

# Liquidity and Asset Prices: A New Monetarist Approach

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## Abstract

When lenders cannot force borrowers to repay debts, assets are often pledged to secure loans. In this paper borrowers lose collateral once they renege on debts, and exclusion of defaulters occurs probabilistically, with a higher probability implying better enforcement. Increased efficiency in enforcement raises loan-to-value ratios, while reducing asset prices. If the rise in loan-to-value ratios is the dominant effect, aggregate liquidity and output increase with the advance in enforcement. Inflation raises the repayment cost by increasing the loan rate, while raising the default cost through exclusion. Consequently, inflation raises loan-to-value ratios and output only when enforcement is sufficiently efficient.

Loan-to-value Ratios; Asset prices; Credit Constraints; Collateral; Liquidity

E41; E50

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# 1 Introduction

The provision of credit helps to satisfy people’s need to finance unanticipated consumption or investment. Credit provision is often constrained if lenders cannot force borrowers to repay their debts, unless the debts are secured. We consider an economy with such a feature, where financial assets are used as collateral to secure loans, to study interactions among asset prices, credit constraints, and aggregate liquidity. In particular, our model features limitations on enforcement, which determine the level of exclusion. We explicitly derive credit constraints and thus, loan-to-value ratios, from the condition that lenders offer to loan only as much as borrowers are willing to repay. Endogenizing loan-to-value ratios can help relax the assumption of the exogenously given, constant loan-to-value ratios that have been made in the previous literature.<sup>1</sup> Determining loan-to-value ratios under various degrees of punishment is also motivated by the evidence that typical loan-to-value ratios vary significantly across countries, and by the argument that cross-country variations in these ratios may partly reflect differences in the technology and institutions to deter default.

Table 1 reports typical loan-to-value ratios (abbreviated as “LTV ratios” in the table), and the average duration and cost of foreclosure procedures for some industrialized countries. Observe that countries with lower LTV ratios usually have longer or more costly foreclosure procedures. For instance, LTV ratios in Italy and Greece are 65% and 70%, respectively. In Italy, the foreclosure process is reported to last about five years; in Greece, though it takes about two years to complete a foreclosure, the cost is estimated at 16%. On the other hand, the Netherlands has LTV ratios as high as 101%, and low figures for the duration and the cost (5 months and 4%, respectively). A similar pattern applies in Austria, Finland, the US and the UK. In Belgium and France, there is a private or public guarantee for mortgage loans, which helps to reduce the total cost of foreclosure.<sup>2</sup> In sum, if one interprets the duration and cost of foreclosure procedures as a measure of the country’s ability to enforce debt repayment, then our observation suggests that economies with

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<sup>1</sup>For example, following Kiyotaki and Moore’s (1997) setup, Iacoviello (2005) assumes a constant fraction,  $1 - \theta$ , of the collateral value as the transaction cost that lenders must pay in order to repossess the borrowers’ assets if borrowers repudiate their debt obligations. That is, the loan-to-value ratio is assumed to be a constant  $\theta$ .

<sup>2</sup>The percentage of the outstanding amount of loans that is covered by public or private guarantee is 18% in Belgium, 13% in Austria, and 44% in France, while there is no such guarantee in Germany, the Netherlands, or Portugal (ECB 2009).

better enforcement have higher LTV ratios.

We introduce banks and a real asset, like the claims to “trees” in the standard Lucas (1978) asset-pricing theory, into a model with limited record keeping, enforcement, and commitment (e.g., Lagos and Wright 2005, and Berentsen, Camera, and Waller 2007). Banks channel funds from people with idle cash to those who need liquidity to finance unanticipated consumption. There are two mechanisms to deter default—a *collateral mechanism*, which requires borrowers to pledge some assets to secure their loans, and banks are entitled to the collateral once borrowers renege on debts, and a *reputation mechanism*, which punishes defaulters by permanent exclusion. To capture the effects of enforcement on loan-to-value ratios, our model combines the collateral mechanism and the reputation mechanism; moreover, we assume that, in the spirit of Kocherlakota and Wallace (1998), exclusion occurs probabilistically, with a higher probability implying better enforcement.

In the current paper, borrowers’ credit limits are affected by the price of the collateralized asset, while the asset price is affected by the amount of liquidity that the asset can generate by backing loans. The asset prices, credit limits, and loan-to-value ratios are determined simultaneously in equilibrium. If banks can force debt repayment at no cost, the asset is priced at the discounted sum of dividends. Under limited enforcement, the asset commands a price higher than the fundamental value if credit constraints bind. This liquidity premium of the asset arises because the marginal benefit of loans is higher than the borrowing cost when credit rationing occurs, and the premium increases if credit rationing becomes more severe.

Increased efficiency of enforcement raises loan-to-value ratios by relaxing borrowing constraints. Advances in the enforcement technology, however, reduces the asset price, because collateral becomes a less important commitment device for borrowing. When the technology’s efficiency is above some threshold, the punishment of exclusion is substantial enough to make the rise in the loan-to-value ratio a dominant effect. As a result, aggregate liquidity, output, and welfare increase with advances in the technology. Therefore, imposing restrictions on the access to future credit may be beneficial to the society only when it constitutes a substantial punishment on defaulters.

In our environment, higher inflation exerts adverse effects on output by reducing the incentive to produce, as is standard in monetary models. The binding credit constraints, however, constitute an additional transmission mechanism of monetary policy. Inflation raises the loan rate and, thus,

the repayment cost, so the loan-to-value ratio has to be reduced to control for the incentive to default. If exclusion is feasible, inflation relaxes the credit constraint by increasing the cost of default, because defaulters need to bring enough money to self-insure against consumption shocks. This disciplinary effect of inflation works through exclusion and, therefore, when enforcement is strong enough for inflation to impose a sufficient penalty, the positive effect can outweigh negative effects so that loan-to-value ratios, liquidity, and output rise.<sup>3</sup>

The rest of the paper is organized as follows. Section 2 describes the model. In section 3 we derive the equilibrium conditions. Section 4 derives the equilibrium under limited enforcement. We further discuss comparisons among mechanisms, our theoretic contribution to the existing literature, and the empirical relevance of our findings. Section 5 concludes. All proofs and omitted derivations of equations are contained in the online supplementary appendix.

## 2 The Model

The basic model is based on Lagos and Wright (2005) and Berentsen, Camera, and Waller (2007). There is a  $[0, 1]$  continuum of infinitely lived agents. Time is discrete and continues forever. Each period is divided into two subperiods, and in each subperiod trades occur in competitive markets. There are two consumption goods, one produced in the first subperiod, and the other (called the *general good*) in the second subperiod. Consumption goods are perishable and perfectly divisible. The discount factor across periods is  $\beta \in (0, 1)$ .

In the beginning of the first subperiod, an agent receives a preference shock that determines whether he consumes or produces. With probability  $n$  an agent can produce but cannot consume, while with probability  $1 - n$  the agent can consume but cannot produce. We refer to producers as sellers and consumers as buyers. This is a simple way to capture the uncertainty of the opportunity to trade. Consumers get utility  $u(q)$  from  $q$  consumption. Producers incur disutility  $c(q)$  from producing  $q$  units of output. Assume  $u(0) = c(0) = 0$ ,  $u'(q) > 0$ ,  $c'(q) > 0$ ,  $u'(0) = \infty$ ,  $u''(q) < 0$  and  $c''(q) \geq 0$ . To motivate a role for fiat money, we assume that all goods trades are anonymous,

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<sup>3</sup>The result that higher inflation may increase output in an environment with credit arrangements also appear in some previous studies, e.g., Berentsen, Camera, and Waller (2007), Aiyagari and Williamson (2000), and Levine (1991). The distinction is that we identify how the level of exclusion influences the effect of inflation on loan-to-value ratios.

and there is no public record of individuals' trading histories.

