Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing

Williamson (1987)

November 2011
This paper relies on monitoring costs to exhibit equilibrium credit rationing in a credit market with asymmetric information and costly monitoring.

An advantage of this approach is that debt contracts are derived as optimal arrangements between borrowers and lenders; such contracts economize on monitoring costs.

Given that the optimal contract is a debt contract, the probability that monitoring occurs and the expected cost of monitoring to the lender increase with the loan interest rate.
Model: lenders and entrepreneurs

- 2 periods: (0, 1)
- A countable infinity of agents indexed by $i$:
  - lenders: $\alpha$
  - entrepreneurs: $1 - \alpha$, $\alpha > \frac{1}{2}$
- A lender $i$ is endowed with 1 unit of indivisible investment good in period 0.
  - lent to an entrepreneur.
  - invested in a project that yield a certain return $t_i$ units of the consumption good in period 1.
    (Assume lenders face different opportunity returns, to generate an upward-sloping supply of funds.)
An entrepreneur $i$ has no endowment but has access to an investment project that produces a random return $\tilde{w}_i$ units of consumption good in period 1, if funded with 1 unit of investment good.

$$\tilde{w}_i \sim_{iid} f(\cdot). \quad f(\cdot) \text{ is the pdf in } [0, \bar{w}], \quad \bar{w} > 0.$$ 

The realization of $\tilde{w}_i$, denoted by $w_i$ is costly observable only to agent $i$, though all agents know $f(\cdot)$.

A lender can observe a particular $w_i$ by spending $\gamma$ units of effort in monitoring, with monitoring decision made in period 1.
Let $r$ denote the market expected return faced by the lender who is indifferent between investing at certain return and lending.

Given $r$, consider an entrepreneur who offers identical contract to potential lenders in exchange for one unit of the investment good.

Features of the contract:
- monitoring in some states
- payments, whether or not monitoring occurs
The contract must satisfy...

(i) the states in which monitoring occurs or not

\[
\begin{cases}
\text{if } w^s \in S \subset [0, \bar{w}], \text{ then monitoring occurs} \\
\text{if } w^s \notin S, \text{ then monitoring does not occur}
\end{cases}
\]

where \( w^s \in [0, \bar{w}] \) is the signal that the entrepreneur emits to the lender, when she observes her return \( w \).

(ii) payment schedule \( R \)

\[
R = \begin{cases}
R(w), & \text{if } w^s \in S \\
K(w), & \text{if } w^s \notin S
\end{cases}
\]

where \( R(\cdot) \) and \( K(\cdot) \) are functions on \([0, \bar{w}]\).

Williamson (1987) Costly Monitoring, Loan Contracts, and Equilibrium Credit Ratio
Payment schedule

- If $w^s \notin S$ (monitoring does not occur)
  $\Rightarrow$ the entrepreneur chooses $w^s$ to minimize $K(w)$
  $\Rightarrow$ a constant payment: $x$

- When monitoring occurs, the payment, $R(w)$, must be incentive compatible

$$\begin{cases} 
  w^s \in S, & \text{if } R(w) < x \\
  w^s \notin S, & \text{if } R(w) \geq x 
\end{cases}$$
Let $A = \{ w : R(w) < x \}$ and $B = \{ w : R(w) \geq x \}$.

The optimal contract is a payment schedule, $\{ R(w), x \}$, which maximizes the entrepreneur’s expected utility while giving the lender a level of expected utility of at least $r$.

$$\max_{\{R(w), x\}} \left\{ \int_A [w - R(w)] f(w)dw + \int_B [w - x] f(w)dw \right\}$$

$$s.t. \int_A [R(w) - \gamma] f(w)dw + \int_B x f(w)dw \geq r.$$
Proof of the Proposition – 1

