

$$x_i' \;=\; x_i^*$$

$$g_i' \;=\; g_i^* + \Delta w_i$$

$$G' \;=\; G^*$$

$$U_i(x_i', G') \;=\; U_i(x_i^*, G^*)$$

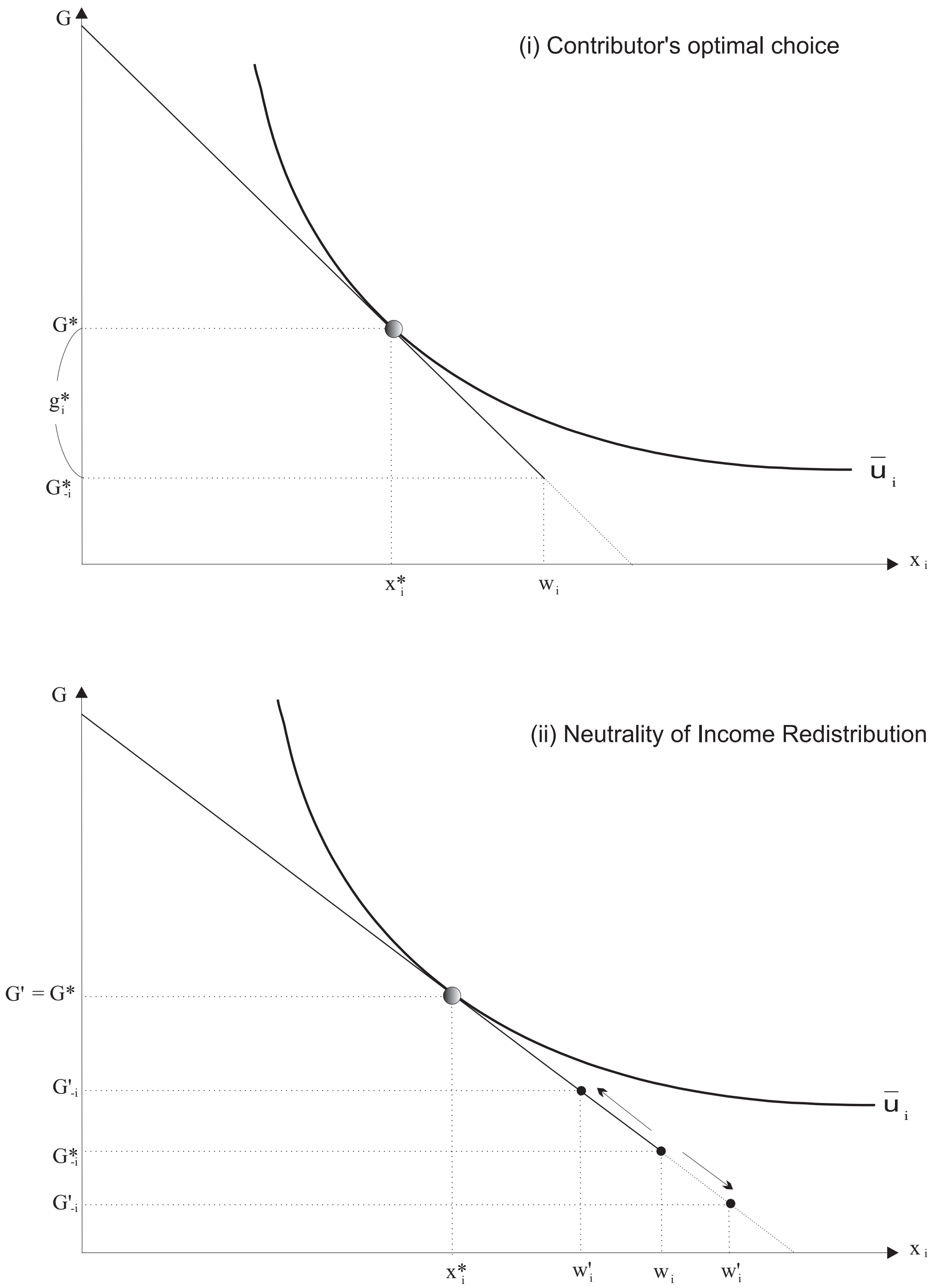



Figure 2: The Neutrality Hypothesis

# 5. Neutrality of Forced Contributions

## 5.1. Warr [JPuE 1982]

- Three consumers: rich are altruistic

$$U^1 = u^1(c^1, c^3), \;\; U^2 = u^2(c^2, c^3)$$
$$U^3 \; = \; u^3(c^3)$$

- Voluntary transfer:

$$c^i \; = \; I^i - v^i, \;\; i = 1, 2$$
$$c^3 \; = \; I^3 + [v^1 + v^2]$$

- Original Nash equilibrium:

$$\max_{v^1} \;\; u^1(c^1, c^3) \quad \text{s.t.} \quad c^1 = I^1 - v^1$$
$$\max_{v^2} \;\; u^2(c^2, c^3) \quad \text{s.t.} \quad c^2 = I^2 - v^2$$

▷ Interior FOC:

$$u^i_i(c^i, c^3) \; = \; u^i_3(c^i, c^3); \;\; i = 1, 2$$

- NE is not PO: with 1 and 2 both contributing \$1 more

$$du^i \; = \; 2u^i_3 - u^i_i \; > \; 0, \;\; i = 1, 2$$

▷ Pareto improvement (both better off)!

