Aggregate Demand Management in Search Equilibrium

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Equilibrium is analyzed for a simple barter model with identical risk-neutral agents where trade is coordinated by a stochastic matching process. It is shown that there are multiple steady-state rational expectations equilibria, with all non–corner solution equilibria inefficient. This implies that an economy with this type of trade friction does not have a unique natural rate of unemployment.

I. Introduction

Some economists attribute fluctuations in unemployment to misperceptions of prices and wages. Others attribute such fluctuations to lags in adjustment of prices and wages (including staggered contracts). It seems to be a shared view that there would be no macroeconomic unemployment problems if prices and wages were fully flexible and correctly perceived. This paper introduces a third cause for macro unemployment problems—the difficulty of coordination of trade in a many-person economy. That is, once one drops the fictional Walrasian auctioneer and introduces trade frictions, one can have macro unemployment problems in an economy with correctly perceived, flexible prices and wages.

Using a barter model with identical, risk-neutral individuals where trade is coordinated by a stochastic matching process, this paper

examines rational expectations steady-state equilibria. The model is shown to have two properties: multiple steady-state equilibria and local inefficiency of all non–corner solution equilibria. The source of local inefficiency is a trading externality, while the source of multiple equilibria is the positive feedback working through this externality. The externality comes from the plausible assumption that an increase in the number of potential trading partners makes trade easier. The positive feedback is that easier trade, in turn, makes production more profitable.

These results are demonstrated in a model where the trade process is mechanistic, with the production decision as the only control variable. Nevertheless, these results seem robust. Individuals control search intensity and advertising and have reputations for offering good deals. Once all individuals have optimized on these control variables affecting trading opportunities, profitability would still be increased by the availability of more potential trading partners. That is, the externality (and positive feedback) from increased willingness to produce is not correctable by privately available actions given frictions in coordinating trade.

To see the importance of this finding, consider Friedman's (1968) definition of the natural rate of unemployment as the level occurring once frictions are introduced into the Walrasian equations. This paper argues that the result of actually modeling a competitive economy with trade frictions is to find multiple natural rates of unemployment. This implies that one of the goals for macro policy should be to direct the economy toward the best natural rate (not necessarily the lowest) after any sufficiently large macro shock.

The basic model[1] is presented in Sections II–IV. Then, a simple static model is presented in Section V to illustrate the workings of the basic externality. Optimal policy in the dynamic model is analyzed in Sections VI–VIII. An example is worked out in Section IX. A summary description of the model and discussion of its implications are in Section X.

II. Basic Model

We use a highly artificial model of the production and trade processes to highlight the workings of a general equilibrium search model. All individuals are assumed to be alike. Instantaneous utility satisfies

$$U = y - c, \tag{1}$$

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[1] The model closest to this in structure is that of Hellwig (1976), who shows that his search model converges to a Walrasian model as the rate of arrival of trade opportunities rises without limit.

where y is the consumption of output and c is the cost of production (disutility of labor). The utility function is chosen to be linear as part of the simplification that leads to the conclusion that trade bargains will not vary across pairs who are trading. In addition, the absence of risk aversion permits us to ignore the absence of implicit or explicit wage insurance. Lifetime utility is the present discounted value of instantaneous utility. Since trade and production take place at discrete times, lifetime utility satisfies

$$V = \sum_{i=1}^{\infty} e^{-rt_i} U_{t_i}. \tag{2}$$

Individuals are assumed to maximize the expected value of lifetime utility, with the times of work and consumption as random variables.

Rather than modeling production as going on continuously, we assume that the arrival of production opportunities is a Poisson process. With arrival rate a, each individual learns of production opportunities. Each opportunity has y units of output and costs c ($c \geq \underline{c} > 0$) units to produce. We assume that y is the same for all projects but that c varies across projects with distribution G. Each opportunity is a random draw from G, with costs known before the decision on undertaking the project. Each project undertaken is completed instantly.

There are two restrictions on individual behavior. (1) Individuals cannot consume the products of their own investment but trade their own output for that produced by others. This represents the advantage of specialized production and trade over self-sufficiency. (2) Individuals cannot undertake a production project if they have unsold produced output on hand. This extreme assumption on the costs of inventory holding is also part of the simplification of the determination of trade bargains. The fact that all trades involve individuals with y units to sell implies that all units are swapped on a one-for-one basis and promptly consumed.[2] It is assumed that there is no credit so that all trade is between individuals with inventories to trade.

