# Banking Panics, Information, and Rational Expectations Equilibrium

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#### ABSTRACT

This paper shows that bank runs can be modeled as an equilibrium phenomenon. We demonstrate that some aspects of the intuitive "story" that bank runs start with fears of insolvency of banks can be rigorously modeled. If individuals observe long "lines" at the bank, they correctly infer that there is a possibility that the bank is about to fail and precipitate a bank run. However, bank runs occur even when no one has any adverse information. Extra market constraints such as suspension of convertibility can prevent bank runs and result in superior allocations.

BANKING PANICS WERE A recurrent phenomenon in the United States until the 1930s. They have re-emerged as a source of public concern and much theoretical research recently. In this paper, we provide an information-theoretic rationale for bank runs. The traditional "story" is that contagion is an important aspect of bank runs. The idea seems to be that when the general public observes large withdrawals from the banking system, fears of insolvency grow resulting in even larger withdrawals of deposits.

In our model, some individuals withdraw because they get information that future returns are likely to be low. Uninformed individuals observing this also have an incentive to liquidate their investments. In addition, some individuals need to withdraw deposits for other than informationally based reasons. Thus, if the random realization of such a group of individuals is unusually large, then the uninformed individuals will be misled and will precipitate a run on the bank. The technology is such that a large volume of withdrawals involves liquidation costs. Consequently, runs on the bank do impose social costs.

A mechanism that may reduce these costs in our model is to suspend convertibility if withdrawals are high. However, those individuals who need to withdraw their assets for liquidity reasons are worse off ex post. Our model provides a rationalization for restrictions in demand deposits that were widespread prior to 1929. Friedman and Schwartz [7] suggest that restrictions of payments ensured

\* Chari, Federal Reserve Bank of Minneapolis; Jagannathan, J. L. Kellogg Graduate School of Management, Northwestern University, and University of Minnesota. The authors are deeply indebted to Larry E. Jones. We also wish to thank the seminar participants at Northwestern University, the University of Chicago, and the Twentieth Annual Conference of the Western Finance Association, where earlier versions of this paper were presented. Chari's research was supported by the Banking Research Center, Northwestern University. Jagannathan's research was supported by a grant from the McKnight foundation to the University of Minnesota. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. that "the panic(s) had a reasonably small effect on the banking structure  $\cdots$  and gave time for the immediate panic to wear off." However, "they were regarded as anything but a satisfactory solution by those who experienced them, which is why they produced such strong pressure for monetary and banking reform" (ibid., p. 329).

Our model is closely related to Diamond and Dybvig [5]. They model banks as providing insurance for individuals who are uncertain about their liquidity needs. Investment in assets with long maturities yields higher return than in short maturities. The optimal contract yields a higher level of consumption for those who withdraw early than the technological return. Consequently bank runs occur when every agent believes that all other agents will withdraw early. Essential to this story is that the bank must honor a sequential service constraint. We do not impose such a constraint. Jacklin [10] in a very similar framework to ours addresses the question of the choice between deposit and equity contracts given that individuals may get information about future returns. Again, a key characteristic is that banks are not allowed to make deposit contracts contingent upon the number of people who desire to withdraw.

Gorton [8] models bank runs as precipitated by the perception on the part of depositors that the return on currency exceeds that on deposits. Banks are better informed about the state of their investments. The driving feature of the model is the assumption that interest rates on deposits cannot be raised or lowered by the banks. Thus, the optimal ex ante agreement specifies that when depositors' expectations about future returns are wrong, the bank suspends convertibility. This occurs only because the bank is assumed not to be able to change the interest payments on deposits in the interim period when information about the state of the bank's investments is revealed (fully to the banks, imperfectly to the depositors).

Bhattacharya and Gale [3] consider a variant of the Diamond-Dybvig model in which there are many intermediaries, each of which has access to information only about the proportion of the population who withdraw from it at the interior stage. They demonstrate that there are welfare gains from setting up an institution such as a central bank or, at any rate, a market for intermediaries to trade in the interim period.

Bhattacharya and Jacklin [4] in a paper similar in some ways to ours consider the choice between deposit and equity contracts in an environment in which some agents receive superior information about future expected returns in the interim period. Their interest is primarily in characterizing the relationship between the riskiness of the underlying stream of returns and the desirability of equity contracts over deposit contracts. Our interest is in developing a model of the information revealed to depositors by the withdrawal decision of *other* depositors.

