Inside Money, Market, and Specialization

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Abstract

We depict an economy with trade frictions where people choose between a random-matching trading sector and organized markets, which resemble Walrasian markets. Merchants in the organized markets issue bills of exchange to producers, and under the no-defection condition all merchants honor the bills issued by other merchants. When trade frictions are moderate and the discount rate is small, bills of exchange serve as the medium of exchange in the organized markets and unorganized sector, where people's trading histories are private information. The existence of organized markets and inside money induces a higher equilibrium level of specialization relative to barter. In this case, higher inside liquidity is accompanied by a higher level of specialization.

Key words: Inside Money; Market; Intermediation; Specialization
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1 Introduction

Search monetary models, by studying trading procedures more closely, have shown success in explaining the mechanisms underlying the use of medium of exchange. In search models, agents trade bilaterally in a random manner, which depicts the least organized form of economic activity. In contrast, Walrasian models postulate the existence of markets, and sellers and buyers can always locate the relevant markets. While it is not completely satisfactory to assume that all trades are perfectly coordinated as depicted in Walrasian models, in real world we do observe a system of trade-facilitating intermediary, such as middlemen, dealers and banks. Also, as specialist traders coordinate transactions, they may create tradable objects to facilitate exchange, which sometimes circulate as media of exchange in diverse locations and among third parties.[1]

To address the above issues, we consider a model with a double-coincidence problem, as in Kiyotaki and Wright (1993), where agents choose between two trading arrangements: an 'unorganized sector' in which agents meet bilaterally in random, and 'organized markets' set up by merchants, which resemble Walrasian markets. Merchants do not produce; they make profits from reselling goods at a higher price than the original purchase price. When deciding which sector to conduct trade, people take into account the trade-off between the saving in waiting time and the higher commodity price in the organized markets.

Merchants issue *bills of exchange* to buy commodities from producers. We assume a technology that keeps merchant's record of transaction and punishment on defecting merchants is feasible.[2] Under the no-defection condition, all merchants in the organized markets honor the bills issued by other merchants. An agent, once acquiring a bill of exchange, can spend it in the organized markets or in the unorganized sector, where agents' trading histories are private information. Thus, bills of exchange are used in the organized markets, and possibly circulate in the unorganized sector also, and they are liabilities of merchants. The extent of organized markets, amount of inside money and merchant's profits are all determined endogenously.

Equilibria are characterized by whether there are active organized markets and whether

[1] Bills of exchange were commonly used in transactions as a medium of exchange in north England during the late 18th and early 19th centuries, as described in Ashton (1945)

[2] Assumptions in random matching models include the lack of commitment and private information concerning trading histories. To study some form of credit in a random matching model, one needs to weaken the above assumptions (see Calvacanti and Wallace 1999).

private liabilities are used as a general medium of exchange. As long as the discount rate is small, all transactions take place in the organized markets, regardless of the degree of trade frictions. This case resembles a Walrasian equilibrium. When trade frictions are moderate, both trading sectors are active and bills of exchange circulate as a general medium of exchange. The reason is this. If trade frictions are low, conducting trade in the unorganized sector is so attractive that it may not generate sufficient profits for the intermediation business. If trade difficulties are high, it would be too time-consuming for bill holders to encounter trade partners in the unorganized sector, and the use of bills would be confined in the organized markets.

The relationship between specialization and the extent of market has long been recognized by economists. Because the present model depicts the existence and features of market and inside money, it provides us a framework to study how endogenously arised intermediation (market and money) ameliorates trade frictions and affects the level of specialization.[3] We model specialization as in Camera et al. (2003), that people are allowed to choose the set of goods they produce, and producing a smaller set of goods involves a lower production cost but reduces the probability of matching with a trade partner. It is found that, when all transactions take place in the organized markets (the market reaches its greatest extent), the economy achieves complete specialization. If both sectors are active and the use of bills is limited in the organized markets, the equilibrium level of specialization is identical to that in the pure barter economy. The mere existence of organized markets does not necessarily induce a higher level of specialization relative to barter. A key factor may lie in the extent of circulation of the medium of exchange. Indeed, we find that if bills circulate in both sectors, the equilibrium level of specialization is higher relative to barter. A distinctive feature is that, the object that plays the role of medium of exchange here is inside money – private liabilities that accompany the rise of organized markets. It is also found that lower trade frictions induce higher inside liquidity and more specialization.

This paper is closely related to Calvacanti and Wallace (1999) in that notes issued by agents

[3]There are articles using search models to examine specialization. Shi (1997) models specialization as producing the goods desired by others less costly than producing his own desired goods. In Shi (2005) an agent specializes more if he makes his product more desirable to the consumers. Camera et al. (2003) show how the use of fiat money affects specialization. Other approaches that examine the relationship between transaction cost and specialization include Yang and Borland (1991) and Yang and Shi (1992). Except Shi (2005) none of the above explicitly study the relationship between specialization and the extent of market which, however, is represented by the society's imperfect ability to update agents' transaction records.

with public transaction records may circulate as a medium of exchange in a random matching economy. As for the role of intermediation, a related study by Shevchenko (2004) considers middlemen that hold a variety of inventories to satisfy customers with heterogenous tastes. The main feature that differentiates the current paper from theirs is that, specialist traders in this economy organize markets as an alternative trading arrangement to search, and issue trade credit to facilitate transactions. We thus can study the links between agent's decision of participating in the competing trading arrangements, potential merchants' incentives to enter the intermediation business, the extent of market and circulation of inside money. This allows us to contribute to the literature by explicitly showing the effect of the extent of market and inside liquidity on economic activity such as specialization.[4]

The rest of the paper is organized as follows. Section 2 introduces the basic model. Section 3 characterizes stationary equilibria. In section 4 we consider specialization. Section 5 studies an economy with outside money. Section 6 concludes.

