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COSTLY MONITORING, FINANCIAL INTERMEDIATION, AND EQUILIBRIUM CREDIT RATIONING

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This paper establishes a link between equilibrium credit rationing and financial intermediation, in a model with asymmetrically informed lenders and borrowers, costly monitoring, and investment project indivisibilities. Intermediation is shown to dominate borrowing and lending between individuals, and these financial intermediaries exhibit several of the important features of intermediaries as we know them. Equilibrium interest rates and the aggregate quantity of loans respond quite differently to changes in taste and technology parameters, depending on whether or not there is rationing in equilibrium.

1. Introduction

The purpose of this paper is to analyze an environment with asymmetrically informed borrowers and lenders which for our purposes has two features of primary interest: (1) financial intermediation arises endogenously as the dominant vehicle for borrowing and lending in equilibrium, and (2) an equilibrium may exhibit credit rationing. In the model, the costly monitoring of lenders by borrowers and large-scale investment projects imply that there exist increasing returns to scale in lending and borrowing which can be exploited by financial intermediaries. Costly monitoring and universal risk neutrality yield debt contracts as optimal arrangements between lending institutions and borrowers. This in turn generates an asymmetry in these agents' payoff functions, which then permits equilibria with credit rationing. All borrowers are identical, ex ante, but in equilibrium some of these borrowers may receive loans while others do not. The system behaves quite differently in response to changes in underlying parameters which characterize technology and preferences, depending on whether or not there is credit rationing in equilibrium.

There are two literatures which are directly related to this paper, both of which study the financial market implications of informational asymmetries. First is the literature which has attempted to explain credit rationing as an

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equilibrium phenomenon, examples of which are Jaffee and Russell (1976), Keeton (1979), Stiglitz and Weiss (1981) and Gale and Hellwig (1984). The second related literature studies the role and reasons for the existence of financial intermediaries and includes the work of Boyd and Prescott (1985), Chan (1983), Diamond (1984) and Ramakrishnan and Thakor (1984).

The credit rationing literature cited above is clearly intended to apply to intermediated credit markets, and much of the empirical support for the existence of credit rationing [see Keeton (1979)] refers specifically to the activities of financial intermediaries, and more specifically to banks. However, the lending institutions featured, for example, in the framework of Stiglitz and Weiss (1981) take on the features of 'banks' mainly by assumption, if at all.[1] By contrast, in the environment studied by Boyd and Prescott (1985), for example, a primitive environment is specified in which financial intermediaries emerge endogenously. These intermediaries exhibit several stylized features which are associated with real-world intermediaries. However credit rationing, in the sense in which it occurs in this paper and in Stiglitz and Weiss (1981), does not occur in the framework of Boyd and Prescott (1985) or in the other papers in the intermediation literature cited above. A contribution of this paper is the construction of a model which motivates financial intermediation from first principles *and* permits equilibria with credit rationing.

At least two types of credit rationing are examined in the literature. In Jaffee and Russell (1976) and Gale and Hellwig (1984), credit is rationed in the sense that an individual borrower receives a smaller loan than she would like, given the quoted interest rate. In this paper we study a different type of rationing. Here, borrowers are identical ex ante, but some receive loans and others do not. Stiglitz and Weiss (1981) call this 'true' rationing, and they show how it can arise in equilibrium due to moral hazard and adverse selection in credit markets. Keeton (1979) also showed how this second type of rationing could arise as the result of a moral hazard problem.

In our model, credit rationing does not arise due to moral hazard or adverse selection, in contrast to Keeton (1979) and Stiglitz and Weiss (1981). Here, a borrower and a lender are asymmetrically informed, ex post, concerning the return on the borrower's investment project, and monitoring of the lender is costly. The optimal contract between a lender and a borrower is a debt contract, whether the lender is an intermediary or a direct lender, and the lender monitors only in the event of default. An increase in the loan interest rate increases the expected return to the lender, but also results in an increase in the probability that the borrower defaults, thus increasing the expected cost of monitoring to the lender. It may not be possible, then, for the loan interest

[1] I.e., contracts in the Stiglitz and Weiss (1981) model are assumed to be debt contracts; the form of the contract is not derived from features of the problem. In addition, Stiglitz–Weiss 'banks' are *assumed* to borrow from one group of agents and to lend to another group, and these groups need not be large, as is the case with real-world intermediaries.

rate to adjust to clear the market, so that some borrowers do not receive loans in equilibrium.

Debt contracts are also derived as the solution to a bilateral contracting problem in Gale and Hellwig (1984), in an environment which is in some ways more general than ours.[2] However, Gale and Hellwig restrict their attention to the contracting problem, and do not consider intermediation or the characteristics of market equilibrium, as we do here. In Diamond (1984), debt contracts are optimal, but the costs of default are non-pecuniary costs to the borrower, rather than monitoring costs to the lender, as in our model. In this respect, our framework resembles that considered by Townsend (1979) where monitoring decisions are made ex post, and monitoring only occurs in the 'default' state.

In our model, there is a duplication of effort which occurs in an equilibrium with direct lending when intermediation is not permitted, in that each borrower borrows from several lenders, and each of these lenders monitors in the case of default. A financial intermediary, which borrows from a large number of lenders and lends to a large number of borrowers, eliminates this duplication. Intermediation drives direct lending out of the system in equilibrium. As in Boyd and Prescott (1985) and Diamond (1984), diversification is critical to the function that intermediation performs, in spite of the fact that all agents are risk-neutral with respect to consumption realizations.

As in Diamond (1984), there is a sense in which intermediaries perform a 'delegated monitoring' role. However, a crucial difference in our framework from Diamond's is that monitoring decisions are made ex post. Monitoring occurs only in the default state, and the probability that monitoring occurs is determined endogenously. As a result, it is possible for credit rationing to exist in equilibrium. In contrast, credit rationing is not a feature of equilibrium in Diamond (1984), and since monitoring decisions are made ex ante, monitoring will either occur with zero probability or with certainty.