## I. The Model

We consider an environment where people live for three periods: a planning period, time 1, and time 2. There is a single commodity. An investment decision is made during the planning period that yields a sure return at time 1. If resources are reinvested in period 1 they generate a random return at time 2. If resources are not reinvested, there is a liquidation cost that depends upon the level of consumption. There are a large number of individuals (technically, a continuum on the interval [0, 1] on which the Lebesque measure is induced), each of whom has access to the blueprint technology.

## A. Technology

The idea behind the technology specified here is straightforward. Individuals invest in the planning period. They receive a random but high expected return in period 2 if the investment is not liquidated in period 1. The return in period 1 is affected by an *exogenously* imposed externality. If many individuals wish to consume in period 1, then each individual's consumption is low. If only a few individuals wish to consume in period 1, then the total return on investment is 1-i.e., a unit invested in period 0 will yield one unit of output in period 1. Our attempt here is to capture some notion of liquidity. The idea is that investments are, at least in part, illiquid but they can be transformed into consumption goods at a cost that depends upon aggregate amount of consumption.

An investment plan for an individual is a pair of numbers  $(k_0, k)$  representing investment in periods 0 and 1, respectively. Realized output is a pair of numbers  $(y_1, y_2)$  in periods 1 and 2, respectively. Investment decisions are costly to liquidate in period 1. In particular, the cost of liquidation depends upon the aggregate investment decisions made in the economy. Let K represent the aggregate volume of investment. Then, output for any individual's technology is

$$y_1 = k_0 - k \qquad \text{if} \quad K \ge K$$
  
$$y_1 = (1 - a)(k_0 - k) \qquad \text{otherwise} \qquad (1)$$

where  $0 \le a \le 1$  and  $\overline{K}$  are exogenously specified.

Output in period 2 is random and is given by

$$y_2 = \tilde{R}k, \tag{2}$$

where R is a random variable which takes the value H with probability p and L with probability (1 - p), with H > L. For convenience, we set L = 0.

#### **B.** Preferences

All agents in this economy are risk neutral and maximize expected utility of consumption. There are two types of individuals in the economy. Type-1 agents care only about consumption in period 1. Type-2 agents derive utility from consumption in both periods 1 and 2. The utility functions of the respective types are given by

$$U^{1}(c_{1}, c_{2}) = c_{1} + \beta c_{2}$$

$$U^{2}(c_{1}, c_{2}) = c_{1} + c_{2}$$
(3)

where the pair  $(c_1, c_2)$  represents consumption levels of the commodity in periods 1 and 2, respectively, and  $\beta$ , the discount factor, is positive and arbitrarily close to zero.

No individual knows his or her type at the planning period. A random fraction  $\tilde{t}$  of individuals are of type 1. The random variable  $\tilde{t}$  can take on only finitely many values. For ease of exposition we assume that  $\tilde{t}$  can take on one of three values,  $t \in \{0, t_1, t_2\}$  with probabilities  $r_0, r_1$ , and  $r_2$ , respectively. Setting the first element to zero is without loss of generality.

# C. Endowments

All agents are endowed with one unit of the good at the planning period.

#### D. Information

At the beginning of time 1, a random fraction  $\tilde{\alpha}$  of type-2 agents receive information about prospective time-2 returns. We will assume that this information is perfect. The fact that individuals are risk neutral implies that this assumption is innocuous. The fraction  $\tilde{\alpha}$  of type-2 individuals who receive this information can take on two values,  $\alpha \in \{0, \tilde{\alpha}\}$ . The probability that  $\tilde{\alpha} = \bar{\alpha}$  is qand the probability that  $\tilde{\alpha} = 0$  is (1 - q).

We will assume that the random variables  $\tilde{t}$ ,  $\tilde{R}$  and  $\tilde{\alpha}$  are independent of each other. Let  $\theta \equiv (t, R, \alpha)$ , the triplet that represents the state of the world. We will denote the set of all possible values for  $\theta$  by  $\Theta$ .

No individual at the planning period knows whether he or she will be informed or not. Furthermore, the realization of  $\tilde{t}$ ,  $\tilde{\alpha}$  and the information received by the informed agents (if any) of the economy are not observable by other agents in the economy. The only information that is public is the aggregate investment decision. To put it differently, what is observed is the fraction of the population that chooses to continue investing, rather than the reasons for doing so.