Thus individuals have 0 or y units for sale. The former are looking for production opportunities and are referred to as unemployed. The latter are trying to sell their output and are referred to as employed.

The basic difference between individuals in these two different states is that the latter have purchasing power while the former do not.[3] If production were modeled as time consuming, then individuals

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[2] Dropping the simplifications of risk neutrality, barter, and identical inventory holdings, we would need to solve for a distribution of trade prices, which would complicate the analysis. Assuming that all trade is one-for-one, we do not basically change the model by allowing simultaneous searches for trade and production and thus inventory accumulation.

[3] This is the counterpart in search equilibrium of effective demand considerations in

would be in one of three states—unemployed, producing, or trading. Commencing production only adds to demand with a lag. In that sense, those producing are similar to those unemployed. However, it remains the case that the decision to switch from searching for production to engaging in production is the driving force in the model. A similar model can be constructed with no unemployment and varying production intensity. It seems appropriate to associate varying levels of production intensity (coming from varying levels of profitability) with varying levels of unemployment. In a more general setting, there would also be varying hours of work and varying labor intensity on the job.

The trading process is such that for each individual the arrival of potential trading partners is a Poisson process with arrival rate $b(e)$, $b' > 0$, where e is the fraction of the population employed in the trading process, that is, the fraction of the population with inventories available for trade. The presence of lags in the trading process represents primarily the time needed to sell goods. Thus the average length of time of consumer goods in inventories is assumed to increase as the rate of sales declines. For example, a trader might meet with others at a constant rate and find that, for any meeting, there is a probability that the potential trading partner has a unit to sell, that is, is employed. The probability that a potential partner is in the market is a function of the fraction of the population employed, e, with the probability increasing in e. With undirected search for trading partners the probability of finding a trading partner in any meeting would equal e. In a more complicated setting, the greater the stock of available inventories the easier it is to find the particular goods that one wants.

The economy is assumed to be sufficiently large that the expected values of potential production and trade opportunities are realized. The employment rate falls from each completed transaction, as a previously employed person becomes eligible to undertake a production opportunity, and rises whenever a production opportunity is undertaken. Assuming that all production opportunities with costs below c^* are undertaken, we have the time derivative of the employment rate satisfying

$$\dot{e} = a(1 - e)G(c^*) - eb(e). \qquad (3)$$

That is, each of the $1 - e$ unemployed (per capita) has the flow probability a of learning of an opportunity and accepts the fraction

$G(c^*)$ of opportunities. Each of the e employed (per capita) faces the probability b of having a successful trade meeting and being freed to seek a new opportunity.[4]

In a steady state, we have the equilibrium rate of unemployment by setting $\dot{e}$ equal to zero. Setting (3) equal to zero, we see that the steady-state employment rate rises with c^*:

$$\left.\frac{de}{dc^*}\right|_{\dot{e}=0} = \frac{a(1-e)G'(c^*)}{b(e) + eb'(e) + aG(c^*)} > 0. \qquad (4)$$

We turn next to the determination of c^*.

III. Individual Choice

As modeled, the only decision to be made is which production opportunities to undertake. Assuming a steady-state equilibrium, we can describe this decision as a simple dynamic programming problem. Let us denote the expected present discounted value of lifetime utility for employed and unemployed by W_e and W_u. Then, the utility discount rate times each of these values equals the expected value of the flow of instantaneous utility plus the expected capital gain from a change in status,

$$rW_e = b(y - W_e + W_u),$$
$$rW_u = a\int_0^{c^*}(W_e - W_u - c)dG(c). \qquad (5)$$

With probability b, an employed person has a trade opportunity giving rise to instantaneous consumption y and a change in status to unemployed. Each unemployed person accepting a production opportunity has an instantaneous utility $-c$ and a change in status to employed.