The financial intermediaries which arise endogenously in our model share several of the important features of financial intermediaries as we know them: (1) They issue securities which have payoff characteristics which are different from those of the securities they hold. (2) They write debt contracts with borrowers. (3) They hold diversified portfolios. (4) They process information. Also, they ration credit in equilibrium, which some would characterize as an empirical fact, as do Stiglitz and Weiss (1981).

The fact that there can be equilibria with and without credit rationing in our model implies that the system responds dichotomously to changes in underlying parameters. For example, a shift in the alternative rates of return faced by lenders (characterized as a shift in the supply curve of available funds faced by intermediaries) will bring about changes in interest rates in equilibrium when

[2] Gale and Hellwig allow monitoring costs to be state-dependent and they consider situations in which the net wealth of borrowers may be non-zero.

there is no rationing, However, if there is rationing, interest rates will remain unchanged, but the number of borrowers who do not receive loans *will* change. As in Williamson (1984b), this is consistent with the thrust of the availability doctrine [see Roosa (1951)] in that monetary policy (which could affect alternative rates of return in our model) can have real effects without changing interest rates in lending markets.

The remainder of the paper is organized as follows. In section 2 we present the model. Section 3 contains an examination of a regime with direct lending where intermediation is prohibited. We show that debt contracts are optimal and define an equilibrium. In section 4 we study the features of a regime with financial intermediation. Again, debt contracts are optimal. We define an equilibrium, and discuss existence and uniqueness. We show that intermediation dominates direct lending and that credit rationing may be a feature of the equilibrium. Section 5 contains a discussion of some comparative statics experiments. The final section is a summary and conclusion.

2. The model

This model is a version of that presented in Williamson (1984b), with modifications designed to permit a role for financial intermediation.

There are two periods, period zero, the planning period, and period one, when consumption takes place. There exists a single consumption good, which is perishable between period zero and period one.

There is a countable infinity of agents, each of whom is either a lender or an entrepreneur. We specify the population in terms of the probabilities of drawing agents of each type from the population, and write equilibrium conditions for the economy as a whole in per capita terms.[3] If we draw an agent at random from the population, then

$$\Pr[\text{agent is a lender}] = \alpha,$$

$$\Pr[\text{agent is an entrepreneur}] = 1 - \alpha,$$

where $0 < \alpha < 1$.

Each lender is endowed with a single indivisible unit of the consumption good in period zero which may be lent to entrepreneurs (directly or indirectly) or invested at a certain rate of return. To generate an upward-sloping supply curve for loanable funds, different lenders face different certain rates of return. That is, if an individual lender invests x units of the consumption good in period zero, she receives a return of s units of the consumption good in period

[3] Boyd and Prescott (1985) take a similar approach for the same reason. I.e., in section 4 we study equilibrium with 'large' intermediaries. To do this while retaining competitive assumptions requires an economy of infinite size.

one, where s is given by

$$s = tx, \qquad 0 \le x \le 1,$$
$$= t, \qquad x \ge 1.$$

I.e., certain return investment projects have a capacity of one unit of input. We have $t \in [\underline{t}, \bar{t}]$, with $0 < \underline{t} < \bar{t} < \infty$. If we were to draw an agent at random, then

$$\Pr[t \le t' | \text{agent is a lender}] = \int_{\underline{t}}^{t'} h(t)\,\mathrm{d}t, \qquad t' \ge \underline{t},$$

where $h(\cdot)$ is a probability density function which is positive and continuous on $[\underline{t}, \bar{t}]$.

Each entrepreneur receives a zero endowment in period zero, and has access to an investment project which yields a random return of $K\tilde{w}$ units of the consumption good in period one to an input of K units in period zero, and zero units otherwise. Here, K is an integer with $K \ge 2$, and $\tilde{w}$ is a random variable. Project returns are independent across entrepreneurs and, for each entrepreneur, $\tilde{w}$ is distributed according to the probability density function $f(\cdot)$ and probability distribution function $F(\cdot)$. The function $f(\cdot)$ is positive and differentiable on $[0, \overline{w}]$, where $\overline{w} > 0$, and is zero otherwise. We have $\alpha > (1-\alpha)K$, so that the demand for credit is at least potentially satisfied.

The realization of $\tilde{w}$, denoted w, is costlessly observable only to the entrepreneur, but all agents know $f(\cdot)$. In period one, an individual lender can expend effort to learn the return(s) on any project(s). It requires c units of effort to observe the return on any one project, where $c > 0$. Lenders are endowed with an unbounded quantity of effort and maximize the expected value of $U(x, e)$, where x is period one consumption and e is effort with $x, e \ge 0$. We have

$$U(x, e) = x - e. \tag{2.1}$$

Entrepreneurs receive an endowment of zero units of effort and are risk-neutral with respect to consumption realizations. Entrepreneurs maximize the expected value of $V(x)$ where x is consumption and $x \ge 0$. We have

$$V(x) = x. \tag{2.2}$$

Two important features of the model are the nature of the informational asymmetry and the timing of monitoring decisions. As in Gale and Hellwig (1984), Townsend (1979) and Diamond (1984), agents are asymmetrically informed (in the absence of monitoring) ex post. This contrasts with the environments of Boyd and Prescott (1985) and Stiglitz and Weiss (1981), which feature ex ante information asymmetries. In our model, as in Diamond (1984), costly monitoring generates a role for financial intermediation. However, our monitoring technology is different from Diamond's, where monitoring decisions are made ex ante, and similar to that of Gale and Hellwig (1984) and Townsend (1979), where monitoring decisions are made ex post. The fact that agents make monitoring decisions conditional on observations in period one will be seen to be crucial in permitting equilibria with financial intermediation *and* credit rationing.

In our model and those of Boyd and Prescott (1985) and Diamond (1984), a role for intermediation arises in spite of the fact that all agents are risk-neutral with respect to consumption realizations. Gale and Hellwig (1984) and Stiglitz and Weiss (1981) also restrict their analyses to models with risk-neutral agents, though these authors do not study financial intermediation. As in Gale and Hellwig (1984), risk neutrality simplifies the contracting problem considerably.