#### E. Parameter Restrictions

In order to ensure that individuals have a nontrivial signal-extraction problem upon observing the bank's balance sheet, we clearly need "confounding." Assume that

$$t_1 = \bar{\alpha} \tag{4}$$

$$t_2 = t_1 + \bar{\alpha}(1 - t_1). \tag{5}$$

Further, assume that, absent any information, it is desirable to continue the investment. Thus:

$$pH + (1-p)L > 1. (6)$$

In some of what follows, results do depend upon the magnitude of  $\overline{K}$ . In general, any concave transformation of investment into consumption will suffice for the results. It will be assumed, for reasons that will become apparent, that

$$\bar{K} = 1 - t_2. \tag{7}$$

#### II. Equilibrium

The decision problem in the planning period is trivial since no individual cares about period-0 consumption. All individuals therefore choose to invest one unit in period 0. The decision problem at time 1 of type-1 agents is also trivial. They consume all their resources by liquidating all their investment. The sequencing of the decisions of other agents is as follows. At time 1, t is first realized and every individual knows his or her own type. A random fraction  $\alpha \in \{0, \bar{\alpha}\}$  of type-2 agents receive the informative signal. Agents then decide how much of their investment to liquidate. The decision problem of type-2 agents who get the informative signal is also trivial. Given the fact that H > 1 (see equation (6)), they will liquidate their investment only if they get information that the return on the project will be low, i.e.,  $\tilde{R} = L$ . Before making the investment decision, agents observe K, the aggregate investment level at date 1. All uninformed type-2 agents will realize that the equilibrium level of K is correlated with the signal received by the informed agents and hence "reveal," albeit imperfectly, their signal. It is important to realize that the aggregate investment could be low *either* because the value of t is high or because some type-2 agents have received information that prospective returns are low. It is this confounding that is crucial to our results. Agents take this into account in their decisions.

The problem faced by the representative type-2 uninformed agents is:

$$\max_{k} c_{1} + \int c_{2} dF(\theta \mid K)$$
(8)

subject to

$$c_1 = (1 - a)(1 - k)$$
 if  $K < \overline{K}$  and  $1 - k$  otherwise  
 $c_2 = w + kR$ 

where w is the period-2 endowment and  $F(\theta | K)$  denotes the distribution of  $\theta$  conditional on knowing the aggregate investment level K at time 1.

Let the solution to this problem be denoted by k = k(K). As pointed out earlier, for the informed type-2 agent k = 1 if  $\tilde{R} = H$  and k = 0 if  $\tilde{R} = L$ . We will denote the informed agent's investment at time 1 as  $k^{I}(R)$ .

The aggregate demand for investment at time 1,  $K_D$ , for any level of observed aggregate investment, K, can now be defined.

$$K_D = \alpha (1 - t) k^I(R) + (1 - \alpha) (1 - t) k(K).$$
(9)

Of course, in equilibrium  $K_D = K$ . The first term on the right-hand side is the aggregate investment decision of the informed type-2 agents. The second term on the right is the aggregate investment decision of the uninformed type-2 agents. Recall that type-1 agents have no desire to invest and that the informed type-2 agents will liquidate their investment at time 1 if and only if their signal is  $\hat{R} = L$ . Table I gives  $K_D$  for every state of the world  $\theta$ .

Definition 1: Rational-Expectations Equilibrium. A rational-expectations equilibrium is: (i) an aggregate investment function  $K(\theta)$  that specifies the

Sta	te of Nature	The aggregate investment demand function	
No.	$\theta \equiv \{t, R. \alpha\}$	$K_D(\cdot)$ defined in equation (9)	
1	(0, R, 0)	<i>k</i> ( <i>K</i> )	
2	$(0, H, \bar{\alpha})$	$\bar{\alpha} + (1 - \bar{\alpha})k(K)$	
3	$(0, L, \bar{\alpha})$	$(1-\tilde{\alpha})k(K)$	
4	$(t_1, R, 0)$	$(1-t_1)k(K)$	
5	$(t_1, H, \bar{\alpha})$	$(1-t_1)\{\bar{\alpha}+(1-\bar{\alpha})k(K)\}$	
6	$(t_1, L, \bar{\alpha})$	$(1-t_1)(1-\bar{\alpha})k(K)$	
7	$(t_2, R, 0)$	$(1-t_2)k(K)$	
8	$(t_2, H, \bar{\alpha})$	$(1-t_2)\{\bar{\alpha}+(1-\bar{\alpha})k(K)\}$	
9	$(t_2, L, \bar{\alpha})$	$(1-t_2)(1-\bar{\alpha})k(K)$	