An unemployed person accepts any opportunity that raises expected utility. Thus we have the criterion

$$c^* = W_e - W_u = \frac{by + a\int_0^{c^*}cdG}{r + b + aG(c^*)}, \qquad (6)$$

where the second equality comes from taking the difference between the two equations in (5) and solving for $W_e - W_u$. The level of aggregate demand, measured as the number of traders seeking to

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[4] We are aggregating the individually experienced process, $b(e)$, over all individuals in the process, rather than (equivalently) the rate of meetings, each of which frees two traders to seek new opportunities.

purchase, affects production decisions since the probability of a sale increases with the employment rate. Differentiating (6) we have

$$\frac{dc^*}{de} = \frac{(y - c^*)b'}{r + b + aG} > 0,$$

$$\frac{d^2c^*}{de^2} = \frac{(y - c^*)b'' - 2b'(dc^*/de) - aG'(dc^*/de)^2}{r + b + aG}.$$

(7)

To see that dc^*/de is positive, we note that (with positive interest) no one would undertake a project with less output than input ($y > c^*$) and $b' > 0$. With $b'' \leq 0$, d^2c^*/de^2 is also negative. Armed with (3) and (6) we can describe steady-state equilibrium.

IV. Steady-State Equilibrium

A steady state is marked by optimal production decisions (6) and a constant rate of employment, with (3) set equal to zero. In each of these equations e and c^* are positively related, which allows the possibility of multiple steady-state equilibria. Except when the shutdown of the economy ($e = 0$) is the unique equilibrium, there will be multiple equilibria. To see this, we note that c^* goes to zero as e (and so b) goes to zero. Also, $c^*(e)$ is bounded above since c^* is less than y for any finite b. Steady-state employment rates are bounded above by the employment level reached if all production opportunities are accepted ($G = 1$). As drawn in figure 1, it is assumed that there is no upper bound to the support of G. The steady-state employment rate equals zero for c^* below $\underline{c}$, the lower bound of possible production costs.

If agents expect the current unemployment rate to be permanent, then the economy is always on the optimal steady-state production decision curve, (6), with movement determined by the $\dot{e}$ equation. Then, the equilibria in figure 1 with the highest employment rate and with a rate of zero are stable. Since G does not necessarily have nice properties, there can be more equilibria than shown.

If the model were extended to allow random shocks to the aggregate economy, the presence of multiple steady-state equilibria implies that the economy can get stuck at the "wrong" steady-state equilibrium after the shock has gone away. Similarly, the presence of multiple steady-state rational expectations equilibria implies the existence of multiple rational expectations paths from some initial positions.

V. Static Model

The dynamic model used above seems useful for understanding both the workings of the externality and the design of policy. Given that
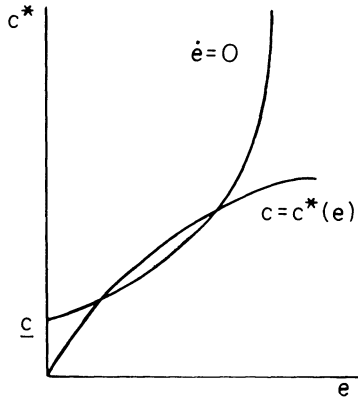
FIG. 1

model to motivate the equilibrium trade possibilities, one can describe the externality more simply in terms of a static model. Let us consider an aggregate cost function

$$c = f(y), \tag{8}$$

with $f' > 0, f'' > 0$. Let $p(y)$ be the probability of making a sale as a function of the aggregate output level. Unsold output is assumed to be wasted, so that welfare satisfies

$$U = yp(y) - c. \tag{9}$$

If individuals view p as a parameter, equilibrium occurs at a level of production satisfying

$$p(y) = f'(y). \tag{10}$$

For efficiency, the aggregate relationship between sales probability and production level must be recognized, which gives an optimality condition

$$p(y) + yp'(y) = f'(y). \tag{11}$$

By subsidizing the cost of production (financed by lump-sum taxation) the decentralized economy can be induced to produce at a point which satisfies the social optimality condition.

It is straightforward to extend this static model to include public goods. This extension will show the presence of a multiplier process and the need to consider multiplier effects in the absence of other demand management policies. Let g be the quantity of output used for public consumption and $V(g)$ the concave addition to social welfare from public consumption. We assume lump-sum tax finance, so the cost of public consumption is added to the cost of production for

private consumption. The probability of a sale is assumed to increase with aggregate demand, $y + g$. Private consumption equals total sales less public consumption. Thus, social welfare can be written as

$$U = yp(y + g) - g + V(g) - c. \tag{12}$$

The equilibrium production decision with p taken to be a parameter can now be written as

$$p(y + g) = f'(y). \tag{13}$$

Implicitly differentiating (and thus assuming equilibrium y continuous in g) we have

$$\frac{dy}{dg} = -\frac{p'}{p' - f''}. \tag{14}$$

To sign this expression we need to appeal to the stability argument that the relevant equilibria have the marginal cost of production, $f'(y)$, rising more rapidly than the probability of a sale, $p(y)$. With $p' - f'' < 0$, we have $dy/dg > 0$.