3. Equilibrium with direct lending

We first look at a regime where intermediation is prohibited, so that the only contractual arrangements are those between individual lenders and entrepreneurs.

Let r denote the certain market return, which is an endogenous variable, but which is treated as a fixed parameter by all agents.[4] First, consider a single entrepreneur who wishes to fund her project. This entrepreneur must then offer contracts to K lenders. Without loss of generality, we assume that these contracts must be identical for each lender. Contracts must specify that one unit of the consumption good will be transferred from each lender to the entrepreneur in period zero, and will also specify the states of the world in which monitoring will occur and the payment schedules in the cases in which monitoring occurs and in which it does not. We consider only pure strategy contracts, i.e., attention is restricted to non-stochastic monitoring.[5]

Following the realization of $\tilde{w}$, the entrepreneur emits a signal, w^{d}, to the K lenders, where $w^{\mathrm{d}} \in [0, \bar{w}]$. The contract specifies that, if $w^{\mathrm{d}} \in S \subset [0, \bar{w}]$, then monitoring occurs, while if $w^{\mathrm{d}} \notin S$, then monitoring does not occur. The payment to each lender will then be

$$\begin{aligned} R &= R(w), \quad && w^{\mathrm{d}} \in S, \\ &= K(w^{\mathrm{d}}), \quad && w^{\mathrm{d}} \notin S, \end{aligned}$$

[4] In equilibrium, this will be the certain return faced by the lender who is on the margin between investing at her certain return and lending to an entrepreneur.

[5] In the case of stochastic monitoring, we would define the 'state' to include extraneous information, such as the outcome of a lottery which has no effect on technology, preferences or endowments. The optimal contract may not be different if we were to allow for stochastic monitoring, but we have not been able to prove this.

where $R(\cdot)$ and $K(\cdot)$ are functions which must obey the following feasibility conditions

$$0 \leq R(w) \leq w, \tag{3.1}$$

$$0 \leq K(w^{\mathrm{d}}) \leq w. \tag{3.2}$$

If the entrepreneur chooses $w^{\mathrm{d}} \notin S$, then clearly she will always choose $w^{\mathrm{d}^*} = \arg\min K(w^{\mathrm{d}})$. Therefore, when monitoring does not occur the payment the entrepreneur makes to each lender is a constant, denoted by

$$\bar{R} = \min_{w^{\mathrm{d}}} K(w^{\mathrm{d}}).$$

It remains to determine the optimal payment schedule when monitoring occurs, $R(w)$. This payment schedule must be incentive compatible, i.e., we must have

$$w^{\mathrm{d}} \in S \qquad \text{if} \quad R(w) < \bar{R}, \tag{3.3}$$

$$w^{\mathrm{d}} = w^{\mathrm{d}^*} \qquad \text{if} \quad R(w) > \bar{R}, \tag{3.4}$$

and

$$w^{\mathrm{d}} \in S \quad \text{or} \quad w^{\mathrm{d}} = w^{\mathrm{d}^*} \quad \text{if} \quad R(w) = \bar{R}. \tag{3.5}$$

Conditions (3.3)–(3.5) allow us to determine the realizations of $\tilde{w}$ over which monitoring occurs, given $R(w)$. Let B and B^{c} be non-intersecting subsets of $[0, \bar{w}]$, with $B + B^{\mathrm{c}} = [0, \bar{w}]$, $B = \{w\colon R(w) < \bar{R}\}$ and $B^{\mathrm{c}} = \{w\colon R(w) \geq \bar{R}\}$. I.e., for $w \in B$, monitoring occurs, and for $w \in B^{\mathrm{c}}$, monitoring does not occur. The optimal contract is a payment schedule – 'interest rate' pair $(R(w), \bar{R})$ which maximizes the entrepreneur's expected utility, while giving each lender a level of expected utility of at least r, the market interest rate:

$$\max_{(R(w),\bar{R})} K\left\{\int_B [w - R(w)] f(w)\,\mathrm{d}w + \int_{B^{\mathrm{c}}} (w - \bar{R}) f(w)\,\mathrm{d}w\right\}, \tag{3.6}$$

subject to

$$\int_B [R(w) - c] f(w)\,\mathrm{d}w + \int_{B^{\mathrm{c}}} \bar{R} f(w)\,\mathrm{d}w \geq r.$$

First, note that the constraint in (3.6) must be binding, since otherwise $R(w)$

and $\overline{R}$ could decrease, with the sets B and B^c remaining unchanged, the constraint would still be satisfied, and the value of the objective function would increase.

Proposition 1. *The optimal payment schedule is* $R(w) = w$.

Proof. Suppose not, and that $(R'(w), \overline{R}')$ is the optimal contract. Let

$$B' = \{w\colon R'(w) < \overline{R}'\} \quad \text{and} \quad B'^{c} = \{w\colon R'(w) \geq \overline{R}'\}.$$

Then, from (3.6),

$$\int_{B'} [R'(w) - c] f(w)\,\mathrm{d}w + \int_{B'^{c}} \overline{R}' f(w)\,\mathrm{d}w = r.$$

Now consider another payment schedule $R''(w)$ with $R''(w) \geq R'(w)$ for all $w \in [0, \overline{w}]$, $R''(w) > R'(w)$ for some $w \in B'$, and $R''(\cdot)$ continuous and monotone increasing on $[0, \overline{w}]$. Then, there is some $\overline{R}''$ with $0 < \overline{R}'' < \overline{R}'$ such that

$$\int_{B''} [R''(w) - c] f(w)\,\mathrm{d}w + \int_{B''^{c}} \overline{R}'' f(w)\,\mathrm{d}w = r,$$

$$B'' = \{w\colon R''(w) < \overline{R}''\}, \qquad B''^{c} = \{w\colon R(w) \geq \overline{R}''\}.$$

The change in the objective function in (3.6) as the result of changing the contract from $(R'(w), \overline{R}')$ to $(R''(w), \overline{R}'')$ is

$$Kc\left[\int_{B'} f(w)\,\mathrm{d}w - \int_{B''} f(w)\,\mathrm{d}w\right] > 0,$$

as $B'' \subset B'$ and $B' - B'' \neq \phi$. We therefore have a contradiction. Q.E.D.