In the second subperiod, agents get utility  $U(x)$  from  $x$  consumption, with  $U'(x) > 0$ ,  $U'(0) = \infty$ ,  $U'(\infty) = 0$  and  $U''(x) \leq 0$ . Agents can produce one unit of the general good with one unit of labor, which generates one unit of disutility. This setup allows us to introduce an idiosyncratic preference shock and incorporate a banking sector while keeping the distribution of asset holdings analytically tractable.

There are two types of infinitely lived assets in the economy: fiat money and a real asset like the claims to Lucas (1978) trees.<sup>4</sup> A government is the sole issuer of fiat money. The money stock evolves deterministically at a gross rate  $\gamma$  by means of lump-sum transfers,  $M_t = \gamma M_{t-1}$ , where  $\gamma > 0$ , and  $M_t$  denotes the per capita currency stock in period  $t$ . Agents receive lump-sum transfers of money  $T_t = (\gamma - 1)M_{t-1}$  in the second subperiod. There is a fixed supply,  $A$ , of Lucas trees. Each tree generates a constant flow of  $\rho > 0$  units of the general good at the beginning of the second subperiod, before the asset is traded. One can think of agents as trading claims on trees.<sup>5</sup>

Competitive banks channel funds from people with idle cash to those with liquidity needs. They accept nominal deposits and make nominal loans. Sellers in the first subperiod can deposit their money holdings in banks at the nominal interest rate,  $i_d$ , and are entitled to withdraw funds in the second subperiod. Buyers may borrow money from banks at the nominal loan rate,  $i$ , and need to repay their loans in the second subperiod. We assume that loans and deposits are not rolled over, and so all financial contracts are one-period contracts.<sup>6</sup> Moreover, banks have zero net worth, and there are no operating costs or reserve requirements.

In this economy, banks have limited ability to force repayment of debts. The possibility of strategic default generates an endogenous borrowing constraint; i.e., a bank's loan contract allows

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<sup>4</sup>To concentrate on the role of the real asset as collateral, our model features fiat money as the unique means of payment. In Online Supplementary Appendix C we consider an extension in which the real asset can be used as collateral and as a means of payment. We illustrate that the price of an asset reflects its dual role in overcoming the frictions caused by an insufficiency of payment instruments and credit market imperfections.

<sup>5</sup>One may think that the real asset can serve as collateral to back trade credit between private agents in an economy with anonymous trade, such as is considered in, e.g., Shi (1996) and Li (2001). Notice that Shi (1996) assumes creditors stay put so debtors can find creditors and make repayment, whereas Li (2001) assumes a technology that allows for communication only between a pair of debtor and creditor. Our model does not assume the aforementioned technology for repayment, and so the lack of record keeping of goods trades obstructs the use of trade credit.

<sup>6</sup>With the assumption on the linear utility costs of production in the second subperiod, agents do not gain by spreading the repayment of loans or redemption of deposits across periods.

agents to borrow only as much as they have incentives to repay. There are two mechanisms to punish defaulters. First, borrowers need to pledge some assets as collateral to secure loans, and banks are entitled to the collateral once borrowers renege on debts (referred as the collateral mechanism).<sup>7</sup> Besides seizing collateral, defaulters can be excluded from the banking sector (referred as the reputation mechanism).<sup>8</sup> In the spirit of Kocherlakota and Wallace (1998), we capture the efficiency of enforcement by the probability with which a defaulter is excluded permanently from the banking sector, and a higher probability implies better enforcement.<sup>9</sup> Specifically, at the second subperiod of each period, after banks seize a defaulter's collateral, an agent's default record is updated with probability  $\zeta$ , and he is excluded permanently. With probability  $1 - \zeta$  the updating does not occur, and a defaulter starts the next period as a nondefaulter. Therefore, one can interpret  $\zeta$  as a measure of the degree of punishment, or a measure of financial development, as it reflects the technology and infrastructure of the financial system to control for the moral hazard problem.<sup>10</sup>

### 3 Equilibrium

Let  $\phi_t$  and  $\psi_t$  denote the values of money and the real asset in terms of the general good produced in the second subperiod, respectively. We study symmetric stationary equilibria in which the real value of asset holdings is constant:  $\phi_t M_t = \phi_{t-1} M_{t-1}$ , which implies  $\frac{\phi_{t-1}}{\phi_t} = \gamma$ ; the inflation rate equals the money growth rate. Because we focus on the stationary equilibria, it is reasonable to consider a constant price of the real asset given its fixed supply; hence,  $\psi_{t-1} = \psi_t = \psi$  for all  $t$ . In

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<sup>7</sup>Borrowers are assumed to pledge all their assets as collateral. This assumption is innocuous, since if agents are credit unconstrained, the asset is priced at the fundamental value and the amount of the real asset pledged as collateral does not matter for economic activity, whereas if agents are constrained, they prefer to pledge all of their assets to receive credit.

<sup>8</sup>Restricting defaulters from future access to credit relies on the efficiency of collecting information and sharing of agents' repayment histories among banks. Commercial and consumer credit bureaus provide mechanisms for the exchange of payment performance data. Empirical evidence suggests an important link between the existence of information exchanges and credit availability (see Berger and Udell 2006). Other institutions that may affect the lending technology include the infrastructure of the legal, judicial, and bankruptcy environments, and regulations.

<sup>9</sup>Kocherlakota and Wallace (1998) show in a random-matching environment that the magnitude of the lag with which past actions are updated affects the allocations and the use of money and credit. Bernhardt (1989) argues that loans are optimal in a close-knit economy, while in a larger economy information concerning the repayment of loans may diffuse too slowly to deter default, so money is essential.

<sup>10</sup>Using the probability of exclusion to capture the efficiency of technology and institutions to deter default is also motivated by the regulation on the length of time a bankruptcy is permitted to stay in one's credit report. For example, in the US, personal bankruptcy may appear on one's credit for 10 years, but usually, credit records cover only the most recent 7 years.

the following discussions, to simplify notations we let variables corresponding to the next period be indexed by  $+1$ , and variables corresponding to the previous period be indexed by  $-1$ .

Let  $V(m, a)$  denote the expected value from trading in the first subperiod with  $m$  units of money and  $a$  units of real assets at time  $t$ . Let  $W(m, a, \ell, d)$  denote the expected value from entering the second subperiod with  $m$  units of money,  $a$  units of real assets,  $\ell$  loans and  $d$  deposits at time  $t$ , where loans and deposits are in the units of fiat money. We study a representative period  $t$  and work backwards from the second to the first subperiod, using a similar approach as in Berentsen, Camera, and Waller (2007) to characterize equilibria.