2 The basic model

Time is discrete and continues forever. The economy is populated by a $[0, 1]$ continuum of infinitely-lived agents. Let T be the set of goods. These goods are divisible but not storable once divided. Goods come in units of size one. Each agent i consumes goods in a subset $T_i \subset T$ and cannot consume goods not in T_i. Let $u > 0$ be the instantaneous utility from consuming an agent's consumption good and r his discount rate. When agent i consumes q units of his consumption goods he enjoys utility qu. He can produce just one good at a time, which is not in T_i, at a cost in terms of disutility c. Assume that agents must consume in order to produce, and the resulting implication is that agents can hold only one unit of asset at a time.

The set of agents is symmetric in the sense that the same number of agents consume each good and the same number of agents produce each good. This leads to the following: whenever you

[4] A related study on how trade frictions affect agent's choice of trading arrangements is Camera (2000). He assumes a costly matching technology that provides deterministic double-coincidence matches as an alternative to monetary exchange, but he does not study the existence and extent of market. Papers considering trade frictions to study market structure are also related. Howitt and Clower (2000), who consider transactions coordinated by specialist traders and show a fully developed market emerges where transactors follow simple adaptive rules. In Kultti (2002) incompleteness of markets arise endogenously because sellers optimally choose separate locations.

meet someone, there is a probability x that he consumes what you produce. The probability that the randomly encountered partner also produces what you consume is x. Thus, the probability of a 'double coincidence of wants' is x^2, which also measures the difficulty of direct barter.

In this economy agents meet bilaterally and at random. The potential trade partner's type and inventory are observable. Agents are unable to commit to future actions, proposed transfers cannot be enforced, and trade history is private information to the agent (except a subset of agents, see below). Transactions thus take the form of barter or may be facilitated by some tangible asset. Because goods are not storable once divided, barter trade is one-for-one swap.

Every agent is endowed with a production opportunity. A fraction $K \in (0, 1)$ of the agents are randomly chosen to be endowed with the ability to organize a market of consumption goods. Those agents are potential merchants. A potential merchant will enter as long as the expected profits are at least as large as the expected returns to a producer. Once a potential merchant enters, he must give up the production technology. Thus, merchants buy and sell goods but they do not produce. Each merchant sets up a store of the commodity that he wishes to consume. Given the assumption on preference and symmetry, there are many merchants in the market of a particular commodity, and we consider merchants run business under competition. Once the organized markets exist, an agent can always locate the market of the commodity that he wishes to buy or sell without search.

Merchants issue bills of exchange to buy commodities from producers.[5] Agents holding bills may buy goods in the organized markets or in the unorganized sector, where agents' trading histories are private information. Bills of exchange are indivisible. Because merchants compete for customers, the quantity of goods that a bill can buy, q, is determined by the competitive condition – merchant's profits equal the opportunity cost, which is the expected value to a producer.

The timing within a period is as follows. Producers and bill holders who decide to trade in the unorganized sector encounter pairwise meetings, and trade occurs in a double coincidence meeting of producers or single coincidence meeting where the consumer has an asset. Agents

[5]Our results do not depend on the assumed exclusive privilege for merchants to print bills of exchange. If we assume everyone has the technology to print bills, since agents in the unorganized sector cannot be monitored and cannot commit to future actions, in a symmetric equilibrium no one will produce goods for a bill issued by non-merchants.

who wish to trade in the organized markets arrive at a store in the markets and contact the merchant sequentially. After all arrivals, producer each produces a good, gets a unit of bill in return and leaves the markets. Consumers and the merchant consume their shares of the goods, q and $1 - q$ units of the goods, respectively, and bills of exchange are destroyed.[6] To rule out credit, we assume that agents who arrive in the organized markets cannot communicate with each other, though each can communicate with the merchant.

We assume that, once the organized markets exist, there is a technology that keeps merchant's record of transaction and punishment on defecting merchants is feasible. We will show below that this monitoring and enforcement technology ensures that all merchants in the organized markets honor the bills issued by other merchants. We also assume that this technology keeps record of bills issued by merchants and, therefore, a merchant can neither issue more than one unit of bill to a producer to compete for customers, nor can he redeem bills with newly-issued bills. Note that this technology keeps only merchant's transaction record; an ordinary producer-consumer's trading history remains private information. Credit arrangements are thus infeasible and a tangible asset is necessary for transactions in the organized markets.[7] As argued in Kocherlakota and Wallace (1998) incomplete record-keeping technology gives a role to a medium of exchange.

3 Symmetric stationary equilibrium

We focus on symmetric equilibria where strategies and distributions are time-invariant, and agents in identical states (regardless of their consumption types) choose identical actions.

Strategies

Let P denote the proportion of agents who can produce goods (called *producers*) and B the proportion of agents who hold bills of exchange (called *consumers*). An agent is in one of the following states: being a producer, a consumer or a merchant. Producers need to decide where

[6] This is unlike Rocheteau and Wright (2005), where sellers and buyers have a probability of not getting an opportunity to trade even in a competitive equilibrium, because they consider the search-type frictions in a Walrasian market.

