台 灣 大 學 經 濟 學 研 究 所

公 共 經 濟 學 期 中 考 試

假設兩消費者 $(1,2)$ 均有所得

$$I_1 \ = \ I_2 \ = \ 300$$

且兩人之效用函數各爲:

$$U_1(x_1, G) \ = \ x_1 \cdot G$$

$$U_2(x_2, G) \ = \ x_2 \cdot G^2$$

其中 x_i 爲 i 之私有消費, G 爲兩人共享之公共財, 且兩商品之價格均爲 1。

1. (20%) 請解此兩人團體之最適公共財數量 G^*。

   (Ans) Note that:

   $$\text{MRS}_1^{G,x} \ = \ \frac{x_1}{G}, \ \ \text{MRS}_2^{G,x} \ = \ \frac{2x_2}{G},$$

   so Samuelson foc requires:

   $$G \ = \ x_1 \ + \ 2x_2$$

   In addition, we have the aggregate budget constraint:

   $$x_1 \ + \ x_2 \ + \ G \ = \ 600$$

   Any combination (x_1^*, x_2^*, G^*) satisfying the above two equations will do. □

2. (20%) 若此公共財是由兩人自發性捐獻所產生, 且令 g_i 爲 i 之捐獻量, 則同時捐獻之 Nash 均衡下公共財總量

   $$\hat{G} \ = \ g_1 \ + \ g_2$$

   將是多少?

   (Ans) In an interior equilibrium, we must have:

   $$\text{MRS}_1^{G,x} \ = \ \text{MRS}_2^{G,x} \ = \ 1,$$

which yields:

$$x_1 \;=\; 2x_2 \;=\; g_1 + g_2$$

Also we know the individual budget constraints:

$$x_1 \;+\; g_1 \;=\; x_2 \;+\; g_2 \;=\; 300$$

Solving simultaneous theses equations, we get:

$$\hat{x}_1 = 240,\ \hat{x}_2 = 120,\ \hat{g}_1 = 60,\ \hat{g}_2 = 180,\ \hat{G} = 240 \quad \square$$

3. (20%) 若兩人爲循序捐獻, 請解出 1 爲先捐者 (leader) 時之均衡公共財總量 $\bar{G}_1$。

(Ans) The follower firm 2's Nash reaction function can be obtained by solving:

$$\text{MRS}_2^{G,x} \;=\; \frac{2x_2}{g_1+g_2} \;=\; \frac{2[300-g_2]}{g_1+g_2} \;=\; 1$$

as:

$$g_2(g_1) \;=\; \frac{600-g_1}{3}$$

Then, firm 1, as the leader, will choose g_1 to:

$$\max_{g_1} \quad x_1 \cdot G \;=\; [300-g_1][g_1 + g_2(g_1)]$$

Therefore, in the Stackelberg equilibrium, leader firm 1 will free ride on firm 2:

$$\bar{g}_1 \;=\; 0,\ \ \bar{g}_2 \;=\; 200,\ \ \bar{G} \;=\; 200 \quad \square$$

4. (20%) 承前一小題, 但換成 2 爲先捐者 (leader), 請解出此 Stackelberg 均衡之公共財總量 $\bar{G}_2$。

(Ans) Now follower firm 1's Nash reaction function can be obtained by solving:

$$\text{MRS}_1^{G,x} \;=\; \frac{x_1}{g_1+g_2} \;=\; \frac{300-g_1}{g_1+g_2} \;=\; 1$$

as:

$$g_1(g_2) \;=\; \frac{300-g_2}{2}$$

Then for the leading firm 2, its goal is to:

$$\max_{g_2} \quad x_2 \cdot G^2 \;=\; [300-g_2][g_1(g_2)+g_2]^2 \;\sim\; [300-g_2][300+g_2]^2$$

The Stackelberg equilibrium is hence:

$$\bar{g}_2 \;=\; 100,\ \ \bar{g}_1 \;=\; 100,\ \ \bar{G} \;=\; 200 \quad \square$$

5. (20%) 試考慮 Lindahl 協商之情形。假設兩人可輕易進行公共財 G 之成本分攤協商, 而公共財只有在兩人對其數量有共識時才能依協商之價格分攤方式被提供。請問 Lindahl 均衡之兩人分攤價格 (p_1, p_2) 將是多少?

(Ans) Equilibrium consumer demands, given Lindahl prices, must be equal:

$$G_1 = \frac{150}{p_1} = G_2 = \frac{200}{p_2}.$$

In addition, we need

$$p_1 + p_2 = 1.$$

Therefore:

$$p_1 = \frac{3}{7}, \; p_2 = \frac{4}{7} \quad \square$$