Public Choice

1. Introduction

- Social/public choice: the process of social/collective decision-making

- Elements:

  √ Candidates/alternatives/options: choice set A

  √ Voters: i

  √ Individual preference/ranking:

$$\{R_i\}$$

- Preference aggrgation mechanism:

  – Social decision rule: collective ranking R of all alternatives

    – Aggregation of individual preference $\{R_i\}$

    – Process: Indv Ranking $\{R_i\}$ in, Social Ranking R out

E Beauty contest, sports event

– Social choice function (SCF): a single choice

$$a \in A$$

– Process: Indv Ranking $\{R_i\}$ in, Social Choice a out

E Political election, travel destination choice

- Saari [1988] story: choice of drinks in department meeting

15 voters	1st	2nd	3rd
6	Milk	Juice	Beer
5	Beer	Juice	Milk
4	Juice	Beer	Milk

– "Milk" chosen initially as most favored (M6 : B5 : J4)

– "Beer" served in meeting for lack of Milk

– But people found "Juice" (10) is actually preferred to "Beer" (5)

– Further: "Milk" least favored by pairwise comparision:

$$J9 : M6, \quad B9 : M6$$

2. Unanimity rule

- Wicksell [1896]
- Consistent with Pareto criterion

  ▷ Bill passed must make everyone better off!

- Problems:

  √ (Theory) Social ranking not "complete". Agreement rarely reached.

  √ (Reality) Distribution/jealousy issue not considered.

  ▷ Some may prefer non-Paretian situation.

  √ (Reality) Everyone has veto power, transaction costs high

  ▷ Outcome subject to negotiation and strategic behaviors.

- Unanimity with compensation/side-payment

  ▷ Buying votes is illegal?

3. Majority Voting

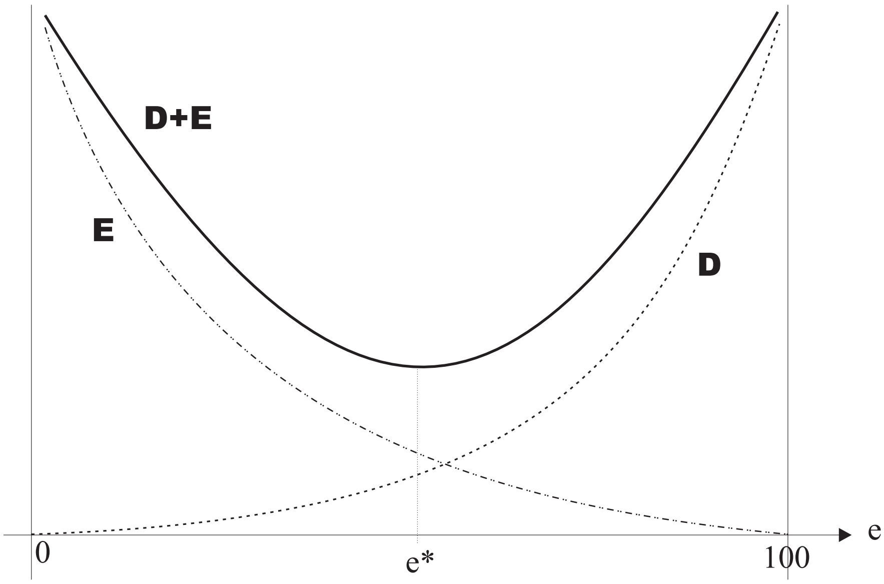
- Relative majority: $\eta\%$ ($\geq$ 50%) for agreement

- Constitutional choice: [Buchanan-Tullock 1962][1]

$$\min_{\eta} \quad \text{ETSC} \equiv D + E$$

√ External costs (外部成本) E: damages imposed on minority

√ Decision costs (交易成本) D: costs for reaching decisions



- Condorcet winner: pairwise comparision
  - Binary agenda for 2 or more options
  - The winner against any other candidate

[1] J.M. Buchanan and G. Tullock, Chapter 6 in *The Calculus of Consent – Logical Foundations of Constitutional Democracy*, 1962, University of Michigan Press.