[7] This precludes the possibility that merchants keep track of who produced and who consumed and communicate that to each other so that an agent could go into the organized market and produce a good for some merchant in a period and consume in the next period.

to sell their products; consumers decide where to spend the bills for consumption goods. Let S_p and S_b be the equilibrium probability that a producer and a consumer, respectively, chooses to trade in the unorganized sector. Thus, at a point of time, PS_p and BS_b are measures of producers and bill holders, respectively, who trade in the unorganized sector; $P(1-S_p)$ and $B(1-S_b)$ are measures of producers and bill holders, respectively, who trade in the organized markets. Agents resume the decision process after production and consumption are completed.

In the unorganized sector, a producer may encounter another producer and if there is a double coincidence of wants, a barter trade takes place. A producer may also encounter a bill holder, and in that situation he must decide whether to accept the bill as payment for his production. Let Σ denote the probability that a random producer accepts a bill in the unorganized sector, and σ an individual's best response. In equilibrium, $\sigma = \Sigma$.

Value functions

Let $V_j, j = p, b$ and f, denote the end-of-period expected life-time utility to a producer, bill holder and merchant, respectively. Let w_p and w_b denote the expected payoff to a producer and a bill holder, respectively, trading in the unorganized sector:

$$\begin{aligned} w_p &\equiv PS_p x^2(u-c) + BS_b x \max_{\sigma} \sigma(V_b - V_p - c, 0) \\ w_b &\equiv PS_p x \Sigma(u + V_p - V_b). \end{aligned}$$

A producer encounters a double-coincidence trade opportunity with probability $PS_p x^2$. He meets a consumer who wants his good with probability $BS_b x$, and in that situation he receives the gain from accepting a bill of exchange.

The Bellman's equations satisfy:

$$\begin{aligned} rV_p &= \max_{s_p}\{w_p,\ V_b - V_p - c\} && (1) \\ rV_b &= \max_{sb}\{w_b,\ qu + V_p - V_b\} && (2) \\ rV_f &= [P(1-S_p)(1-q)u]/K. && (3) \end{aligned}$$

Equation (1) and (2) set the flow return to a producer and a bill holder, respectively. Because merchants compete for customers, in a symmetric equilibrium they earn identical expected profits. Equation (3) shows that each merchant gets an equal share of business, $P(1-S_p)/K$ units of goods, each of which brings in profits $(1-q)u$.

Best response and steady state conditions

An agent chooses his strategies taking as given those of others, value functions and the expected terms of trade. Let (s_p, s_b, σ) denote an individual's best response when he takes as given others' strategies (S_p, S_b, Σ). A producer's decision as whether to trade in the unorganized sector is described by

$$s_p \begin{cases} = 1 & \text{if } w_p > V_b - V_p - c \\ \in [0,1] & \text{if } w_p = V_b - V_p - c \\ = 0 & \text{if } w_p < V_b - V_p - c. \end{cases} \tag{4}$$

A bill-holder's decision as whether to trade in the unorganized sector is

$$s_b \begin{cases} = 1 & \text{if } w_b > qu + V_p - V_b \\ \in [0,1] & \text{if } w_b = qu + V_p - V_b \\ = 0 & \text{if } w_b < qu + V_p - V_b. \end{cases} \tag{5}$$

A similar best response condition holds for producer's strategy σ as whether to accept a bill offered in the unorganized sector. From equation (1) we see that the organized markets would not exist if $V_b - V_p - c < 0$, because producers' expected payoff from search is bounded by the gains of barter. If organized markets are inactive, no bills would be issued, and so the strategy σ is irrelevant. Thus, in equilibrium if bills of exchange ever exist, they will be accepted in the unorganized sector ($\Sigma = 1$). Circulation of bills outside the organized markets thus is determined by bill holder's strategy.

We assume that competition among merchants results in no-surplus condition; that is, the expected profits equal to the expected payoff to a producer. The value of bills of exchange in the organized markets, q, is thus determined by

$$V_f = V_p. \tag{6}$$

It remains to check whether a merchant has an incentive to defect by not honoring bills of exchange issued by other merchants. Assume that defection by a merchant is punished by having the defecting merchant get the payoff from autarky, which is zero. By defection, we mean that a merchant consumes all the goods in inventory and does not redeem bills presented to him. If a merchant defects, he gets utility $P(1 - S_p)u/K$. If a merchant chooses to stay in business, he enjoys utility $P(1 - S_p)(1 - q)u/K$ and the continuation value V_f. The no-defection condition thus implies

$$P(1 - S_p)u/K \leq P(1 - S_p)(1 - q)u/K + V_f \tag{7}$$

The steady state requires the outstanding bills of exchange be constant; i.e., the amount of bills issued equals the amount of bills redeemed every period,

$$P(1 - S_p) = B(1 - S_b). \tag{8}$$

Since the creation and redemption of bills involve the exchange of goods, (8) can be interpreted as a condition that equates goods supplied and goods demanded in the markets. Finally,

$$P + B + K \equiv 1. \tag{9}$$

Definition 1 *A symmetric stationary equilibrium with active organized markets is a vector of value functions* $V = (V_p, V_b, V_f)$, *trading strategies* $S = (s_p, s_b, \sigma)$, *price* q, *and steady state distribution* $p = (P, B)$ *such that (i) given* S, q, *and* p,*value functions* V *satisfy (1) – (3); (ii) given* V, S, *and* p, *price* q *satisfies (6); (iii) given* V, q, *and* $(s_p, s_b) = (S_p, S_b)$, $\sigma = \Sigma = 1$, *strategies* S *satisfy (4) and (5); (iv) no-defection condition (7) is satisfied; (v)* p *satisfy (8) and (9).*

3.1 Existence of equilibria

First, a stationary equilibrium without organized markets always exists. If it is believed that no potential merchants would set up the markets, then $S_p = 1$ is the unique best response. If all producers sell products in the unorganized sector, no potential merchants would enter, and no bills of exchange would be issued. Hence $V_p > 0$ and $V_f = 0$ sustain the equilibrium strategies. Only barter takes place in this economy.