The proof of Proposition 1 is adapted from Williamson (1984b) and a proof of a similar proposition in a different environment (which is in some ways more general) is in Gale and Hellwig (1984). Proposition 1 states that the optimal contract is what Gale and Hellwig call a 'standard debt contract'. I.e., either each lender receives a fixed payment, $\overline{R}$, or the entrepreneur defaults, each lender monitors, and the entrepreneur receives a zero return (bankruptcy). The 'interest rate', $\overline{R}$, is then sufficient to characterize the contract, where $\overline{R}$ solves

$$\max_{\overline{R}} K \int_{\overline{R}}^{\overline{w}} (w - \overline{R}) f(w)\,\mathrm{d}w, \tag{3.7}$$

subject to

$$\int_0^{\bar{R}} wf(w)\,\mathrm{d}w + \bar{R}(1 - F(\bar{R})) - cF(\bar{R}) = r.$$

Assuming that the following condition holds for $0 \leq \bar{R} \leq \bar{w}$:

$$f(\bar{R}) + cf'(\bar{R}) > 0, \tag{3.8}$$

then the expected utility of each lender is a concave function of $\bar{R}$, reaching a maximum for some $\bar{R} \in [0, \bar{w})$, while the expected utility of the entrepreneur is decreasing in $\bar{R}$. As in Stiglitz and Weiss (1981) and as we discuss in more detail in Williamson (1984b), the asymmetry in the payoff functions of lenders and entrepreneurs leads to the possibility of credit rationing in equilibrium. In contrast to Stiglitz and Weiss (1981), this asymmetry did not come about due to moral hazard or adverse selection,[6] and the form of the contract is derived rather than assumed. Debt contracts are also optimal in the environments specified by Gale and Hellwig (1984) and Diamond (1984), though in Diamond's paper bankruptcy does not impose costs of monitoring on the lender, but instead implies non-pecuniary costs for the borrower.

Definition 1. An equilibrium is a loan interest rate $\bar{R}^*$, a certain market interest rate r^* and an aggregate loan quantity q^*, which satisfy:

1) $\bar{R}^*$ solves (3.7) given $r = r^*$.

2) $q^* = \alpha \int_{\underline{t}}^{R^*} h(t)\,\mathrm{d}t$.

3) Either (a) $q^* = (1-\alpha)K$, or (b) $q^* < (1-\alpha)K$ and $1 - F(\bar{R}^*) - cf(\bar{R}^*) = 0$.

There are two types of equilibria, those with rationing (RA equilibria) given by 3b) and those without rationing (NRA equilibria) given by 3a). In an RA equilibrium, all entrepreneurs offer the same contracts on the market, while lenders who wish to accept one of these contracts choose an entrepreneur at random. If after all lenders have chosen an entrepreneur, a given entrepreneur is paired with a positive number of lenders, but this number is insufficient to fund the project, then these lenders choose another entrepreneur at random. This process continues until all lenders are allocated to entrepreneurs and any projects that are funded are fully funded. It is then possible for some

[6] Actually, it is not quite correct to say that moral hazard is not an important element here. Indeed, monitoring is necessary since there is otherwise an incentive for the borrower to misreport her project return. However, moral hazard enters in a very different manner in the Stiglitz–Weiss paper.

entrepreneurs not to receive loans, since if 3b) holds there is no contract which a rationed entrepreneur can offer which will bid loans away from other entrepreneurs, or draw additional lenders into the market.

4. Financial intermediation

In both the RA and NRA equilibria of the previous section, duplication of effort occurs, in that each entrepreneur borrows from K lenders, who each monitor the entrepreneur when default occurs. Potentially, a group of lenders could act as individual monitoring agents or auditors, who monitor entrepreneurs and sell the information to lenders, thus exploiting the economies of scale in monitoring entrepreneurs. However, such a market for information would fail since the value of information does not diminish with use. Also, the information may not be reliable due to incentives to cheat on the part of the auditors, and because entrepreneurs have an incentive to make side-payments to auditors to induce them to reveal incorrect information [see Campbell and Kracaw (1980)].[7]

These problems can be solved, however, if a proportion of lenders act as intermediaries. Each intermediary is an individual lender who acts to maximize expected utility by issuing financial claims to other lenders (depositors) and lending K units of the consumption good to each entrepreneur she contracts with. One entrepreneur is then matched with one lending institution, though an intermediary may lend to more than one entrepreneur. The expected utility cost of monitoring entrepreneurs is covered by the expected return on the intermediary's portfolio, i.e., the value of information is captured in a private good [see Leland and Pyle (1977)]. As in Diamond (1984), there remains the problem that the intermediary's depositors will need to monitor the intermediary when it defaults. By holding a diversified portfolio, the intermediary can reduce or eliminate (with a large number of independent risks) the expected utility loss to depositors due to monitoring.

Though the monitoring technology and the timing of monitoring decisions are different in this model than in Diamond (1984), the way we model intermediation is quite similar. Intermediaries are single agents, though we do not identify separate intermediary agents, as does Diamond, but instead allow lenders to function as intermediaries, as depositors, or as investors in projects with certain returns. In contrast to the approach of modelling intermediaries as single agents, intermediation in Boyd and Prescott (1985) is performed by multi-agent coalitions. This approach is tractable in their framework, as the production of information is assumed to be public.

[7] The problem of information reliability may go away in repeated games where auditors can establish reputations, and side payments to auditors would likely be illegal. However, the appropriability problem remains.

