台灣大學經濟學研究所

公共經濟學期中考試

1. 兩消費者 $(1, 2)$ 之效用函數各是:

$$U_1(x_1, y) = x_1 y; \quad U_2(x_2, y) = x_2 y^2$$

其中 x 爲各人之私有消費 (價格爲一), 而 y 是一加總式公共財 (價格爲 p)。令兩人之所得各爲:

$$I_1 = 200; \quad I_2 = 300$$

- (10%) 此兩人團體之公共財 Pareto 最適量 y^* 是多少?

  (Ans) PO y^* must satisfy Samuelson foc

$$\sum_i \text{MRS}_i = \frac{x_1}{y} + \frac{2x_2}{y} = p$$

  and aggregate budget constraint:

$$x_1 + x_2 + py = I_1 + I_2 = 500$$

  There are infinite solutions. □

- (10%) 由此兩人自願捐獻之 Nash 均衡公共財總量 $\bar{y}$ 是多少?

  (Ans) From individual incentives:

$$\text{MRS}_1 = \frac{x_1}{y_1 + y_2} = p, \quad \text{MRS}_2 = \frac{2x_2}{y_1 + y_2} = p$$

  and individual budget:

$$x_1 + py_1 = 200, \quad x_2 + py_2 = 300$$

  we have:

$$y_1 = 0, \; y_2 = \frac{200}{p} = \bar{y}, \; x_1 = 200, \; x_2 = 100 \quad \square$$

- (10%) 若兩人可當面協商公共財成本之分攤方式, 則 Lindahl 均衡之個人價格 p_i 將是多少?

(Ans) Demand for y for two consumers are:

$$y_1 = \frac{200}{2p_1}, \ \ y_2 = \frac{2 \times 300}{3p_2}$$

Hence $p_2 = 2p_1$. Also we must have $p_1 + p_2 = p$, so:

$$p_1 = \frac{p}{3}, \ \ p_2 = \frac{2p}{3}, \ \ y = y_1 = y_2 = \frac{300}{p}$$

Note that Lindahl equilibrium must be Pareto optimal since it satisfies Samuelson foc. But PO may not necessarily be Lindahl. □

- (10%) 若兩人互不知道對方之效用函數與所得, Lindahl 協商是否仍然能夠達到效率性?

  (Ans) Yes. □

2. 一個經濟體中有 n 位同質之消費者 (homogeneous consumers) 和兩個互補商品 (complements): 私有財 x 和加總式之公共財 y。假設衆人之效用函數皆是:

$$U_i(x_i, y) = \min\{x_i, y\}, \quad \forall i = 1, \ldots, n$$

而所得皆爲 I。請問:

- (10%) Pareto 最適之公共財總量 y^*?

  (Ans) For PO, must have:

$$y = x_1 = \cdots = x_n$$

  Then with aggregate budget:

$$y + x_1 + \cdots + x_n = nI$$

  Hence:

$$y^* = x_1 = \cdots = x_n = \frac{nI}{n+1} \quad \square$$

- (10%) 對稱 Nash 均衡之公共財總量 $\hat{y}$?

(Ans) Each consumer i would also like $y = x_i$.
Equilibrium is then again:

$$\hat{y} = x_1 = \cdots = x_n = \frac{nI}{n+1} \quad \square$$

- (10%) 當總人數 n 增加時, 搭便車程度會上升還是下降?

  (Ans) Since $y^* = \hat{y}$, free-riding index $\delta(n) = 1$ for any $n \geq 2$. $\square$

3. 承續第二題之設定, 但現在假設兩商品是完全替代品 (perfect substitutes)。故衆人之效用函數皆是:

$$U_i(x_i, y) \;=\; 2x_i + y, \quad \forall\, i = 1, \ldots, n$$

請問:

- (10%) 此時 Pareto 最適之公共財總量 y^*?

  (Ans) $y^* = nI$, $x_i = 0$ $\square$

- (10%) 對稱 Nash 均衡之公共財總量 $\hat{y}$?

  (Ans) $\hat{y} = 0$, $x_i = I$ $\square$

- (10%) 當總人數 n 增加時, 搭便車程度會上升還是下降?

  (Ans) Free-riding index $\delta(n) = \hat{y}/y^* = 0$ for any $n \geq 2$. $\square$

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公 共 經 濟 學 期 末 考 試

1. (40%) 假設兩人 ($i = 1, 2$) 有相同的原賦 $w_1 = w_2 = 10$, 且有相同的 Cobb-Douglas 效用函數:

$$U_i(x_i, y) \;=\; x_i^{0.6} y^{0.4}$$

其中 x_i 爲 i 之私有消費量, 而 y 爲公共財總量。在此兩人經濟體中, 公共財爲兩人捐獻之和 $y = y_1 + y_2$, 故兩人之預算限制式均爲 $w_i = x_i + y_i$。

今考慮一個兩階段賽局, 兩人在首期同時決定是否要參與。而在第二期中自願參與者再決定其捐獻金額, 不參與者便可搭便車坐享其成。

- 試解出此賽局在四種可能參與狀況 (1 可參加或不參加, 2 可參加或不參加) 下之兩人效用。

  (Ans) If none participates, both get $U_1 = U_2 = 0$ (with $x_1 = x_2 = 10$ and $y = 0$). With i participating only, i gets $U_i = 5.10$ (with $x_i = 6$ and $y = 4$), and j gets $U_j = 6.93$ (with $x_j = 10$ and $y = 4$). When both are in, their Nash utility levels are $U_1 = U_2 = 6.38$. If costless bargaining is possible, their Lindahl utility will be $U_1 = U_2 = 6.73$. Therefore, we have the following normal-form game:

(U_1, U_2)	2 in	2 out
1 in	(6.73, 6.73)	(5.10, 6.93)
1 out	(6.93, 5.10)	(0, 0)

- 請問此賽局之 SPE 均衡爲何? 是否具有效率性?

  (Ans) There are two equilibria: (1 in, 2 out) and (1 out, 2 in). □

2. (30%) 以 $C(S)$ 表示一消費者在面對選項集合 S 會選出的偏好集合。試以下例說明可以合理化 (rationalize) 此消費者選擇行爲 $C(\cdot)$ 之偏好 R 未必是

quasi-transitive:

$$\{x\} = C(\{x, y, z\})$$

$$\{x\} = C(\{x, y\})$$

$$\{y\} = C(\{y, z\})$$

$$\{x, z\} = C(\{x, z\})$$

(Ans) Since ${}_xP^*_y$, ${}_yP^*_z$, but ${}_xI^*_z$, we know $C(\cdot)$ is rationalizable, but R^* is only acyclic, not quasi-transitive. □

3. (30%) 試解出在 Harbaugh [PC 1996] 的投票與說謊模型中, 有正投票率的兩個均衡解投票率。

(Ans) The equilibrium requires:

$$\frac{\text{Area}(V)}{a^2} \;=\; \frac{n_V}{n}$$

Let the voting rate

$$\frac{n_V}{n} \;\equiv\; \eta$$

We have:

$$\left[a - \frac{t}{r}\frac{1}{\eta}\right]\left[a - \frac{t}{r}\right] \;=\; a^2\eta$$

So:

$$\eta \;=\; \frac{a[a-\frac{t}{r}] \pm \sqrt{a^2[a-\frac{t}{r}]^2 - 4a^2\frac{t}{r}[a-\frac{t}{r}]}}{2a^2}$$

assuming

$$a \;>\; \frac{5t}{r}$$

for real solutions. □