Currency and Checking Deposits as Means of Payment

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Abstract

We consider a record keeping cost to distinguish checking deposits from currency in a model where means-of-payment decisions and liquidity of assets are modelled explicitly. An equilibrium exists where checks are used only in big transactions while cash is used in all transactions. Higher inflation or lower reserve requirements raise the deposit interest rate, lower the currency deposit ratio and thereby increase the money multiplier and money supply. Monetary policy has differential impacts on the terms of trade in transactions using different means of payment. During high inflation, individuals economize on the holdings of nominal assets and use checks more frequently, implying higher liquidity of $M1$.

Key words: Currency; Deposits; Record keeping cost; Inflation; Reserve requirements
JEL classification: E40; E43; E51; E52

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1 Introduction

An important feature in the real-world economies is that multiple means of payment are used in transactions. Casual observations suggest that cash is very often used for everyday small-value purchases while checks are used for larger-value payments. The payment instruments have various characteristics that influence people's means-of-payment decisions. For example, carrying large amounts of cash is costly due to the risk of loss or theft, and the forgone interest earnings from holding other assets. Bank deposits may pay you interest but they often have fees or minimum balance requirements. How do individuals' means-of-payment decisions affect the liquidity properties of various assets? And how do changes in the rate of inflation affect the means-of-payment decisions, equilibrium portfolios, and the terms of trade in transactions using different means of payment?

The goal of this paper is to study the above issues in a model where the means-of-payment decisions and liquidity of assets are considered explicitly. The framework is based on Lagos and Wright (2005), with the addition of a preference shock that generates various types of transactions to a representative agent. The decentralized trading arrangement in the model makes media of exchange essential, and also allows us to depict the expected payoff of a payment instrument in facilitating exchange, and derive its liquidity property. While many features distinguish cash from noncash instruments of wealth transfer, we focus on the aspect of record keeping cost – the saving of monitoring resources from using cash instead of the alternatives employing the record keeping technology.[1] Although the basic framework can be used to analyze a variety of assets, we consider here the payment instruments based on bank deposits, such as checks and debit cards, as the alternatives to cash.[2] For this purpose, we introduce a banking sector that accepts deposits and provides interbank settlement services.

We first illustrate the working of the model by considering two types of transactions in terms of the purchase value. If the record keeping cost is not too large nor too small, checks are used only in big transactions while cash is used in all types of transactions. A certain

[1] Kocherlakota (1998) shows that the coexistence of credit and money requires imperfect knowledge of individuals' histories. Along this line, Kocherlakota and Wallace (1998) consider that individual histories are made public only with a lag, and Cavalcanti and Wallace (1999) consider that a subset of agents has public histories.

[2] The recent rising trend of using debit cards in US for payments is remarkable: debit card transactions grew from \$8.3 billion in 2000 to \$15.6 billion in 2003, and \$25.3 billion in 2006 (see Gerdes 2008).

degree of ‘illiquidity’ associated with deposits is necessary for the coexistence of both means of payment, since the interest-bearing feature implies a higher rate of return of bank deposits than currency. In this equilibrium, higher inflation raises the nominal deposit interest rate and reduces agents’ real wealth. Individuals adjust portfolios by substituting out of currency and into bank deposits. The quantity traded in all transactions are reduced by a higher rate of inflation, though the impacts are not uniform. Lower reserve requirements raise both the nominal and real deposit interest rates. Consequently, individuals’ real wealth increases, with a larger proportion in deposits, and the banking sector expands.

The money multiplier and monetary aggregate $M1$ are derived endogenously in this model. Higher inflation or lower reserve requirements reduce the currency deposit ratio and, thereby, increase the money multiplier and money supply. We also show that, as people are more likely to engage in unexpected small transactions, demand for money is higher, resulting in a higher currency deposit ratio and a lower money multiplier. This provides a microfoundation for the precautionary demand for money.

Depending on the record keeping cost, there may exist other types of equilibria. If the record keeping cost is sufficiently large to preclude deposits to be used as a means of payment, individuals may hold deposits simply as a store of value. If the cost is sufficiently low, deposits substitute out of currency as the only means of payment. Since banks hold the base money, monetary authority can still affect the economic activity through changing the growth rate of base money and the required reserve ratio. Interestingly, under the Friedman rule and zero reserve requirements the banking sector will be active only when the record keeping cost is not too large nor too small. Intuitively, holding cash is not costly under Friedman rule, deposits would not be valued unless it generates high enough returns from paying interests and facilitating trades.

The model is extended to consider a more general setup of preference shocks. An individual’s means-of-payment decision is described by a threshold of the valuation of consumption good above which agents use checks.[3] If higher inflation reduces agents’ real deposit balances, this critical value would be lower, meaning that checks are used more frequently. The increase in the

[3] In terms of how an agent’s spending strategy depends on his valuation on seller’s good, our result is similar to what is found by Berentsen and Rocheteau (2002): An agent spends entire money holdings if his valuation for the seller’s good is above some critical value, and spends a fraction of his money holdings if his valuation is below.

liquidity of bank liabilities is an endogenous response of individuals against higher inflation. For holding currency as well as deposits to be incentive compatible during high inflation, individuals not only economize on the holdings of all nominal assets but also use checks more frequently. Consequently, the liquidity of $M1$ is higher under a higher inflation rate.

The idea of considering a record keeping cost to distinguish bank deposits and cash as means of payment has been put forth in a Walrasian model by Prescott (1987). Search-theoretic models have been used to study competition among means of payment and the resulting policy implications.[4] Among this literature, Calvacanti et al. (1999), Williamson (1999) and Li (2006), for example, consider banks to issue private money that competes with fiat money. These papers, however, cannot answer questions as how inflation affects the equilibrium portfolios and liquidity of assets, due to the assumption of indivisible money and restrictions on asset holdings. A more related paper by He et al. (2008) considers the safe-keeping role of banks. An important distinction is that we focus on the role of record keeping cost in getting the bank liabilities into the economy, and derive endogenously money multiplier and monetary aggregate. Banks in the current paper are essentially a costly commitment technology that allows anonymous agents with private trading histories to issue claims to pay for the purchases in the decentralized markets. Our analysis thus adds to the literature on the coexistence of money and credit by further displaying how they may be used in different types of transactions, the effects of monetary policy on various terms of trade, and the liquidity differential of money and the alternative means of payment.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 discusses the optimal portfolio choices and the means-of-payment decisions. In section 4 we discuss the existence and properties of various equilibria. Section 5 considers a more general setup of the preference shocks and a cost of accepting checks. Section 6 concludes.

[4] For example, Matsuyama et al. (1993), Head and Shi (2003) consider competition between local and foreign currencies, Lagos and Rocheteau (2006), Shi (2004), Telyukova and Wright (2008), and Lester et al. (2008) study the competition (and coexistence) of money and other assets such as bonds, capital and credit. But none of the above considers the use of different means of payment in different types of transactions.

2 The Basic Model

The basic framework we use is the divisible money model developed in Lagos and Wright (2005). People trade goods in the market characterized by bilateral random matching, while they visit a centralized market periodically to adjust asset holdings. This model allows us to introduce an idiosyncratic preference shock and incorporate a banking sector while keeping the distribution of balances of currency and deposits analytical tractable.

Time is discrete and there is a $[0, 1]$ continuum of infinitely-lived agents. Each period is divided into two subperiods, that differ in terms of economic activity. All consumption goods are nonstorable and perfectly divisible. In the first subperiod people specialize in production and consumption and there is no double coincidence of wants. Agents meet anonymously according to a random bilateral matching process. When two agents meet, agent i wants something that agent j can produce but not vice versa with probability σ; agent j wants something agent i can produce but not vice versa with probability σ; and neither wants what the other produces with probability $1 - 2\sigma$, where $0 < \sigma < 1/2$. An idiosyncratic preference shock arrives to an agent that determines the utility from consuming goods. An agent consuming q units of his consumption good in period t gets utility $\eta_t u(q)$, where η_t is an i.i.d. preference shock with $\eta_t \in \{\delta, 1\}$, $0 < \delta < 1$, and $\Pr[\eta_t = 1] = \lambda$, $\Pr[\eta_t = \delta] = 1 - \lambda$. Producers incur disutility $v(q)$ from producing q units of output. Assume $u(0) = v(0) = 0, u' > 0, v' > 0$, $u'(0) = \infty$, $u'' < 0, v'' \geq 0$. Trading histories of agents are private information to the agent. There is no commitment or public memory so all trade must be *quid pro quo*.

In the second subperiod there is a frictionless centralized market and all agents can produce and consume a consumption good (called 'general good'), getting utility $U(x)$ from x consumption, with $U'(x) > 0, U'(0) = \infty$, $U'(\infty) = 0$ and $U''(x) \leq 0$. Agents can produce one unit of the good with one unit of labor which generates one unit of disutility. The discount factor across dates is $\beta \in (0, 1)$.