Suppose that a given intermediary contracts to fund m entrepreneurs, indexed by $j=1,2,\ldots,m$, that it contracts with $mK-1$ lenders to act as 'depositors', and that it invests its single unit of the consumption good in the intermediary. Without loss of generality, assume that the intermediary writes identical contracts with each of the (ex ante) identical entrepreneurs. As in the regime with contracting between individuals, entrepreneur j pays the intermediary a gross rate of return of $\bar{R}$ in the event that monitoring does not occur. Let $R(w_j)$ denote the payment per unit of the consumption good lent, in the event that the intermediary monitors entrepreneur j. The contract must be incentive-compatible, as in (3.3), (3.4) and (3.5) (simply replace w^{d} by w_j^{d} and w by w_j), and we let $B=\{w_j\colon R(w_j)<\bar{R}\}$ and $B^{\mathrm{c}}=\{w_j\colon R(w_j)\geq\bar{R}\}$. As in the previous section, we let r denote the certain market return. The total return to the intermediary (before compensating depositors) from the m loan contracts is

$$\pi_m = K\sum_{j=1}^{m}\min\{R(w_j),\bar{R}\}. \tag{4.1}$$

By the strong law of large numbers, we obtain

$$\operatorname*{plim}_{m\to\infty}\frac{1}{mK}\pi_m=\int_B R(w_j)\,\mathrm{d}w_j+\bar{R}\int_{B^{\mathrm{c}}}f(w_j)\,\mathrm{d}w_j. \tag{4.2}$$

Let N denote the number of borrowers with the intermediary who default in period one. Then Nc is the utility cost to the intermediary of monitoring these borrowers. Since N is a binomial random variable with parameters $(m,\ \int_B f(w_j)\,\mathrm{d}w_j)$, therefore, by the strong law of large numbers,

$$\operatorname*{plim}_{m\to\infty}\frac{Nc}{mK}=\frac{c}{K}\int_B f(w_j)\,\mathrm{d}w_j. \tag{4.3}$$

Therefore, if the following weak inequality holds,[8]

$$\int_B R(w_j)f(w_j)\,\mathrm{d}w_j+\bar{R}\int_{B^{\mathrm{c}}}f(w_j)\,\mathrm{d}w_j-(c/K)\int_B f(w_j)\,\mathrm{d}w_j\geq r, \tag{4.4}$$

then as the intermediary grows large ($m\to\infty$) it can guarantee a certain return of r to its depositors and attain a level of expected utility of at least r. Given the contract $(R(w_j),\bar{R})$, a finite-sized intermediary must write contracts with its depositors which involve monitoring and, given the certain market

[8] In fact, condition (4.4) must hold if intermediaries willingly engage in the contract $(R(w_j),\ \bar{R})$ with borrowers.

return r, depositors must be compensated for these monitoring costs by the intermediary. This compensation lowers expected utility for the intermediary, so that, with $(R(w_j), \bar{R})$ given, expected utility is higher for a large (i.e., infinite-sized) intermediary, which depositors need not monitor, than for an intermediary of finite size. As in Diamond (1984), the costs of delegated monitoring go to zero in the limit as the intermediary grows large.

Given that intermediaries will choose to grow large (since this is expected utility-maximizing) for any contract $(R(w_j), \bar{R})$, it remains for us to determine the optimal payment schedule, $R(w_j)$, for a large intermediary. Not surprisingly, this is identical to the optimal payment schedule for the direct lending case.

Proposition 2. The optimal payment schedule for an intermediary is $R(w_j) = w_j$.

Proof. Since depositors need not monitor the intermediary as $m \to \infty$, and using (4.2) and (4.3), the probability limit as $m \to \infty$ of the total return per intermediary member (the members are the depositors and the intermediary agent) is given by

$$V(R(w_j), \bar{R}) = \int_B R(w_j) f(w_j)\,\mathrm{d}w_j + \bar{R}\int_{B^c} f(w_j)\,\mathrm{d}w_j$$

$$-(c/K)\int_B f(w_j)\,\mathrm{d}w_j.$$

The optimal contract maximizes the expected utility of the borrower, subject to $V(R(w_j), \bar{R}) \geq r$. That is

$$\max_{(R(w_j), \bar{R})} K\left[\int_B [w_j - R(w_j)] f(w_j)\,\mathrm{d}w_j + \int_{B^c}(w_j - \bar{R}) f(w_j)\,\mathrm{d}w_j\right], \tag{4.5}$$

subject to

$$\int_B R(w_j) f(w_j)\,\mathrm{d}w_j + \bar{R}\int_{B^c} f(w_j)\,\mathrm{d}w_j - (c/K)\int_B f(w_j)\,\mathrm{d}w_j \geq r.$$

By simply replacing w by w_j and c by c/K in (3.6), the proof of the proposition is immediate from the proof of Proposition 1. Q.E.D.

As for the case with direct lending, the optimal contract between an intermediary and a borrower is a debt contract and, similarly, the constraint in

(4.5) is binding. Therefore, the 'loan interest rate', $\bar{R}$, solves

$$\max_{\bar{R}} K\int_{\bar{R}}^{\bar{w}} w_j f(w_j)\,\mathrm{d}w_j, \tag{4.6}$$

subject to

$$\int_0^{\bar{R}} w_j f(w_j)\,\mathrm{d}w_j + \bar{R}(1-F(\bar{R})) - (c/K)F(\bar{R}) = r.$$

Letting $U(\bar{R})$ denote the expected utility attained by each depositor and by the intermediary when the loan interest rate is $\bar{R}$, we have, from (4.6),

$$U(\bar{R}) = \int_0^{\bar{R}} w_j f(w_j)\,\mathrm{d}w_j + \bar{R}(1-F(\bar{R})) - (c/K)F(\bar{R}). \tag{4.7}$$

Condition (3.8) then guarantees that $U(\bar{R})$ is a concave function on $[0,\bar{w}]$.

We can now define an equilibrium as follows.

Definition 2. An equilibrium with intermediation is a loan interest rate $\bar{R}^*$, a certain market interest rate r^*, and an aggregate loan quantity q^* which satisfy:

1) $\bar{R}^*$ solves (4.6).
2) $q^* = \alpha\int_{\underline{t}}^{r^*} h(t)\,\mathrm{d}t$.
3) Either (a) $q^* = (1-\alpha)K$, or (b) $q^* < (1-\alpha)K$ and $1 - F(\bar{R}^*) - (c/K)f(\bar{R}^*) = 0$.