- **Plurality rule**: simultaneous voting[2]
  - For 3 or more candidates.
  - Condorcet winner may lose:

(9 voters)	1st	2nd	3rd
2	A	B	C
3	B	A	C
4	C	A	B

▷ C is the Plurality winner; A is Condorcet winner

  - Strategic behavior[3]

- **May's Theorem**: with only 2 candidates[4]

Only majority rule can satisfy the following:

√ **Anonymity**: symmetry among all voters (treated equally)

√ **Neutrality**: symmetry among all candidates

√ **Decisiveness**: a winner will always be picked

√ **Positive responsiveness**: more votes, more likely to win ■

[2] Hindriks-Myles, 2006, MIT press, p.319.
[3] For example, people may vote for 2nd choice, if they feel their top choice has no chance to win.
[4] Hindriks-Myles, 2006, MIT press, p.306.

- Voting paradox [Condorcet 1785]:

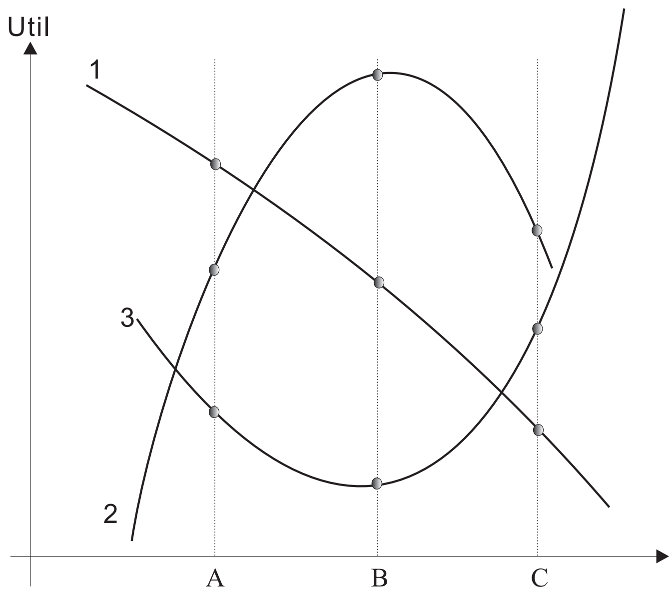
Ranking	1st	2nd	3rd
Voter 1	A	B	C
Voter 2	B	C	A
Voter 3	C	A	B

  – Voting cycles:

$$A \succ_{1,3} B \succ_{1,2} C \succ_{2,3} A$$

  ▷ Outcome subject to "agenda manipulation"

  – Single-peaked preferences (單峰偏好) [Black]: 1-dim choice



– Single-crossing preferences (SC):[5]

∗ Def: On a 1-dim line, for 2 voters $a < b$, and 2 options $x < y$:

$$U^a(y) > U^a(x) \quad \Rightarrow \quad U^b(y) > U^b(x)$$

and

$$U^b(x) > U^b(y) \quad \Rightarrow \quad U^a(x) > U^a(y) \quad \Box$$

∗ If voter preferences satisfy SC, then there is no cycle.

∗ Condorcet winner is preferred option of the median voter M.[6]