### The second subperiod

In the second subperiod an agent produces  $h$  goods and consumes  $x$ , repays loans, redeems deposits, and adjusts his holdings of fiat money and real assets. He solves the following problem:

$$\begin{aligned} W(m, a, \ell, d) &= \max_{x, h, m_{+1}, a_{+1}} U(x) - h + \beta V(m_{+1}, a_{+1}) \\ \text{s.t. } x + \phi m_{+1} + \psi a_{+1} &= h + \phi(m + T) + (\psi + \rho)a + \phi(1 + i_d)d - \phi(1 + i)\ell. \end{aligned}$$

A unit of the real asset brought to the second subperiod is worth  $\psi + \rho$  units of the general good, because it generates a dividend  $\rho$  and can be resold in the market at price  $\psi$ . If an agent has deposited  $d$  in the first subperiod, he receives  $(1 + i_d)d$  units of money, and if he has borrowed  $\ell$ , he should repay  $(1 + i)\ell$ . Substituting  $h$  from the budget constraint into the objective function we obtain

$$\begin{aligned} W(m, a, \ell, d) &= \phi(m + T) + (\psi + \rho)a + \phi(1 + i_d)d - \phi(1 + i)\ell \\ &\quad + \max_{x, m_{+1}, a_{+1}} \{U(x) - x - \phi m_{+1} - \psi a_{+1} + \beta V(m_{+1}, a_{+1})\}. \end{aligned}$$

The first order conditions are  $U'(x) = 1$  and

$$\phi \geq \beta V_m(m_{+1}, a_{+1}), \quad \text{"=" if } m_{+1} > 0, \quad (1)$$

$$\psi \geq \beta V_a(m_{+1}, a_{+1}), \quad \text{"=" if } a_{+1} > 0, \quad (2)$$

where  $V_m(m_{+1}, a_{+1})$  and  $V_a(m_{+1}, a_{+1})$  are the marginal values of an additional unit of money and the real asset, respectively, taken into the first subperiod of  $t + 1$ . The optimal choice of  $x^*$  satisfies

$U'(x^*) = 1$  for all agents. Conditions (1) and (2) determine the portfolio  $(m_{+1}, a_{+1})$ , independent of the initial holdings of  $m$  and  $a$ . Therefore, the distribution of holdings of money and the real asset is degenerate at the beginning of period  $t + 1$ . The envelope conditions are

$$W_m = \phi, \quad (3)$$

$$W_a = \psi + \rho, \quad (4)$$

$$W_\ell = -\phi(1 + i), \quad (5)$$

$$W_d = \phi(1 + i_d). \quad (6)$$

### The first subperiod

Let  $q_b$  and  $q_s$  denote the quantities consumed by a buyer and produced by a seller, respectively, and  $p$  denote the nominal price of the good. Because sellers do not take loans and buyers do not make deposits, in what follows we let  $\ell$  denote loans taken out by buyers, and  $d$  deposits of sellers. For notational simplicity we also drop these arguments in  $W(m, a, \ell, d)$  when no confusion is caused. An agent holding a portfolio of  $(m, a)$  entering the first subperiod has the expected lifetime utility

$$V(m, a) = (1 - n)[u(q_b) + W(m + \ell - pq_b, a, \ell)] + n[-c(q_s) + W(m - d + pq_s, a, d)]. \quad (7)$$

An agent may be a buyer with probability  $1 - n$ , spending  $pq_b$  units of money to get  $q_b$  consumption, or he may be a seller with probability  $n$ , receiving  $pq_s$  units of money from  $q_s$  production.

As agents trade in a centralized market, they take the price  $p$  as given. A seller solves

$$\begin{aligned} \max_{q_s, d} \quad & -c(q_s) + W(m - d + pq_s, a, d) \\ \text{s.t.} \quad & d \leq m. \end{aligned}$$

Let  $\lambda_d$  denote the multiplier on the deposit constraint. The first order conditions are

$$\begin{aligned} -c'(q_s) + pW_m &= 0, \\ -W_m + W_d - \lambda_d &= 0. \end{aligned}$$

Using (3) and (6), the first order conditions become

$$\begin{aligned} p &= \frac{c'(q_s)}{\phi}, \\ \lambda_d &= \phi i_d. \end{aligned} \quad (8)$$



Equation (8) implies that a seller's production is such that the marginal cost of production,  $\frac{c'(q_s)}{\phi}$ , equals the marginal revenue,  $p$ . The production  $q_s$  is independent of the seller's initial portfolio brought to the first subperiod. Moreover, for any  $i^d > 0$ , the deposit constraint binds, and sellers deposit all money balances.

A buyer's problem is

$$\begin{aligned} \max_{q_b, \ell} \quad & u(q_b) + W(m + \ell - pq_b, a, \ell) \\ \text{s.t.} \quad & pq_b \leq m + \ell, \quad \ell \leq \bar{\ell}. \end{aligned}$$

The buyer faces the budget constraint that his spending cannot exceed his money holdings,  $m$ , plus borrowing,  $\ell$ . He also faces the credit constraint that his borrowing is bounded above by the credit limit,  $\bar{\ell}$ . Let  $\lambda$  and  $\lambda_\ell$  be the multipliers on the buyer's budget constraint and borrowing constraint, respectively. From (3), (5) and (8), we rewrite the first order conditions of the buyer's problem as

$$u'(q_b) = c'(q_s)\left(1 + \frac{\lambda}{\phi}\right), \quad (9)$$

$$\phi i = \lambda - \lambda_\ell. \quad (10)$$

If  $\lambda = 0$ , (9) reduces to  $u'(q_b) = c'(q_s)$ , which implies trades are efficient.

If  $\lambda > 0$ , the budget constraint binds, and buyers spend all their money. Combining (9) and (10) we obtain

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i + \frac{\lambda_\ell}{\phi}.$$

If the borrowing constraint does not bind,  $\lambda_\ell = 0$  and

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i, \quad (11)$$

which implies that buyers borrow up to the point at which the marginal benefit of an additional unit of borrowed money,  $\frac{u'(q_b)}{c'(q_s)}$ , equals the marginal cost,  $1 + i$ .<sup>11</sup> If  $\lambda_\ell > 0$ , the borrowing constraint binds,  $\ell = \bar{\ell}$ , and

$$\frac{u'(q_b)}{c'(q_s)} > 1 + i.$$

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<sup>11</sup> Given the market price  $p$ , one unit of borrowed money buys  $\frac{1}{p}$  units of the good, which generates utility  $\frac{u'(q_b)}{p}$ . To compare the utility from consumption with the nominal cost of borrowing, one needs to convert the utility in terms of money; i.e., dividing  $\frac{u'(q_b)}{p}$  by  $\phi$ , which becomes  $\frac{u'(q_b)}{c'(q_s)}$  by using (8).

Buyers wish to borrow more money, but banks may not be willing to lend because of the concern about default. The buyer thus borrows  $\bar{\ell}$  and spends all his money to consume, so  $q_b = (m + \bar{\ell})/p$ .

In the first subperiod, banks accept deposits from the sellers and make loans to the buyers. If banks have full enforcement on repayment, the borrowing constraint does not bind. When enforcement is limited, banks choose the credit limit  $\bar{\ell}$  to ensure voluntary repayment. The zero-profit condition for competitive banks is  $i = i_d$  (we derive solutions to the bank's problem in the online supplementary appendix).

In a symmetric equilibrium, the goods market clearing condition in the first subperiod is

$$nq_s = (1 - n)q_b. \quad (12)$$

The market clearing conditions for loans, money, and the real asset are  $(1 - n)\ell = nd$ ,  $m = M_{-1}$ , and  $a = A$ , respectively.

The marginal values of money and the real asset at time  $t$  are

$$V_m(m, a) = \phi[(1 - n)\frac{u'(q_b)}{c'(q_s)} + n(1 + i_d)], \quad (13)$$

$$V_a(m, a) = (1 - n)\phi[\frac{u'(q_b)}{c'(q_s)} - (1 + i)]\frac{\partial \ell}{\partial a} + \psi + \rho, \quad (14)$$

respectively. The benefits of holding an additional unit of money include the expected gains from spending the money on goods as a buyer, and the interest payments from making deposits as a seller. The benefits of carrying an additional unit of the real asset include the gains of consumption accrued from the loans minus the debt repayment, and the resale price and the dividend that the holder of the asset is entitled to when entering the second subperiod.