There are three types of equilibria with organized markets. When all producers trade in the organized markets, so do consumers, $S_p = 0, S_b = 0$. If producers are indifferent between trading in the organized and unorganized sectors, $S_p \in (0, 1)$, the use of bills may be limited in the organized markets, $S_b = 0$, or may circulate also in the unorganized sector as a medium of exchange, $S_b \in (0, 1)$. Note that $S_b = 1$ is not consistent with the existence of organized markets, because in steady state bills of exchange must be created and redeemed.

The following proposition concerning the existence of equilibrium that all trades take place in the organized markets – a fully organized markets equilibrium.

Proposition 1 *For sufficiently small r there exists a fully organized markets equilibrium.*

Proof: Given $(S_p, S_b) = (0,0)$, $P = B = (1-K)/2$ by (8). The strategy $s_p = 0$ is the best response if and only if $V_b - V_p - c > 0$, which is satisfied iff $r < (u-c)/c$. The strategy is the best response iff $qu + V_p - V_b > 0$, which holds given $r < (u-c)/c$. This implies $V_p > 0$ and $V_b > 0$. Equation (6) thus can be solved for $q = \frac{2cK(1+r)+(1-K)(2+r)u}{[2+(1-K)r]u}$. One can show that for any $K \in (0,1)$, $q \in (0,1)$ iff $r < (u-c)/c$. Thus, $V_f > 0$. Also, the no-defection condition (7) is satisfied iff $q \leq \frac{1}{1+r}$, which requires $r \leq \frac{-2cK-(1-K)u+\sqrt{(1+K)u[2cK+(1-K)u]}}{2cK+(1-K)u}$. ■

The fully organized markets equilibrium resembles a Walrasian equilibrium, underlying which the mechanism is interpreted as follows. Specialized traders organize markets for various commodities. Each producer sells product in the market of his production good, receives a bill of exchange, and redeems it next period for his consumption good. A distinction from Walrasian equilibrium is that here it involves the use of a medium of exchange. The reason is that, in the present model though there is public knowledge regarding merchants' trading histories, an ordinary producer-consumer's trading history remains private information. Credit arrangements are thus infeasible and a tangible asset is necessary for transactions in the organized markets.

Now consider the equilibrium where consumers always spend bills in the organized markets but producers find it equally profitable to trade in the organized and unorganized sectors, $S_p \in (0,1), S_b = 0$. Only barter takes place in the unorganized sector.

Proposition 2 *If trade frictions are very low or very high and the discount rate is small, both sectors are active and the use of bills of exchange is limited in the organized markets.*

Proof: See Appendix.

Figure 1 illustrates existence of equilibria.[8] The key element for the existence of this equilibrium lies in bill holders' incentives to trade only in the organized markets. When x is small, it's hard to have a single-coincidence match for bill holders and so always making purchase in the organized markets is incentive compatible. When x is large, barter is easy, and so bill holders like to redeem the bills in the organized markets and become a producer as soon as possible to take advantage of easy barter trade. This is so particularly when the discount rate is big,

[8]Parameter values in the examples of figures 1 are $u = 1$, $K = .1$, $c = .5$, and in figure 2, $r = .05$.

because a higher rate of discounting makes the time-consuming exchange process more troublesome. Indeed, we find that, as r is sufficiently big, always trading bills in the organized markets is the best response for all x. However, for this equilibrium to exist merchant's no-defection condition requires the discount rate be small, so that the discounted utility would be high enough for merchants to stay in business.

In this equilibrium, the value of bills and merchants' profits increase as trade becomes easier in the unorganized sector. One perhaps surprising result is that the extent of organized markets (i.e., the measure of producers who trade in the organized markets, $P(1-S_p)$) increases as trade frictions in the competing sector fall. The intuitive reason is as follows. When trade frictions fall, agents' expected returns from trading in the unorganized sector rise, and so merchants need to offer a higher q to attract customers. Merchant's profits would be sustained only if there is more business, to compensate the lower profit margin. This is possible only if more producers supply goods to the organized markets; i.e., more transactions take place in the organized markets.

Note that in the model considered here, the extent of organized markets does not affect trading efficiency in the markets, but it affects trade difficulties in the unorganized sector. As more transactions take place in the organized markets, it is more difficult to conduct trade in the unorganized sector, because the probability of meeting a trade partner depends on the number of traders. Thus, agent's decision to trade in the organized markets creates a negative externality to those who trade in the unorganized sector.

We now consider the equilibrium where bills circulate as a general medium of exchange. We discuss this equilibrium with observations from numerical experiments. Given other parameters, the best response conditions for $s_p \in (0,1)$ and $s_b \in (0,1)$ hold when x is not too low or too high (see figure 1). When trade frictions are low, conducting trade in the unorganized sector is so attractive that it may not generate sufficient profits for the intermediation business. If trade difficulties are high, it would be too time-consuming for bill holders to encounter trade opportunities in the unorganized sector. The existence of this equilibrium also requires discount rate be small for the no-defection condition to hold. As in the previous equilibrium, merchant's profits and the extent of organized markets increase when trade becomes easier.

In summary, as long as the discount rate is small, merchants arise to set up markets, honor the bills issued by other merchants, and privately-issued trade credit is used as a medium of exchange in the organized markets. If trade frictions are moderate, bills of exchange circulate in the unorganized sector, where people's trading histories are private information.

