台 灣 大 學 經 濟 學 研 究 所

公 共 經 濟 學 期 中 考 試

1. 試考慮 Gerber-Wichardt [JPubE 2009] 之「押金機制」。假設兩消費者 $(1,2)$ 之效用函數均是:

$$U_i(x_i, y) \;=\; x_i y$$

其中 x_i 爲各人之私有消費 (價格爲一), 而 y 是一加總式公共財 (價格亦爲一)。令兩人之所得各爲:

$$I_1 \;=\; 200; \;\; I_2 \;=\; 300$$

今公共財須由兩人自由捐獻產生:

$$y \;=\; y_1 + y_2$$

- (10%) Nash 均衡之各人捐獻量 (y_1, y_2) 與公共財總量 $\hat{y}$ 各是多少?

  (Ans) By individual foc, in equilibrium:

$$\text{MRS}_i^{y,x} \;=\; \frac{x_i}{y} \;=\; 1$$

  hence:

$$x_1 \;=\; x_2 \;=\; y \;=\; y_1 + y_2$$

  Along with the individual budget constraints:

$$x_1 + y_1 \;=\; I_1$$

$$x_2 + y_2 \;=\; I_2$$

  we have:

$$\hat{y}_1 = \frac{2I_1 - I_2}{3} \;=\; \frac{100}{3}, \;\; \hat{y}_2 = \frac{2I_2 - I_1}{3} \;=\; \frac{400}{3}$$

$$\hat{y} \;=\; \hat{y}_1 + \hat{y}_2 \;=\; \frac{I_1 + I_2}{3} \;=\; \frac{500}{3}$$

$$\hat{x}_1 \;=\; \hat{x}_2 \;=\; \hat{y} \;=\; \frac{I_1 + I_2}{3} \;=\; \frac{500}{3}$$

$$\hat{U}_1(I_1, I_2) \;=\; \hat{U}_2(I_1, I_2) \;=\; \hat{x}_i\hat{y} \;=\; \frac{[I_1 + I_2]^2}{9} \;=\; \frac{250,000}{9} \quad \square$$

- (10%) Pareto 最適之共財總量 y^* 應是多少?

(Ans) Any PO level y^* must satisfy Samuelson foc:

$$\sum_i \text{MRS}_i^{y,x} \;=\; \frac{x_1 + x_2}{y} \;=\; 1$$

then with aggregate budget:

$$[x_1 + x_1] \;+\; y \;=\; I_1 + I_2$$

we have:

$$y^* \;=\; x_1^* + x_2^* \;=\; \frac{I_1 + I_2}{2} \;=\; 250 \quad \square$$

- (20%) 如果你想要利用「押金機制」讓兩人之捐獻總量符合最適量, 兩人押金 $(\bar{d}_1, \bar{d}_2)$ 之可能範圍爲何? 相對應之捐獻量 (y_1^*, y_2^*) 應如何規定?

(Ans) The deposit design can support any Pareto-superior (x_1^*, x_2^*, y^*).
Take the "equal utility allocation" ($x_1^* = x_2^* = y^*/2$, $U_1^* = U_2^*$) as example:

$$x_1^* \;=\; x_2^* \;=\; \frac{I_1 + I_2}{4}$$

$$y_1^* = I_1 - x_1^* = \frac{3I_1 - I_2}{4}, \quad y_2^* = I_2 - x_2^* = \frac{3I_2 - I_1}{4}, \quad y^* = \frac{I_1 + I_2}{2}$$

$$U_1^* \;=\; U_2^* \;=\; x_i^* y^* \;=\; \frac{[I_1 + I_2]^2}{8} \;>\; \hat{U}_1 \;=\; \hat{U}_2$$

Now to implement this goal, we must set deposits (d_1, d_2) such that:

$$U_1^* \;\geq\; \hat{U}_1(I_1 - d_1 \,|\, y_2^*)$$

$$U_2^* \;\geq\; \hat{U}_2(I_2 - d_2 \,|\, y_1^*)$$

To calculate the "deviation" utility $\hat{U}_1(I_1 - d_1 \,|\, y_2^*)$, note that 1 will choose:

$$x_1 \;=\; y \;=\; y_1 + y_2^* \;=\; y_1 + \frac{3I_2 - I_1}{4}$$

subject to her lower (deviation) budget:

$$x_1 + y_1 \;=\; I_1 - d_1$$

Hence:

$$x_1 \;=\; y_1 + \frac{700}{4}, \;\; x_1 + y_1 = 200 - d_1$$

and we get:

$$x_1 = \frac{1500 - 4d_1}{8}, \;\; y_1 = \frac{100 - 4d_1}{8}, \;\; y = \frac{1500 - 4d_1}{8}$$

Finally for d_1:

$$U_1^* = \frac{500^2}{8} \;\geq\; \hat{U}_1(I_1 - d_1 \,|\, y_2^*) = \frac{[1500 - 4d_1]^2}{64}$$

therefore:

$$I_1 = 200 \;\geq\; d_1 \;\geq\; 21.45$$

We can solve for d_2 in the same way. □

2. A 與 B 兩人是室友。A 喜歡抽煙, 但 B 厭惡煙味; 相反地, B 喜歡聽搖滾樂, 但 A 卻討厭搖滾樂的噪音。令 s 表示 A 的抽煙量, y 表示 B 聽音樂的時間; 另外以 x_i $(i = A, B)$ 表示 A 與 B 的個人私有消費。假設兩人的所得皆為 1000 元, 而抽煙的單位成本為 $p_s = 20$, 聽音樂的單位成本為 $p_y = 5$。兩人的效用函數各是:

$$U_A \;=\; x_A + 200 \ln s - 10y$$
$$U_B \;=\; x_B + 150 \ln y - 20s$$

- (15%) 請解出 Pareto 最適之 s^*, y^* 與 x_i^*?