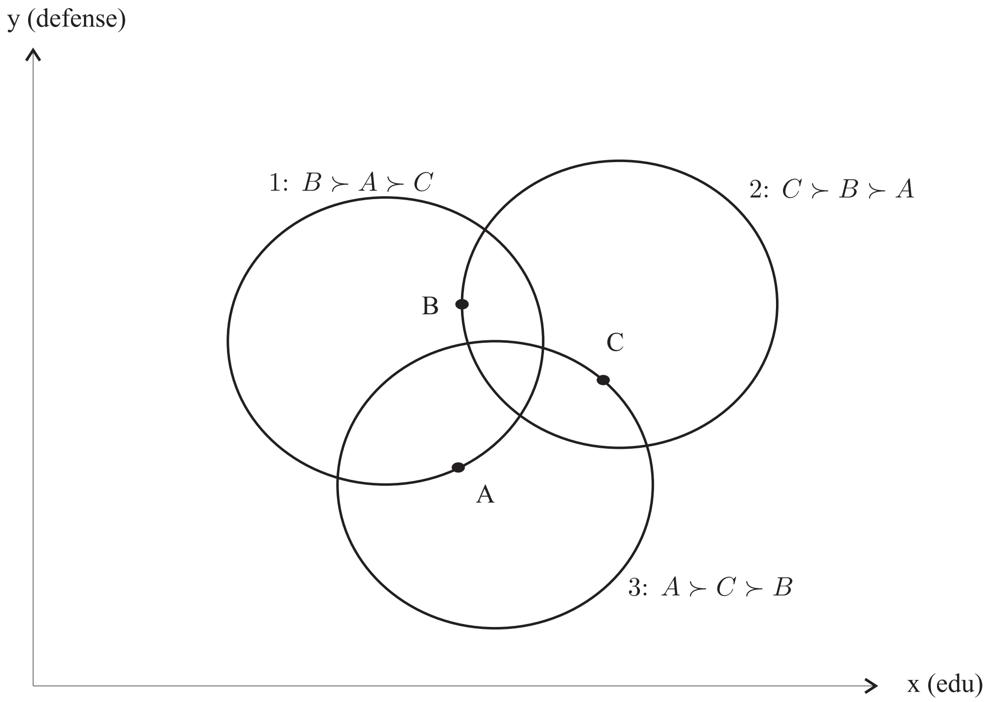
– Cycle probability 1-2%; not detectable when it arises!

[5]Hindriks-Myles, 2006, MIT, pp.310.

[6]Because, for any 2 options $x < y$, if M prefers x, then all voters to his left will also prefer x. If M prefers y, then all voters to his right must also prefer y. $\Box$

– 2-dim voting cycle

$$A \succ_{1,3} C \succ_{2,3} B \succ_{1,2} A$$



E 3 people dividing \$1: no Condorcet winner!

Round	A	B	C
1	1/3	1/3	1/3
2	1/2	1/2	0
3	2/3	0	1/3
4	0	1/2	1/2
...	...	...	...

E Bundled voting: no Condorcet winner!

Voter value	A	B	C
1	500	−100	−100
2	−100	500	−100
3	−100	−100	500

▷ Cycle: (n,n,n) → (y,y,y) → (y,y,n) → (n,y,n) → (n,n,n)[7]

- Independence from Irrelevant Alternatives (IIA) may be violated

E Example:

#voters / ranking	1st	2nd	3rd
9	A	B	C
4	B	C	A
6	C	B	A

– With all 3 candidates: (A9 : B4 : C6) ⇒ A elected

– If C drops out: (A9 : B10) ⇒ B elected

[7] Any proposal changing a "y" to "n" will pass with two votes. But then (n,n,n) will be defeated by a proposal replacing any two "y" with two "n".

– Need IIA to avoid sabotage (攪局)[8]

- Outcome may be Pareto inferior!

Ranking	1st	2nd	3rd	4th	5th	6th	7th
Voter 1	A	B	C	D	E	F	G
Voter 2	C	D	A	F	G	B	E
Voter 3	D	A	G	B	C	E	F

▷ Possible outcome: $A \to D \to C \to B \to G \to F \to E$

▷ E is Pareto inferior to (A, B, C, D) !

- Voter preference intensity not considered:

▷ Logrolling (選票互換): vote trading/exchange

– (Yes) Voter intensity revealed: compromise means efficiency!

(Project)	A	B	C	NetValue	M.V.	logrolling
Hospital	200	-50	-55	95	n	y (1,2)
Library	-40	150	-30	80	n	y (1,2), (2,3)
Park	-120	-60	400	220	n	y (2,3)

– (No) Special-interest gains may outweight general losses!

[8]For example: Taipei city mayor election 1998, Presidential election 2000.