In a stationary equilibrium where agents hold money and real assets, use (1) and (2) lagged one period to eliminate  $V_m(m, a)$  and  $V_a(m, a)$  from (13) and (14), respectively. Then, using the conditions of stationarity,  $\frac{\phi_{-1}}{\phi} = \gamma$  and  $\psi_{-1} = \psi$ , we obtain

$$\frac{\gamma - \beta}{\beta} = (1 - n)[\frac{u'(q_b)}{c'(q_s)} - 1] + ni_d, \quad (15)$$

$$\frac{1 - \beta}{\beta}\psi = \rho + (1 - n)\phi[\frac{u'(q_b)}{c'(q_s)} - (1 + i)]\frac{\partial \ell}{\partial a}. \quad (16)$$

Equation (16) illustrates a key feature of our model: the extra loan amount secured by a marginal unit of the real asset provides a link between the asset price and its role in generating liquidity.

Recall that if the credit constraint does not bind, the marginal benefit of receiving an additional dollar of a loan equals the cost,  $\frac{u'(q_b)}{c'(q_s)} = 1 + i$ . The second term in the right side of (16) vanishes, and the asset price is determined only by the dividend flows. If agents are credit constrained,  $\frac{u'(q_b)}{c'(q_s)} > 1 + i$ , the asset price is also influenced by the extent to which the real asset relaxes credit constraints. We will elaborate more in Section 4

**Equilibrium with full enforcement.** When banks can force repayment at no cost, default is not possible, and agents face no borrowing constraints.

**Proposition 1** *Under full enforcement, the asset price is the present value of dividends; i.e.,  $\psi = \psi^u$  where*

$$\psi^u = \frac{\beta \rho}{1 - \beta}. \quad (17)$$

To find an equilibrium, substituting  $i = i_d$  and (11) into (15) we obtain  $i = i^u$ , where

$$i^u = \frac{\gamma - \beta}{\beta}. \quad (18)$$

From (12), (15) and (18), the output  $q_b = q_b^u$ , where  $q_b^u$  satisfies

$$\frac{\gamma - \beta}{\beta} = \frac{u'(q_b^u)}{c'(\frac{1-n}{n}q_b^u)} - 1. \quad (19)$$

Under full enforcement, a monetary equilibrium with credit is a list  $(\psi^u, i^u, q_b^u)$  satisfying (17) – (19).

## 4 Limited enforcement

When the bank's ability to force repayment is limited, borrowers have an incentive to renege on debts. If default occurs, banks seize collateral, and they exclude defaulters from the banking sector with probability  $\zeta$ .<sup>12</sup> The enforcement technology, however, cannot exclude defaulters from the asset market so they can still trade money and real assets. Moreover, banks are not able to seize a defaulter's assets or income in the future. To illustrate the logic underlying the model, we first

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<sup>12</sup>One may justify the exclusion of defaulters in an environment as considered in Gomis-Porqueras and Sanches (2011), where a costly technology allows agents to report their trades and identities to the credit center. This creates public information regarding private trades, which can enforce the repayment of private liabilities.

consider an economy under the collateral mechanism ( $\zeta = 0$ ), so the only punishment on defaulters is seizing collateral. We then study an economy under the combined mechanism, in which defaulters lose their collateral and are excluded with probability  $\zeta \in (0, 1]$ .

#### 4.1 Collateral mechanism

Consider an economy in which the only punishment on defaulters is seizing collateral. A buyer entering the second subperiod who repays his loan and holds no money has the expected discounted utility:

$$W(m, a) = U(x^*) - h_b + \beta V(m_{+1}, a_{+1}),$$

where  $h_b$  is a buyer's production in the second subperiod if he repays the loan. If a borrower defaults, the benefit is enjoying more leisure since he doesn't have to produce to repay the debt, while the cost is the loss of his collateral. He then starts the next period as a nondefaulter. Thus, a defaulter would choose the same portfolio as nondefaulters, and his expected value from the next period is also  $V(m_{+1}, a_{+1})$ . The expected discounted utility of a deviating buyer entering the second subperiod is

$$\widehat{W}(m, a) = U(\widehat{x}) - \widehat{h}_b + \beta V(m_{+1}, a_{+1}), \quad (20)$$

where the hat indicates a deviator's optimal choice.

For the existence of equilibrium with credit, borrowers must voluntarily repay their loans, which requires  $W(m, a) \geq \widehat{W}(m, a)$ . Banks offer loan contracts such that borrowers will repay their debts, so the real borrowing constraint  $\phi \bar{\ell}$  satisfies  $W(m, a) = \widehat{W}(m, a)$ , which leads to

$$\phi \bar{\ell} = \frac{(\psi + \rho)a}{1 + i}. \quad (21)$$

The real credit limit in (21) is obtained from equating the benefits of not repaying loans,  $(1 + i)\phi \bar{\ell}$ , to the cost of losing collateral,  $(\psi + \rho)a$ , which includes the resale price and dividends accrued from possessing the asset.

For a given  $\bar{\ell}$ , an agent's demand for loans may be less than the credit limit imposed by banks, so that the borrowing constraint does not bind. Hence,  $\ell < \bar{\ell}$  in an unconstrained equilibrium; otherwise,  $\ell = \bar{\ell}$ . From the market clearing conditions for loans and money, and using the fact that sellers deposit all money holdings if  $i > 0$ , i.e.,  $d = m = M_{-1}$ , one finds  $\ell = \frac{n}{1-n}M_{-1}$ . Because

what is important for economic activity is the real value of loans,  $\phi\ell$ , in the following discussions we will focus on the loan amount in real terms:  $\phi\ell = \frac{n}{1-n}\phi M_{-1}$ .

**Definition 1** *A monetary equilibrium with unconstrained credit is a list  $(\psi^u, i^u, q_b^u)$  satisfying (17), (18) and (19), and  $0 < \phi\ell = nc'(q_s^u)q_b^u < \phi\bar{\ell}$ , where  $\phi\bar{\ell}$  satisfies (21), and  $q_s^u = \frac{1-n}{n}q_b^u$ .*

Even if enforcement is limited, the asset price is determined by the discounted dividend streams as long as credit constraints do not bind. Moreover, the asset price, loan rate, and allocations are identical to those in the economy with full enforcement.<sup>13</sup>

If agents are credit constrained, banks charge a nominal loan rate,  $i$ , and lend out  $\ell = \bar{\ell}$  to induce voluntary repayment. From (21), the extra loan amount that the marginal unit of the asset can generate is

$$\frac{\partial \ell}{\partial a} = \frac{\psi + \rho}{\phi(1+i)}. \quad (22)$$

Using (15) and substituting (22) into (16), we obtain the following definition of equilibrium.

**Definition 2** *Under a collateral mechanism, a monetary equilibrium with constrained credit is  $(q_b, i, \psi)$  satisfying*

$$\frac{\gamma - \beta}{\beta} = (1-n)\left[\frac{u'(q_b)}{c'(q_s)} - 1\right] + ni \quad (23)$$

$$\frac{1-\beta}{\beta}\psi = \rho + (1-n)\left[\frac{u'(q_b)}{c'(q_s)} - (1+i)\right]\frac{\psi + \rho}{1+i} \quad (24)$$

*such that  $nc'(q_s)q_b = \phi\bar{\ell}$ , where  $\phi\bar{\ell}$  satisfies (21), and  $q_s = \frac{1-n}{n}q_b$ .*

We define the loan-to-value ratio as the ratio of the real loan amount to the real value of collateral,  $\frac{\phi\ell}{\psi a}$ , which can be interpreted as the rate at which the asset generates liquidity to lubricate economic activity.

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<sup>13</sup>Substituting market clearing conditions of money and loans into the binding budget constraint, we obtain  $pq_b = m + \ell = M_{-1} + \frac{n}{1-n}M_{-1}$ . Hence,  $(1-n)pq_b = M_{-1}$ . Substituting  $M_{-1} = \frac{1-n}{n}\ell$  into the previous expression, we have  $\ell = npq_b$ . Using  $p = \frac{c'(q_s)}{\phi}$ , we obtain  $\phi\ell = nc'(q_s)q_b$ , shown in the definition of equilibrium. In our online supplementary appendix we have derived the existence condition for the unconstrained equilibrium. Martin and Monnet (2011) also derive an equation similar to (21), and a similar existence condition for the unconstrained equilibrium.