Competitive banks open in the second subperiod. Banks accept deposits from agents and allow them to write checks to pay for purchases in the decentralized market. We assume that banks have a commitment technology – they take deposits and settle financial transactions without defaulting on the interbank debt. If an agent accepts a check for payment in the decentralized market, he presents the check to a bank when arriving in the centralized market.

The balance of the receiving party's account is credited while that of the agent who wrote a check is debited. Agents adjust balances in deposits and currency in the centralized market. The banking system has a technology for record keeping on financial histories but not the trading histories in the goods markets of agents. Therefore, individuals cannot issue trade credit; only cash and bank liabilities such as checks drawn on interest-bearing demand deposits are available means of payment.

Currency

A government is the sole issuer of fiat currency. We assume no costs associated with holding or using currency (but one can incorporate the costs of transportation, risk of loss, theft, and counterfeiting in this model). Currency stock evolves deterministically at a gross rate γ by means of lump-sum transfers, or taxes if $\gamma < 1$, $M_t = \gamma M_{t-1}$, where $\gamma > 0$ and M_t denote the per capita currency stock in period t. Agents receive lump-sum transfers of new money $T_t = (\gamma - 1)M_{t-1}$ in the centralized market. Let ϕ_t denote the value of money in terms of the general good. We denote the real transfer $\tau_t = \phi_t T_t$. For notational ease variables corresponding to the next period are indexed by $+1$, and variables corresponding to the previous period are indexed by -1. We study equilibria in a stationary economy in which the real value of asset holdings is constant. In particular, $\phi M = \phi_{-1} M_{-1}$, which implies $\frac{\phi_{-1}}{\phi} = \gamma$.

Checking deposits

There are two features of checking deposits that distinguish it from currency: the interest payments and the record keeping cost. Assume that banks take deposits and invest in a technology that turns one unit of general good into $R \geq 1$ units of general good in the next period centralized market. Suppose that the investment technology is available only to banks and is not accessible to individuals. This simplifying assumption is not necessary for generating the main results.[5] The required reserve ratio is $\mu \in [0, 1)$. Banks make investments and hold reserves, of which the proceeds are used as interest payments on deposits. The use of bank deposits as a means of payment involves resources, as it relies on the technology of monitoring and record keeping. We use a fixed cost, p_c, paid in the centralized market whenever an individual uses bank

[5] Suppose that agents also had an access to such a technology. If the inflation rate and reserve requirements are in the proper ranges, they would still want to invest some of their resources in the banks since this would allow them to write checks in the decentralized meetings. The setup that banks provide an insurance against the random need of consumption is in the spirit of Diamond and Dybvig (1982).

deposits to make payments in the decentralized market, to capture this record keeping cost.[6] Although the private information problem regarding checks is an important factor for whether they may be widely accepted, for simplicity we assume an enforcement technology that ensures no returned checks due to insufficient funds. This is less a problem to debit cards, because funds are immediately deducted from buyers' accounts at the time of making payments.

We now determine the deposit interest rate i_d (see Freeman and Kydland (2000) for a similar setup). Suppose the reserve requirements are binding, then banks invest $(1-\mu)$ fraction of one dollar deposit (which is worth $(1-\mu)\phi$ units of general good) in the investment technology to get $(1-\mu)\phi R$ goods, and μ fraction in fiat money reserves to get real return $\mu\phi_{+1}$ goods, in the next period. Suppose that banks have zero net worth. The zero-profit condition thus implies

$$1+i_d = [(1-\mu)\phi R + \mu\phi_{+1}]/\phi_{+1},$$

which determines the interest rate on deposits as follows:

$$i_d = (1-\mu)(\gamma R - 1). \tag{1}$$

The deposit rate i_d is affected by the inflation rate γ, required reserve ratio μ, and the rate of return on the investment technology R. If $R \leq \frac{1}{\gamma}$; i.e., the return on the real investment is lower than the return on holding money, banks would invest all the deposits in fiat money and $i_d = 0$. This implies that the banking sector has no advantage over individuals' storage technology, so agents would not make any deposits. Therefore, the banking sector would be inactive. In the following discussions we will restrict our attention to the situation in which $R > \frac{1}{\gamma}$ so that banks are active, and they hold no more than the required reserves. The real deposit rate (in terms of general good) r_d satisfies

$$1 + r_d = (1-\mu)R + \mu\frac{1}{\gamma}.$$

Obviously, higher inflation raises the nominal interest rate but lower the real rate, while lower reserve requirements raise both the nominal and real rates.

Timing of events is as follows. At the beginning of the first subperiod, agents receive a preference shock. Then, agents meet at random and trade in a single-coincidence meeting with terms of trade determined by bargaining. In the second subperiod agents trade goods in the

[6] One can also interpret this cost as the fee for keeping a checking account. For example, checking accounts often have fees, minimum balance requirements or a limit on how many checks people can write each month.

centralized market, settle financial claims with banks, receive lump-sum transfers, and adjust the balances of currency and deposits.

3 Equilibrium

In this economy the preference shock and random matching generate different trading histories across agents. An agent may encounter a meeting in which he is a seller or a buyer with high or low marginal utility, or he may not have any trading opportunity. In a single-coincidence meeting if the buyer has high (low) marginal utility and is willing to buy large (small) amounts of goods, then it is called a type h (l) transaction, or simply a big (small) transaction. A buyer chooses the means to pay for the purchases. Let $I_j, j = h, l$, be the indicator function, of which the value is 1 if a buyer pays for a type j transaction with checks, and 0, otherwise. Due to different trading histories in the decentralized market, agents begin the second subperiod with different portfolios. Since agents can produce one unit of the general good with one unit of labor which generates one unit of disutility, they optimally redistribute the asset holdings so that all agents carry identical portfolios out of the centralized market.[7] That is, under the quasilinear utility assumption the distribution of asset holdings is degenerate at the beginning of a period.

A representative agent begins a period with a portfolio comprised of m units of currency and d units of deposits. Let $V(m, d)$ denote the expected life-time utility of an agent beginning a period with portfolio (m, d) before the preference shock is realized. Denote $z = \phi[m + (1 + i_d)d]$ the worth of an agent's portfolio when entering the centralized market, and $W(z)$ his expected life-time utility. Since the centralized market is frictionless, it is the total value z and not the composition of portfolio that is relevant. In what follows we look at a representative period t and work backwards from the second to the first subperiod.

3.1 The value functions and bargaining

In the second subperiod, agents produce ℓ goods, consume x, and adjust the balances of currency and deposits in the centralized market. The expected utility of an agent holding portfolio value

[7] One can consider ex ante heterogeneity among agents in preferences, discount factors and productivity. Thus, agents may choose different balances of money and deposits out of the centralized market; however, agents of the same type choose identical portfolios.

z entering the second subperiod is

$$
\begin{aligned}
W(z) &= \max_{x,\ell,m_{+1},d_{+1}\geq 0}\{U(x)-\ell+\beta V(m_{+1},d_{+1})\} \\
\text{s.t. } x &= \ell+z+\tau-p_c I_j-\phi(m_{+1}+d_{+1}) \qquad (2)
\end{aligned}
$$

where m_{+1} and d_{+1} are the balances of currency and deposits taken into period $t+1$. Note that the cost p_c must be paid if an agent uses checks to make payments in the first subperiod decentralized market. Due to different trading histories and means of payment decisions in the decentralized market, agents begin the second subperiod with different portfolios. We could have denoted the portfolio z_{ij} where $i = s, b, n$ identifies an agent who was a seller, buyer and non-trader in the decentralized market, respectively, and $j = h, l$ denotes the transaction type. We use z instead of z_{ij} since the initial portfolio brought to the centralized market does not have real consequences, due to quasilinear preferences. For notational ease, an agent's value function entering the second subperiod is denoted as $W(z)$ instead of $W_{ij}(z)$, but it should be noted that $W(z)$ is affected by the means of payment decision, reflected in the term, $p_c I_j$.