(Project)	A	B	C	NetValue	M.V.	logrolling
Hospital	200	-110	-105	-15	n	y (1,2)
Library	-40	150	-120	-10	n	y (1,2), (2,3)
Park	-270	-140	400	-10	n	y (2,3)

- 64% mojority rule [Caplin-Nalibuff, Econometrica 1988]
  - In k-dim elections, incumbent can garantee only: Figure 1

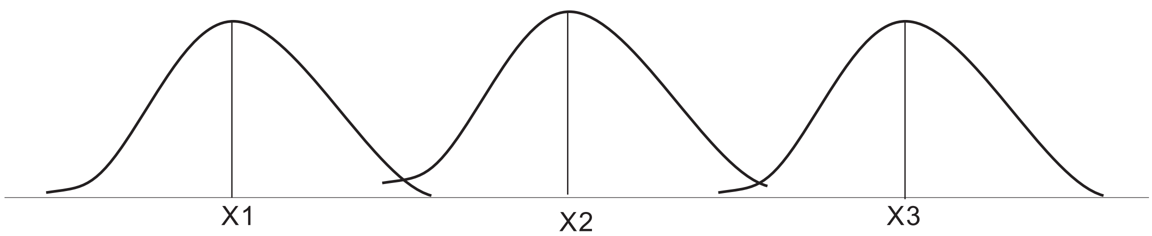
$$\sigma_k = \left(\frac{k}{k+1}\right)^k$$

▷ For example: $\sigma_1 = 1/2$, $\sigma_2 = 4/9$

  - In real-life elections, a challenger will get at least:

$$\sigma_\infty = \lim_{k\to\infty}\left[1-\left(\frac{k}{k+1}\right)^k\right] = 1-\frac{1}{e} \approx 64\% \quad \square$$

- Median Voter Theorem (中值選民定理)[9]
  - M.V. outcome reflects preference of the median voter:



  - X_2 is Condorcet winner (by pairwise comparison)
  - Outcome usually inefficient

[9] Holcombe pp.175–76; Hyman p.165.