- Forced transfer by government $t$ ($t < v^1, t < v^2$):

$$\frac{dc^i}{dt} \; = \; \frac{d[I^i - v^i]}{dt} \; = \; -1 - \frac{dv^i}{dt}$$
$$\frac{dc^3}{dt} \; = \; 2 + \frac{dv^1}{dt} + \frac{dv^2}{dt}$$

- New transfer equilibrium:

$$\frac{dv^1}{dt} = \frac{dv^2}{dt} = -1$$

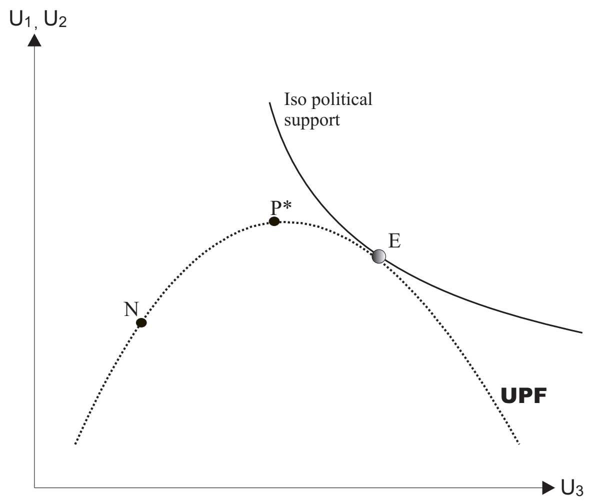
▷ Complete crowding-out, full offsetting

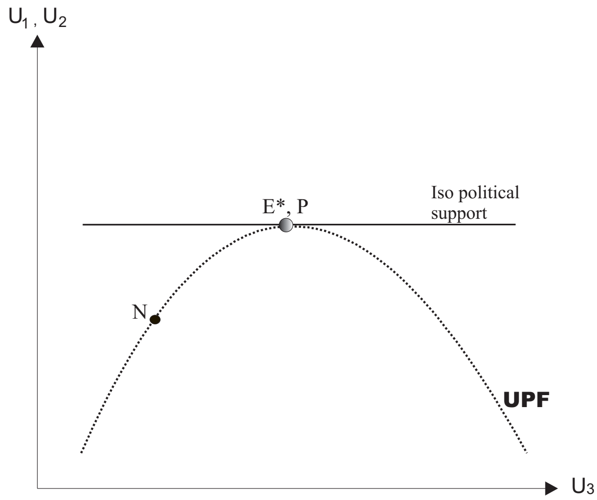
## 5.2. Political Support Model: Roberts [JPE 1984]

- Congress goal:

$$\max \quad P(u_1, u_2, u_3)$$

- Stylized facts




## 5.3. Data Testing: Andreoni-Payne [JPuE 2011/v95]

- Two possible effects of government grants to charity:
  - Classical crowding-out: donors reduce contributions
  - Fundraising crowding-out: charities reduce fundraising efforts

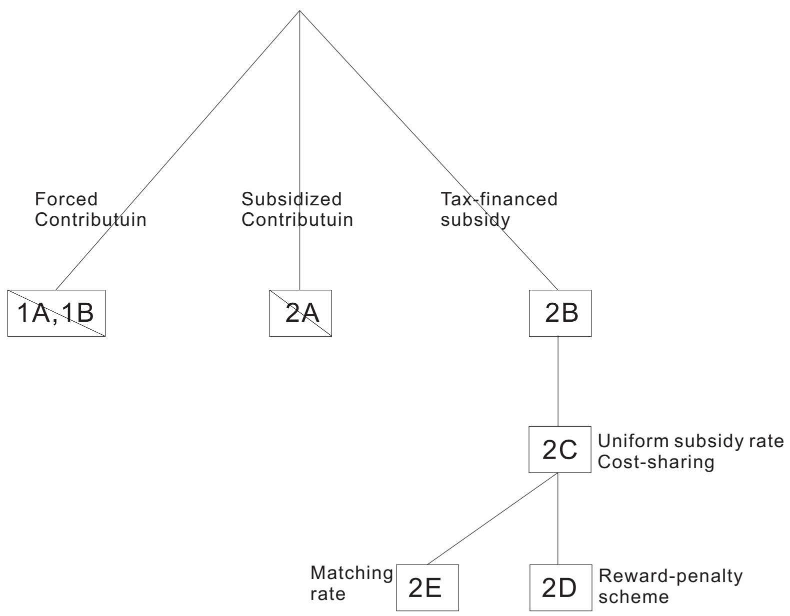
- Panel data of tax returns from 8,000 charities:
  - Classical crowding-out: 30% to slight crowding-in
  - Fundraising crowding-out: 70–100%
- Policy implication: requiring charities to match govt grants