 Table I

 Aggregate Demand Function for Investment

aggregate investment K for each state of nature  $\theta$ ; (ii) an investment demand function k(K) for each uninformed type-2 agent such that:

- (a)  $K(\theta) = \alpha(1-t)k^{I}(R) + (1-\alpha)(1-t)k(K(\theta))$ , for all  $\theta$ ,
- (b) k(K) solves the maximization problem in equation (8),
- (c) if  $\alpha(1-t)k^{I}(R) + (1-\alpha)(1-t)k(\cdot) = \alpha'(1-t')k^{I}(R) + (1-\alpha')(1-t')k(\cdot)$  for any two states  $\theta = \{t, R, \alpha\}$  and  $\theta' = \{t', R', \alpha'\}$ , for all functions  $k(\cdot)$ , then  $K(\theta) = K(\theta')$ .

Condition (a) is a consistency requirement that is analogous to the familiar market-clearing condition. It merely requires that aggregate outcome be the same as the sum of individual decisions. It is trivial to verify that the assumed symmetry of individual decisions is in fact the outcome of optimizing behavior on the part of agents. Condition (b) needs no explanation. Condition (c) requires some explanation. The analogue of this condition is found in rational-expectations equilibria in competitive markets. In such a case several authors have found it necessary to impose a condition that essentially states that the price function must be measurable with respect to the market excess-demand function of agents.<sup>1</sup> In order to see the need for such a condition as the one we have imposed, note that without this condition, the market could reveal more information than anybody in the economy possesses. For example, K = 0 whenever  $\tilde{R} = L$  and K = 1 - t whenever  $\tilde{R} = H$ , regardless of the value of  $\alpha$ , is consistent with conditions (a) and (b) of the definition of the equilibrium. This is obviously absurd and reflects the fact that there is no mechanism in conditions (a) and (b) alone describing how information is aggregated in a competitive environment. We could alternatively impose the requirement that equilibrium outcomes be measurable with respect to the join of all the information possessed by all the agents in the economy. What we seek to capture in this model, however, is the notion that equilibrium outcomes reflect the information that individuals possess through their decisions rather than through some arbitrary process. It is appropriate to view the right side of equation (9) as an aggregate excess-demand function for

<sup>&</sup>lt;sup>1</sup> See Admati [1], Anderson and Sonnenschein [2], and Diamond and Verrecchia [6].

investment. With this interpretation, it seems appropriate that if the aggregate excess-demand functions are the same for two states of the world, then the equilibrium outcomes should also be the same.

Definition 2: Panic Equilibrium. A rational-expectations equilibrium is a panic equilibrium if the equilibrium aggregate investment is zero for at least one state in which  $\alpha = 0$ , i.e.,  $\theta = \{t, R, 0\}$ .

In other words, in a panic equilibrium every one liquidates his or her investment at date 1 even though no one has any information about the return next period.

It is worthwhile to contrast the panic-equilibrium outcome with the fullinformation outcome. In this case, given that pH + (1-p)L > 1, it follows that individuals would want to continue investing when no one receives the informative signal. Hence the panic-equilibrium outcome can occur only if there is confounding between a large number of individuals unexpectedly desiring to liquidate their investments for "transactions" reasons and the possibility that some individuals have received information that returns are expected to be poor. Theorem 1 below establishes sufficient conditions for all rational-expectations equilibria to be panic equilibria.

**THEOREM 1.** A. Given restriction (6), if the following conditions hold, there exists a rational-expectations equilibrium that is also a panic equilibrium:

$$\frac{r_1(1-q)pH}{r_0(1-p)q+r_1(1-q)} > 1$$
(10)

$$\frac{r_2 p H}{r_1 (1-p)q + r_2 (1-q) + r_2 p q} < (1-a).$$
(11)

B. In this economy, every rational-expectations equilibrium is also a panic equilibrium.

**T T** 

Proof: See the Appendix. Q.E.D.