Substituting ℓ from the budget constraint, (2) is rearranged as

$$W(z)=z+\tau-p_c I_j+\max_{x,m_{+1},d_{+1}\geq 0}\{U(x)-x-\phi(m_{+1}+d_{+1})+\beta V(m_{+1},d_{+1})\}.$$

The first-order conditions are $U'(x)=1$, which implies $x=x^*$ for all agents, and

$$\phi \geq \beta V_m(m_{+1},d_{+1}), \quad = \text{ if } m_{+1}>0, \qquad (3)$$

$$\phi \geq \beta V_d(m_{+1},d_{+1}), \quad = \text{ if } d_{+1}>0. \qquad (4)$$

Conditions (3) and (4) determine (m_{+1}, d_{+1}), independent of x and z. That is, the optimal choice of (m_{+1}, d_{+1}) is independent of the initial portfolio when entering the centralized market. The envelope conditions are

$$\frac{\partial W(z)}{\partial m} = \phi \qquad (5)$$

$$\frac{\partial W(z)}{\partial d} = \phi(1+i_d). \qquad (6)$$

Agents enter the decentralized market at the beginning of a period, in which each meeting is bilateral and at random. In such a meeting, the seller cares only about the total value, not the composition, of the assets that he receives. For simplicity we assume that the buyer determines the means of payment in a transaction (we derive the means-of-payment decisions below). In a single-coincidence meeting $j = h, l$ between a buyer with portfolio (m, d) and a seller with portfolio $(\widetilde{m}, \widetilde{d})$, the terms of trade are $(q_j, y_j) \in \mathbb{R}^2_+$ where q_j is the quantity of good traded and y_j represents the transfer of asset value from the buyer to the seller. The terms of trade (q_j, y_j) are determined by generalized Nash bargaining, in which the buyer has bargaining power $\theta > 0$, and threat points are given by the continuation values.[8]

Let $a_{I_j,j}$ denote the value of assets available in a transaction j, given the means-of-payment decision I_j. Thus,

$$a_{I_j,j} = \begin{cases} z & \text{if } \; I_j = 1 \\ \phi m & \text{if } \; I_j = 0. \end{cases}$$

Assume that the buyer's preference for the consumption good and the value of assets available in a transaction are known to the seller in a match, so they bargain under complete information. Consider a meeting in which the buyer has high marginal utility of consumption. The terms of trade (q_h, y_h) solves

$$\max_{q_h, y_h \leq a_h} [u(q_h) + W(z - y_h - p_c I_h) - W(z)]^{\theta} [-v(q_h) + W(\widetilde{z} + y_h) - W(\widetilde{z})]^{1-\theta}.$$

Given $W(z + y_h) = W(z) + y_h$, the bargaining problem can be rewritten as

$$\max_{q_h, y_h \leq a_h} [u(q_h) - y_h - p_c I_h]^{\theta} [-v(q_h) + y_h]^{1-\theta}.$$

Solving the bargaining problem we find that the value of assets that the buyer need transfer to the seller in exchange for quantity $q_h \in [0, q_h^*]$ of good is $b_{I_h,h}(q_h)$, where

$$b_{I_h,h}(q_h) = \frac{\theta v(q_h) u'(q_h) + (1-\theta)[u(q_h) - p_c I_h] v'(q_h)}{\theta u'(q_h) + (1-\theta) v'(q_h)} \tag{7}$$

and q_h^* solves $u'(q) = v'(q)$. Similarly, for small transactions, the buyer spends $b_{I_l,l}(q_l)$ in exchange for $q_l \in [0, q_l^*]$, where

$$b_{I_l,l}(q_l) = \frac{\theta v(q_l) \delta u'(q_l) + (1-\theta)[\delta u(q_l) - p_c I_l] v'(q_l)}{\theta \delta u'(q_l) + (1-\theta) v'(q_l)}$$

[8] One can think of this setup as a two-stage game where in the first stage the buyer makes a take-it-or-leave-it offer in bargaining over I_j (i.e., buyer has full power in determining the means of payment), and in the second stage they bargain over (q_j, y_j) with the bargaining power θ to the buyer. This arrangement may not be efficient, but we wish to simplify the analysis by using the Nash bargaining solution. In general, the buyer-seller pair could bargain over q_j, y_j and the use of means of payment I_j jointly. Since $I_j \in \{0, 1\}$, the bargaining set is not convex so one cannot simply apply Nash bargaining solution. To resolve the problem of nonconvexity, one can allow probability mixtures on the outcomes of negotiation (see, e.g., Berentsen et al. 2002).

and q_l^* solves $\delta u'(q) = v'(q)$.[9]

The bargaining solution satisfies

$$q_j(a_{I_j,j}) = \begin{cases} q_j^* & \text{if } \ a_{I_j,j} \geq b_{I_j,j}(q_j^*) \\ b_{I_j,j}^{-1}(a_{I_j,j}) & \text{if } \ a_{I_j,j} < b_{I_j,j}(q_j^*). \end{cases} \tag{8}$$

If $a_{I_j,j} \geq b_{I_j,j}(q_j^*)$ the buyer spends $b_{I_j,j}(q_j^*)$ in exchange for quantity $q = q_j^*$. If $a_{I_j,j} < b_{I_j,j}(q_j^*)$, the buyer spends $a_{I_j,j}$ in exchange for q that solves $b_{I_j,j}(q_j) = a_{I_j,j}$. Note that $b'_{I_j,j}(q_j) > 0$ for all $q_j < q_j^*$. Also note that the bargaining solution is independent of the seller's portfolio, though it depends on $a_{I_j,j}$ which, in turn, depends on the buyer's means-of-payment decision. We assume that $\frac{u'(q)}{b'_{I_h,h}(q)}$ is strictly decreasing in q, so that we have $a_{I_h,h} < b_{I_h,h}(q_h^*)$ when $I_h = 1$.[10] This implies that the total value of an agent's portfolio is less than the amount that is required to buy the socially efficient quantity in a big transaction, and the buyer will spend all his asset. One can show that $q_j \leq \overline{q}_j$ where $\overline{q}_j$ is the q that maximizes the buyer's surplus $\delta_j u(q_j) - b_{I_j,j}(q_j)$, where $\delta_h = 1$ and $\delta_l = \delta$, and $\overline{q}_j \leq q_j^*$ with strict inequality unless $\theta = 1$.

The value function $V(m, d)$ satisfies the following Bellman's equation:

$$\begin{aligned} V(m,d) \;\; = \;\; & \sigma\lambda\{u[q_h(a_{I_h,h})] + W(z - y_h - p_c I_h)\} + \sigma(1-\lambda)\{\delta u[q_l(a_{I_l,l})] + W(z - y_l - p_c I_l)]\} \\ & + \sigma\lambda\{-v[q_h(\widetilde{a}_{I_h,h})] + W(z + \widetilde{a}_{I_h,h}\} + \sigma(1-\lambda)\{-v[q_l(\widetilde{a}_{I_l,l})] + W(z + \widetilde{a}_{I_l,l})\} \\ & + (1 - 2\sigma)W(z), \end{aligned}$$

where the first two terms represent the payoff to buying $q_j(a_{I_j,j})$ units for $a_{I_j,j}$ in transaction $j = h, l$, and the last two terms represent the payoff to selling $q_j(\widetilde{a}_{I_j,j})$ units for $\widetilde{a}_{I_j,j}$. Given the bargaining solution we rewrite $V(m, d)$ as

$$\begin{aligned} V(m,d) \;\; = \;\; & \sigma\lambda\{u[q_h(a_{I_h,h})] - b_{I_h,h}[q_h(a_{I_h,h})] - p_c I_h\} + \sigma(1-\lambda)\{\delta u[q_l(a_{I_l,l})] - b_{I_l,l}[q_l(a_{I_l,l})] - p_c I_l\} \\ & + \sigma\lambda\{-v[q_h(\widetilde{a}_{I_h,h})] + \widetilde{a}_{I_h,h}\} + \sigma(1-\lambda)\{-v[q_l(\widetilde{a}_{I_l,l})] + \widetilde{a}_{I_l,l}\} + W(z). \end{aligned} \tag{9}$$

[9] Assuming that buyers make take-it-or-leave-it offer in the bargaining of means-of-payment and (q_j, y_j), the results are qualitatively the same as in the current model, except that $b_{I_j,j}(q_j) = v(q_j)$, which is independent of the cost p_c.

[10] Lagos and Wright (2005) show that this assumption holds if θ is close to 1, and it holds for any θ if $v(q)$ is linear and $u'(q)$ is log concave.

3.2 The optimal portfolio choices

To find an agent's optimal portfolio, we first derive the expected marginal value of each asset in the decentralized market:

$$\begin{aligned}
V_m(m,d) &= \sigma\lambda\{u'[q_h(a_{I_h,h})] - b'_{I_h,h}[q_h(a_{I_h,h})]\}\frac{\partial q_h(a_{I_h,h})}{\partial m} \\
&\quad +\sigma(1-\lambda)\{\delta u'[q_l(a_{I_l,l})] - b'_{I_l,l}[q_l(a_{I_l,l})]\}\frac{\partial q_l(a_{I_l,l})}{\partial m} + \frac{\partial W(z)}{\partial m} \\
V_d(m,d) &= \sigma\lambda\{u'[q_h(a_{I_h,h})] - b'_{I_h,h}[q_h(a_{I_h,h})]\}\frac{\partial q_h(a_{I_h,h})}{\partial d} \\
&\quad +\sigma(1-\lambda)\{\delta u'[q_l(a_{I_l,l})] - b'_{I_l,l}[q_l(a_{I_l,l})]\}\frac{\partial q_l(a_{I_l,l})}{\partial d} + \frac{\partial W(z)}{\partial d}.
\end{aligned}$$