## 6. Govt Financing/Subsidizing of PG Contribution

Notation:

- Consumer:
  - Wealth: $m_i$
  - Private consumption: $c_i$
  - Individual PG contribution: $g_i$
- Pre-tax Nash equilibrium:

$$(g_1^0, \cdots, g_n^0)$$



## 6.1. Forced Contributions

**1A** Lump-sum Head Tax: Warr [JPuE 1982]

- Govt policy: individual head tax $\tau_i$
- Total PG:

$$G = \sum_i g_i + \sum_i \tau_i$$

- Consumer budget:

$$c_i + g_i = m_i - \tau_i$$

- Full neutrality: unique Nash (if $\tau_i \leq g_i^0$)

$$c_i^* = c_i^0$$

$$g_i^* = g_i^0 - \tau_i$$

$$G^* = G^0$$

**1B** Labor Taxation: Bernheim [AER 1986]

- Consumer income: $m_i(l_i)$, $l_i \equiv$ individual labor supply
- Labor tax:

$$t_i(l), \quad l \equiv (l_1, \cdots, l_n)$$

- Total PG:

$$G = \sum_i g_i + \sum_i t_i(l)$$

- Consumer budget:

$$c_i + g_i = m_i(l_i) - t_i(l)$$

- Pre-tax Nash equilibrium:

$$(g_1^0, \cdots, g_n^0, l_1^0, \cdots, l_n^0)$$

$$G^0 = \sum_i g_i^0$$

- Full neutrality:

$$c_i^* = c_i^0$$

$$l_i^* = l_i^0$$

$$g_i^* = g_i^0 - t_i(l^0)$$

$$G^* = G^0$$

▷ Unique Nash: for any $t_i(l)$ with

$$t_i(l^0) \leq g_i^0$$

## 6.2. Subsidized Contributions

2A Individual Subsidy Rate and Head Tax: Andreoni [JPuE 1988, Sec 3.1]

- Govt policy:
  - Subsidy rate: $\beta_i \in [0,1]$
  - Fixed head tax: $\tau_i$
- Govt PG addition: residual revenues

$$T = \sum_i \tau_i - \sum_i \beta_i g_i$$

▷ Government budget balanced

▷ Total PG:

$$G = \sum_i g_i + T = \sum_i [1-\beta_i] g_i + \sum_i \tau_i$$

- Consumer budget:

$$c_i + [1-\beta_i] g_i = m_i - \tau_i$$

! For \$1 contribution, consumer can still only raise $G$ by 1

$$\mathrm{MRS}_i^{G,x} = 1$$

- Full neutrality:[1] unique Nash for any $\{\beta_i, \tau_i\}$ with $\tau_i \le g_i^0$

$$c_i^* = c_i^0, \quad g_i^* = \frac{g_i^0 - \tau_i}{1-\beta_i}$$

$$G^* = \sum_i g_i^* + T = G^0$$

[1]Note that the relevant part of $i$'s budget remains same if all other consumers act according to the offsetting rule. PG attributable to $i$ is $[1-\beta_i]g_i^* + \tau_i = g_i^0$.

## 2B Individual Subsidy Rate and Head Tax: A general framework

- *Tax-financed subsidy*:
  - Individual subsidy rate:
$$\beta_i$$
  - Linked head tax:
$$\tau_i$$
- Government budget balanced in equilibrium:
$$\{\beta_i, \tau_i\}, \quad \forall\, i$$

  such that:
$$\sum_i \beta_i g_i \;=\; \sum_i \tau_i$$

  ▷ Total PG:
$$G \;=\; \sum_i g_i$$

- New equilibrium:
  - Budget illusion: "myopic" consumers
    - ▷ Consumers take $(\beta_i, \tau_i)$ as fixed/given
  - Consumer budget:
$$c_i \;+\; [1-\beta_i]g_i \;=\; m_i - \tau_i$$
    - ▷ Consumer choice:
$$\text{MRS}_i \;=\; 1-\beta_i$$

- For Samuelson efficiency:

$$\sum_i \mathrm{MRS}_i \;=\; n - \sum_i \beta_i \;=\; 1$$

Hence:

$$\sum_i \beta_i \;=\; n - 1$$

$\boxed{\text{E}}$ Equal cost sharing:

$$\beta_i \;=\; \frac{n-1}{n}, \;\; 1 - \beta_i \;=\; \frac{1}{n} \quad \square$$

$\boxed{\text{N}}$ For \$1 contribution, consumer can raise $G$ by

$$\frac{1}{1-\beta_i}$$

## 2C Uniform Subsidy and Cost Sharing: Andreoni-Bergstrom [PC 1996]

- Govt policy: *tax-financed subsidy*
  - Uniform subsidy rate:
$$\beta \in [0,1]$$
  - Individual cost share:
$$s_i$$
  - Individual tax total:
$$s_i \beta G$$
- Govt budget balanced:
$$\sum_i s_i = 1$$
- Total PG:
$$G = \sum_i g_i$$
- Consumer budget:
$$c_i + [1-\beta]g_i = m_i - s_i \beta G$$