Turning to the first-order condition for the optimal level of public consumption we have

$$\frac{dU}{dg} = yp' - 1 + V' + (p + yp' - f')\frac{dy}{dg} = 0. \tag{15}$$

Using the equilibrium condition, (13), we can write this as

$$V' = 1 - yp'\left(1 + \frac{dy}{dg}\right) = 1 + \frac{yp'f''}{p' - f''} < 1. \tag{16}$$

For a contrast let us consider an economy where the exogenous fraction $(1 - p)$ of output produced is lost in the distribution network. Then dy/dg would be zero and the marginal benefit of public consumption should be equated with the marginal cost of forgone consumption, which is one. Thus there is higher optimal public consumption when the profitability of production increases with increased government demand, and greater production yields a trade externality.

VI. Long-Run Stimulation Policy

To explore policy in the dynamic model we will assume that the government has sufficient policy tools to control production decisions. (In this barter economy, one cannot distinguish between aggregate demand policy and aggregate supply policy.) Below we will consider a production-cost subsidy to induce private decisions that sustain the optimal steady state. In this section we will examine a small perma-

nent change in c^* away from a steady-state equilibrium with no intervention. In the next section we will examine the optimal path for $c^*(t)$ from an arbitrary initial position. In steady-state equilibrium, we have a flow of utility per capita satisfying

$$Q(t) = eb(e)y - a(1 - e)\int_0^{c^*} cdG, \qquad (17)$$

where $eb(e)$ is the rate of sales, with consumption of y per sale, and $a(1 - e)G$ is the rate of production, with an average cost of $\int_0^{c^*} cdG/G$ per project undertaken. For social welfare we are interested in the present discounted value of Q:

$$W = \int_0^\infty e^{-rt}Q(t)dt. \qquad (18)$$

When the economy starts at a steady-state equilibrium ($\dot{e} = 0$), the change in W (along the dynamic path of economy) resulting from a permanent change in c^* satisfies (for a derivation of [19] see Diamond [1980])

$$r\frac{\partial W}{\partial c^*} = -a(1 - e)c^*G'(c^*)$$
$$+ \left[y(b + eb') + a\int_0^{c^*} cdG\right]\frac{a(1 - e)G'(c^*)}{r + b + eb' + aG(c^*)}. \qquad (19)$$

The first term represents the increase in production costs at the steady-state employment rate, while the second represents the change in both output and production costs along the employment trajectory induced by the change in production rule. At an equilibrium without intervention (where [6] holds), we can write this as

$$r\frac{\partial W}{\partial c^*} = -a(1 - e)c^*G' + \frac{[yeb' + c^*(r + b + aG)]a(1 - e)G'}{r + b + eb' + aG}$$
$$= \frac{a(1 - e)G'eb'}{r + b + eb' + aG}(y - c^*) > 0. \qquad (20)$$

Thus, without intervention, there is locally too little activity in the economy.[5] This permanent increase in c^* raises expected lifetime utility for every person as well as raising aggregate welfare. The efficiency argument does not apply at the equilibrium with no economic activity ($e = 0$) since $G'(c)$ is zero for $c < \underline{c}$.

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[5] In a partial equilibrium model of job matching, I have argued (Diamond 1981) that equilibrium has too rapid job filling for efficiency. I have not integrated that model with this one. If such an integration were done in a model with a single decision (by having a be a function of $1 - e$, e.g.), these would be offsetting externalities. If such an integration were done in a model with two decisions, two externalities may prove to be simultaneously present rather than offsetting.

VII. Short-Run Stabilization Policy

Continuing with the assumption that the government can control production decisions, we can examine the optimal policy for an arbitrary initial position. That is, the optimal stabilization policy satisfies

$$\max_{c^*(t)} \int_0^\infty e^{-rt} Q(t) dt,$$

$$\text{where } Q(t) = e(t)b[e(t)]y - a[1 - e(t)]\int_0^{c^*(t)} cdG, \qquad (21)$$

$$\dot{e}(t) = a[1 - e(t)]G[c^*(t)] - e(t)b[e(t)],$$

$$e(0) = e_0.$$

Writing the optimal policy as $c^{**}(t)$, we get (the Euler equation)

$$\dot{c}^{**}(t) = rc^{**} - (y - c^{**})(b + eb') + a\int_0^{c^{**}} (c^{**} - c)dG. \qquad (22)$$

Setting $\dot{c}^{**}(t)$ equal to zero and differentiating, we have

$$\left.\frac{dc^{**}}{de}\right|_{\dot{c}^{**}=0} = \frac{(y - c^{**})(2b' + eb'')}{r + b + eb' + aG}. \qquad (23)$$

With $b'' \leq 0$, this expression is not necessarily positive, except near the origin. The phase diagram is shown in figure 2 under the assumption that the state with lowest unemployment is the optimum for any initial position.