From the bargaining solution, $\frac{\partial q_j}{\partial m} = \frac{\phi}{b'_{I_j,j}(q_j)}$ and $\frac{\partial q_j}{\partial d} = \frac{\phi(1+i_d)}{b'_{I_j,j}(q_j)}$ are the change in the quantity traded if the buyer brings an additional unit of money and deposit, respectively, to the market. Let $r_{I_h,h}(q_h) = \frac{u'(q_h)}{b'_{I_h,h}(q_h)} - 1$ and $r_{I_l,l}(q_l) = \frac{\delta u'(q_l)}{b'_{I_l,l}(q_l)} - 1$ denote the rate of liquidity return of an asset from conducting big and small transactions, respectively. Given (5) and (6) we rewrite the above equations as

$$V_m(m,d) = \phi[\sigma\lambda r_{0,h}(q_h) + \sigma(1-\lambda)r_{0,l}(q_l) + 1] \tag{10}$$

$$V_d(m,d) = \phi(1+i_d)[\sigma\lambda I_h r_{I_h,h}(q_h) + \sigma(1-\lambda)I_l r_{I_l,l}(q_l) + 1]. \tag{11}$$

Using (3), (4), (10) and (11), a representative agent's optimal portfolio choices must satisfy

$$\phi_{-1} \geq \beta\phi[\sigma\lambda r_{0,h}(q_h) + \sigma(1-\lambda)r_{0,l}(q_l) + 1], \ = \text{ if } m > 0 \tag{12}$$

$$\phi_{-1} \geq \beta\phi(1+i_d)[\sigma\lambda I_h r_{I_h,h}(q_h) + \sigma(1-\lambda)I_l r_{I_l,l}(q_l) + 1], \ = \text{ if } d > 0. \tag{13}$$

Condition (12) states that if people choose to hold currency, the cost of acquiring an additional unit of currency must equal the expected discounted payoff from facilitating all kinds of transactions in the decentralized market. The right-hand side of condition (13) is the expected discounted payoff of an additional unit of deposits from facilitating transactions, given the means-of-payment decisions.

3.3 The means-of-payment decisions

When deciding the means of payment in a transaction, an individual compares the cost p_c with the net payoffs from using checks rather than cash only. To derive the means of payment decision, in this subsection we reserve the notation q_j for the quantity in exchange when $I_j = 1$, and use q_j^0 to denote the quantity when $I_j = 0$.

Let $b_{0,h}(q_h^0)$ denote the value of $b_{I_h,h}(q_h)$ with $I_h = 0$ in (7). In a big transaction, if an agent uses only cash, his payoff is $u[q_h^0(\phi m)] - b_{0,h}[q_h^0(\phi m)]$. If he spends all assets, the payoff is $u[q_h(z)] - b_{1,h}[q_h(z)]$ with $I_h = 1$ in (7). The net benefit of using checks instead of cash only is

$$\Delta_h = \{u[q_h(z)] - b_{1,h}[q_h(z)]\} - \{u[q_h^0(\phi m)] - b_{0,h}[q_h^0(\phi m)]\}. \tag{14}$$

Hence, $I_h = 1$ if $p_c < \Delta_h$. Let w denote an individual's wealth. We establish in the Appendix the following condition on the means-of-payment decision by using the notion of the liquidity return $r_{I_h,h}(q_h)$:

$$I_h = 1 \text{ if } p_c < \int_{\phi m}^{z} r_{1,h}(q_h)dw. \tag{15}$$

A similar argument can be applied to find the condition on I_l, but note that the bargaining solution (8) implies that at most the buyer spends $b_{I_l,l}(q_l^*)$ in exchange for q_l^* in a small transaction. Let $d_l = \min\{z, b_{I_l,l}(q_l^*)\}$. We have

$$I_l = 1 \text{ if } p_c < \int_{\phi m}^{d_l} r_{1,l}(q_l)dw. \tag{16}$$

Given p_c, if δ is sufficiently small, it is possible that the net payoff from using checks to finance small transactions is not large enough to compensate the cost, and so agents use only cash to pay for the purchases.

Definition 1 *A stationary equilibrium is a list of value functions (V, W), individuals' choices (m, d, I_h, I_l), terms of trade (q_h, q_l, y_h, y_l), a sequence of prices $\{\phi\}$ and a deposit rate i_d that solve (2) and (9), satisfy the bargaining solution (8), the optimal portfolio choices (12) and (13), the means-of-payment decisions (15) and (16), market clearing for fiat money $m + \mu d = M_{-1}$, and bank's zero profit condition (1).*

There are four types of potential equilibria, characterized by the means-of-payment decisions (I_h, I_l), where I_h and I_l can take a value in the set $\{0, 1\}$. The decisions (I_h, I_l) should be consistent with the holdings of asset (m, d) in equilibrium. We can rule out the case $(I_h, I_l) = (0, 1)$, since if it pays to bear the cost p_c to use checks in small transactions, it must do so in big transactions. If the cost p_c is sufficiently large to preclude the use of deposits as means of payment, $(I_h, I_l) = (0, 0)$, and the economy would rely on currency as the unique means of payment. Individuals may or may not hold deposits, and if they do, deposits are held simply as a store of value. On the other hand, if p_c is small enough so that $(I_h, I_l) = (1, 1)$, we may see a 'cashless' society. If $(I_h, I_l) = (1, 0)$, then checks are used only in big transactions and currency is used in all transactions. We characterize all possible equilibria in the next section.

4 Coexistence of currency and checking deposits

We first characterize a monetary equilibrium in which checks are used only in big transactions and currency is used in all transactions, and then briefly discuss other types of equilibria. The following lemma establishes the result on deposits as a means of payment when currency circulates to facilitate transactions.

Lemma 1 *When currency is used as a medium of exchange, if $i_d > 0$ then $I_h = 1$ implies $I_l = 0$.*

Lemma 1 implies that it is not feasible for both currency and checks to be used in *all* transactions if the deposit rate is strictly positive. A certain degree of 'illiquidity' associated with checks is necessary for the coexistence of both means of payment, due to the interest-bearing feature of deposits. This result also provides an explanation for the rate-of-return dominance puzzle.

Given the conditions on the means-of-payment decisions and Lemma 1, we characterize monetary equilibrium as follows.

Proposition 1 *In a monetary equilibrium with $i_d > 0$, if p_c is not too large nor too small, currency is used in all transactions and checks are used only in big transactions. The quantity (q_h, q_l) solves*

$$\sigma\lambda r_{1,h}(q_h) = \frac{\gamma}{\beta(1+i_d)} - 1 \tag{17}$$

$$\sigma(1-\lambda) r_{0,l}(q_l) = \frac{\gamma i_d}{\beta(1+i_d)}, \tag{18}$$

the price ϕ and the portfolio (m, d) solve

$$\begin{aligned} b_{1,h}(q_h) &= \phi[m + (1 + i_d)d] && (19) \\ b_{0,l}(q_l) &= \phi m && (20) \\ m + \mu d &= M_{-1}, \end{aligned}$$

and i_d is given by (1).

In this equilibrium, individuals hold both currency and deposits. Therefore, (12) and (13) hold at equality, which give us (17) and (18). From (17) and (18) one finds $\sigma(1-\lambda)r_{0,l}(q_l) = i_d[1 + \sigma\lambda r_{1,h}(q_h)]$; that is, the expected liquidity return from holding one more unit of currency must equal its opportunity cost – the interest payments plus the expected liquidity return from facilitating big transactions that deposits could have derived.

From (18), if $i_d > 0$ then $r_{0,l}(q_l) > 0$, which implies that people will not carry more currency than needed to achieve the quantity that maximizes buyer's surplus, $\overline{q}_l$. From (17) if $\gamma \geq \beta(1+i_d)$, then $r_{1,h}(q_h) \geq 0$, and agents do not hold more assets than needed to buy the quantity that maximizes buyer's surplus $\overline{q}_h$. The condition $\gamma \geq \beta(1+ i_d)$ requires $\gamma \geq \underline{\gamma}$ where

$$\underline{\gamma} = \frac{\beta\mu}{1 - \beta(1-\mu)R}. \tag{21}$$

Intuitively, when inflation is high to offset what would have been earned from working harder today for more savings, people keep their deposits for the sole purpose of transaction. If $\gamma < \underline{\gamma}$, agents would hold more deposits than needed to achieve the quantity $\overline{q}_h$.[11]

The next proposition shows the effects of monetary policy on individuals' equilibrium portfolios, and the endogenously determined money multiplier and monetary aggregate.

Proposition 2 *In the equilibrium in which currency is used in all transactions and checks are used only in big transactions:*

1. *Higher inflation raises the deposit rate i_d, and reduces q_l and q_h. Individuals' real wealth and real money balances also decline.*

[11] If agents can keep some of the deposits in savings account that could not be used in transaction, then agents will hold checking deposits no greater than what needed to buy $\overline{q}_h$; also see Lester et al. (2008).