Remark 1: In a rational-expectations equilibrium a panic (i.e., K = 0 even when no one has any adverse information) can never occur in states in which some agents receive the informative signal (i.e.,  $\alpha = \overline{\alpha}$ ) and R = H. Hence any panic equilibrium must involve K = 0 in some subsets of states from the collection  $\{1, 4, 7\}$  in Table I. Using the fact that K(3) = K(4) and K(6) = K(7) to satisfy the measurability condition (c), it can be verified that the only other equilibria in this economy are those in which either (a) K = 0 in states 1, 3, 4, 6, 7, and 9; or (b) K = 0 in states 1, 6, 7, or 9; or (c) K = 0 in states 3, 4, 6, 7, and 9. In a sense there is "minimal" panic in the equilibrium we have described in Table II. For this reason if extra market arrangements like "suspension of convertibility" improve on the allocations of the equilibrium described in Table II, they will also improve on the allocations obtained in the other equilibria in this economy.

Remark 2: The most serious problem with the model presented here is the absence of markets for trading in asset  $claims^2$ . It is possible to show that the rational-expectations equilibrium allocations can be supported as a competitive

<sup>&</sup>lt;sup>2</sup> We are indebted to Sandy Grossman for pointing this out to us.

State					Investment K in a Rational Expectations Equilibrium			
No.	t	R	a	Probability	Without Suspension of Convertibility	With Suspension of Convertibility		
1	0	no	info	$\pi_1$	1	1		
2	0	H	ā	$\pi_2$	1	1		
3	0	L	ā	$\pi_3$	$1 - t_1$	$1 - t_1$		
4	$t_1$	no	info	$\pi_4$	$1 - t_1$	$1 - t_1$		
5	$t_1$	H	ā	$\pi_5$	$1 - t_1$	$1 - t_1$		
6	$t_1$	L	ā	$\pi_6$	0	$1 - t_1$		
7	$t_2$	no	info	$\pi_7$	0	$1 - t_1$		
8	$t_2$	HG	ā	$\pi_8$	$1 - t_2$	$1 - t_1$		
9	$t_2$	L	ā	$\pi_9$	0	$1 - t_1$		
(a) (b)	Notes: (a) $t_2 = t_1 + (1 - t_1)\bar{\alpha}$ (b) $\pi_1 = r_0(1 - g); \pi_2 = r_0pq; \pi_3 = r_0(1 - p)q; \pi_4 = r_1(1 - q); \pi_5 = r_1pq; \pi_6 = r_1(1 - p)q; \pi_7 = r_2(1 - q); \pi_8 = r_2pq;$							
$\pi_5 = r_1 pq; \ \pi_6 = r_1(1-p)q; \ \pi_7 = r_2(1-q); \ \pi_8 = r_2 pq; \pi_9 = r_2(1-p)q.$ (c) K is the equilibrium aggregate investment in the economy in ime 1.								

A "Minimal" Panic Equilibrium for the Economy
Described in the Text

equilibrium in asset markets where short sales and forward contracts are prohibited.

The model presented above has no apparent role for a bank or other financial intermediary. A few comments are appropriate about our modeling choices. Our intent here has been to develop an explicit model where the observed decisions of *other* agents are relevant for the decision making of uninformed agents. Obviously, banks play many roles other than providing insurance to depositors. Among other things, they monitor the actions of their debtors and provide means of payments. For our purposes, therefore, we think it misleading to focus on the liquidity services that banks provide. Instead, we chose to focus on the information content of the "line length" and the withdrawal decisions of other depositors. From that perspective, also, risk aversion on the part of depositors would have caused us to focus on the optimal insurance contract. Since these issues have been addressed extensively in the literature we chose a framework that would focus on issues of information.

The most troublesome (to us) issue is the fact that liquidation costs are exogenously imposed rather than generated from deeper elements of technology and preferences. Obviously, a richer model would have many intermediaries, perhaps geographically isolated, and would generate liquidation costs from the difficulties of monitoring a particular intermediary's portfolio choice. One interpretation of our model is that we focus on runs on a particular bank. Our preferred interpretation is that this is a first step in modeling information-based bank runs. The obvious extension is to consider interbank trades in the interim stage.

The relationship between the equilibria discussed in this section and observed banking panics is discussed in Section III below.