  Comparing the equation for $\dot{c}^{**} = 0$, (22), and the private choice of c^* in a steady state, (6), we see that the former is always above the latter as a function of e. That is, superimposing figures 1 and 2, we see that the $\dot{c}^{**} = 0$ curve lies above the $c = c^*(e)$ curve.
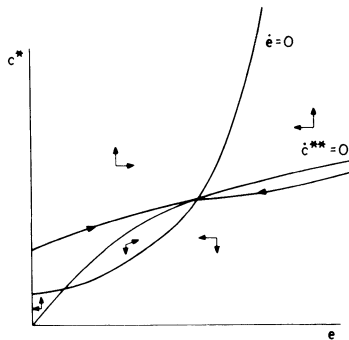

FIG. 2

VIII. Subsidizing Production

The asymptotically optimal steady state is described by setting $\dot{c}^{**}$, in (22), equal to zero (or, alternatively, by setting $\partial W/\partial c^*$, in [19], equal to zero). By subsidizing the cost of production, individuals can be induced to select this cutoff cost. In this section we derive the equation for this subsidy. We assume that the subsidy is financed by a lump-sum tax (payable in labor) that falls on the employed and unemployed equally.

With a subsidy of s per project completed, the individually optimal cutoff rule becomes

$$c^* - s = W_e - W_u = \frac{by + a\int_0^{c^*}(c - s)dG}{r + b + aG(c^*)}. \qquad (24)$$

The asymptotically optimal level satisfies

$$c^{**} = \frac{by + eb'y + a\int_0^{c^{**}}cdG}{r + b + eb' + aG(c^{**})}. \qquad (25)$$

Equating the expressions for c^* and c^{**} and solving, we have

$$s = \frac{eb'\left[ry + a\int_0^{c^*}(y - c)dG\right]}{(r + b)(r + b + eb' + aG)}. \qquad (26)$$

This subsidy level is positive, as can be seen from (22), which implies that $y > c^*$ when $\dot{c}^{**}$ equals zero.[6]

IX. An Example

As an example, assume that $b(e) = eb$ and that all projects cost the same, $\underline{c}$. In this case there will be three steady-state equilibria provided that $\underline{c} < y/[1 + (r/b\bar{e})]$, where $\bar{e}$ is the solution to $be^2 = a(1 - e)$. For this case the curves determining equilibria and optima are shown in figure 3.

It is interesting to consider the optimal plan in more detail. Let the

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[6] Laurence Weiss suggested calculating the effect of unemployment compensation, financed by a tax on output. Such a policy can be fitted into the model by giving each unemployed person a probability of receiving an output bundle just equal to the after-tax output level of a project. Such a policy moves in the wrong direction, since the incentive to production of having more potential trading partners is smaller than the disincentives coming from the sum of output taxation and unemployment subsidization.
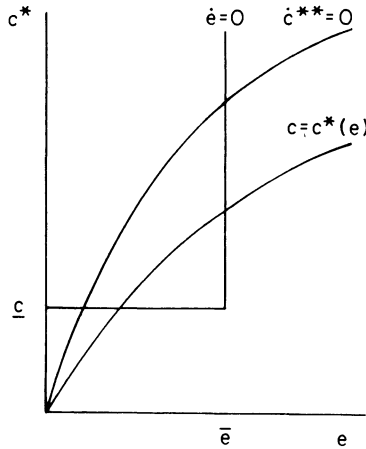
FIG. 3

rate of change of the employment rate be the control variable. Then, social welfare can be written as

$$W = \int_0^\infty e^{-rt}\{b(y - \underline{c})[e(t)]^2 + \underline{c}\dot{e}(t)\}dt, \tag{27}$$