2. *Lower reserve requirements raise i_d and q_h, but reduces q_l. Individuals' real wealth is increased, with a larger proportion in deposits, and real money balances decline.*

3. *Let $M1 = m + d$ denote the money supply. Then $M1 = M_{-1}[1 + \frac{d(1-\mu)}{m+\mu d}]$ where M_{-1} is the monetary base and $1 + \frac{d(1-\mu)}{m+\mu d}$ is the money multiplier. Lower reserve requirements reduce the currency deposit ratio and thus, raise the money multiplier and money supply.*

Any policies or factors that affect the deposit interest rates will change the equilibrium portfolios and the intratemporal terms of trade. Proposition 2 shows that higher inflation reduces the quantities traded in all transactions, but the impacts are not uniform. Using $u(q) = \sqrt{q}, v(q) = q$ as an example and letting $\theta \to 1$, we find

$$\frac{dq_h}{d\gamma} - \frac{dq_l}{d\gamma} = \frac{4(\frac{i_d \sqrt[3]{q_l}}{\eta(1-\lambda)} - \frac{\sqrt[3]{q_h}}{\lambda})}{\beta(1+i_d)\sigma},$$

which may be positive if λ is big. That is, inflation reduces q_h less than it does to q_l if λ, the probability of conducting small transactions, is sufficiently big. This example implies that inflation could have differential welfare impacts on people who rely on different means of payment.[12] From numerical examples we also find that as inflation goes up, individuals adjust their portfolio in such a way that they hold less cash but more deposits, resulting in a lower currency deposit ratio, higher money multiplier and money supply.

Changes in the reserve requirements have even more differential impacts on the terms of trade. From Proposition 2, as the required reserve ratio is lower, q_h increases but q_l declines. This policy in fact allows banks to take more advantage on the investment technology so that they can raise the deposit rate and attract more deposits. Lower reserve requirements thus expand the size of the banking system.

The interest-bearing feature of bank deposits makes it more attractive than cash during high inflation. Note that in this model agents adjust holdings of currency and deposits in the centralized market and can use both assets in the subsequent period. One can consider that check clearing takes time, which would introduce additional frictions for bank deposits to be

[12] Due to the minimum balance requirements to open and maintain a checking account, people with lower income or wealth may not have checking accounts, so they rely on cash as the means of payment. The above result can be interpreted as an implication for the distributional effects of monetary policy.

used in transactions. This time-consuming feature of check clearing may cause people to switch from deposits and into more liquid assets such as cash to ameliorate the loss of purchasing power. This 'flight to liquidity' should be observed, unless the rate of returns on the alternative asset can compensate the loss.

The precautionary demand for money

In this equilibrium fiat money is dominated by bank deposits in the rate of return, yet both coexists. To avoid the cost of using checks in small transactions, people are willing to forgo some interest payments by holding currency. The forgone interest earnings may be interpreted as the 'insurance premium' for not using checks to pay for the unexpected small-value purchases. Currency's liquidity value thus derives mainly from facilitating the unexpected small transactions, and we call it the 'precautionary demand for money'. The following result provides a microfoundation for the precautionary demand for money.

Proposition 3 *In the equilibrium with currency and checking deposits as means of payment, if agents are more likely to conduct small transactions, the demand for currency is higher and checking deposits lower, resulting a higher currency deposit ratio, lower money multiplier and money supply.*

We now discuss other types of equilibria.

Deposits are held simply as a store of value

If the record keeping cost p_c is sufficiently large so that (15) and (16) do not hold, there can exist a monetary equilibrium with $(I_h, I_l) = (0, 0)$, and currency is the unique medium of exchange. If individuals hold deposits, equations (12) and (13) hold at equality, which requires $\gamma = \underline{\gamma}$, where $\underline{\gamma}$ is defined in (21). If $\gamma > \underline{\gamma}$, individuals do not hold deposits, since inflation is so high that it does not pay to work hard today to save in order to get the interest payments next period. If $\gamma < \underline{\gamma}$, condition (13) does not hold so the equilibrium does not exist. In summary, if the record keeping cost is sufficiently large to preclude deposits to be used as a means of payment, it will be held simply as a store of value when $\gamma = \underline{\gamma}$. If inflation is higher than the threshold $\underline{\gamma}$, the only nominal asset that people are willing to hold is currency, in order to carry out transactions in the decentralized market. Under this situation, the banking sector becomes inactive.

A 'cashless' economy

If the record keeping cost is sufficiently small, then $(I_h, I_l) = (1, 1)$, checking deposits are used in all transactions. For this to be an equilibrium, currency would not be used as a medium of exchange, as indicated by Lemma 1. The conditions for agents' optimal portfolio choices are (12) holding at strict inequality (i.e., $\gamma > \beta$), and (13) holding at equality. In this equilibrium, the only means of payment is deposits. We characterize this equilibrium in the following proposition.

Proposition 4 *If p_c is sufficiently low, individuals do not hold currency, and deposits are the only means of payment in the economy. The quantity (q_h, q_l), value of the base money ϕ, and the holdings of deposits d solve*

$$\begin{aligned}
\frac{\gamma}{\beta(1+i_d)} - 1 &= \sigma\lambda r_{1,h}(q_h) + \sigma(1-\lambda) r_{1,l}(q_l) \\
b_{1,h}(q_h) &= \phi(1+i_d)d \\
b_{1,l}(q_l) &= \min[b_{1,l}(q_l^*), \phi(1+i_d)d] \\
\mu d &= M_{-1},
\end{aligned}$$

and $i_d > 0$ is given by (1).

Note that even though $m = 0$, this is not a nonmonetary equilibrium. Banks hold the base money since they operate under the reserve requirements. The base money may take the form of reserves in the central bank and is not necessarily a tangible object like vault cash. Money thus functions as a unit of account, rather than a medium of exchange. In this 'cashless' economy, since banks hold the base money, monetary authority can still affect the economic activity through changing the lump-sum transfers to banks and the required reserve ratio. Those measures affect the value of base money ϕ and the deposit interest rate.[13] The effects of changes in γ and μ are similar to those in the equilibrium with currency and deposits as means of payment (proof is similar to that of Proposition 2 and is omitted).

One may wonder whether the banking sector is still active under Friedman rule ($\gamma = \beta$) and zero reserve requirements ($\mu = 0$). The key factors are whether the return on real investment is higher than the return of holding money, and the record keeping cost p_c. As discussed in Section

[13] Banks have to hold the base money as reserves, so ϕ can also be interpreted as the price to buy and sell reserves in the interbank funds market

2, if $R \leq \frac{1}{\gamma}$, banks invest all deposits in fiat money and $i_d = 0$. Under this situation, agents have no incentives to hold deposits since the interest payments are zero and it is costly to use deposits as a means of payment. Hence, the banking sector disappears regardless of Friedman rule and zero reserve requirements. If $R > \frac{1}{\gamma}$, $i_d = R\beta - 1 > 0$ (since $\gamma = \beta$ and $\mu = 0$). If the record keeping cost p_c is not too large nor too small, agents hold deposits and currency, and use checks in big transactions (see Proposition 1).[14] If p_c is sufficiently large to preclude deposits to be used as a means of payment, (13) does not hold under Friedman rule, so the equilibrium where deposits are held solely as a store of value does not exist. Similarly, if p_c is sufficiently small, Friedman rule implies (12) holds at equality so the 'cashless' equilibrium does not exist, either. Therefore, the banking sector is active under the Friedman rule and zero reserve requirements only when the record keeping cost is not too large nor too small. Intuitively, holding cash is not costly under Friedman rule, deposits would not be valued unless it generates high enough returns from paying interests and facilitating trades.

5 Extensions

We extend the basic model by considering a more general setup of preference shocks. This allows us to study further the effects of monetary policy on the use of means of payment and liquidity of assets. We also consider a cost of accepting checks, and briefly discuss how it affects the determination of a payment instrument in a transaction.

5.1 A more general setup of preference shocks

An agent consuming q units of his consumption good in the decentralized market gets utility $\eta u(q)$, where η is a random variable drawn from the distribution $F(\eta)$ with support $[\ \underline{\eta}, 1\ ]$, and $0 < \underline{\eta} < 1$. The preference shock is independent across time and agents. We maintain the assumption that buyers have full power in determining the means of payment in a transaction. The terms of trade (q_η, y_η) are determined by generalized Nash bargaining, in which the buyer has bargaining power $\theta > 0$, and the threat points are given by the continuation values. Our focus is to find the critical value η_b such that buyers use checks to make payments in transactions

[14] Since under Friedman rule $\gamma < \beta(1 + i_d)$, (17) implies $r_{1,h}(q_h) < 0$, agents hold more deposits than needed to achieve the quantity that maximizes buyer's surplus $\overline{q}_h$.

with $\eta \geq \eta_b$.