# III. Equilibria With Suspension of Convertibility

The model described in Section II is best thought of as a production economy with no apparent role for a financial intermediary. We describe below a mechanism that can, under appropriate circumstances, yield allocations that are superior in terms of ex ante expected utility to the allocations associated with the rational-expectations equilibrium and the market equilibrium discussed in Section II. The tradeoff for the equilibrium in Section II is between the fact that liquidation costs are sometimes incurred, while at the full-information optimum these costs would not have been incurred (see row 7, Table II). A mechanism is described below that can dominate the equilibrium allocations. This mechanism is closely linked to "suspension of convertibility."

The idea is that individuals pool their resources and entrust them to a "bank" at the planning period. The observable magnitude is the assets of the bank. These assets correspond to K at date 1. At date 1, individuals "queue" up at the bank to make their withdrawals. The bank is permitted to ration depositors at date 1. We assume that the conditions of Theorem 1 hold. Given the results in Theorem 1 and the remarks following the theorem, suspension of convertibility is meaningful only if no more than the first  $t_1$  investors are permitted to withdraw at will at time 1 but every one else must wait. Suspending convertibility at a level of withdrawal higher than  $t_1$  will be of no help in preventing panic in state 7.

It is then readily verified that the only equilibrium with suspension of convertibility, when the bank suspends convertibility whenever withdrawals reach a level of  $t_1$ , will be the one described in Table II.

Clearly, there is gain in the state when there is no information available (state 7), but a loss because either (a) investments are made even though prospective returns are low (states 6 and 9), or (b) some type-1 agents who get little utility from time-2 consumption are prevented from withdrawing when the return is known to be high (state 8).

Suspension of convertibility will improve ex ante expected utility if

$$\sum_{j=1}^{9} \pi_j [U_s(j) - U_e(j)] > 0, \qquad (12)$$

where  $\pi_j$  denotes the probability that the event described by state j, j = 1, 2, ..., 9 will occur, and  $U_s$  and  $U_e$  denote utilities with and without suspension of convertibility, respectively. Substituting the expressions for  $\pi_j$ , j = 1, 2, ..., 9, we get

$$r_{2}(1-q)[t_{1}+\beta(t_{2}-t_{1})pH+(1-t_{1})pH-(1-a)]$$

$$> r_{1}(1-p)q[(1-a)-t_{1}]+r_{2}(1-p)[(1-a)-t_{1}]$$

$$+ (t_{2}-t_{1})(1-\beta H)r_{2}pq. \quad (13)$$

Inequality (13) has been written so as to highlight the potential conflict between the possibility that suspension of convertibility can gain in ex ante terms and the fact that, in order for the equilibrium to involve runs on banks without suspension of convertibility the conditions of Theorem 1 must hold. This establishes Theorem 2.

**THEOREM** 2. If inequalities (10), (11), and (13) hold, the equilibrium with suspension of convertibility yields higher ex ante utility than the rational-expectations equilibrium<sup>3</sup> of Section II.

It is important to note that suspension of convertibility improves upon the rational-expectations equilibrium outcomes primarily because of the specification of the liquidation technology. The liquidation costs have been set up so that sudden or large-scale withdrawals are costly.

Of course, such suspension of convertibility accompanied by random assignment of withdrawal rights leaves some individuals worse off than others who are identically situated. In a sense, therefore, the fact that suspension of convertibility was consistently practiced in every bank run but that there were many ex post complaints about this state of affairs is explained by this model.

The assumption that individuals are risk neutral also plays a role in our evaluation of the welfare consequences, since consumption is different across otherwise identical individuals with suspension.

The role of a financial intermediary in the model we consider is solely to allow agents to coordinate their strategies. Intermediaries clearly perform many other functions, so our description of the role of a bank must be viewed primarily as a means of focusing attention on one issue. Within this perspective, it is optimal to ration consumption at date 1. One means of rationing consumption is to form a "mutual fund" that is required as part of the rules of the game to suspend convertibility under certain circumstances. This policy inevitably carries ex post regret. Those who are rationed out get less consumption than their lucky colleagues.