**Proposition 2** *In a constrained monetary equilibrium under a collateral mechanism, the asset price is  $\psi = \psi_1$ , where*

$$\psi_1 = \frac{\beta B \rho}{1 - \beta B}, \quad (25)$$

and

$$B = 1 + (1 - n) \left[ \frac{u'(q_b)}{c'(q_s)} \frac{1}{1 + i} - 1 \right],$$

and the loan-to-value ratio is  $\theta = \theta_1$ , where

$$\theta_1 = \frac{1 + \frac{\rho}{\psi}}{1 + i}. \quad (26)$$

The asset-pricing equation in (25) shows how asset prices, borrowing constraints, and credit market imperfections are intertwined. Denote  $\beta B$  the “effective” discount factor, which takes into account credit market frictions. As agents are credit constrained,  $\frac{u'(q_b)}{c'(q_s)} > 1 + i$  and  $B > 1$ ; therefore,  $\psi_1 > \psi^u$ . The liquidity premium—the difference between the price of an asset and its fundamental value—arises from its role in relaxing the borrowing constraint, and the premium is increased by the discrepancy between the marginal benefit of the loan,  $\frac{u'(q_b)}{c'(q_s)}$ , and the marginal cost,  $1 + i$ . That is, the more severe the credit rationing, the higher the liquidity premium. The loan-to-value ratio,  $\theta_1$ , depends positively on the dividend-price ratio of the asset pledged as collateral, and negatively on the loan rate. For a given asset price, an increase in the loan rate makes agents more likely to default since the repayment cost is increased. Hence, banks should lend less by setting a lower loan-to-value ratio to deter default.

**Proposition 3** *Monetary policy has similar effects on the loan rate, allocations, and prices in a constrained and unconstrained equilibrium :  $\frac{\partial i}{\partial \gamma} > 0$ ,  $\frac{\partial q_b}{\partial \gamma} < 0$ ,  $\frac{\partial \phi \ell}{\partial \gamma} < 0$ ,  $\frac{\partial p}{\partial \gamma} > 0$ . However, in a constrained equilibrium,  $\frac{\partial \theta}{\partial \gamma} < 0$  and  $\frac{\partial \psi}{\partial \gamma} \leq 0$  iff  $\frac{-u'' q_b}{u'} \leq 1$ .*

Higher inflation raises the loan rate, and thus the repayment cost, which reduces loan-to-value ratios and real loan amounts in a constrained equilibrium. Therefore, aggregate liquidity and output fall. The effects of inflation on the asset price depend on the coefficient of relative risk aversion. Higher inflation raises the borrowing cost, which dampens the demand for the asset as collateral. Inflation also reduces real balances and real loan amounts, so risk-averse agents may

increase their demand for collateral to borrow in order to smooth consumption. If the coefficient of relative risk aversion is less than one, the former effect dominates and, consequently, the demand for assets diminishes and asset prices fall with inflation. The predictions of Proposition 3 imply that the loan-to-value ratio can be positively correlated with the price of the collateralized asset and aggregate liquidity.<sup>14</sup>

**Proposition 4** *The effects of changes in the supply of the real asset and dividend flows are:*

1. *A change in the asset supply does not affect the loan rate and allocations in an unconstrained equilibrium, but it has real effects in a constrained equilibrium:  $\frac{\partial q_b}{\partial A} > 0$ ,  $\frac{\partial i}{\partial A} > 0$ ,  $\frac{\partial \phi}{\partial A} > 0$ ,  $\frac{\partial p}{\partial A} < 0$ ,  $\frac{\partial \psi}{\partial A} < 0$ ,  $\frac{\partial \theta}{\partial A} = 0$ .*
2. *A change in the asset's dividend flows affects only the asset price in an unconstrained equilibrium:  $\frac{\partial \psi}{\partial \rho} > 0$ ; however, in a constrained equilibrium:  $\frac{\partial q_b}{\partial \rho} > 0$ ,  $\frac{\partial i}{\partial \rho} > 0$ ,  $\frac{\partial \phi}{\partial \rho} > 0$ ,  $\frac{\partial p}{\partial \rho} < 0$ ,  $\frac{\partial \psi}{\partial \rho} > 0$  if  $\left| \frac{\partial B/B}{\partial \rho/\rho} \right| < 1 - \beta B$ ,  $\frac{\partial \theta}{\partial \rho} = 0$ .*

In contrast to frictionless asset-pricing models, the asset price depends negatively on the supply of the asset if credit rationing occurs. Although an increase in the asset supply does not affect the loan-to-value ratio, it exerts positive effects on aggregate liquidity and output by increasing the collateralizable assets. Higher dividends have a positive effect on the fundamental element of the asset price, and a negative general equilibrium effect on the asset price, by reducing the severity of credit rationing (because  $\frac{\partial B}{\partial \rho} < 0$ ). When the positive effect dominates, i.e., the elasticity of the severity of credit rationing with respect to the dividend flows is sufficiently small, higher dividend flows lead to a higher asset price.

## 4.2 Combined collateral mechanism and reputation mechanism

We now study an economy in which banks take defaulters' collateral and exclude them permanently from the banking system with probability  $\zeta \in (0, 1]$ . Consider a defaulter who will be excluded. Because a defaulter cannot make deposits or receive any credit in the future, he needs to bring

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<sup>14</sup>Along the lines of Kiyotaki and Moore (1997) and Chen (2001), Chen and Wang (2007) use a panel transaction data set from Taiwan to find that the value of collateralized assets has positive and significant effects on loan amounts and that the leverage effect of collateral is procyclical to asset price cycles.

enough money to execute trades. Also, he will not choose to hold real assets in a constrained equilibrium. The reason is that the asset price incorporates extra benefits from backing loans, yet no such gains accrue to deviators because they cannot borrow in the future. As a result, a deviator would choose a different portfolio from nondeviators, and trade at a different quantity,  $\tilde{q}_b$ . Let  $\tilde{V}(\tilde{m}_{+1}, \tilde{a}_{+1})$  denote a deviator's expected discounted utility from entering the next period, where the tilde indicates the optimal choice. For a defaulter who will be excluded, his expected discounted utility in the second subperiod is

$$\tilde{W}(m, a) = U(\tilde{x}) - \tilde{h}_b + \beta \tilde{V}(\tilde{m}_{+1}, \tilde{a}_{+1}).$$

With probability  $1 - \zeta$  a defaulter faces only the punishment of losing collateral. His expected utility in the second subperiod is  $\widehat{W}(m, a)$ , from (20), the same as under the collateral mechanism. With probability  $\zeta$  a defaulter will be excluded, and has the expected discounted utility,  $\tilde{W}(m, a)$ . Thus, the expected discounted utility of a deviator entering the second subperiod is

$$\overline{W}(m, a) = \zeta \tilde{W}(m, a) + (1 - \zeta) \widehat{W}(m, a).$$

Existence of an equilibrium with credit requires that borrowers voluntarily repay loans; i.e.,  $W(m, a) \geq \overline{W}(m, a)$ . From  $W(m, a) = \overline{W}(m, a)$ , we solve for the real borrowing constraint:

$$\phi \bar{\ell} = \frac{(1 - \beta + \beta \zeta) \rho a + (1 - \beta)(1 - \zeta) \psi a}{(1 - \beta)(1 + i)} + \frac{\beta \zeta}{(1 - \beta)(1 + i)} \left\{ (1 - n) \Psi + \frac{\gamma(1 - \beta)}{\beta} c'(q_s) [\tilde{q}_b - (1 - n) q_b] \right\}, \quad (27)$$

where

$$\Psi = u(q_b) - u(\tilde{q}_b) - c'(q_s)(q_b - \tilde{q}_b) \geq 0.$$

The real borrowing constraint  $\phi \bar{\ell}$  shown in (27) comprises the loss from losing collateral and the long-term loss from being excluded from the banking sector, which is the difference of the expected discounted gains from trade between nondeviators and deviators. From (27), the extra loan amount generated by a marginal unit of the real asset is

$$\frac{\partial \ell}{\partial a} = \frac{(1 - \beta + \beta \zeta) \rho + (1 - \beta)(1 - \zeta) \psi}{\phi(1 - \beta)(1 + i)}. \quad (28)$$

Using an equation similar to (15) for defaulters who are excluded from the banking system, and substituting (28) into (16), we have the following definition of equilibrium.