We denote $b_{I_\eta,\eta}(q_\eta)$ the real value of asset that the buyer transfers to the seller in transaction η, where η is drawn from $F(\eta)$ and

$$b_{I_\eta,\eta}(q_\eta) = \frac{\theta v(q_\eta)\eta u'(q_\eta) + (1-\theta)[\eta u(q_\eta) - p_c I_\eta]v'(q_\eta)}{\theta\eta u'(q_\eta) + (1-\theta)v'(q_\eta)}. \tag{22}$$

The bargaining solution is (8) with $j = \eta$, where $a_{I_\eta,\eta} = z$ if $I_\eta = 1$, and $a_{I_\eta,\eta} = \phi m$, otherwise. The bargaining solution implies that people do not buy more than q^*_η; i.e., at most he spends $b_{I_\eta,\eta}(q^*_\eta)$ in exchange for q^*_η, no matter which payment instrument he uses. Thus, in a sufficiently small transaction, it is possible that buyer's currency holding is more than what is needed to buy q^*_η, and there is no need to use checks. Let η' denote the transaction in which buyer's currency is just enough to buy the social optimal quantity $q^*_{\eta'}$. Hence, to look for η_b we need only consider transactions with $\eta > \eta'$. Moreover, let η'' denote the transaction in which the buyer's real wealth z is just enough to buy the social optimal quantity $q^*_{\eta''}$. Whether η_b is greater than η'' depends on, among other things, the record keeping cost. We present the results here for the case $\eta_b > \eta''$, but the other case can be studied in a similar way.

Let $c_\eta = \min\{z,\, b_{1,\eta}(q^*_\eta)\}$. Let $b_{0,\eta}(q^0_\eta)$ denote the value of $b_{0,\eta}(q_\eta)$ with $I_\eta = 0$ in (22). In transaction η if the buyer spends only cash, his payoff is $\eta u[q^0_\eta(\phi m)] - b_{0,\eta}[q^0_\eta(\phi m)]$; if he uses checks, the payoff is $\eta u[q_\eta(c_\eta)] - b_{1,\eta}[q_\eta(c_\eta)]$ with $I_\eta = 1$ in (22). The net benefit of using checks rather than currency only is

$$\triangle_\eta = \{\eta u[q_\eta(c_\eta)] - b_{1,\eta}[q_\eta(c_\eta)]\} - \{\eta u[q^0_\eta(\phi m)] - b_{0,\eta}[q^0_\eta(\phi m)]\}.$$

The critical value η_b satisfies $p_c = \triangle_{\eta_b}$. As in the basic model, there can exist a variety of equilibria, characterized by whether deposits and currency are used as means of payment. Here we focus on the case that the record keeping cost is not too big nor too small so that there exists a critical value $\eta_b \in (\eta', 1)$ satisfying $\triangle_{\eta_b} = p_c$. In Appendix we show that $\triangle_\eta \geq 0$ and is increasing in η when $\eta \geq \eta'$. Thus, agents use checks to make payments in larger-value purchases, i.e., in transaction $\eta \geq \eta_b$. Let $r_{I_\eta,\eta}(q_\eta) \equiv \frac{\eta u'(q_\eta)}{b'_{I_\eta,\eta}(q_\eta)} - 1$ denote the rate of liquidity return of an asset from financing a type η transaction. In the following discussion we use the liquidity return $r_{I_\eta,\eta}(q_\eta)$ to express the means-of-payment decision. That is, η_b satisfies

$$p_c = \int_{\phi m}^{z} r_{I_\eta,\eta_b}(q_{\eta_b})dw. \tag{23}$$

In summary, in a transaction $\eta \leq \eta'$, the buyer spends $b_{I_\eta,\eta}(q^*_\eta)$ of cash in exchange for q^*_η, and he spends all currency in exchange for quantity $q_\eta = b^{-1}_{0,\eta}(\phi m)$ in a transaction $\eta \in (\eta', \eta_b)$. In a transaction $\eta \in [\eta_b, 1]$ the buyer spends all his asset in exchange for $q_\eta = b^{-1}_{1,\eta}(z)$.

To derive an individual's optimal portfolio, note that when the asset holdings are more than enough to buy the socially efficient quantity q^*_η, an additional unit of asset would not increase the purchase of goods, so $\frac{\partial q^*_\eta}{\partial m} = \frac{\partial q^*_\eta}{\partial z} = 0$. Also note that currency is used in all transactions in the equilibrium we consider here. The optimal portfolio choices thus satisfy

$$\begin{aligned}
\phi_{-1} &\geq \beta\phi[\sigma \int_{\eta'}^{1} r_{0,\eta}(q_\eta)dF(\eta) + 1], \;=\; \text{if } m > 0, \\
\phi_{-1} &\geq \beta\phi(1+i_d)[\sigma \int_{\eta_b}^{1} r_{1,\eta}(q_\eta)dF(\eta) + 1], \;=\; \text{if } d > 0.
\end{aligned}$$

The value $(\eta', q^*_{\eta'})$ satisfies

$$\begin{gathered}
b_{0,\eta'}(q^*_{\eta'}) = \phi m \\
\eta' u'(q^*_{\eta'}) = v'(q^*_{\eta'})
\end{gathered} \tag{24}$$

The asset transferred from the buyer to the seller and the quantity of goods in exchange satisfy

$$b_{I_\eta,\eta}(q_\eta) = \begin{cases} b_{0,\eta}(q^*_\eta) \text{ and } q_\eta = q^*_\eta \text{ for } \eta \in [\,\underline{\eta}, \eta'] \\ \phi m \text{ and } q_\eta = b^{-1}_{0,\eta}(\phi m) \text{ for } \eta \in (\eta', \eta_b) \\ z \text{ and } q_\eta = b^{-1}_{1,\eta}(z) \text{ for } \eta \in [\eta_b, 1]. \end{cases} \tag{25}$$

In an equilibrium with $m > 0, d > 0$ the conditions on the optimal portfolios are

$$\sigma \int_{\eta_b}^{1} r_{1,\eta}(q_\eta)dF(\eta) + 1 = \frac{\gamma}{\beta(1+i_d)} \tag{26}$$

$$\sigma \int_{\eta'}^{\eta_b} r_{0,\eta}(q_\eta)dF(\eta) = \frac{\gamma i_d}{\beta(1+i_d)}. \tag{27}$$

Conditions (25) – (27), together with the market clearing condition for fiat money and bank's zero profit condition (1), can be solved for q_η and (m, d, ϕ, i_d).[15]

[15] If $\eta_b < \eta''$, then in a transaction $\eta \in [\eta_b, \eta'']$ the buyer spends $b_{I_\eta,\eta}(q^*_\eta)$ in exchange for q^*_η. When $\eta > \eta''$, the buyer spends all his asset in exchange for $q_\eta = b^{-1}_{1,\eta}(z)$. One can solve for $(\eta'', q^*_{\eta''})$ from

$$\begin{aligned}
b_{1,\eta''}(q^*_{\eta''}) &= \phi[m + (1+i_d)d] \\
\eta'' u'(q^*_{\eta''}) &= v'(q^*_{\eta''}).
\end{aligned}$$

The asset transferred from the buyer to the seller and the quantity of goods in exchange satisfy

The next result shows the effects of monetary policy on the means-of-payment decisions, equilibrium portfolios and liquidity of assets.[16]

Proposition 5 *In the economy with a distribution of preference shock, $F(\eta)$, currency is used in all transactions, and checks are used only in transaction $\eta \geq \eta_b$.*

1. *Higher inflation reduces the quantity traded in all transactions. Individuals' real wealth and real money balances also decline. If the real deposit balances are reduced by inflation, so is η_b, which implies checks are used more frequently and liquidity of deposits is higher.*

2. *Lower reserve requirements raise individuals' real wealth, real deposits, and the quantity traded using checks. The critical value η_b is higher, so checks are used less frequently.*

What is interesting here perhaps is the result that individuals may choose a lower η_b, implying higher liquidity of bank deposits, during high inflation. For holding currency as well as deposits to be incentive compatible during high inflation, individuals may economize on the holdings of nominal assets and also use checks more often, reducing the liquidity differential of both assets. The increase in the liquidity of bank deposits is an endogenous response of individuals against higher inflation. The empirical implication is that the liquidity of $M1$ is increased by a higher inflation rate.

Lower reserve requirements induce agents to use checks less frequently, reducing the liquidity of bank deposits. This is also a result of individuals' optimal response: as the return on deposits is raised by the policy, its liquidity must be lower for agents to be willing to hold both assets. Given that individuals' real wealth and deposit rate are increased by the policy, changes in the real money balances depends on the magnitudes of the two opposing forces. If the effect of an increase in total wealth dominates that of an increases in the deposit rate; i.e., the income effect dominates the substitution effect, real money balances would be higher.

$$b_{I_\eta,\eta}(q_\eta) = \begin{cases} b_{0,\eta}(q^*_\eta) \text{ for } \eta \in [\,\underline{\eta}, \eta'] \\ \phi m \text{ and } q_\eta = b^{-1}_{0,\eta}(\phi m) \text{ for } \eta \in (\eta', \eta_b) \\ b_{1,\eta}(q^*_\eta) \text{ for } \eta \in [\eta_b, \eta''] \\ z \text{ and } q_\eta = b^{-1}_{1,\eta}(z) \text{ for } \eta \in (\eta'', 1] \end{cases}$$

[16] We define liquidity as the ratio of the amounts of an asset used as means of payment in transactions to the total stock of the asset, in a specified period of time.