## **IV.** Conclusion

In a sense this paper is an extended example. Expanding the number of states yields no major changes in the results, so we have chosen to restrict the number of states in order to make the results more transparent. We have established that bank runs can be modeled as an equilibrium phenomenon in a model in which all equilibria have bank runs. Previous work (Diamond and Dybvig) generates bank runs as one of a series of possible multiple equilibria. Multipleequilibria models in which only some of the equilibria involve bank runs of course suffer from the problem that they have limited predictive power. We have demonstrated that some aspects of the intuitive "story" that bank runs start with fears of insolvency of particular banks and then spread to other sectors can be

<sup>&</sup>lt;sup>3</sup> It is possible to show that the set of parameters that satisfy the conditions of Theorem 2 are nonempty. For example, for the parameter values given in the Appendix, the left side of inequality (13) is 0.02229 and the right side of the inequality is 0.01887.

rigorously modeled. The essence of our model is that if individuals observe long "lines" at banks, they correctly infer that there is a possibility that the bank is about to fail and precipitate a bank run. Bank runs occur *even if* no one has any adverse information about future returns.

In making explicit this "story," it has been necessary to abstract from the important issues of what exactly banks do. Instead, our focus is on a particular signal-extraction problem in which agents reasonably infer poor prospects for a bank from the withdrawal decisions of other depositors. Obviously, this model can be embedded in a framework where insurance is desirable and the choice between equity and deposit contracts studied (as, for example, in Jacklin [10] and in Bhattacharya and Jacklin [4]). Our focus has been to argue that runs can occur even in environments where banks provide services other than insurance.

We have also argued that suspension of convertibility can improve upon the equilibrium allocations. This occurs because in the model considered here there are two sources of social costs in bank runs. One is the cost involved in liquidating fixed investments, the other is the fact that bank runs occur in some states even though returns are high and are known by some individuals to be high. Essentially, the fear induced by a large number of withdrawers, even though their withdrawals are not informationally based, causes a run on the bank.

The conditions required to ensure that there are bank runs are not crucially dependent upon risk neutrality. It is quite possible, however, that suspension of convertibility might no longer lead to an improvement of ex ante welfare. An important direction in which this model needs to be extended is to incorporate the linkage between failures of particular banks and runs on the banking system as a whole. We have imposed a liquidation cost on the technology to capture the idea that failures of many banks are more costly than failures of a few. It is not clear to us why this might be so. In addition, the aggregate risks in this model are exogenously imposed. The interaction between bank runs and business cycles needs to be modeled explicitly.

#### Appendix

#### Proof of Theorem 1, Part A

For convenience, we denote states by the row numbers in Table II. For example, state 1 will refer to the state described in row 1 of Table II, i.e.,  $\theta = \{0, R, \alpha\}$ . (The reader will find it useful to refer to Table II at this stage.)

The outcomes in Table II can be thought of as follows. All uninformed type-2 agents continue to invest unless they see an aggregate investment of zero. The information partitions of uninformed type-2 agents in the conjectured equilibrium are:

K = 1	implies $\theta \in \{1, 2\}$
$K=1-t_1$	implies $\theta \in \{3, 4, 5\}$
$K=1-t_2$	implies $\theta \in \{8\}$
K = 0	implies $\theta \in \{6, 7, 9\}$ .

Since pH + (1-p)L > 1, it is optimal for the uninformed agent not to liquidate if he or she observes K = 1, i.e., in states 1 and 2.

The left side of inequality (10) is the expected future consumption from maintaining the investment when  $\theta = \{3, 4\}$ . Since R = H in state 5, the expected future consumption conditional on  $\theta \in \{3, 4, 5\}$  is greater than the left side of inequality (10). Since the left side of inequality (10) is greater than 1, it is optimal not to liquidate if the agent observed  $K = 1 - t_1$ , because this implies  $\theta = \{3, 4, 5\}$ .

The left side of inequality (11) is the expected future consumption from maintaining the investment when  $\theta \in \{6, 7, 8\}$ . Since the aggregate investment K = 0 in states 6, 7, and 9, the agent will only get (1 - a) units if he or she liquidates his or her investment whenever the agent observes K = 0. However, since the left side of inequality (11) is less than (1 - a), it is still optimal to liquidate if the agent observed K = 0, i.e.,  $\theta \in \{6, 7, 9\}$ , since  $\tilde{R} = L$  in state 9 while state 8 has  $\tilde{R} = H$ . Hence the outcomes described in Table II constitute an equilibrium. Since all agents liquidate in state 7, which is a state in which no one has any adverse information about the next-period return, it is a panic equilibrium.