▷
$$c_i + (1-\beta[1-s_i])g_i = m_i - s_i \beta G_{-i}$$

N Special case of 2B:
$$\beta_i = \beta[1-s_i], \;\; \tau_i = s_i \beta G_{-i}$$

- Nash equilibrium: individual interior foc

$$\text{MRS}_i \;=\; 1 - \beta[1 - s_i]$$

- For Samuelson efficiency ( $\sum$ MRS $= 1$):

$$\sum_i \text{MRS}_i \;=\; n - n\beta + \beta \;=\; 1$$

▷ Just let

$$\beta \;=\; 1$$

Thm1 Unique Nash for any $\beta < 1$ and $\{s_i\}$

Thm2 $G$ rises with $\beta$.

▷ $\exists$ unique efficient $G$ with $\beta = 1$, but $\{g_i\}$ not unique.

## 2D Reward/Penalty Scheme: Falkinger [JPuE 1996]

- Govt subsidy:

$$r_i \;=\; \beta[g_i - \bar{g}_i] \gtreqless 0$$

where

$$\bar{g}_i \;\equiv\; \frac{G_{-i}}{n-1}$$

▷ Reward for contribution above average, penalty if below average.

- Govt budget always balanced, since $\sum_i r_i = 0$

- Total PG:

$$G \;=\; \sum_i g_i$$

- Consumer budget:

$$c_i \;+\; g_i \;=\; m_i \;+\; r_i$$

▷

$$c_i + [1-\beta]g_i \;=\; m_i - \beta\bar{g}_i \;=\; m_i - \frac{\beta G_{-i}}{n-1}$$

N Special case of 2C: with $\tilde{\beta}$ and $s_i = 1/n$

$$\begin{aligned} c_i + g_i &= m_i + \beta[g_i - \tfrac{G_{-i}}{n-1}] \\ &= m_i + \tfrac{n-1}{n}\tilde{\beta}[g_i - \tfrac{G_{-i}}{n-1}], \quad \tilde{\beta} \equiv \tfrac{n}{n-1}\beta \\ &= m_i + \tilde{\beta}g_i - \tfrac{\tilde{\beta}}{n}G \end{aligned}$$

- Individual interior foc:

$$\text{MRS}_i \;=\; 1 - \beta$$

▷

$$\sum_i \text{MRS}_i \;=\; n - n\beta$$

▷ For efficiency: let

$$\beta = \frac{n-1}{n}$$

Thm1 If $0 < \beta < \frac{n-1}{n}$, then $\exists$ unique Nash, and $G^*$ increases with $\beta$.

Thm2 If $\beta = \frac{n-1}{n}$, then equilibrium not unique.

- Efficiency can be approximated with

$$\beta \to \frac{n-1}{n}$$

- Intuition: incentives for simultaneous cooperation in PD game.

## 2E Individual Subsidy and Tax Rates: *tax-financed subsidy*

- Govt policy:
  - Individual subsidy rate:

$$\beta_i$$

  - Linked matching rate:

$$t_i$$

- Consumer budget:

$$c_i + [1 - \beta_i]g_i \ = \ m_i - t_i G_{-i}$$

N Special case of 2B

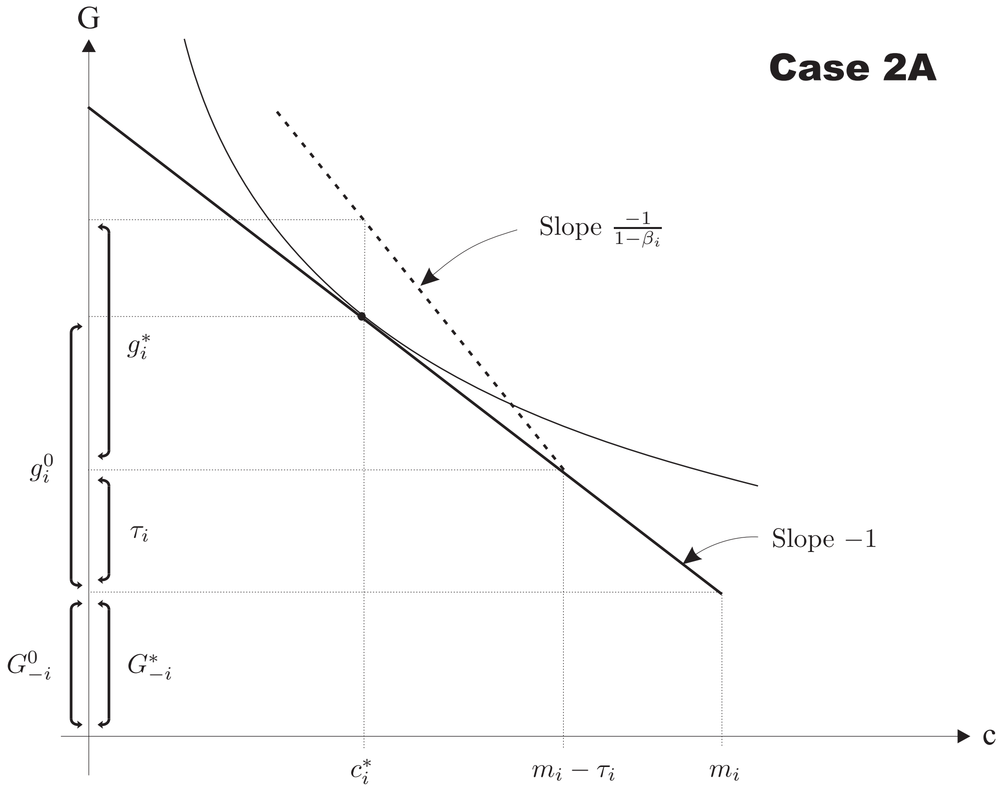
E Guttman [AER 1978]