As is the case for equilibria with direct lending (Definition 1), there are two types of equilibria: NRA equilibria characterized by 3a) and RA equilibria characterized by 3b). In both types of equilibria, lenders with $t \le r^*$ are indifferent in equilibrium between acting as depositors and acting as intermediaries. Those lenders with $t > r^*$ invest in their certain return projects.

In an RA equilibrium, each entrepreneur offers a loan contract on the market at an interest rate $\bar{R}^*$, and each intermediary makes loans to entrepreneurs who are chosen at random. However, for each entrepreneur there is a probability $q^*/(1-\alpha)K$ of receiving a loan and, given 3b), if an entrepreneur does not receive a loan, then there is no loan interest rate she can offer which would bid loans away from other entrepreneurs, or which would draw additional lenders into the market to act as intermediaries and depositors. Also, as we will show in the next section, in an RA equilibrium there is no contract that entrepreneurs who do not receive loans could offer directly to lenders that would give these lenders a level of expected utility greater than

r^*. I.e., the intermediation process cannot be circumvented by entrepreneurs who are rationed out of the market.

The fact that lenders who participate in the loan market (lenders for whom $t \leq r^*$) are indifferent in equilibrium between acting as intermediaries and acting as depositors is of some interest. In the framework studied by Boyd and Prescott (1985), agents in the model are also free to choose the activities they engage in, though Boyd and Prescott's framework is more general in this respect than ours. Here, there are assumed attributes of agents which differentiate 'lenders' from 'entrepreneurs' at the outset.

4.1. NRA equilibrium

Given (4.6) and Definition 2, an equilibrium without rationing is the solution to the following two equations: intermediaries and depositors attain a level of expected utility of r from (4.6),

$$\int_0^{\bar{R}} w_j f(w_j)\,\mathrm{d}w_j + \bar{R}(1 - F(\bar{R})) - (c/K)F(\bar{R}) = r, \tag{4.8}$$

and market clearing,

$$(1-\alpha)K = \alpha \int_{\underline{t}}^{r} h(t)\,\mathrm{d}t. \tag{4.9}$$

Eqs. (4.8) and (4.9) solve for equilibrium $\bar{R}$ and r. From (4.6), the following condition also holds in equilibrium:

$$1 - F(\bar{R}) - (c/K)f(\bar{R}) \geq 0. \tag{4.10}$$

4.2. RA equilibrium

From Definition 2, an equilibrium with rationing is the solution to (4.8) and the two following equations: the equilibrium loan rate, $\bar{R}$, gives the maximum level of expected utility to each intermediary, i.e., it is 'bank-optimal', using the terminology of Stiglitz and Weiss (1981)

$$1 - F(\bar{R}) - (c/K)f(\bar{R}) = 0, \tag{4.11}$$

and from condition 2) in Definition 2,

$$q = \alpha \int_{\underline{t}}^{r} h(t)\,\mathrm{d}t. \tag{4.12}$$

Eqs. (4.8), (4.11) and (4.12) solve for equilibrium $\bar{R}$, r, and q.

4.3. Existence and uniqueness of equilibrium

From (4.7) and (4.8) an equilibrium, whether NRA or RA, must satisfy

$$U(\bar{R}) = r. \tag{4.13}$$

Given (3.8), there is some unique $\bar{R}_{\max}$ which maximizes $U(\bar{R})$ on $[0, \bar{w}]$. Also, let $\hat{r}$ denote the 'market-clearing' interest rate, ie.e, the certain market interest rate which solves

$$\alpha \int_{\underline{t}}^{\hat{r}} h(t)\,\mathrm{d}t = (1-\alpha)K. \tag{4.14}$$

We can then state the following proposition without proof (for brevity).

Proposition 3. If $U(\bar{R}_{\max}) \geq \underline{t}$, then an equilibrium exists and it is unique. Further, if $\underline{t} \leq U(\bar{R}_{\max}) < \hat{r}$, then the equilibrium is RA and if $U(\bar{R}_{\max}) \geq \hat{r}$, then the equilibrium is NRA.

Proposition 3 states that there will be rationing in equilibrium (if the equilibrium exists) if $\hat{r} > U(\bar{R}_{\max})$. Since $U(\bar{R}_{\max})$ is finite, there exist functions $h(\cdot)$ and intervals $[\underline{t}, \bar{t}]$ such that this condition holds [see (4.14)].

4.4. Dominance of direct lending by intermediation

Let $U_{\mathrm{D}}(\bar{R})$ denote the counterpart of $U(\bar{R})$ for the regime with direct lending, i.e., $U_{\mathrm{D}}(\bar{R})$ is the expected utility attained by a direct lender as a function of the loan interest rate. Then from (3.7),

$$U_{\mathrm{D}}(\bar{R}) = \int_0^{\bar{R}} w f(w)\,\mathrm{d}w + \bar{R}(1 - F(\bar{R})) - cF(\bar{R}). \tag{4.15}$$

Then from (4.7), since $K \geq 2$, $U(\bar{R}) > U_{\mathrm{D}}(\bar{R})$ for all $\bar{R} \in (0, \bar{w}]$. Suppose then that an equilibrium $(\bar{R}^*, r^*, q^*)$ exists for the regime with direct lending of section 3. Then intermediaries can enter the lending industry, offering entrepreneurs loan contracts at the interest rate $\bar{R}^*$, and attain a level of expected utility greater than r^* for themselves and their depositors, since $U(\bar{R}^*) > r^* = U_{\mathrm{D}}(\bar{R}^*)$. Note also that if an equilibrium $(\bar{R}^{**}, r^{**}, q^{**})$ exists for the regime with intermediation, then direct lending cannot upset this, since $U_{\mathrm{D}}(\bar{R}^{**}) < r^* = U(\bar{R}^{**})$, i.e., in equilibrium lenders prefer to act as intermediary agents or as depositors, rather than as direct lenders. Therefore, intermediation will drive direct lending out of the system.