#### Proof of Theorem 1, Part B

Recall that  $\bar{\alpha} = t_1$  and  $(1 - t_1)(1 - \bar{\alpha}) = (1 - t_2)$ . The measurability condition (c) in our definition of the equilibrium together with the definition of the function  $K_D(\cdot)$  given in equation (9) and Table I then immediately implies that the equilibrium investment must be the same in states 3 and 4 and states 6 and 7.

Suppose that the equilibrium involves no panic. In such a case (by definition) it must be that K(t, R, 0) > 0 for all t, i.e., the aggregate investment should be strictly positive in states 1, 4, and 7. However, the argument above then implies that  $K(t, L, \bar{\alpha}) > 0$  if t = 0 or  $t = t_1$ . In other words, the aggregate investment should be strictly positive in states 3 and 6 as well. Let K(i) denote the value of K in each state  $i, i = 1, 2, \dots, 9$ . We have already argued that in any equilibrium with no panics k(K(6)) = k(K(7)). Since there are no panics, k(K(6)) = k(K(7))> 0. It follows from Table I that  $K(j) \neq K(6), j = 1, 2, 3, 4, 5, \text{ and } 9$ . To see why this is true, suppose K(6) = K(1). This would imply from the definition of equilibrium that  $(1 - t_1)(1 - \bar{\alpha})k(K) = k(K)$ . This yields an obvious contradiction since  $(1 - t_1)(1 - \bar{\alpha}) = (1 - t_2) > 0$ . A similar argument holds for the other cases. It is possible that K(6) = K(7) = K(8). This will be true if k(K(6)) = 1. In such a case if an uninformed agent observes  $K = K(6) = 1 - t_2$ , he or she will infer that the only possible states are 6, 7, and 8. However, in this case, the expected consumption from continuing to invest is given by the left side of inequality (11). This is less than the right side of inequality (11), which is the utility that can be attained by liquidating the investment. Hence all uninformed agents will wish to consume in period 1 in the event that  $K = 1 - t_2$ . Therefore k(K(6)) = 1 cannot be an equilibrium.

If 0 < k(K(6)) < 1, the only confusion is between states 6 and 7. But recall that R = H in state 8. Hence the expected future consumption conditional on  $\theta \in \{6, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{6, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{6, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{6, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{6, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{6, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditional on  $\theta \in \{1, 7\}$  must be less than the expected future consumption conditin the expected future co

{6, 7, 8}. We have already shown that individuals will wish to liquidate if  $\theta \in \{6, 7, 8\}$ . Therefore k(K(6)) > 0 cannot be optimal.

It follows that any equilibrium must involve panic in the sense that agents choose to liquidate even though nobody has any information.<sup>4</sup> Q.E.D.

<sup>4</sup> The reader can verify that there are parameter values satisfying the restrictions imposed in Theorem 1. An example is  $\beta = 0.0000001$ ; H = 2.31; p = q = 0.5;  $r_0 = 0.1875$ ;  $r_1 = 0.75$ ;  $r_2 = 0.0625$ ;  $\bar{\alpha} = 0.05$ ;  $t_1 = 0.25$ ;  $t_2 = t_1 + \bar{\alpha}(1 - t_1) = 0.2875$ ; a = 0.66. The value of the left side of inequality (11) is 1.026 > 1, and the left side of inequality (12) is 0.308 < (1 - a) = 0.34.

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#### DISCUSSION

JOSEPH WILLIAMS<sup>\*</sup>: This paper is an interesting addition to the rapidly growing literature on banking panics. Like Bhattacharya and Jacklin [1], Bryant [2], and Gorton [4], the assets of the representative bank are risky. Some depositors have private information about the future return on these assets, while others try to infer this information from the number of depositors who line up to withdraw their funds. As suggested in Bryant [2] and formalized first in this paper, this inference is confounded by the unknown number of depositors who withdraw funds solely to finance their current consumption. In the resulting equilibrium with rational expectations, all depositors sometimes withdraw their funds even though no depositor has negative private information. This panic is possible without the sequential service constraint familiar from previous papers because the bank's assets are illiquid. If withdrawals exceed a critical level, then the bank must liquidate loans and thereby incur a transaction cost. In this case, suspending convertibility can improve depositors' welfare by blocking the forced liquidation of loans in runs unrelated to private information.

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