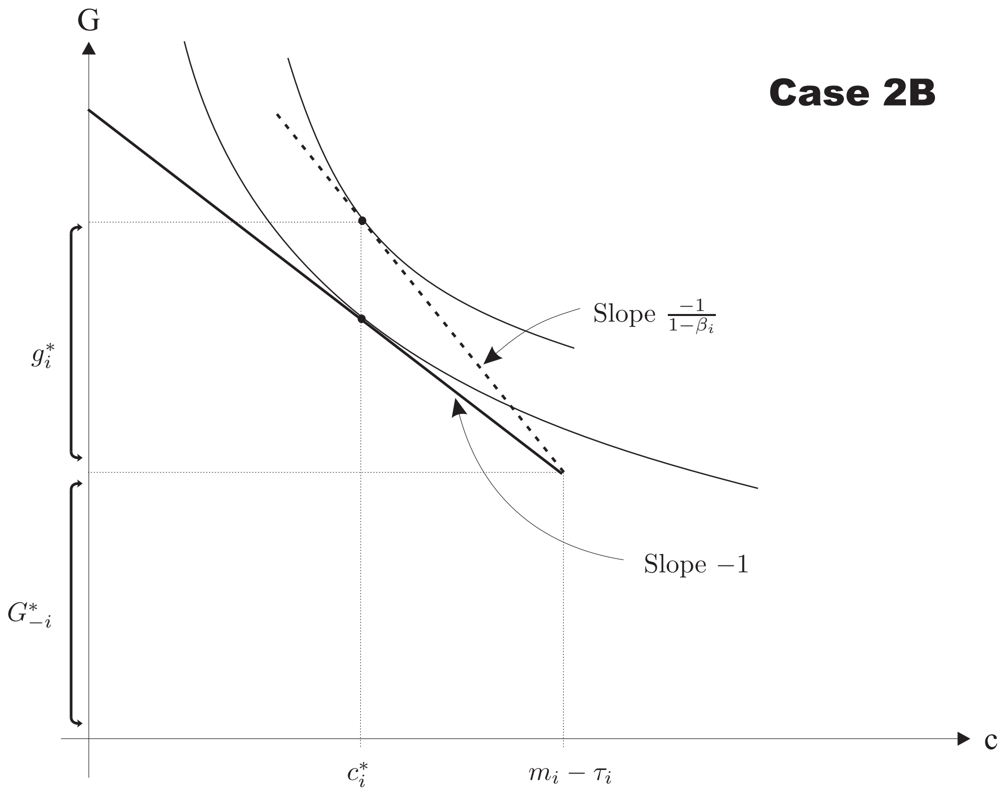
- Govt budget balance in equilibrium:

$$\sum_i \beta_i g_i \ = \ \sum_i t_i G_{-i}$$

▷

$$\sum_i [\beta_i + t_i] g_i \ = \ \left[ \sum_i t_i \right] G$$


G
Case 2A
Slope $\frac{-1}{1-\beta_i}$
$g_i^*$
$g_i^0$
$\tau_i$
Slope $-1$
$G_{-i}^0$
$G_{-i}^*$
c
$c_i^*$
$m_i - \tau_i$
$m_i$


G
Case 2B
Slope $\frac{-1}{1-\beta_i}$
$g_i^*$
Slope $-1$
$G_{-i}^*$
c
$c_i^*$
$m_i - \tau_i$

# 7. The Rotten Kid Theorem (RKT)

## 7.1. Becker [1974, 1981]

- Household:
  - Head: wealth $I_H$, consumption $x_H$
  - Kids $i$ $(= 1, \ldots, n)$: income $I_i$, consumption $x_i$