**Definition 3** Under a combined mechanism, a monetary equilibrium with constrained credit is  $(q_b, \tilde{q}_b, i, \psi)$  satisfying (23) and

$$\frac{\gamma - \beta}{\beta} = (1 - n) \left[ \frac{u'(\tilde{q}_b)}{c'(q_s)} - 1 \right], \quad (29)$$

$$\frac{1 - \beta}{\beta} \psi = \rho + (1 - n) \left[ \frac{u'(q_b)}{c'(q_s)} - (1 + i) \right] \frac{(1 - \beta + \beta\zeta)\rho + (1 - \beta)(1 - \zeta)\psi}{(1 - \beta)(1 + i)}, \quad (30)$$

such that  $nc'(q_s)q_b = \phi\bar{\ell}$ , where  $\phi\bar{\ell}$  satisfies (27), and  $q_s = \frac{1-n}{n}q_b$ .

**Proposition 5** In a constrained monetary equilibrium under a combined mechanism with probability of exclusion,  $\zeta$ , the asset price is  $\psi = \psi_2$ , where

$$\psi_2 = \frac{\beta B_2 \rho}{1 - \beta B_3},$$

and

$$\begin{aligned} B_2 &= 1 + (1 - n) \left( 1 + \frac{\beta\zeta}{1 - \beta} \right) \left[ \frac{u'(q_b)}{c'(q_s)} \frac{1}{1 + i} - 1 \right], \\ B_3 &= 1 + (1 - n)(1 - \zeta) \left[ \frac{u'(q_b)}{c'(q_s)} \frac{1}{1 + i} - 1 \right], \end{aligned}$$

and the loan-to-value ratio is  $\theta = \theta_2$ , where

$$\theta_2 = \frac{(1 - \beta + \beta\zeta)\frac{\rho}{\psi} + (1 - \beta)(1 - \zeta)}{(1 - \beta)(1 + i)} + \frac{\zeta\beta}{(1 - \beta)(1 + i)\psi a} \left\{ (1 - n)\Psi + \frac{\gamma(1 - \beta)}{\beta} c'(q_s) [\tilde{q}_b - (1 - n)q_b] \right\}. \quad (31)$$

In Proposition 5,  $B_2 > 1$ ,  $B_3 > 1$ , and  $B_2 > B_3$ . Therefore,  $\psi_2 > \psi^u$ ; the real asset commands a liquidity premium in a constrained equilibrium, again due to the role in relaxing the borrowing constraint. The loan-to-value ratio,  $\theta_2$ , captures the current-period loss of collateral and the expected long-term loss of being excluded from the banking sector.

**Effects of changes in enforcement, asset supply, and dividends.** We first discuss how the improved technology in enforcement affects asset prices, allocation, and welfare, in a constrained equilibrium. At the beginning of a period before the preference shock is realized, the expected lifetime utility of the representative agent is

$$(1 - \beta)\Omega = [(1 - n)u(q_b) - nc(q_s) + U(x) - x].$$

We use  $\Omega$  as the measure of welfare.

Figure 1 shows that an advance in enforcement (i.e., an increase in  $\zeta$ ) raises the loan-to-value ratio, reduces the asset price, and has non-monotonic effects on real loan amounts, loan rates, output, and welfare: all values decline and then rise over the range  $\zeta \in [0, 1]$ , with the highest levels at  $\zeta = 1$ .<sup>15</sup> The improved efficiency in enforcement reduces the asset price because the more severe punishment of exclusion makes collateral a less important commitment device for borrowing.<sup>16</sup> When financial development is in the primitive stage, an improvement in  $\zeta$  results in a small increase in the loan-to-value ratio. As the effect of increased loan-to-value ratios is dominated by the effect of decreased asset prices, aggregate liquidity and output fall. When  $\zeta$  is above some threshold, the punishment of exclusion is substantial enough to make the rise in the loan-to-value ratio a dominant effect, so aggregate liquidity and output rise.

From numerical examples we find that, an increase in the asset supply or dividend flows raises the real loan amount, interest rate and output, as in the economy under the collateral mechanism, but it may lower the loan-to-value ratio if the probability of exclusion is sufficiently high. An increase in  $\rho$  leads to a higher asset price, and an increase in  $A$  results in more collateralizable assets. Moreover, an increase in  $\rho$  or  $A$  raises the real loan amount. It turns out that when  $\zeta$  is high enough, the former effect dominates, and so the loan-to-value ratio falls as  $\rho$  or  $A$  increases.

**Monetary policy implications.** In this economy inflation affects output through various channels. First, inflation reduces the incentive to produce by lowering the value of money. Moreover, inflation exerts two opposite effects through credit constraints: it raises the loan rate, and thus the repayment cost, so the loan-to-value ratio has to be reduced to control for the incentive to default; on the other hand, because defaulters need to bring enough money to self-insure against consumption shocks, inflation raises the cost of default. This disciplinary effect of inflation works through exclusion. Therefore, under the combined mechanism, when enforcement is strong enough for inflation to exert sufficient punishment on defaulters, the positive effect can outweigh negative

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<sup>15</sup>In Figure 1, the functional forms are  $u(q_b) = \frac{q_b^{1-\eta}}{1-\eta}$ ,  $c(q_s) = q_s$ , and  $U(x) = C \log(x)$ . The parameter values are  $\eta = 0.2$ ,  $n = 0.6$ ,  $\beta = 0.95$ ,  $\gamma = 1.016$ ,  $C = 2.537$ ,  $\rho = 0.001$ , and  $A = 2.5$ .

<sup>16</sup>The immediate implication is that the asset prices under a collateral mechanism (where  $\zeta = 0$ ) are higher than the asset prices under a combined mechanism.

effects so that loan-to-value ratios, aggregate liquidity, and output rise.

Our numerical examples show that higher inflation lowers the asset price, whereas it raises loan-to-value ratios, real loan amounts, and output only when  $\zeta$  is above a certain threshold.<sup>17</sup> That is, if the efficiency of enforcement is above a certain threshold, the positive impact dominates the negative impacts, and so inflation raises loan-to-value ratios; otherwise, it reduces loan-to-value ratios. Even though the asset price (i.e., the value of collateral) is reduced by inflation, the real loan amount may increase due to a higher loan-to-value ratio. This result runs counter to previous theoretic models with exogenously given, constant loan-to-value ratios, which predict a positive correlation between the loan amount and the value of collateral. This distinction implies that whether to derive credit constraints explicitly under various technologies to deter default matters for policy implications.<sup>18</sup>

To discuss the optimal money growth rate in this economy, we need to be explicit about how enforcement may restrict the implementation of monetary policy (see, e.g., Berentsen, Camera, and Waller 2007). If there is full enforcement, the central bank can levy taxes to extract money from the economy; i.e., it can run deflation. Thus, the Friedman rule ( $\gamma = \beta$ ) achieves the efficient allocation.<sup>19</sup> Agents can perfectly self-insure against consumption shocks by holding money at no cost. In contrast, if limited enforcement implies that the central bank does not have the power to extract money, then deflation cannot be implemented, and  $\gamma \geq 1$ . Under limited enforcement, therefore, the optimal money growth rate may depend on the probability of exclusion,  $\zeta$ .