  (Ans) By Samuelsom foc:

  $$\frac{200}{s} - 20 = 20, \;\; \frac{150}{y} - 10 = 5$$

  we have:

  $$s^* = 5, \;\; y^* = 10$$

  This is where:

  $$\text{MB}_A(s) \;=\; \text{MD}_B(s), \;\; \text{MB}_B(y) \;=\; \text{MD}_A(y)$$

  Hence:

  $$x_A^* + x_B^* \;=\; 2000 - 20 \cdot 5 - 5 \cdot 10 \;=\; 1850 \quad □$$

- (15%) 若兩人無法協商, 則此時之解 $\bar{s}$ 與 $\bar{y}$ 爲多少?

  (Ans) Consumer A's utility-max problem is:

  $$\max_{s,x_A} \; U_A \quad \text{s.t.} \quad x_A + 20s = 1000$$

  while B's goal is:

  $$\max_{y,x_B} \; U_B \quad \text{s.t.} \quad x_B + 5y = 1000$$

  Therefore:

  $$\bar{s} = 10, \;\; \bar{y} = 30 \quad \square$$

- (15%) 若兩人非得到對方的同意無法從事 s 與 y 之活動, 請問此時之 s 與 y 將是多少? 此時 A 與 B 對 s 與 y 成本分擔各多少?

  (Ans) This is the Lindahl equilibrium.

  Let individual prices for s and y are (p_A^s, p_A^y) and (p_B^s, p_B^y), respectively, with the supply-side constraints:

  $$p_A^s + p_B^s = 20, \;\; p_A^y + p_B^y = 5$$

  Their util-max problems are now:

  $$\max_{x_A,s_A,y_A} \; U_A = x_A + 200\ln s_A - 10y_A \quad \text{s.t.} \quad x_A + p_A^s \cdot s_A + p_A^y \cdot y_A = 1000$$

  $$\max_{x_B,s_B,y_B} \; U_B = x_B + 150\ln y_B - 20s_B \quad \text{s.t.} \quad x_B + p_B^s \cdot s_B + p_B^y \cdot y_B = 1000$$

  subject to the equilibrium condition:

  $$s_A = s_B, \;\; y_A = y_B$$

  Then we have:

  $$p_A^s = 40, \; p_B^s = -20$$
  $$p_A^y = -10, \; p_B^y = 15$$

  and efficient levels of the activities:

  $$s^* = s_a = s_B = 5, \;\; y^* = y_A = y_B = \; 10$$

  That is, for each unit of s consumption, A has to compensate B \$20 (total payment \$100). And for each unit of y consumption, B must compensate A \$10 (total payment \$100). $\square$

- (15%) 今兩人可自由從事其 s 與 y 之活動無須對方首肯, 請解出此時之 s 與 y, 以及此時之成本分擔方式?

  (Ans) Same as above:

  $$p_A^s = 40,\ p_B^s = -20$$

  $$p_A^y = -10,\ p_B^y = 15$$

  and

  $$s^* = 5,\ \ y^* = \ 10$$

  But now compensation goes in opposite direction as s (y) drops below $\bar{s}$ ($\bar{y}$). That is, B will pay A \$20 for each unit of s reduction below $\bar{s} = 10$ (total payment \$100). And for each unit of y reduction below $\bar{y} = 30$, B must compensate A \$10 (total payment \$200). Note that p_A^s and p_B^y are their respective opportunity cost of consumption. □

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公 共 經 濟 學 期 末 考 試

1. (50%) Heintzelman et al. [JEEM 2009][1] 的思考結合了我們在課堂中討論過的「搭便車行爲」,「團體競賽」,「外部性」,「機制設計」等特性。

   請說明他們的兩階段賽局設計有何功能? 賽局均衡爲何? 其結果和我們討論過的文獻模型有何新奇之處?

2. (50%) Bergstrom et al. [Econometrica 1982] 的公共財需求估計程序針對受訪者徵詢其對於實際居住地區各項公共支出之意見, 問其希望支出金額要「增加」,「減少」, 還是「不變」。

   試以任一種你熟悉的統計計量軟體 (如 SAS 或 Limdep) 說明你的估計變數和程序要如何設定和進行。

[1] "Putting Free-riding to Work: A Partnership Solution to the Common-property Problem," *Journal of Environmental Economics and Management*, 2009, 57:309–320.