- Head's goal: max HH welfare with *non-negative* transfers

$$\max_{\{t_i\}} \quad U_H(x_1, \cdots, x_n, x_H) \quad \text{s.t.} \quad \begin{cases} x_H + \sum_{i=1}^n t_i = I_H \\ x_i = t_i + I_i \\ t_i \ \geq \ 0, \ \forall\, i \end{cases}$$

- Assuming *interior* solution $(t_i > 0)$:

$$\max_{\{x_i\}} \quad U_H(x_1, \cdots, x_n, x_H) \quad \text{s.t.} \quad x_H + \sum x_i = I_H + \sum I_i$$

▷ *Social income*:

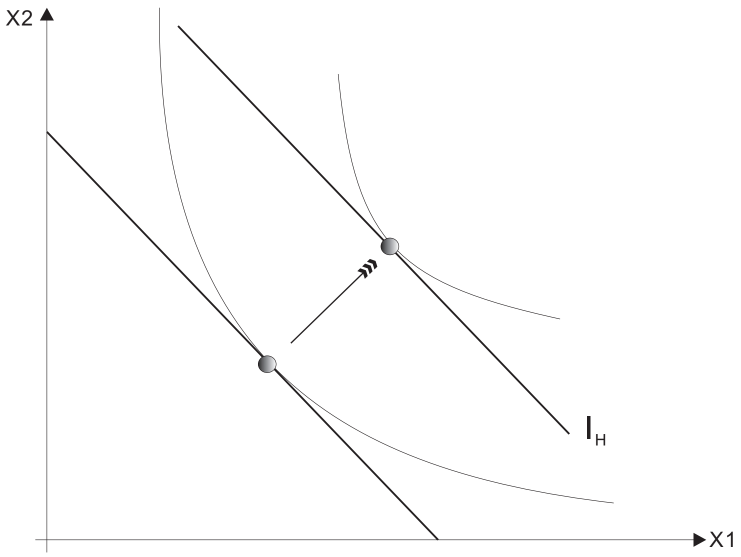
$$I_H \ + \ \sum_i I_i$$

- HH interior foc:

$$\frac{\partial U_H}{\partial x_i} \ = \ \frac{\partial U_H}{\partial x_j}; \ \ \forall\, i, j$$

- Kids will be *well-behaved*:
  - Will maximize HH wealth ($I_H + \sum I_i$) for higher own $x_i$
  - Will not steal from parents/siblings

E Concubines, party members



## 7.2. Bergstrom [1989]

Counter Examples

- Example #1: Lazy rotten kids

Kids: work $y_i$, leisure $[1 - y_i]$

$$U_i \;=\; x_i[1 - y_i]$$

Head:

$$\max_{x_1, x_2} \; U(x_1, x_2 \,|\, y_1, y_2) \;\equiv\; \sqrt{U_1} + \sqrt{U_2} \;=\; \sqrt{x_1[1 - y_1]} + \sqrt{x_2[1 - y_2]}$$

$$\text{s.t. } \; x_1 + x_2 \;=\; I_0 \;+\; w[y_1 + y_2]$$

foc:

$$\frac{x_1}{x_2} \;=\; \frac{1 - y_1}{1 - y_2}$$

▷ No kid will want to work!

- Example #2: Night light (Becker [1974, p.1078])

Husband: income $I$, night reading hours $y$

$$u_h \;=\; x_h[y + 1]$$

Wife:

$$u_w \;=\; x_w \cdot e^{-y}$$

Husband's goal:

$$\max \;\; U = u_h \cdot u_w^a \quad \text{s.t.} \quad x_h + x_w = I$$

hence:

$$x_h^* = \frac{I}{1+a}, \quad x_w^* = \frac{aI}{1+a}, \quad y^* = \frac{1}{a} - 1$$

▷ Wife should cut the wire!

- Example #3: Prodigal son (Lindbeck-Weibull [JPE 1988])

  2-period: $t = 1, 2$

  Kid:

$$U_k = C_k^1 \cdot C_k^2$$

Head: max HH utility

$$U_H = [C_p^1 C_p^2] \cdot [U_k]^\alpha = [C_p^1 C_p^2][C_k^1 C_k^2]^\alpha$$

FOC:

$$\frac{C_p^2}{C_k^2} = \frac{1}{\alpha}$$

▷ Kid consumes too much in period 1.[2]

[2] This is the Samaritan's Dilemma.

## The 2-stage game $\Gamma$ Rotten Kids Play

- $n$ kids: individual action $a_i$ ($\in A_i$)

$$a \equiv (a_1, a_2, \ldots, a_n)$$

- Consumption of kid $i$:

$$m_i(a)$$

- HH budget:

$$M(a) \equiv \sum_i m_i(a)$$

- Kid utility:

$$u_i(a, m_i)$$

- Two stages:
  - $\boxed{\text{S1}}$ Kids choose $a_i$ first
  - $\boxed{\text{S2}}$ Head makes transfer

$$\max_{\{m_i\}} \; U(u_1, u_2, \ldots, u_n \,|\, a)$$

## Analysis

- RKT holds if SPE of the game $\Gamma$ accords with head's goal:

$$\max_{\{a_i, m_i\}} \; U(u_1(a, m_1), \ldots, u_n(a, m_n))$$

▷ Kids are "well-behaved"