From numerical examples we find that, under limited enforcement, the optimal money growth rate is a hump-shaped function of  $\zeta$ . Recall that inflation exerts a penalty on defaulters through exclusion. When the probability of exclusion is low, the money growth rate should be set low to reduce the negative impacts of inflation on the incentive to produce. As  $\zeta$  increases, inflation can impose a larger positive impact through exclusion, which dominates the negative impacts, and thus, the optimal money growth rate increases. When  $\zeta$  is sufficiently high, mild inflation can exert a sufficient penalty to curb the incentive to default, and thus, the money growth rate should decrease

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<sup>17</sup>Using the set of parameters for Figure 1, we find that the threshold for  $\zeta$  above which higher inflation increases output and welfare is 0.7.

<sup>18</sup>From numerical examples we find that, if the coefficient of relative risk aversion is above some threshold, asset prices rise with inflation. This threshold is greater than one, and it may increase with the degree of punishment.

<sup>19</sup>Substituting  $\gamma = \beta$  into (18) and (19), we obtain  $i^u = 0$  and  $q_b^u = q^*$ .

to minimize the negative impacts.

### 4.3 Comparisons among mechanisms

There are studies on credit arrangements that consider the collateral mechanism (e.g., Ferraris and Watanabe 2008, Martin and Monnet 2011) or that incorporate permanent exclusion as the only punishment (e.g., Berentsen, Camera, and Waller 2007). By combining the collateral mechanism and the reputation mechanism with probabilistic exclusion, we are able to obtain insights that would have been missing when each mechanism is studied in isolation. For instance, we illustrate how loan-to-value ratios and the optimal money growth rate depend on the level of enforcement. From identifying the relationship between inflation and the level of exclusion in curbing the incentive to default, we show that inflation raises loan-to-value ratios and output only when enforcement is sufficiently efficient. Moreover, we find that, when equilibria under the collateral mechanism, the reputation mechanism, and the combined mechanism coexist, the one under the combined mechanism entails the highest welfare, if the probability of exclusion is not too low.<sup>20</sup> Therefore, the economy under the combined mechanism can achieve allocations that are not achievable otherwise.

One may wonder to what extent seizing collateral can substitute for exclusion in the sense that the two mechanisms result in identical allocations. To answer this question, we consider an economy under a collateral mechanism, and the other under a reputation mechanism, in which banks require no collateral but defaulters can be excluded from the banking sector forever with probability  $\zeta'$ . From numerical exercises, we find a value of  $\zeta' \in (0, 1)$  with which the former economy replicates the allocations in the economy with the collateral mechanism.

The current paper focuses on the role of assets as collateral in relaxing borrowing constraints. There are other arrangements for relaxing liquidity constraints; e.g., agents who have consumption needs can sell assets to banks instead of making loans, or they can sell assets in the asset market to others with idle cash. Do these alternative financing arrangements lead to equivalent allocations? We answer this question in the following proposition.

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<sup>20</sup>For the set of parameters in Figure 1, we find that the equilibrium under the combined mechanism entails the highest welfare when  $\zeta > 0.6$ .

**Proposition 6** *Three arrangements—borrowing money from banks under the collateral mechanism, selling assets to banks, and selling assets in the financial market—result in the identical asset price and allocation.*

The first equivalence result in Proposition 6 implies that a bank’s asset portfolio, whether it consists of loans or securities, is irrelevant to economic activity. The second equivalence result implies that the institutions that provide liquidity—banks or asset markets—do not matter. Notice that when comparing different arrangements, we assume away transaction costs or asymmetric information problems regarding assets. Also, the equivalence result does not hold if exclusion of defaulters is possible.<sup>21</sup>

#### 4.4 Discussion

In this subsection we discuss our theoretic contribution to the literature and the empirical relevance of our findings. In terms of resorting to credit market imperfections to motivate credit or liquidity constraints, the current paper is related to Kiyotaki and Moore (1997, 2005) and Holmstrom and Tirole (1998), though we derive credit constraints explicitly under various specifications of the technology to deter default.<sup>22</sup> Since we identify the factors that determine loan-to-value ratios implied by the endogenously derived credit constraints, our results may help relax the assumption of the exogenously given, constant loan-to-value ratios that have been made in the previous literature.

In some monetary models that assume limitations in record keeping, enforcement, and commitment, assets are used as a means of payment or collateral to secure debt. Compared to the studies that consider fiat money and other assets as competing means of payment, our model features the

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<sup>21</sup>Berentsen and Waller (2011) show that allocations in an economy with private debt and permanent exclusion are equivalent to those in an economy where agents can trade government bonds, under a properly chosen fee for participating in the asset market.

<sup>22</sup>In Kiyotaki and Moore (1997) financial contracts are imperfectly enforceable and creditors protect themselves from the threat of repudiation by collateralizing borrowers’ debt. Many studies have followed Kiyotaki and Moore’s setup, in which collateral constitutes a binding constraint on loans. For example, Chen (2001) shows that banks’ capital-asset ratios and entrepreneurs’ net worth jointly determine the constraints on banks’ lending and entrepreneurs’ borrowing, and thus determine aggregate investment. Kiyotaki and Moore (2005) assume constraints on debt issuance and resaleability of private claims due to limited commitment so that borrowers can sell an exogenous fraction of their capital to finance investment. In Holmstrom and Tirole (1998) the investment is subject to moral hazard, in that an entrepreneur may choose a lower probability of success, which provides him with a private benefit. Hence, the entrepreneur can borrow from outside investors only a small fraction of the expected returns of the firm.

complementarity of money and other assets that are used as collateral to secure loans.<sup>23</sup> This complementarity feature also appears in Berentsen and Monnet (2008) and Martin and Monnet (2011), who study the central bank’s lending facilities in which assets are used as collateral. When money and other assets are substitutes as means of payment, an increase in inflation lowers the return on money and causes agents to move out of cash and into other assets, and consequently, the prices of these assets are driven higher. By contrast, in our model, inflation may reduce asset prices when enforcement is sufficiently efficient. The reason is that higher inflation raises the borrowing cost, which dampens the demand for the asset as collateral.

As for the empirical relevance of our findings, one prediction of our model is that inflation raises loan-to-value ratios only if the enforcement ability is high enough. We thus attempt to suggest that, when searching for the effect of inflation on loan-to-value ratios across countries, one needs to classify countries by the efficiency of enforcement. It is expected that, for countries with high enforcement ability, inflation should have a positive effect on loan-to-value ratios, whereas the opposite holds for countries with low enforcement ability. On the other hands, if one controls for the inflation, then the enforcement ability has a positive effect on loan-to-value ratios.<sup>24</sup>

Finally, our simulation yields the figure of loan-to-value ratios for the Netherlands as reported in Table 1; however, we are not able to generate the figures of loan-to-value ratios for other countries with plausible parameter values.<sup>25</sup> We contemplate a possible reason as follows. Our model considers only strategic default. In reality, defaults occur for mortgage loans, and usually unemployment is the primary cause of defaults (Ahearne et al. 2005). One can add income shocks into our model

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<sup>23</sup>Studies on the coexistence of fiat money and other assets as competing means of payment include, e.g., Geromichalos, Licari, and Suarez-Lledo (2007), Lagos and Rocheteau (2008), Jacquet and Tan (2012), and Lagos (2011). Studies that focus on how the recognizability of an asset or government regulations affect the acceptability of the asset in exchange for goods include, for instance, Lester, Postlewaite, and Wright (2012), Rocheteau (2011), Li, Rocheteau and Weill (2012), Lagos (2010).