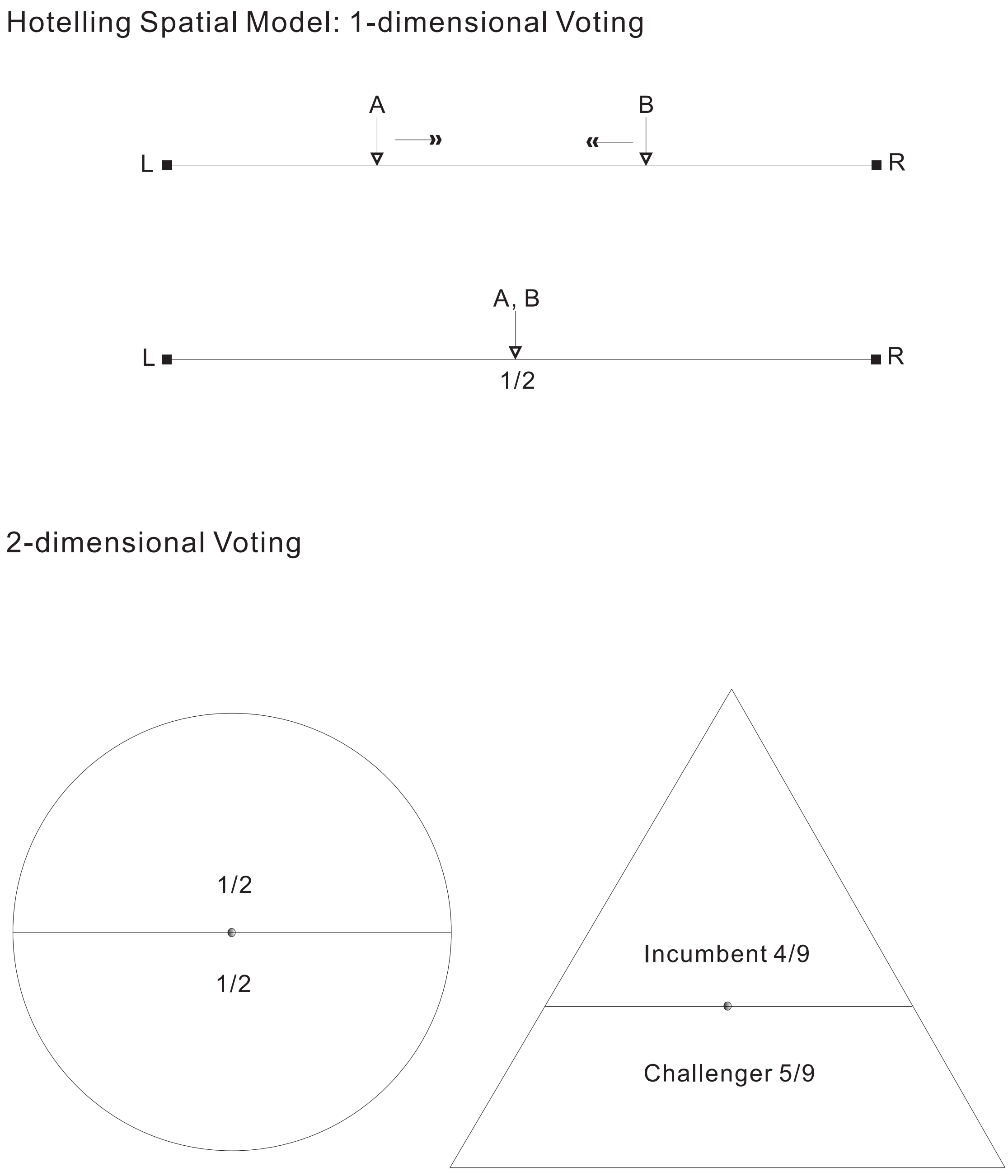


Figure 1: Justification for 2/3 majority rule

4. Borda Count

- Counting pocedure: choose one with lowest count ⇒ no cycles

#voters	Keynes	Becker	Chair
10 Macro	1	2	3
10 Micro	2	1	3
1 Chair	2	3	1
Rank / Score	1(32)	2(33)	3(61)

▷ May set rank values to reflect relative weights (eg, 1,2,3,10,...)

- Problem: Strategic manipulation

E 10 Micros now claim [Chair as 2nd, Keynes as 3rd]

#voters	Keynes	Becker	Chair
10 Macro	1	2	3
10 Micro	3	1	2
1 Chair	2	3	1
Rank / Score	2(42)	1(33)	3(51)

- Problem: IIA violated, different outcomes w/w.o. chair

#voters	Keynes	Becker
10 Macro	1	2
10 Micro	2	1
1 Chair	1	2
Rank / Score	1(31)	2(32)

5. Approval Voting

- Can vote for any number of alternatives, each vote counts as 1.

- Voter flexibility.

- Outcome indeterminacy:

#voters / ranking	1st	2nd	3rd
6	x	z	y
5	y	z	x
4	z	y	x

  - x wins: if everyone votes only for 1st choice (x6 : y5 : z4)
  - y wins: if group 3 votes for top 2 choices (x6 : y9 : z4)
  - z wins: if everyone votes for top 2 choices (x6 : y9 : z15)

  ▷ Condorcet winner may not be picked.

6. Runoff Voting

- Top 2 winners in Round 1 will enter Round 2.

- Condorcet winner may not win.

- Positive Responsiveness may be violated.

Count	1st	2nd	3rd
6	a	b	c
5	c	a	b
4	b	c	a
2	b	a	c

7. Elimination

- Everyone votes for the candidate you dislike most.

  ▷ The candidate who receives least votes get elected.

- May have cycle.

- IIA violated.

Count	1st	2nd	3rd	4th
9	A	B	C	D
4	B	C	D	A
6	C	D	A	B
5	D	A	B	C

  - 4 candidates: (A4 : B6 : C5 : D9) $\Rightarrow$ A elected.

  - If B withdraws: (A10 : C5 : D9) $\Rightarrow$ C elected.

8. Collective Choice Depends on Voting Mechanism

E 7 voters, 4 alternatives:

V1	V2	V3	V4	V5	V6	V7
A	A	A	B	B	C	C
B	B	B	C	C	D	D
C	C	C	D	D	A	A
D	D	D	A	A	B	B

- Plurality rule: A*(3) : B(2) : C(2) : D(0)
- Borda count: A(17) : B(16) : C*(15) : D(22)
- Approval (2 votes): A(3) : B*(5) : C(4) : D(2)
- Pairwise comparision: cycle, no Condorcet winner

$$A \succ_{5:2} B \succ_{5:2} C \succ_{7:0} D \succ_{4:3} A$$

9. Arrow's Impossibility Theorem [1951]

- Axiomatic approach
- No social decision rule can guarantee satisfaction of the following:
  - Universality (全域性): voters may have any preference patterns.
  - Consistency (一致性): social preference is transitive, no cycle.