- The iff condition:
  - Gorman utility function:

$$u_i(m_i, a) \; = \; A(a)m_i + B_i(a), \;\; \forall i$$

  - Utility possibility set (UPS): UP$(a)$ is a *simplex*

$$\sum_i u_i \; = \; A(a)M(a) + \sum_i B_i(a) \; = \; K(a)$$

▷ Transferable utility

- Examples revisited:
  - Lazy rotten kids:

$$U_i(m_i, y) \; = \; m_i + B_i(y_i)$$

▷

$$\text{UP}(y_1, \ldots, y_n) \; = \; \sum U_i \; = \; M(y_1, y_2, \ldots, y_n) + \sum_i B_i(y_i)$$

  - Night light:

$$u_h(m_h, y) \; = \; A(y)m_h + B_h(y)$$

$$u_w(m_w, y) \; = \; A(y)m_w + B_w(y)$$

▷

$$\text{UP}(y) \; = \; u_h + u_w \; = \; A(y)I + [B_h(y) + B_w(y)]$$

– Prodigal son:

$$U_i(C_i^1, C_i^2) \;=\; C_i^2 + B_i(C_i^1), \;\; \forall\, i$$

▷

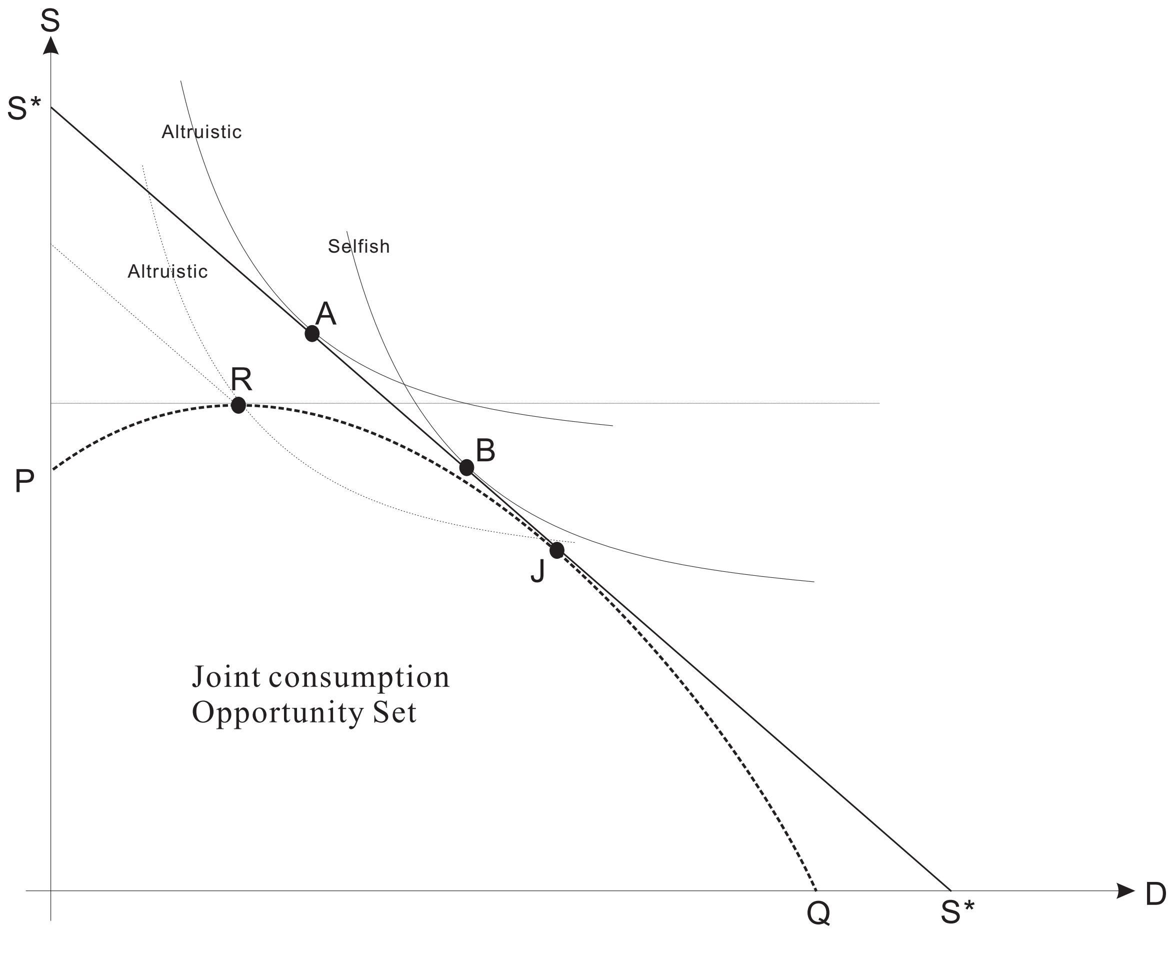
$$\text{UPF} \;=\; \sum_i C_i^2 + \sum_i B_i(C_i^1) \;=\; \left[I - \sum_i C_i^1\right] + \sum_i B_i(C_i^1)$$