4.5. Remarks

In our model, as in that of Stiglitz and Weiss (1981), credit rationing may be a characteristic of loan market equilibrium; all entrepreneurs are identical, ex ante, but in equilibrium it may be the case that some receive loans and others do not. A unique feature of our framework is that this phenomenon arises in an *intermediated* loan market. The costly monitoring of borrowers by lenders creates the possibility of equilibrium credit rationing, while costly monitoring in conjunction with large-scale investment projects implies a role for financial intermediation. The fact that financial intermediation and equilibrium credit rationing are linked in our model is important since (1) most lending in developed economies occurs through intermediaries and (2) credit rationing is usually associated in the literature with intermediated lending.[9]

Here, as in papers by Boyd and Prescott (1985) and Diamond (1984), financial intermediation dominates direct lending as a means for financing investment projects under asymmetric information. In our model and those of Boyd and Prescott and Diamond, intermediation produces information more efficiently and does this through diversification, in spite of universal risk neutrality.

The financial intermediaries in our model exhibit several important features of intermediaries as we know them. They issue securities with return characteristics which are different from those of the assets they hold, they manage a diversified portfolio, their assets are debt claims, and they process information. Intermediaries in this framework also may ration credit in equilibrium, though credit rationing has much the same status as involuntary unemployment in terms of its acceptance as an empirical fact.

5. Comparative statics

In this section we examine the responses of equilibrium $\bar{R}$, r and q to changes in parameters and distribution functions. These responses are quantitatively different and frequently qualitatively different, depending on whether the equilibrium is RA or NRA.

We carry out three experiments. Experiment 1 is a shift in the schedule of alternative rates of return faced by lenders, interpreted as a downward shift in the supply of funds faced by intermediaries. That is, the function $h(\cdot)$ shifts to $h^*(\cdot)$, where $h^*(t) = h(t-\varepsilon)$, with $\varepsilon > 0$. Experiment 2 is a change in c, the cost of monitoring, and experiment 3 is a mean preserving spread in the distribution of project returns, $f(\cdot)$. That is, in experiment 3 we change $f(\cdot)$ by shifting probability mass from around equilibrium $\bar{R}$ to the tails of the

[9] It is no accident that Stiglitz and Weiss (1981) refer to their lending institutions as 'banks', though Stiglitz–Weiss 'banks' have few bank characteristics which are not imposed.

distribution. The probability density function is changed to

$$f^*(x) = f(x) + \delta g(x),$$

where $0 < \delta \leq 1$ and $g(x)$ is continuous on $[0, \overline{w}]$. Let

$$G(x) = \int_0^x g(t)\,\mathrm{d}t, \qquad 0 \leq x \leq \overline{w}.$$

Then $g(x)$ and $G(x)$ have the following properties:

$$G(\overline{R}^*) = 0, \tag{5.1}$$

$$f(x) + g(x) > 0, \qquad x \in [0, \overline{w}], \tag{5.2}$$

$$\int_0^{\overline{w}} zg(z)\,\mathrm{d}z = 0, \tag{5.3}$$

$$\int_0^x G(z)\,\mathrm{d}z \geq 0, \qquad x \in [0, \overline{w}]. \tag{5.4}$$

Condition (5.3) states that the change in the distribution is mean-preserving. Given (5.1), the quantities of probability mass to the right and to the left of $\overline{R}^*$ (the equilibrium level of $\overline{R}$) do not depend on δ, and (5.4) states that the shift in probability mass increases risk in the sense of Rothschild and Stiglitz (1970).

5.1. NRA equilibrium

An NRA equilibrium is the solution to eqs. (4.8) and (4.9). The quantity of loans, q, will not be affected by any of the above experiments. From eq. (4.9), experiment 1 will bring about an increase in r, the market interest rate. We therefore characterize experiment 1 as a differential change in r, experiment 2 as a differential change in c, and experiment 3 as a differential change in δ, evaluated at $\delta = 0$. Comparative statics results were obtained by totally differentiating eq. (4.8) and solving. We let ∇_1 denote the quantity

$$\nabla_1 = 1 - F(\overline{R}) - (c/K)f(\overline{R}).$$

Given (4.10) and ignoring the borderline case, we have $\nabla_1 > 0$.

Experiment 1. $\mathrm{d}\overline{R}/\mathrm{d}r = 1/\nabla_1 > 0.$

Experiment 2. $\mathrm{d}\overline{R}/\mathrm{d}c = F(\overline{R})/K\nabla_1 > 0.$

Experiment 3. $\mathrm{d}\overline{R}/\mathrm{d}\delta\,|_{\delta=0} = -\int_0^{\overline{R}} wg(w)\,\mathrm{d}w/\nabla_1 > 0.$

5.2. RA equilibrium

An RA equilibrium is the solution to eqs. (4.8), (4.11) and (4.12). Here, in contrast to the NRA equilibrium, changes in the underlying cost parameters and distributions will in general change r, and this will change q, the aggregate quantity of loans, and the quantity of rationing. Since an increase in r leads to an increase in q (except for experiment 1), we need to determine only the sign of the change in r to determine the qualitative effect on q.

We let ∇_2 denote the following quantity:

$$\nabla_2 = f(\bar{R}) + (c/K)f'(\bar{R}).$$

Given (3.8), we have $\nabla_2 > 0$.

Totally differentiating eqs. (4.8) and (4.11) and solving, we obtain:

Experiment 1. $\mathrm{d}\bar{R}/\mathrm{d}\varepsilon = \mathrm{d}r/\mathrm{d}\varepsilon = 0$, $\mathrm{d}q/\mathrm{d}\varepsilon < 0$.

Experiment 2. $\mathrm{d}\bar{R}/\mathrm{d}c = -f(\bar{R})/K\nabla_2 < 0$, $\mathrm{d}r/\mathrm{d}c = -F(\bar{R})/K < 0$, $\mathrm{d}q/\mathrm{d}c < 0$.

Experiment 3. $\mathrm{d}\bar{R}/\mathrm{d}\delta\,|_{\delta=0} = -cg(\bar{R})/K\nabla_2 > 0$, $\mathrm{d}r/\mathrm{d}\delta\,|_{\delta=0} = \int_0^{\bar{R}} wg(w)\,\mathrm{d}w < 0$, $\mathrm{d}q/\mathrm{d}\delta\,|_{\delta=0} < 0$.