3.2 Welfare

We have demonstrated a case in that merchants set up markets, operate under competitive conditions, and associated with the emergence of markets privately-issued trade credit may be used as a general medium of exchange. A natural question is whether the emergence of markets and inside money is welfare improving. We use the weighted average flow returns W as the criterion to discuss welfare issues, where

$$W = r(PV_p + BV_b + KV_f). \tag{10}$$

Proposition 3 *The equilibrium with organized markets and inside money dominates the pure barter equilibrium when x or K is small.*

In a pure barter equilibrium all agents are identical and $W = rV_p = x^2(u - c)$. In the equilibrium with active organized markets and inside money, V_p, V_b, V_f are all decreased by the number of merchants. Hence, this equilibrium dominates the pure barter equilibrium when K or x is small. The reason lies in the assumption that merchants in this economy do not produce; they are trade agencies only. A bigger x makes barter easier and so the benefit of organized markets in facilitating trade may not be big enough to compensate the loss in resources used for providing intermediation services.

4 Specialization

We model specialization as producing a smaller set of goods and doing it more proficiently, similar to Camera et al. (2003). Agents become more specialized when they choose a smaller set of goods to produce, with a lower production cost. This implies a lower probability to meet suitable trade partners in a decentralized trading environment. The trade-off between the saving of cost and difficulties in conducting trade is taken into account by agents when deciding the level of specialization.

Specifically, specialization is modeled as follows. If agent i chooses to expand his production set to $s(y_i)$ then the probability that a randomly encountered agent wants to consume agent i's output is $p(y_i) = x + y_i \leq 1$, where $p'(y_i) = 1$ if $y \leq 1 - x$, and $p'(y_i) = 0$, otherwise. A larger y_i thus implies a broader production set and agent i specializes less. The cost in terms of disutility to producing one unit of goods is $c(y_i) = c + ey_i$, with $e > 0$, implying a higher production cost for a larger y_i chosen. Agents cannot produce a good in his consumption set.

We study symmetric and stationary equilibria. Let $s(Y)$ denote all other agents' production set. Given $s(Y)$, the probability that a randomly encountered agent can produce for agent i is $p(Y) = x + Y \leq 1$. Given that the type of agents is randomly and independently determined, and the set of agents is symmetric, the probability of double coincidence of wants in a match is $p(Y)p(y_i)$. In the following discussions, we use y (which equals Y in a symmetric steady sate equilibrium) to represent the level of specialization.

4.1 Symmetric stationary equilibria

We maintain the assumption that a fraction $K \in (0, 1)$ of agents are potential merchants, and agents who trade in the organized markets can always locate the market of their production and consumption goods without search; i.e., the degree of specialization does not affect trade efficiency in the organized markets. When agents choose the extent of specialization, they take into account whether there are organized markets and whether bills of exchange circulate as a general medium of exchange. Agents' beliefs must be sustained in equilibrium. Once the specialization level is chosen, it cannot be changed.

As we have shown in the previous section, a pure barter equilibrium always exists. We first determine the specialization level in a pure barter economy as the benchmark. The flow value to a producer satisfies

$$rV_p(y, Y) = p(Y)p(y)[u - c(y)].$$

An agent, taking as given the expected degree of specialization of other agents, chooses y to maximize his expected return from trade. Let $\overline{V}(y, Y) = rV_p(y, Y)$. Differentiate $\overline{V}(y, Y)$ with respect to y one gets

$$\overline{V}_y(y, Y) = p(Y)[u - c(y) - e(x + y)],$$

because $p(y) = x + y$ and $p'(y) = 1$. The second derivative $\overline{V}_{yy}(y, Y) < 0$. Hence, $\overline{V}_y(y, Y) = 0$ yields the individual optimal choice

$$y^b = \frac{u - c - ex}{2e}.$$

When $x \leq \min\{\frac{u-c}{e}, \frac{2e-(u-c)}{e}\}$, $y^b \in [0, 1-x]$. Notice that $\frac{\partial y^b}{\partial x} < 0$ and $\frac{\partial y^b}{\partial c} < 0$. If trade difficulties are less severe, agents choose a higher level of specialization; when the 'fixed cost' of production is larger, they tend to specialize more to improve production proficiency.

We now check the degree of specialization in the fully organized markets equilibrium. A producer-consumer, taking as given Y and $q(Y)$, chooses y to maximize his expected flow return,[9]

$$\begin{aligned} \overline{V}(y, Y) &= P\ rV_p + B\ rV_b \\ &= (1-K)[q(Y)u - c(y)]/2. \end{aligned}$$

where $q(Y) = \frac{2c(Y)K(1+r)+(1-K)(2+r)u}{2u+(1-K)ru}$. One can easily find that $\overline{V}_y(y, Y) < 0$ and we have corner solution $y^* = 0$.

Proposition 4 *The economy achieves complete specialization in a fully organized markets equilibrium.*

In this equilibrium because agents can always locate the relevant markets of their production and consumption goods without search, they will fully exploit the benefit of specialization to achieve the greatest production efficiency.

The specialization choice in the equilibrium with $S_b = 0$ is summarized as follows.

Proposition 5 *If both sectors are active and the use of bills of exchange is limited in the organized markets, the extent of specialization is identical to that in a pure barter equilibrium.*

Proof: See Appendix.

This result may sound somewhat surprising because people may expect that the degree of specialization would be higher relative to barter due to the existence of organized markets. The intuitive reason lies in the fact that bills are spent only in the organized markets, where there is no double-coincidence problem and the price $q(Y)$ is not affected by an individual's choice of y. Bill holders, after spend bills in the organized market, become producers, who may trade in the unorganized sector where only barter takes place.