## 7.3. Bruce-Waldman [QJE 1990]

- 2-stage consumption-saving model
- Timing of transfer
- Samaritan's Dilemma: Buchanan 1975

## 7.4. Implication#1: Altruism in Evolution

- Biology: group rationality, genetic fitness, kin selection
- Becker: individual rationality[3]
- Hirshleifer [JEL 1977]




- Khalil [JEBO 2004, 53:89–92]
  - Prosocial preferences:

[3] "Altruism, Egoism, and Genetic Fitness: Economics and Sociology," *JEL*, G. Becker, 1976, 14(3):817–26.

    * Cooperation: for self-image[4]

    * Altruism: for other-interest[5]

  – A group may not be better off with more altruists

    [E] Costly mutual help

    [E] Private taste/benefit info

## 7.5. Implication#2: United Way Donations

- UW will offset individual donation [Bilodeau, JPuE 1992]
  ▷ You might as well give money to UW

- Contribution earmarking [Bilodeau-Slivinski, JPuE 1997]
  ▷ Donor designations
  ▷ Not to discourage individual donotions.

[4]不好意思不做: 給小費, 讓座, 婚宴送禮, 履行交易承諾。
[5]甘心樂意: 給乞丐錢, 匿名捐獻, 服務志工。

# 8. Government by Jury

## 8.1. The Logic

- Voting paradox: why do people vote?

$$\text{Voting Costs} \;>\; \text{Expected Voting Benefits}$$

▷ $D$-term solution:

$$\text{Voting Costs} \;<\; \text{Expected Voting Benefits} + D$$

- Votes not intelligent?

  ▷ PG argument

- Similar PG examples: providing public services
  - Congressman
  - Court judge

- Government by Jury: Bergstrom-Varian [1987]
  - Small congress
  - Random selection by computer
    * Statistical (demographic) advantage
    * Minimize socisl costs: election, gathering
    * Argument against democracy v. GJ

## 8.2. Base Model: Constant Marginal Cost, Equal Benefit

- Population: $m$

  Congress size: $n$

- Congressman:
  - Effort:
  $$e_i$$
  - Constant MC:
  $$c$$
  - Total congress efforts:
  $$E = \sum e_i$$

- Citizen benefits:
$$B(E); \;\; B' > 0, \; B'' < 0$$

- Optimality:
$$\max_E \;\; \pi \equiv \; mB(E) - cE$$

  foc:
$$mB'(E^*) = c$$

## 8.3. Nash Equilibrium

- Individual foc:

$$B'(\hat{E}) = c$$

  - Any combination of $e_i$ for $\hat{E}$ will do
  - Symmetric solution:

$$\hat{e} = \frac{\hat{E}}{n}$$

$$B'(n \cdot \hat{e}) = c$$

- Total Nash effort is independent of $n$:[6]

$$\hat{e}(n) = \frac{\hat{e}(1)}{n}$$

  - Severe free-riding
  - To min TSC, should have few congressmen, picked by lottery
    - ▷ Lack of diversity

[6] Because

$$B'(n \cdot \hat{e}(n)) = B'(n \cdot \frac{\hat{e}(1)}{n}) = B'(\hat{e}(1)) = c$$

### 8.4. Modification#1: Different Benefit $B_i(E)$

- Nash: only one member $k$ makes sole effort $\hat{E}$:

$$B_k'(\hat{E}) \;=\; c$$
$$B_i'(\hat{E}) \;\leq\; c,\ \forall i \neq k$$

- Free-riding more severe
  - Only the highest $B_k$ individual makes sole effort $e_k = \hat{E}$
  - All others make no efforts $e_i = 0$

### 8.5. Modification#2: Increasing Marginal Costs

- Effort cost:

$$C(e_i),\ C' > 0,\ C'' > 0$$

- Symmetric Nash:

$$B'(n \cdot e(n)) \;=\; C'(e(n))$$

- Congress size effect:

$$\frac{de}{dn} \;=\; \frac{-eB''}{nB'' - C''} \;=\; \frac{-e}{n - \frac{C''}{B''}} \;<\; 0$$
$$\frac{dE}{dn} \;=\; \frac{d[n \cdot e(n)]}{dn} \;=\; \frac{e}{1 - n \cdot \frac{B''}{C''}} \;>\; 0$$

- Welfare effect:

$$\pi(n) \;=\; m \cdot B(n \cdot e(n)) - n \cdot C(e(n))$$
$$\left.\frac{d\pi(n)}{dn}\right|_{(m,n)} \;=\; m \cdot B'(E)E'(n) - [C(e) + nC'(e)e'(n)] \;\gtrless\; 0$$

$\triangleright$ Positive if $m$ is large enough, justifying large congress