where $\dot{e}(t)$ is constrained by

$$b[e(t)]^2 - a[1 - e(t)] \leq -\dot{e}(t) \leq b[e(t)]^2. \tag{28}$$

Since the objective is linear in $\dot{e}$, there are two possible asymptotic solutions as one or the other of the constraints on $\dot{e}(t)$ is binding. Thus, asymptotically, either no opportunities are accepted or all of them are. For initial condition e_0, let us write the levels of welfare under these plans[7] as $W_0(e_0)$ and $W_1(e_0)$. For some parameter values one or the other of these two functions is larger for all values of e_0 between zero and one. For some parameter values the functions appear as in figure 4. In this case it is optimal to take all opportunities for $e_0 > e_0'$ and optimal to take no opportunities for $e_0 < e_0'$.

X. Summary and Conclusions

It is common in theoretical economics to use a tropical island metaphor to describe the workings of a model. The island described here has many individuals, not one. When employed, they stroll along the beaches examining palm trees. Some trees have coconuts. All

[7] We are ignoring the possibility that for some parameters it might be optimal to take all opportunities for a range of employment rates above $\bar{e}$ and then switch over to taking no further opportunities.
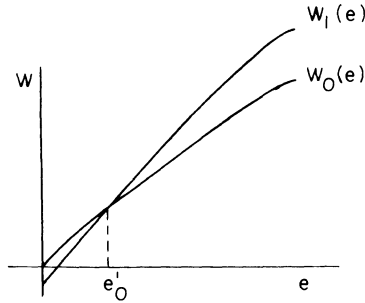
FIG. 4

bunches have the same number of nuts but differ in their height above the ground. Having spotted a bunch, the individual decides whether to climb the tree. There is a taboo against eating nuts one has picked oneself. Having climbed a tree, the worker goes searching for a trade—nuts for nuts—which will result in consumption. This represents, artificially, the realistic aspect of the small extent of consumption of one's own production in modern economies. The ease in finding a trading partner depends on the number of potential partners available. Thus the equilibrium level of production is not efficient if everyone correctly predicts the difficulty of successful trading. Of course, overoptimism can result in the efficient production level. There is no mechanism to ensure that individual by individual, or on average, forecasts of time to completed trade are correct. Errors would be particularly likely on a non-steady-state path.

When a Walrasian auctioneer organizes a competitive equilibrium, there are not unrealized mutually advantageous trading opportunities. In a complex modern economy, there will always remain unrecognized, and so unrealized, opportunities. The complexity of the many-person, many-good trades needed to realize some potential opportunities, together with costs of information, prevent the economy from achieving a full realization. The model employed here has abstracted from the many-good aspect of modern economies. However, the fact of large numbers of different goods should be kept in mind when interpreting the difficulty in completing trades as modeled here. In the presence of unrealized trading opportunities, many government policies will naturally affect the extent to which these opportunities are realized by affecting individual production and trade incentives. Policies can have two distinct goals—inducing small changes in the steady-state equilibrium position to offset externalities and inducing large changes when the economy has settled down at an inefficient long-run equilibrium.

There are several properties of this type of macro search model that seem particularly attractive. Even without lags in the ability of the

government to affect private decisions, the government does not have the power to move instantaneously to a full employment position. Recognizing the costs of starting a production process, we see that there is an optimal rate of convergence to the optimally full employment steady state, which reflects the higher real costs of moving too quickly. Knowledge of private forecasts would be essential to the optimal design of tools to alter private decisions but is not necessary for recognizing a situation calling for intervention (except to the extent that the bases of private forecasts might improve the government forecast).

The model presented here is very special. One cannot draw policy conclusions directly from such a model. There are two purposes for its construction. One is to form a basis for further generalization and study. In particular, it would be interesting to introduce varying technological conditions to examine how government policy should vary with the position of the economy. The second is to provide an example to contrast with models that assume, unrealistically, the existence of a frictionless, instantaneous trade-coordination mechanism and thus the absence of the potential for corrective policies.[8] While the construction of realistic models of trade frictions (and wage rigidities) is needed for good policy analysis, the existence of this simple model indicates the feasibility of constructing consistent micro based models with a role for reactive macro policy.

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[8] In Lucas (1972) and Lucas and Prescott (1974) there are physically separate markets, with each market involving perfect coordination. The efficiency of movements between markets is unaffected by aggregate demand management policies. Thus these extensions of the competitive model have trade frictions but not the externalities from trading efforts modeled here.