We find that the existence and features of this type of equilibrium is similar to that in the basic model; i.e., the equilibrium exists when x is small or x is big, though the existence region

[9] The price $q(Y)$ is determined by a competitive condition similar to (6) and is independent of individuals' choice of specialization.

is larger here. In this equilibrium the degree of specialization $y^* > 0$ and $\frac{\partial y^*}{\partial x} < 0$ when x is small, while $y^* = 0$ when x is large.

We now turn to the equilibrium where bills of exchange circulate as a general medium of exchange in the economy. Given that producers and bill holders are indifferent between trading in both sectors, the value functions satisfy

$$\begin{aligned} rV_p(y,Y) &= PS_p p(Y)p(y)[u - c(y)] + BS_b p(Y)[V_b(y,Y) - V_p(y,Y) - c(y)] \\ rV_b(y,Y) &= PS_p p(Y)[u + V_p(y,Y) - V_b(y,Y)], \end{aligned}$$

Note that the inventory distribution (P, B) and the aggregate strategic variables (S_p, S_b) all depend on Y, and are taken as given when an individual chooses the optimal level of specialization. We solve for the optimal level of specialization y^* from $\overline{V_y}(y, Y) = 0$, and then check whether all the incentive constraints and steady state conditions are satisfied, given $Y = y^*$.

From numerical examples we find $y^* < y^b$: The existence of organized markets and general acceptance of a medium of exchange reduces trade difficulties so that people choose a higher level of specialization relative to barter to improve production efficiency. This result confirms the notion that the existence of markets and money induces specialization. A notable feature is that, the object that plays the role of medium of exchange here is inside money – private liabilities which accompany the rise of organized markets. The reason why the specialization level is different from the case with $S_b = 0$ is that now bill holders trade in single-coincidence matches in the unorganized sector. The existence of inside money, by allowing trade in single-coincidence matches, expands the set of trading opportunities and so increases individuals' incentive to specialize more.

Figure 2 illustrates the effect of changes in trade frictions on the specialization level, welfare and liquidity.[10] As trade frictions become less severe (larger x) agents choose a higher degree of specialization, but $p(Y) = x + Y$ is lower. That is, as trade frictions fall, individuals take this advantage by specializing more to such an extent that the probability of match is actually decreased. If we use the welfare criterion defined in (10), then welfare decreases in x, because $\frac{\partial W}{\partial x} = \frac{\partial W}{\partial P(Y)} \frac{\partial P(Y)}{\partial x} < 0$ when $y > 0$. Recall that welfare increases in x in the basic model with

[10]Parameter values in the examples of figure 3 are $u = 1$, $K = .1$, $e = 1.5$, $r = .05$, $c = .5$. When $x \to .78$, the specialization level $y \to 0$, and the economy performs as in the basic model with no specialization choice when $x > .78$. This equilibrium does not exist when $x < .036$.

no specialization choice (or, in this model when $y^* = 0$ welfare increases in x).

Moreover, the number of bills circulating in the unorganized sector (BS_b) is higher when x is larger. That is, higher inside liquidity is accompanied by a higher degree of specialization. Similar result is also found in Camera et al. (2003), where a larger stock of outside money increases the extent of specialization when liquidity is scarce. The distinctive feature of the present model is that, it is inside liquidity, and is responsive to changes in the characteristics of environment such as trade frictions and production cost.

5 An economy with outside money

In this section we briefly discuss the role of outside money in affecting the performance of inside money. For simplicity we assume no barter in this economy, and producers must pay a cost η (in terms of disutility) to conduct trade in the organized markets.[11] Suppose that initially a fraction M of people are chosen at random and each is endowed with one unit of fiat money. We focus on the equilibria where fiat money is valued.

Let α denote the equilibrium probability that a money holder chooses to trade in the unorganized sector. Whether fiat money is used as a means of payment in the organized markets depends on money holder's strategy. Let q_b and q_m denote the share of goods that merchants give to a customer paying with bill and fiat money, respectively. Let $n = \frac{M(1-\alpha)}{P(1-S_p)}$ denote the fraction of producers receiving fiat money as payment from merchants. This also implies that merchants get a fraction n and $1-n$ of profits from selling goods to money holders and bills holders, respectively. The Bellman's equations now satisfy

$$\begin{aligned} rV_p &= \max_{s_p}\{BS_b x(V_b - V_p - c) + \alpha M x(V_m - V_p - c),\ (1-n)V_b + nV_m - V_p - c - \eta\} \\ rV_b &= \max_{sb}\{PS_p x(u + V_p - V_b),\ q_b u + V_p - V_b\} \\ rV_m &= \max_{\alpha}\{PS_p x(u + V_p - V_m),\ q_m u + V_p - V_m\} \\ rV_f &= P(1-S_p)[(1-n)(1-q_b) + n(1-q_m)]u/K. \end{aligned}$$

[11] In an economy without barter, producers get one unit of bill from trade in the unorganized sector as well as in the organized markets, but trading in the unorganized sector is subject to a time-consuming random matching process. Hence, if there was no cost to trade in the organized markets, all producers will opt to trade there.

The steady state requires goods supplied equal goods demanded in the markets,

$$P(1 - S_p) = B(1 - S_b) + M(1 - \alpha).$$

Finally,

$$P + B + K + M \equiv 1.$$

Instead of describing each type of equilibrium in detail, we summarize the main findings below.[12] First, the equilibrium with inside money can survive more severe trade frictions when fiat money is used in the unorganized sector than otherwise. Thus, outside money overcomes trade frictions to improve, rather than deteriorate, feasibility of inside money. Second, the degree of trade frictions affects agents' incentives to use different means of payment: If trade is easy, it's more likely that both inside and outside money circulate as general media of exchange. If trade is more difficult, inside money is used solely for the transactions in the organized markets and outside money among randomly matched agents, a situation with complete separation in the use of means of payment. Finally, similar to the result found in the previous section, when bills of exchange circulate in both sectors, the society achieves the highest degree of specialization.