The quantitative effects on $\bar{R}$, r and q of each experiment *always* differ, depending on whether the equilibrium is RA or NRA. Also, except for the effect on $\bar{R}$ in experiment 3, variables either move in different directions in the NRA and RA equilibria or remain unchanged in one type of equilibrium while increasing or decreasing in the other. In addition, note the following:

(1) In the RA equilibrium, all experiments have an effect on the aggregate quantity of loans, q, while these effects are absent in the NRA equilibrium.[10] Note also, in this respect, that experiment 1 has no effect on interest rates in the RA equilibrium. This result is consistent with the thrust of the availability doctrine [see Roosa (1951)], i.e., monetary policy can have real effects without affecting interest rates in lending markets. For there to be any real effects, of course, monetary policy must change some real interest rate(s), and just how this might occur would have to be worked out by embedding our model in a general equilibrium framework.

[10] In general, there would be effects on the quantity of loans in the NRA equilibrium if the demand for loans were not inelastic. However, what is important here is that there are effects on the amount of rationing in the RA equilibrium which are absent in the NRA equilibrium.

(2) For experiment 1, in the NRA equilibrium an increase in r not only increases $\bar{R}$, but also increases the difference between $\bar{R}$ and r. This result was also obtained in Williamson (1983, 1984b) and is consistent with the stylized fact that interest rate differentials increase with an increase in all interest rates.

(3) With an increase in the riskiness of entrepreneurs' investment projects in experiment 3, the loan interest rate, $\bar{R}$, increases, i.e., there is a risk premium effect in spite of the fact that all agents are risk-neutral. This occurs due to a corresponding increase in the probability of default for entrepreneurs which increases the expected cost of monitoring.

From Proposition 3, we note that anything that would increase $\hat{r}$ or decrease $U(\bar{R}_{max})$ would make an equilibrium with credit rationing more likely. First, given (4.14), experiment 1 will increase $\hat{r}$. Next, to determine what will bring about decreases in $U(\bar{R}_{max})$, note that in an RA equilibrium $U(\bar{R}_{max})=r$. Therefore, experiments 2 and 3 will reduce $U(\bar{R}_{max})$. We then conclude that increases in alternative rates of return, in monitoring costs, and in project riskiness all increase the likelihood of equilibrium credit rationing.

6. Summary and conclusion

A shortcoming of previous studies in the credit rationing literature [for example, Jaffee and Russell (1976), Keeton (1979) and Stiglitz and Weiss (1981)] is that the lending institutions in these models have few of the features one would associate with real-world intermediaries (other than what is assumed), in spite of the fact that these analyses are often clearly intended to apply to intermediated markets. The main purpose of this paper has been to demonstrate that equilibrium credit rationing can occur in an environment where financial intermediation is motivated from first principles. In fact, in the model considered, intermediation and credit rationing are related phenomena, in that the same set of assumptions can produce both.

There are two types of equilibria in our model, those with credit rationing and those without. Both types of equilibria are possible either with direct lending (if intermediation is prohibited) or in a regime with financial intermediation. Credit may be rationed in the sense of Stiglitz and Weiss (1981), in that all entrepreneurs (potential borrowers) are identical, ex ante, but it may be the case that some receive loans and others do not. In contrast to Stiglitz and Weiss (1981), this does not occur due to moral hazard or adverse selection, though informational asymmetries are crucial. Instead, the costly monitoring of borrowers by lending agents (either intermediaries or direct lenders) implies, given risk neutrality, that debt contracts are optimal. There is therefore an asymmetry in the payoff functions of borrowers and lenders, which creates the possibility of equilibrium credit rationing.

Debt contracts are also derived as the optimal arrangement between borrowers and lenders in Diamond (1984) and Gale and Hellwig (1985), though the costs of 'bankruptcy' in Diamond's paper are non-pecuniary costs to the borrower rather than monitoring costs, as in our model. Debt contracts are an important element in generating equilibrium credit rationing in Stiglitz and Weiss (1981), as they are here. However, Stiglitz and Weiss simply assumed that the contracting arrangement took this form.

Financial intermediation dominates direct lending in our model as a result of costly monitoring and large scale investment projects. As in Diamond (1984), intermediation performs a 'delegated monitoring' role, and intermediaries are single agents. However, in our model we do not distinguish separate intermediary agents at the outset as Diamond does, but instead allow agents in the model to freely choose activities given their endowments and preferences. Such an approach is also taken in Boyd and Prescott (1985), though in their framework intermediaries are multi-agent coalitions. Financial intermediaries in our model share several of the important features of intermediaries as we know them; they issue securities which have payoff characteristics which are different from those of the securities they hold, they write debt contracts with borrowers, they hold diversified portfolios, and they process information.

It was shown that an equilibrium with intermediation, if it exists, is unique and that this equilibrium will be one of two types: either credit is rationed or it is not. An equilibrium can be simply characterized, and it is then relatively straightforward to derive implications concerning the effects on observable variables (market interest rates and the quantity of lending) of changes in the parameters of preferences and technology. Similar implications are not a part of any of the other financial intermediation studies cited here. These implications, which are broadly consistent with the results in Williamson (1984b) for direct lending, follow from the comparative statics experiments in section 5 of the paper and can be summarized as follows.

1) The responses of the endogenous variables are always quantitatively different for the two types of equilibria and are frequently qualitatively different. 2) Quantity effects which are absent in an equilibrium without rationing are a feature of rationing equilibria. 3) There are risk premium effects on the loan interest rate in spite of universal risk neutrality. 4) An increase in all interest rates is consistent with an increase in interest rate differentials. 5) Changes in alternative rates of a return can produce changes in the quantity of lending without interest rate effects.

We note in conclusion that it is not clear, in spite of the fact that some of the results might be interpreted as being consistent with the availability doctrine, that such an interpretation would provide a role for monetary policy as envisioned by proponents of this doctrine [see Roosa (1951)]. To draw normative conclusions for monetary policy, our model would have to be embedded in a more fully-specified dynamic general equilibrium framework.

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