5.2 Costs of accepting checks

Merchants' costs of accepting checks may be caused by the delay of check clearing and returning of checks due to insufficient funds in the payers' accounts. There is usually a day or two between when a merchant receives a check and when the funds in the checking account are actually deducted from the payers and transferred to the merchants' accounts. These problems are less severe under the rapid development of debit cards and other types of electronic payments based on checking deposits, and the legislation on speeding up check clearing.[17] However, to accept electronic payments merchants need card readers and communication devices which are costly to set up and maintain. Moreover, there is usually a merchant fee that sellers have to pay to the payment card providers.

In this subsection we consider instead a cost of accepting checks, p_a, paid in the centralized market. Sellers have to make the decision as whether or not to accept checks in a transaction. Let $\mathbf{1}_\eta$ be an indicator function, whose value is 1 if the seller accepts checks in a type η transaction, and 0, otherwise, where η is draws from $F(\eta)$. Then,

$$W(z) = z + \tau - p_a \mathbf{1}_\eta + \max_{x, m_{+1}, d_{+1} \geq 0} \{U(x) - x - \phi(m_{+1} + d_{+1}) + \beta V(m_{+1}, d_{+1})\}.$$

The bargaining problem becomes

$$\max_{q_\eta, y_\eta \leq z} [\eta u(q_\eta) - y_\eta]^\theta [-v(q_\eta) + y_\eta - p_a \mathbf{1}_\eta]^{1-\theta}.$$

Solving the bargaining problem we find that the total value of assets that the buyer needs to transfer to the seller in exchange for quantity $q_\eta \in [0, q_\eta^*]$ of good is $b_\eta(q_\eta)$, where

$$b_{\mathbf{1}_\eta, \eta}(q_\eta) = \frac{\theta[v(q_\eta) + p_a \mathbf{1}_\eta]\eta u'(q_\eta) + (1-\theta)\eta u(q_\eta) v'(q_\eta)}{\theta \eta u'(q_\eta) + (1-\theta) v'(q_\eta)}. \tag{28}$$

and $a_{\mathbf{1}_\eta, \eta} = z$ if $\mathbf{1}_\eta = 1$, and $a_{\mathbf{1}_\eta, \eta} = \phi m$, otherwise.

The condition on the decision of accepting checks can be found in a similar way as in the previous section. If in a transaction η a seller accepts checks for payments, his payoff is

[17] The Check Clearing for the 21st Century Act was passed in October 2003 in the U.S., which speeds up the check-clearing process. The law permits banks to clear funds electronically instead of waiting for paper checks to make their way around the country, thus eliminating the three- to four-day "float" many consumers have come to count on. Check 21 is intended to increase the speed of check clearing, lower clearing system costs, and reduce the financial system's vulnerability to problems with air and ground travel.

$-v[q_\eta(c_\eta)] + b_{1,\eta}[q_\eta(c_\eta)]$ with $\mathbf{1}_\eta = 1$ in (28). If he accepts only cash, his payoff is $-v[q_\eta^0(\phi m)] + b_{0,\eta}[q_\eta^0(\phi m)]$ with $\mathbf{1}_\eta = 0$ in (28). The difference between both is

$$\Upsilon_\eta = -v[q_\eta(c_\eta)] + b_{1,\eta}[q_\eta(c_\eta)] - \{-v[q_\eta^0(\phi m)] + b_{0,\eta}[q_\eta^0(\phi m)]\}. \tag{29}$$

Thus, sellers accept checks in a transaction in which $\Upsilon_\eta \geq p_a$.

Use of a means of payment when there are costs of using and accepting checks

The use of a particular payment instrument in a transaction may be related to the institutions such as financial infrastructure and the relative bargaining power of customers and merchants. Suppose that there are costs of using and accepting checks. We assume, as in Section 4, the buyer has full power in determining the means of payment in a transaction, and the buyer-seller pair bargains over q_j and y_j. Depending on, among other things, the costs of using and accepting checks, there may be some transactions that both sides prefer different payment instruments. Consider a situation in which the buyer prefers paying checks but seller prefers cash. They may not trade if seller refuses to accept the alternative, more costly, means of payment. However, if seller concedes to accept the more costly means of payment (if doing so yields positive gains from trade), the transaction will be executed. Consequently, the liquidity of bank deposits would be higher than otherwise.

In the real world merchants may wish to accept a variety of payment instruments if doing so brings them more transaction opportunities. The current model assumes a given probability of being a seller or a buyer and, hence, the probability of meeting a customer is not affected by the means of payment that a seller accepts. Nonetheless, the above example demonstrates that by accepting a more costly payment instrument the seller can make transactions, for otherwise, he would lose the sale. This captures the idea that a merchant can increase sales by accepting the payment instruments that are convenient to the customers.

5.3 Discussion: no-surcharge rule

Merchants may incur large cost differentials in accepting various payment instruments.[18] The higher cost of debit cards is reflected in a merchant fee that a seller pays to the card company.

[18] Table A2 in Humphrey et al. (1997) shows that, in US 1994, the costs per transaction (in US$) of cash, check and debit card to the payees are 0.07, 0.43 and 0.3, respectively. The costs per $100 of sale of cash, check and debit card to the payees are 0.52, 1.2 and 0.94, respectively.

Usually this fee is deducted from the merchant's payment by the card provider, and not paid explicitly by the customer. The card providers even prohibit merchants to charge different prices for different methods of payment, i.e., they impose the 'no-surcharge' rule. Our model can also address the debate on the 'no-surcharge' rule.[19]

For simplicity, we consider an economy with $\eta_t = 1$ for all t.[20] Let p_c represent the cost that a buyer has to pay if he uses the debit card to make payments. Suppose the values of purchase (i.e., the value of payment transferred in a transaction) by using different payment methods are identical. We focus on the quantity of good exchanged to compare the per-unit price paid by credit cards versus cash. The bargaining solution implies that the value of assets that the buyer need transfer to the seller in exchange for quantity $q \in [0, q^*]$ of good is $b_{I_c}(q)$, where

$$b_{I_c}(q) = \frac{\theta v(q)u'(q) + (1-\theta)[u(q) - p_c I_c]v'(q)}{\theta u'(q) + (1-\theta)v'(q)},$$

in which $I_c = 1$ if the buyer pays by debit card, and $I_c = 0$ if the buyer pays by cash.

Let q_1 and q_0 denote the quantity in exchange when debit cards and cash are used for payments, respectively. Buyers use debit cards if $u(q_1) - u(q_0) \geq p_c$. This condition is slightly different from (14) because we have controlled for the amount of purchase; i.e., $b_1(q_1) = b_0(q_0)$. From the bargaining solution $b_{I_c}(q)$, $q_1 \geq q_0$ due to the cost p_c. Hence, the buyer gets larger consumption (lower per-unit price) when he uses debit card rather than cash.

Now consider the case in which the card providers charge a merchant fee, p_a. Assume that merchants choose the means of payment in a particular transaction. The total value of assets that the buyer needs to transfer to the seller in exchange for quantity $q \in [0, q^*]$ of good is $b_{\mathbf{1}_a}(q)$, where

$$b_{\mathbf{1}_a}(q) = \frac{\theta[v(q) + p_a \mathbf{1}_a]u'(q) + (1-\theta)u(q)v'(q)}{\theta u'(q) + (1-\theta)v'(q)}.$$

Merchants accept debit cards if $v(q_0) - v(q_1) \geq p_a$. This condition is similar to (29), except that we have controlled for the amount of purchases. From the bargaining solution $b_{\mathbf{1}_a}(q)$, $q_1 \leq q_0$ due to the cost p_a. Since sellers pay merchant fees, they produce less output (charge higher price) to customers making payment by debit cards.

[19] I thank the referee for the suggestion of applying the model to address the debate on the no-surcharge rule.

[20] We do not consider the case where buyers may undertake big or small transactions as in Section 2, for the reason that even if the per-unit prices in big transactions and small transactions are different, it is not easy to argue whether the no-surcharge rule holds. For example, in reality consumers may get discounts from big transactions, regardless of the payment methods.

Monnet and Roberds (2008) define the no-surcharge rule as such that a consumer's per-unit cost of buying goods with payment cards does not exceed the cost of making the same purchase with cash. They find that no-surcharge rule must hold in equilibrium when money growth rate is sufficiently close to the time preference rate (i.e., near the Friedman rule). The reason is that since it is costly to join the trade club that accepts cards as a means of payment, the no-surcharge rule is necessary to induce agents to transact using cards when the cost of holding cash is sufficiently low. In the current model, if the buyer bears the cost of using debit card, his per-unit cost is lower than the cost of making the same purchase with cash. This result is consistent with the no-surcharge rule. The more costly payment method will be used only if buyers receive enough compensation over cash payment. Our result does not depend on the money growth rate since the cost of using debit card is reflected in the bargaining solution. The no-surcharge rule, however, does not hold in the case where merchants have to pay the fee.