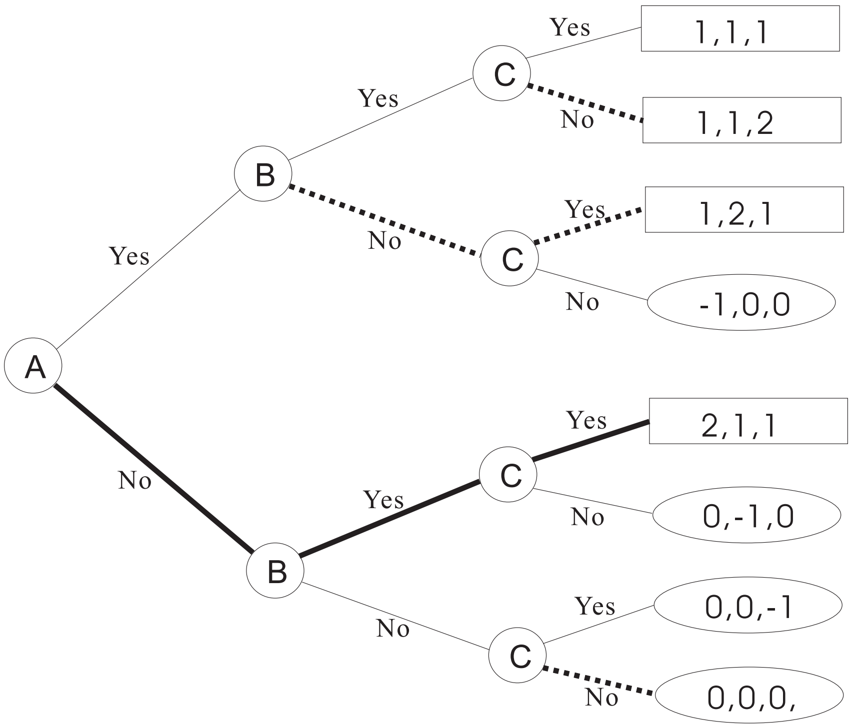
  - Pareto axiom
  - IIA (independence of irrelevant alternatives)
  - Non-dictatorship

- Satherswaite Theorem: strategy-proofness required (instead of IIA)

10. Application: Congress Voting on Own Pay Raise

Payoff	Bill "pass"	Bill "fail"
Vote "yes"	1	-1
Vote "no"	2	0

Congress pay-raise voting:



11. Application: Tie-breaking Power [Farquharson 1969, p.50]

▷ Vote by majority rule, voter 1 can break tie.

Voter	1st	2nd	3rd
1	A	C	B
2	B	A	C
3	C	B	A

Figure 2

Pay-offs:

(3=A)

1/2	A	B	C
A	A	A	A
B	A	B	B*
C	A	C*	C

(3=B)

1/2	A	B	C
A	A	B	A*
B	B	B	B
C	C*	B	C

(3=C)

1/2	A	B	C
A	A	A*	C
B	B*	B	C
C	C	C	C

Elimination of dominated strategies (Round 1):

(3=A)

1/2	A	B	~~C~~
A	A	A	~~A~~
~~B~~	~~A~~	~~B~~	~~B*~~
~~C~~	~~A~~	~~C*~~	~~C~~

(3=B)

1/2	A	B	~~C~~
A	A	B	~~A*~~
~~B~~	~~B~~	~~B~~	~~B~~
~~C~~	~~C*~~	~~B~~	~~C~~

(3=C)

1/2	A	B	~~C~~
A	A	A*	~~C~~
~~B~~	~~B*~~	~~B~~	~~C~~
~~C~~	~~C~~	~~C~~	~~C~~

Elimination of dominated strategies (Round 2):

(3=A)

1/2	~~A~~	B
A	~~A~~	A

(3=B)

1/2	~~A~~	B
A	~~A~~	(B)

(3=C)

1/2	~~A~~	B
A	~~A~~	A*

Equilibrium outcome: B (1 for A, 2 for B, 3 for B), 1 gets worst!

Figure 2: Tie-breaking power may hurt you!