<sup>24</sup>To find the effect of inflation on loan-to-value ratios, we do an empirical analysis for the US by using yearly data for the period of 1963-2010. The loan-to-value ratio,  $\theta$ , is the typical ratio for single-family mortgages reported in Monthly Interest Rate Survey Data of Federal Housing Finance Agency (Table 9). The inflation rate,  $\gamma$ , is the rate of increase of the consumer price index from the Bureau of Labor Statistics. The loan-to-value ratio in each period  $t$  is regressed on the period  $t$  inflation rate:  $\theta_t = c_0 + c_1\gamma_t + \epsilon_t$ . We find that the coefficient  $c_1$  is negative and significant, implying that inflation has negative effects on loan-to-value ratios in the US.

<sup>25</sup>Using the parameter set of Figure 1 with  $\zeta = 0.1$  and  $\gamma = 1.016$  (for the year of 2007, the inflation rate is 1.6% in the Netherlands), we obtain the LTV ratio equal to 101%, as in Table 1. However, to obtain the figures for LTV ratios for other countries in Table 1, we need to set a much higher inflation rate than the actual one. For instance, if we set  $\zeta = 0.01$ , which implies lower enforcement ability than in the Netherlands, and  $\gamma = 1.55$ , then the LTV ratio equals 65%, as reported in Table 1 for Italy. However, the inflation rate is 1.83% in Italy for the year of 2007.

to capture the unintentional default caused by unemployment. To recover possible losses from unintentional defaults, banks need to incur the cost of foreclosure procedures. Therefore, they may set a lower credit limit for a given set of collateral, which results in lower loan-to-value ratios.<sup>26</sup>

## 5 Conclusion

The current paper combines the collateral mechanism and the reputation mechanism with probabilistic exclusion to illustrate how loan-to-value ratios and monetary policy implications depend on enforcement. The key finding is that high loan-to-value ratios are driven by sufficient efficiency in enforcement, while inflation may raise loan-to-value ratios only if the enforcement ability is high enough. Advances in the technology of enforcement that make exclusion more likely raise loan-to-value ratios, while reducing asset prices. When the punishment of exclusion is substantial enough to make the rise in loan-to-value ratios the dominant effect, aggregate liquidity and output increase with the advance in technology. Imposing restrictions on the access to future credit may improve liquidity and allocations only when they constitute a substantial punishment on defaulters. Our result may shed some light on the question of the macroeconomic consequences of financial development and regulations.

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<sup>26</sup>There are other factors, such as the maturity of loans, that may affect loan-to-value ratios (see, e.g., Calza, Monacelli, and Stracca 2013, Ahearne et al. 2005, and ECB 2009 for discussions on the characteristics of the mortgage system).

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Table 1: Loan-to-value ratios and foreclosure cost

Country	BE	DE	GR	ES	FR	IT	NL	AT	PT	FI	UK	US
LTV ratios (%)	80	70	70	72.5	91	65	101	84	71	81	80-90	80
Duration (months)	18	9	24	8	20	56	5	9	24	2.5	<12	<12
Cost (%)	18.7	7.5	16	10	9.5	n/a	4	7.5	8	1.5	n/a	n/a

Sources: Table 2 and Chart 9 in ECB (2009). The LTV ratios for the UK and US are from Table 1 in Calza, Monacelli, and Stracca (2013). The figures for foreclosure duration in the UK and US are estimated from ECB (2009, page 72) “it lasts only a few months in the United States and the United Kingdom, and a year in only exceptional cases.”

Note: a. The LTV ratios in Table 1 are the typical loan-to-value ratios for a first-time home buyer in 2007.

b. According to ECB (2009), the cost of the enforcement procedure is a percentage of either the loan balance or the proceeds of the sale. It refers to the cost of legal, registration, administration, or auctioneer fees.

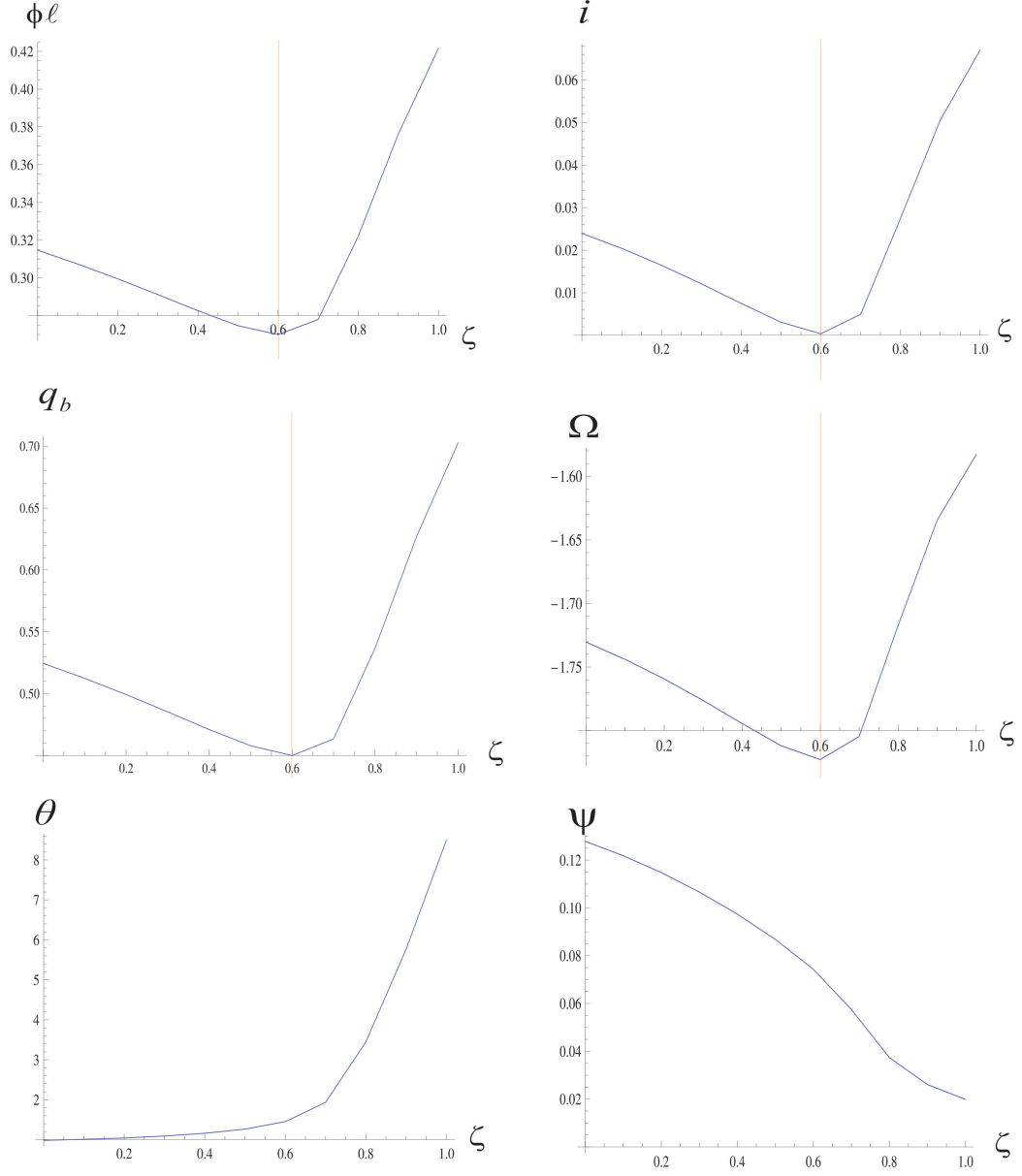


Figure 1: The effects of advances in the technology of enforcement and record keeping