6 Conclusions

We depict an economy with trade frictions, where merchants may arise to set up markets and associated with the emergence of markets, privately-issued trade credit may be used as a general medium of exchange. The trade credit issued by merchants is a kind of inside money; it is used in the organized markets and may also circulate in the unorganized sector where people's trading histories are private information. The existence of organized markets and inside money leads to higher extent of specialization relative to barter. Furthermore, higher inside liquidity is accompanied by a higher level of specialization.

For tractability we use a simple divisible goods setup that allows us to model merchant's profits while every trade in the unorganized sector is one-for-one swap, so one did not need to determine the value of bills in the unorganized sector using bilateral bargaining. The model is

[12] There are four types of equilibria: (1) $S_b \in (0,1), \alpha \in (0,1)$; (2) $S_b = 0, \alpha \in (0,1)$; (3) $S_b \in (0,1), \alpha = 1$; and (4) $S_b = 0, \alpha = 1$. We discuss existence and properties of equilibria by numerical examples. Parameter values are $u = 1$, $M = .1$, $K = .05$, $r = .05$, $c = .5$, $\eta = .25$.

able to illustrate the relationship between different trade patterns, characterized by Walrasian markets and a random-matching trading process. Also for simplicity we assume that a predetermined subset of individuals are potential merchants, and trade efficiency in the organized markets is not affected by the number of merchants. An interesting question would be how trading efficiency of markets is affected by merchants' entrance decision and the size of clientele served by merchants. We leave it for future research.

Appendix

Proof of Proposition 2. Given $s_b = 0$, The value functions $V = (V_p, V_b, V_f)$ are strictly positive and inventory distribution $p = (P, B) \in (0, 1)$ if $s_p \in (0, 1)$. We show it is the case when x is close to 0 or close to 1. From $w_p = V_b - V_p - c$ one can solve for s_p. When K is big, as $x \to 0$, $s_p \to 1$ and $\frac{\partial s_p}{\partial x}|_{x\to 0} < 0$; as $x \to 1$, $s_p \to \frac{2[u-(1+r)c]}{4[u-(1+r)c]+2rc} \in (0, 1)$ and $\frac{\partial s_p}{\partial x}|_{x\to 1} < 0$. Next, we check $q \in (0, 1)$. When K is big, as $x \to 0$, $q \to 1$ and $\frac{\partial q}{\partial x}|_{x\to 0} < 0$; as $x \to 1$, $q \to \frac{cs_p+u(1-2s_p)}{u(1-s_p)} > 0$ since $\frac{\partial s_p}{\partial x}|_{x\to 1} < 0$. We also need to check whether the best response condition (5) for $s_b = 0$ is satisfied. When K is big, as $x \to 0$, $(qu + V_p - V_b - w_b) \to \frac{ru(2-s_p)(1-s_p)}{(1+r)(2-s_p)(1-s_p)} > 0$; as $x \to 1$, $(qu + V_p - V_b - y_b) \to \frac{(2-s_p)(u+cs_p)+u(1-2s_p)}{(1+r)(2-s_p)(1-s_p)} > 0$ since $\frac{\partial s_p}{\partial x}|_{x\to 1} < 0$. When K is big, $q \to \frac{c}{u}$, and $\frac{1}{1+r} \to 1$, as $r \to 0$. That is, as r is sufficiently small, $q \leq \frac{1}{1+r}$ and so the no-defection condition is satisfied. ■

Proof of Proposition 5. In the equilibrium with $S_p \in (0, 1), S_b = 0$, the value functions satisfy

$$\begin{aligned} rV_p(y,Y) &= PS_p p(Y)p(y)[u - c(y)] \\ rV_b(y,Y) &= q(Y)u + V_p(y,Y) - V_b(y,Y). \end{aligned}$$

$\overline{V}_y(y,Y) = P^2 S_p p(Y)[u - c(y) - e(x+y)] + B[\frac{\partial V_p(y,Y)}{\partial y} - \frac{\partial V_b(y,Y)}{\partial y}]$. One can solve for $\frac{\partial V_p(y,Y)}{\partial y} - \frac{\partial V_b(y,Y)}{\partial y} = PS_p p(Y)[u - c(y) - e(x+y)]/(1+r)$. Hence, $\overline{V}_y(y,Y) = 0$ yields $y^* = \frac{u-c-ex}{2e}$. ■

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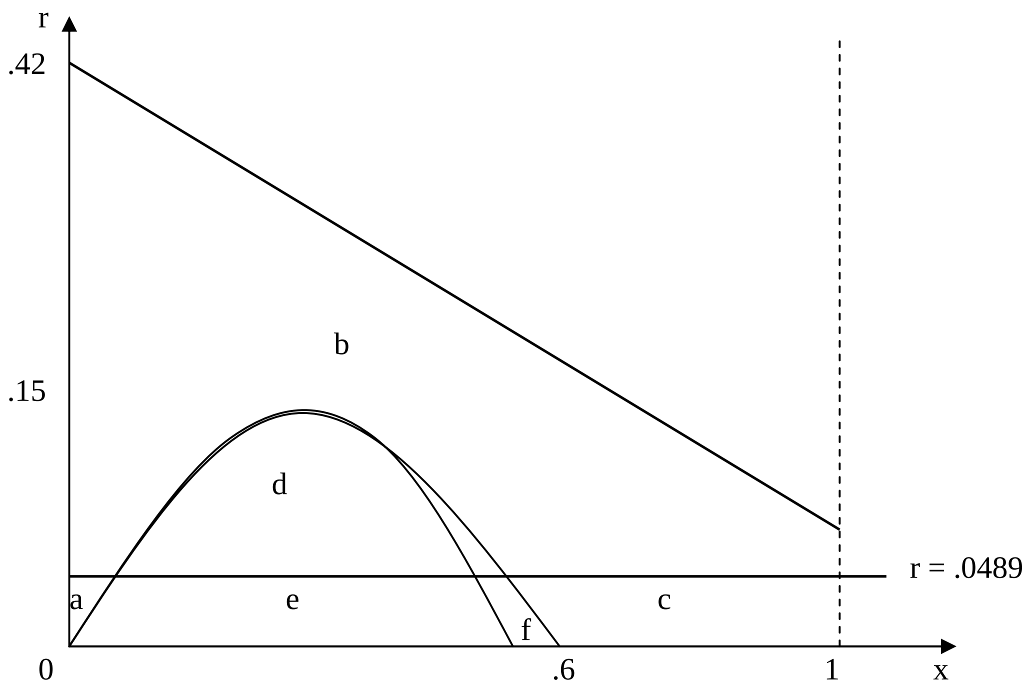

Fullly organized markets equilibrium: areas a, e, f, c

Equilibrium $S_p = (0,1)$, $S_b = 0$: areas a, b, c

Equilibrium $S_p = (0,1)$, $S_b = (0, 1)$: areas d, e

Figure 1 Existence of equilibria with organized markets

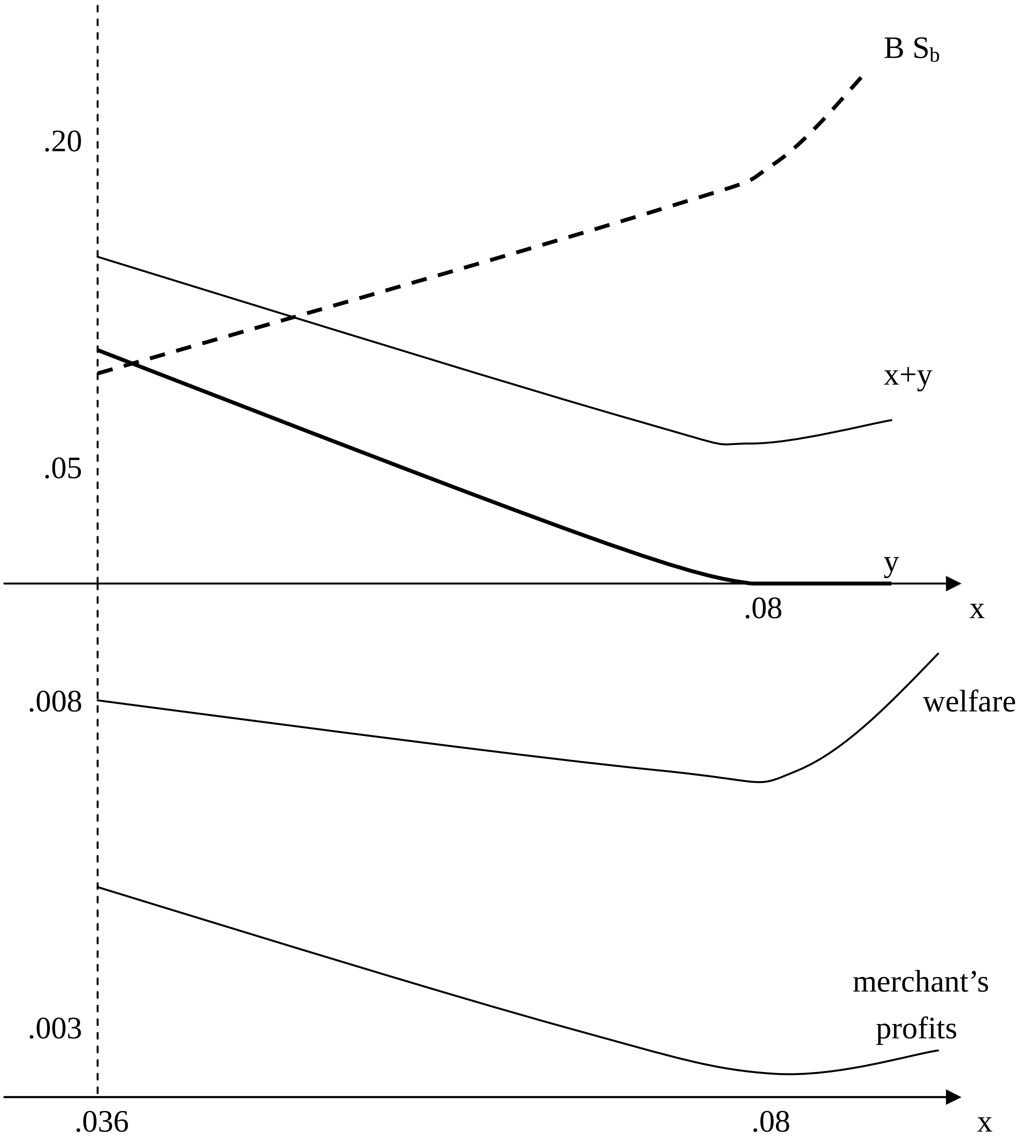

Figure 2 Effects of trade fricitons in equilibrium S_p = (0,1), S_b = (0,1) with specialization choice