6 Conclusion

This paper incorporates banks into a monetary model, in which media of exchange are essential, means-of-payment decisions and liquidity are modeled explicitly, and monetary aggregates including government money and bank liabilities are endogenously determined. We study the effects of monetary policy on the equilibrium portfolios, liquidity and the rate of return distribution of assets. The arrangement studied here is sufficiently explicit that one can examine the costs and benefits associated with modifying the scheme. Moreover, as mentioned in He et al. (2006), much work uses $M1$ and $M2$ empirically, even though it looks like the relevant measure in the models should be $M0$ (e.g., Lagos and Wright 2005, and Lucas 2000). Our model represents an attempt to reconcile theory and practice along this dimension.

In this economy the banking system has a technology that keeps financial records of people but not transaction records in the decentralized market and, therefore, individuals cannot issue trade credit. One can introduce some features into the environment that would give rise to the use of credit as well as currency and checking deposits as means of payment. Another extension is to study issues related to the rate-of-return-dominance puzzle and denomination of assets. We have shown that the record keeping cost makes the interest-bearing deposits less liquid than cash. If one interprets the alternative asset in the model as any financial asset with a record keeping cost, such as government bonds, then such a model can account for the rate-of-return-dominance puzzle.[21] One can also study whether an asset with a higher record keeping cost should have a larger denomination. From the society's view point, it is optimal to allocate the scarce monitoring resources in larger-value transactions. The large denomination may create an additional friction for an asset to be used to facilitate transactions. Thus, we may observe lower liquidity for assets with a larger rate of return and, perhaps, of a larger denomination. In such an economy, banks may engage in the 'denomination intermediation' – buying relatively illiquid high-yield, large denomination assets to issue lower-yield, smaller denomination liabilities that may be more liquid.

[21] Telyukova and Wright (2008) also consider an interest-bearing asset with a fixed liquidation cost in an extension, and show that agents liquidate this asset to make payment only in big transactions.

Appendix

Proof of Lemma 1.

Suppose not, then both means of payment have identical liquidity return from facilitating transactions. Comparing (12) and (13) one finds that checking deposits earn interest whereas currency does not, but the values of both means of payments in the decentralized market are identical, a contradiction.■

We show the condition on an agent's means-of-payment decision. If the buyer does not use checks, $I_h = 0$, the asset value transferred from buyer to the seller is $b_{0,h}(q_h^0)$. The difference in the payoffs of using checks rather than cash is $\Delta_h = \{u[q_h(z)] - b_{1,h}[q_h(z)]\} - \{u[q_h^0(\phi m)] - b_{0,h}[q_h^0(\phi m)]\}$. When an agent uses cash in a big transaction, the quantity he receives is $q_h^0(\phi m) = b_{0,h}^{-1}(\phi m)$, while if paying checks he gets $q_h(z) = b_{1,h}^{-1}(z)$. The two functions solving for q_h^0 and q_h are slightly different due to the cost p_c. One can decompose Δ_h as $\Delta_h = \{u[q_h(z)] - u[q_h(\phi m)] - [b_{1,h}(q_h(z)) - b_{1,h}(q_h(\phi m))]\} + u[q_h(\phi m)] - u[q_h^0(\phi m)] - \{b_{1,h}[q_h(\phi m)] - b_{0,h}[q_h^0(\phi m)]\}$. The term in the first big bracket equals $\int_{q_h(\phi m)}^{q_h(z)} (u' - b')dq_h(w) = \int_{\phi m}^{z} (u' - b')\frac{dq_h(w)}{dw}dw$. But note that $\frac{dq_h(w)}{dw} = \frac{1}{b'}$, so we get $\int_{\phi m}^{z} r_{1,h}(q_h)dw$. The term $b_{1,h}[q_h(\phi m)] - b_{0,h}[q_h^0(\phi m)]$ is zero, as both equals ϕm. Let ε denote $u[q_h(\phi m)] - u[q_h^0(\phi m)] > 0$ because $q_h(\phi m) > q_h^0(\phi m)$. Thus, $\Delta = \int_{\phi m}^{z} r_{1,h}(q_h)dw + \varepsilon$. Since ε is very small, we use condition (15) in the text.■

Proof of proposition 2.

1. Substituting i_d from (1) into (17) and (18), one finds that $\frac{\partial r_{1,h}(q_h)}{\partial \gamma} = \frac{\mu}{\beta\lambda\sigma[\mu+(1-\mu)\gamma R]^2} > 0$ and $\frac{\partial r_{0,l}(q_l)}{\partial \gamma} = \frac{(1-\mu)[-\mu+2\mu\gamma R+(1-\mu)\gamma^2 R^2]}{\beta(1-\lambda)\sigma[\mu+(1-\mu)\gamma R]^2} > 0$. Because $r'_{1,h}(q_h) < 0$ and $r'_{0,l}(q_l) < 0$, we have $\frac{\partial q_h}{\partial \gamma} < 0$ and $\frac{\partial q_l}{\partial \gamma} < 0$. Since $b'_{1,h}(q_h) > 0$ and $b'_{0,l}(q_l) > 0$, given (19) and (20), individuals hold less real wealth z, and less real balances ϕm.

2. From (1), $\frac{\partial i_d}{\partial \mu} < 0$ since $\gamma R > 1$. From (17) and (18), $\frac{\partial r_{1,h}(q_h)}{\partial i_d} < 0$ and $\frac{\partial r_{0,l}(q_l)}{\partial i_d} > 0$, and so $\frac{\partial q_h}{\partial \mu} < 0$ and $\frac{\partial q_l}{\partial \mu} > 0$. Thus, z is higher but ϕm is lower and, therefore, real deposits increase.

3. $M1 = m + d = M_{-1}[1 + \frac{d(1-\mu)}{m+\mu d}]$. A lower $\frac{m}{d}$ raises the money multiplier $1 + \frac{d(1-\mu)}{m+\mu d}$ and $M1$.■

Here we show that $\triangle_\eta$ is increasing in η when $\eta \geq \eta'$, and there exists a critical value η_b satisfying $\triangle_{\eta_b} = p_c$. Given the definition of η', obviously $\triangle_\eta \geq 0$ for $\eta \geq \eta'$. Taking the derivative of $\triangle_\eta$ with respect to η and after some manipulation we have

$$\frac{\partial \triangle_\eta}{\partial \eta} = \{u[q_\eta(c_\eta)] - u[q^0_\eta(\phi m)]\} + \phi\{r_{1,\eta}[q_\eta(c_\eta)]\frac{\partial c_\eta}{\partial \eta} - r_{0,\eta}[q^0_\eta(\phi m)]\frac{\partial(\phi m)}{\partial \eta}\} \geq 0$$

because $u[q_\eta(c_\eta)] - u[q^0_\eta(\phi m)] \geq 0$ when $\eta \geq \eta'$, $\frac{\partial c_\eta}{\partial \eta} \geq 0$ and $\frac{\partial(\phi m)}{\partial \eta} = 0$.■

Proof of proposition 5.

1. First note the RHS of (26) and (27) are increased by γ. Equation (26) requires higher $r_{1,\eta}(q_\eta)$ and/or a lower η_b. Consider first $r_{1,\eta}(q_\eta)$ is increased by the policy. This implies lower total real wealth z, since $\frac{\partial q_\eta(z)}{\partial z} > 0$ and $r'_{1,\eta}(q_\eta) < 0$. We now show that real balances ϕm is decreased by γ. Suppose not. From (24) higher real balances imply a higher η' and lower $r_\eta(q_\eta)$ since $\frac{\partial q_\eta(\phi m)}{\partial(\phi m)} > 0$. Since the RHS of (27) is increased by γ, η_b must be raised in order to satisfy (27). However, given that z is lower and ϕm is higher, $r_{\eta_b}(q_{\eta_b})$ must be increased to satisfy (23), which implies a lower η_b since $\frac{\partial q_\eta}{\partial \eta} > 0$, a contradiction. Hence, inflation reduces real balances. Suppose inflation decreases the real deposits $\phi(1 + i_d)d$ as well. Since $z - \phi m$ is smaller, (23) implies a higher r_{η_b}, i.e., a lower η_b. Second, consider that η_b is decreased by γ. From (27), η' must be lower, which implies lower real balances. Since $r_{\eta_b}(q_{\eta_b})$ is increased by a lower η_b, and ϕm is lower, (23) implies that $z - \phi m$ must be smaller. That is, inflation reduces real deposits.

2. A lower required reserve ratio μ reduces the RHS of (26). Wish a similar argument as in the previous proof, the total wealth z is increased. However, a lower μ increases the RHS of (27). We consider two cases. (i) Suppose ϕm is increased by a lower μ. We have a higher η' and lower $r_\eta(q_\eta)$. Thus, η_b must be increased to satisfy (27). With higher z, ϕm and η_b, the real deposits $\phi(1 + i_d)d$ must be increased to satisfy (23). (ii) Suppose ϕm is decreased by a lower μ. This implies higher real deposits $\phi(1 + i_d)d$ because z is higher. Since $z - \phi m$ is larger, (23) implies a lower r_{η_b}, i.e., a higher η_b. Both cases imply that lower μ increases real deposits as well as η_b.■

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