**Proposition:** The optimal payment schedule is \( R(w) = w \), independent of \( x \).

**Proof**

- *Suppose not, and that \([R'(w), x']\) is the optimal contract.*
- *The constraint must hold with equality; otherwise the value of the object function can be increased by reducing \( R(w) \) for some \( w \) such that the constraint still hold.*

Let \( A' = \{ w : R'(w) < x' \} \) and \( B' = \{ w : R'(w) \geq x' \} \). Hence

\[
\int_{A'} [R'(w) - \gamma] f(w) dw + \int_{B'} x' f(w) dw = r.
\]

Williamson (1987)  
Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing
Proof of the Proposition – 2

Since $R'(w) < w$ for some $w \in A'$, there exists another payment schedule $R''(w)$ with $R''(w) \geq R'(w)$ for all $w$ and $R''(w) > R'(w)$ for some $w \in A'$, with $R''(w)$ continuous and monotone increasing on $[0, \bar{w}]$. There is some $x''$, where $0 < x'' < x'$, such that, with $A'' = \{w : R''(w) < x''\}$ and $B'' = \{w : R''(w) \geq x''\}$,

$$\int_{A''} [R''(w) - \gamma] f(w) \, dw + \int_{B''} x'' f(w) \, dw = r.$$
Proof of the Proposition – 3

The object function can be rewritten as

\[
\int_{A} [w - R(w)] f(w) dw + \int_{B} w f(w) dw - \int_{B} x f(w) dw
\]

\[
= E(w) - \left\{ \int_{A} R(w) f(w) dw + \int_{B} x f(w) dw \right\}
\]

\[
= E(w) - r - \gamma \int_{A} f(w) dw.
\]

Note: \( \int_{A} f(w) dw \) is \( \text{prob}(R(w) < x) \). So the optimal contract is to minimize the expected monitoring cost.
Proof of the Proposition – 4

Because $x'' < x'$ and $R''(w) > R'(w)$ for some $w \in A'$, implying $A'' \subset A'$ and $A' - A'' \neq \emptyset$, the change in the objective function in changing the contract from $[R'(w), x']$ to $[R''(w), x'']$ is then

$$\gamma \left[ \int_{A'} f(w)dw - \int_{A''} f(w)dw \right] > 0,$$

a contradiction. Q.E.D.
Optimal contract is a debt contract

- Either the entrepreneur pays the lender a fixed amount $x$, or he defaults, monitoring occurs, and the lender receives the entire return on the project $w$.
- The default state can be interpreted as bankruptcy and $\gamma$ as a cost of bankruptcy.
Optimal contract is completely characterized by $x$

- For the lender, expected utility is
  \[ \pi_l(x) = \int_0^x w f(w)dw + x[1 - F(x)] - \gamma F(x), \]

  and for the entrepreneur is
  \[ \pi_e(x) = \int_x^{\overline{w}} w f(w)dw - x[1 - F(x)]. \]

- Note: $\pi_l(x)$ is not monotone increasing in $x$, because of $\gamma$. $\pi_e(x)$ is monotone decreasing in $x$.

- $\pi_l'(x) = 1 - F(x) - \gamma f(x)$. $\pi_l'(x) < 0$ because $f(x) > 0$ for $x \in [0, \overline{w}]$; i.e. $\pi_l'(x^*) = 0$ for $x^* < \overline{w}$. 

Williamson (1987) Costly Monitoring, Loan Contracts, and Equilibrium Credit Ratio
Equilibrium

Definition
An equilibrium is a loan interest rate $x^*$, a market expected return $r^*$, and an aggregate loan quantity $q^*$, which satisfy

(i) $x^*$ solves $\max_x \pi_e(x)$ subject to $\pi_l(x) \geq r^*$

(ii) $q^* = \alpha H(r^*)$

(iii) Either $q^* = 1 - \alpha$

or $q^* < 1 - \alpha$ and $\pi'_l(x^*) = 0$

Note: Offering a higher $x$ implies a higher probability of default, with larger expected monitoring costs for the lender.

Williamson (1987) Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing