Abstract

We consider a monetary economy in which lenders cannot force borrowers to repay their debts, and financial assets are used as collateral to secure loans. Asset prices, credit limits, and loan-to-value ratios are determined simultaneously in equilibrium. Assets command a liquidity premium when credit constraints bind, and the premium is higher if credit rationing is more severe. Monetary policies influence macroeconomic outcomes through the channel of endogenously determined credit constraints, and the effects depend on the technology available to deter default. If exclusion constitutes a substantial punishment, aggregate liquidity and output may be raised by inflation because the cost of losing the access to future credit is increased. Asset prices are reduced by inflation due to the complementarity of money and other assets as collateral to secure nominal loans.

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1 Introduction

The provision of credit helps to satisfy people's need to finance unanticipated consumption or investment. Credit provision is often constrained if lenders cannot force borrowers to repay their debts, unless the debts are secured. We consider an economy with such a feature, where financial assets are used as collateral to secure loans, to study the interaction between the asset prices, credit constraints, and aggregate liquidity. In contrast to much of the previous literature, which features exogenously given, constant loan-to-value ratios, we explicitly derive this ratio from the condition that lenders offer to loan only as much as borrowers have incentives to repay. Moreover, we identify the endogenously determined credit constraints as a channel through which monetary policies influence macroeconomic outcomes.

Specifically, we introduce banks and a real asset like the claims to “trees” in the standard Lucas (1978) asset-pricing theory into a model with limited commitment, enforcement, and record keeping, similar to the one in Lagos and Wright (2005) and Berentsen, Camera, and Waller (2007). Banks channel funds from people with idle cash to those who need liquidity to finance unanticipated consumption. Due to the limited enforcement of debt repayment, borrowers are required to pledge some assets to secure their loans. Borrowers’ credit limits are affected by the prices of the collateralized assets, while these prices are affected by the amount of liquidity that the assets can generate by backing loans. The asset prices, credit limits, and loan-to-value ratios are determined simultaneously in equilibrium. Besides requiring collateral, banks may deter default by denying credit in the future. This provides an important route through which monetary policy can affect credit constraints: since defaulters need to hold a sufficient amount of money to self-insure against consumption shocks, inflation raises the cost of default and thus relaxes credit constraints.

We illustrate how asset prices, aggregate liquidity, and credit constraints are intertwined under various specifications of enforcement and record keeping. In a benchmark case, where banks can

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1 This type of economy has been considered in Kiyotaki and Moore (1997), in which a productive input, such as land, serves as collateral. They assume that lenders never allow the size of the debt to exceed the (expected) value of the collateral. Hence, the loan-to-value ratio is one when the credit constraint binds. Many studies have followed their setup, assuming that collateral constitutes a binding constraint on loans. For example, Iacoviello (2005) assumes a constant fraction, $1 - \theta$, of the collateral value as the transaction cost that lenders must pay in order to repossess the borrowers' assets if borrowers repudiate their debt obligations. That is, the loan-to-value ratio is assumed to be a constant $\theta$. More discussions on the literature are provided in footnote 5.

2 This condition is similar to the individual rationality constraints in Kehoe and Levine (1993).
force debt repayment at no cost, the asset is priced at the discounted sum of dividends. Under limited enforcement, the asset commands a price higher than the fundamental value if credit constraints bind. This “liquidity premium” of the asset arises because the marginal benefit of loans is higher than the borrowing cost when credit rationing occurs, and the premium increases if credit rationing becomes more severe. If seizing collateral is the only punishment, a defaulter faces a current-period trade off of the benefit and cost. The loan-to-value ratio is positively influenced by the dividend-price ratio of the asset pledged as collateral, but is negatively related to the loan rate. Because a higher loan rate raises the repayment cost, banks must lower the loan-to-value ratio to control for the incentives to default. If defaulters can be excluded from the credit market, the loan-to-value ratio is also influenced by the expected gain of consumption that will accrue from future credit.

The policy implications differ depending on whether the technology and institutions allow defaulters to be excluded from the banking sector. In our environment, higher inflation exerts adverse effects on output by reducing the incentives to produce, as is standard in monetary models. The binding credit constraints, however, constitute an additional transmission mechanism of monetary policy. When seizing collateral is the only penalty for default, the loan-to-value ratio is lowered by inflation, and aggregate liquidity and output fall. If exclusion is feasible and constitutes a substantial punishment, higher inflation may raise the cost of default to such an extent that the loan-to-value ratio rises, and so do aggregate liquidity and output. Finally, advances in technology that make exclusion more likely have positive effects on aggregate liquidity, output, and welfare, if the technology’s efficiency is above some threshold.\textsuperscript{3} Hence, imposing restrictions on the access to future credit may be beneficial to the society only when they constitute a substantial punishment on defaulters.

Unlike studies that consider the coexistence of fiat money and other assets as competing means of payment (e.g., Lagos and Rocheteau 2008, Geromichalos, Licari, and Suarez-Lledo 2007, Jacquet and Tan 2010), our model features the complementarity of money and other assets that are used

\textsuperscript{3} According to Calza, Monacelli, and Stracca (2009), the loan-to-value ratios vary significantly across EU countries, ranging between 50% in Italy and over 90% in the Netherlands. This may partly reflect differences in the technology and institutions available to deter default across countries.
as collateral to secure nominal loans. When money and other assets are substitutes as a means of payment, an increase in inflation lowers the return on money and causes agents to move out of cash and into other assets. Consequently, the prices of these assets are driven higher and their rates of return lower. By contrast, in our model without exclusion, higher inflation tends to reduce the real value of loans. Thus, the demand for assets as collateral diminishes, and asset prices fall. Even in the case where inflation raises the cost of being excluded so high that real loan amounts and output rise, asset prices still fall. The reason is that when the default cost is raised by a higher inflation rate, collateral becomes a less important commitment device for borrowing.

To explore further the pricing of the asset, we consider an extension in which the real asset may be readily used in some markets to finance consumption. The asset pricing equation thus incorporates the liquidity services provided by the asset in its role as a means of payment and as collateral. The asset commands a liquidity premium if there are not sufficient amounts of means of payment to achieve the first-best allocation, or credit constraints bind. Our model illustrates that the price of an asset reflects its dual role in overcoming the frictions caused by insufficiency of payment instruments and credit market imperfections.

This paper shares the theme of literature that resorts to credit market imperfections to motivate credit or liquidity constraints, such as Kiyotaki and Moore (1997, 2001, 2005) and Holmstrom and Tirole (1998), though we derive credit constraints explicitly under various specifications of the technology to deter default. In monetary models that assume no record keeping, enforcement, and commitment, assets are used as a means of payment or collateral to secure debt. These models are designed to explore what the important characteristics are that make an asset a means of payment or collateral, and how these characteristics affect people’s ability to finance transactions.

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1. We thank Randy Wright for pointing out the feature of complementarity.
2. In Kiyotaki and Moore (1997) financial contracts are imperfectly enforceable and creditors protect themselves from the threat of repudiation by collateralizing borrowers’ debt. Many studies have followed Kiyotaki and Moore’s setup, in which collateral constitutes a binding constraint on loans. For example, Chen (2001) shows that banks’ capital-asset ratios and entrepreneurs’ net worth jointly determine the constraints on banks’ lending and entrepreneurs’ borrowing, and thus determine aggregate investment, and Iacoviello (2005) considers housing collateral to study shocks of monetary policy on the fluctuations of house prices and aggregate quantities. Kiyotaki and Moore (2001, 2005) assume constraints on debt issuance and resalability of private claims due to limited commitment so that borrowers can sell an exogenous fraction of their capital to finance investment. In Holmstrom and Tirole (1998) the investment is subject to moral hazard, in that an entrepreneur may choose a lower probability of success, which provides him with a private benefit. Hence, the entrepreneur can only borrow from the outside investors a small fraction of the expected returns of the firm.
Studies that focus on how the recognizability of an asset affects its acceptability in exchange for goods include, for instance, Lester, Postlewaite, and Wright (2008), Rocheteau (2008), and Li and Rocheteau (2009). Ferraris and Watanabe (2008) consider a productive input as collateral, but their aim is to explore the determination of capital accumulation. The distinction is that our paper emphasizes the role of an asset as collateral and considers various degrees of imperfections in record keeping and enforcement, which allow for partially unsecured debt.6

The paper is organized as follows. Section 2 describes the basic model. Section 3 derives the equilibrium conditions. In section 4 we derive the asset pricing equations, interest rates, and allocations, under various imperfections in the technologies of enforcement and record keeping. Section 5 studies an economy in which the real asset is used as collateral as well as a means of payment. Section 6 concludes.

2 The Model

The basic model is based on Lagos and Wright (2005) and Berentsen, Camera, and Waller (2007). There is a \([0, 1]\) continuum of infinitely lived agents. Time is discrete and continues forever. Each period is divided into two subperiods, and in each subperiod trades occur in competitive markets. There are two consumption goods, one produced in the first subperiod, and the other (called the \textit{general good}) in the second subperiod. Consumption goods are perishable and perfectly divisible. The discount factor across periods is \(\beta \in (0, 1)\).

In the beginning of the first subperiod, an agent receives a preference shock that he either consumes or produces. With probability \(n\) an agent can produce but cannot consume, while with probability \(1 - n\) an agent can consume but cannot produce. (We refer to consumers as buyers and producers as sellers.) This is a simple way to capture the uncertainty of the opportunity to trade. Consumers get utility \(u(q)\) from \(q\) consumption. Producers incur disutility \(c(q)\) from producing \(q\) units of output. Assume \(u(0) = c(0) = 0, u'(q) > 0, c'(q) > 0, u'(0) = \infty, u''(q) < 0\) and \(c''(q) \geq 0\). To motivate a role for a tangible medium of exchange, we assume that all goods trades

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6Studies that consider unsecured credit include, e.g., Berentsen, Camera, and Waller (2007), in which defaulters are excluded from future credit forever, and Gu and Wright (2010), in which exclusion occurs with some probability. Rocheteau and Wright (2010) study asset price dynamics in a model with an essential role for liquid assets in transactions.
are anonymous, and there is no public record of individuals’ trading histories.

In the second subperiod, agents get utility $U(x)$ from $x$ consumption, with $U'(x) > 0, U'(0) = \infty, U'(\infty) = 0$ and $U''(x) \leq 0$. Agents can produce one unit of the general good with one unit of labor, which generates one unit of disutility. This setup allows us to introduce an idiosyncratic preference shock and incorporate a banking sector while keeping the distribution of asset holdings analytically tractable.

There are two types of infinitely lived assets in the economy: fiat money and a real asset like the claims to Lucas (1978) trees. A government is the sole issuer of fiat money. The money stock evolves deterministically at a gross rate $\gamma$ by means of lump-sum transfers, or taxes if $\gamma < 1$, $M_t = \gamma M_{t-1}$, where $\gamma > 0$ and $M_t$ denotes the per capita currency stock in period $t$. Agents receive lump-sum transfers of money $T_t = (\gamma - 1)M_{t-1}$ in the second subperiod. There is a fixed supply, $A$, of Lucas trees. Each tree generates a constant flow of $\rho > 0$ units of general good at the beginning of the second subperiod before the asset is traded. One can think of agents as trading claims on trees.\(^7\)

There are financial intermediaries that potentially can channel funds from people with idle cash to those with liquidity needs. Financial intermediation is performed by competitive banks, which accept nominal deposits and make nominal loans. Agents who are sellers in the first subperiod can deposit their money holdings in banks at the nominal interest rate, $i_d$, and are entitled to withdraw funds in the second subperiod. Those who are buyers may borrow money from banks at the nominal loan rate, $i$, and need to repay their loans in the second subperiod. We assume that loans and deposits are not rolled over, and so all financial contracts are one-period contracts.\(^8\) Moreover, banks have zero net worth and there are no operating costs or reserve requirements.

The efficacy of financial intermediaries in ensuring the repayment of loans depends on the technologies of enforcement, commitment, and record keeping. We first consider as a benchmark an economy of full enforcement, in which banks can force repayment at no cost. Therefore, default is

\(^7\)One may think that the real asset can serve as collateral to back trade credit even in a market with anonymous trade as in, e.g., Shi (1996) and Li (2001). Notice that Shi (1996) assumes that creditors stay put so debtors can find creditors and make repayment, whereas Li (2001) assumes a technology that allows for communication between the debtor and creditor. Our model does not assume the aforementioned technology for repayment, and so the lack of record keeping of goods trades obstructs the use of trade credit.

\(^8\)With the assumption on the linear utility costs of production in the second subperiod, agents do not gain by spreading the repayment of loans or redemption of deposits across periods.
not possible, and agents face no borrowing constraints. In equilibrium the loan rate $i$ clears the loan market. We then consider an environment in which banks have limited ability to force repayment. Banks offer loan contracts that allow agents to borrow only as much as they have incentives to repay. We assume that borrowers pledge all their assets as collateral to secure loans, and banks are entitled to the collateral once borrowers renge on debts. We also consider the technology that allows banks to exclude defaulters from the banking sector, besides seizing collateral. Therefore, our model features partially unsecured credit when exclusion is feasible. Restricting defaulters from future access to credit relies on the efficiency of the enforcement and record-keeping technology, which facilitates the collection of information and sharing of agents’ repayment histories among banks.\footnote{This assumption is innocuous, since if agents are credit unconstrained, the asset is priced at the fundamental value and the amount of the real asset pledged as collateral does not matter for economic activity, whereas if they are constrained, they prefer to pledge all of their assets to receive credit.}

In the spirit of Kocherlakota and Wallace (1998), we capture the efficiency of record keeping by the probability with which agents’ default records are updated and they are excluded from the banking sector.\footnote{Commercial and consumer credit bureaus provide mechanisms for the exchange of payment performance data. Empirical evidence suggests an important link between the existence of information exchanges and credit availability (see Berger and Udell 2006). Other institutions that may affect the lending technology include the infrastructure of the legal, judicial, and bankruptcy environments, and regulations.}

We first study two extreme cases: one in which record keeping is extremely limited so the only punishment on defaulters is seizing collateral, and the other in which banks take defaulters’ collateral and exclude them from the banking system permanently. We then study interim cases, where a defaulter’s record is updated only probabilistically. For each case we derive the conditions that ensure voluntary repayment and show that this may involve binding borrowing constraints, i.e. credit rationing. To concentrate on the role of the real asset as collateral, our basic model features fiat money as the unique means of payment. This assumption can be justified as follows: fiat money is perfectly recognizable, but agents can counterfeit the real asset at zero cost. The banking system has the technology to fully recognize the true quality of assets, but individuals do not have access to this technology. Consequently, sellers do not accept the real asset for payments...
In section 5 we consider an extension in which the real asset can be used as collateral and as a means of payment.  

3 Equilibrium

Let \( \phi_t \) and \( \psi_t \) denote the values of money and the real asset in terms of the general good produced in the second subperiod, respectively. We study symmetric stationary equilibria in which the real value of asset holdings is constant. In particular, \( \phi_t M_t = \phi_{t-1} M_{t-1} \), which implies \( \frac{\phi_{t-1}}{\phi_t} = \gamma \); the inflation rate equals the money growth rate. Because we focus on the stationary equilibria, it is reasonable to consider a constant price of the real asset given its fixed supply; hence, \( \psi_{t-1} = \psi_t = \psi \) for all \( t \). In the following discussions, to simplify notations we let variables corresponding to the next period be indexed by \(+1\), and variables corresponding to the previous period be indexed by \(-1\).

Let \( V(m, a) \) denote the expected value from trading in the first subperiod with \( m \) units of money and \( a \) units of real assets at time \( t \). Let \( W(m, a, \ell, d) \) denote the expected value from entering the second subperiod with \( m \) units of money, \( a \) units of real assets, \( \ell \) loans and \( d \) deposits at time \( t \), where loans and deposits are in the units of fiat money. We use a similar approach as in Berentsen, Camera, and Waller (2007) to characterize equilibria. We study a representative period \( t \) and work backwards from the second to the first subperiod.

The Second subperiod

In the second subperiod an agent produces \( h \) goods and consumes \( x \), repays loans, redeems...
deposits, and adjusts his holdings of fiat money and real assets. He solves the following problem:

\[
W(m, a, \ell, d) = \max_{x, h, m+1, a+1} U(x) - h + \beta V_1(m+1, a+1)
\]

s.t. \( x + \phi m+1 + \psi a+1 = h + \phi (m + T) + (\psi + \rho) a + \phi (1 + i_d) d - \phi (1 + i) \ell \).

A unit of the real asset brought to the second subperiod is worth \( \psi + \rho \) units of the general good, because it generates a dividend \( \rho \) and can be resold in the market at the price \( \psi \). If an agent has deposited \( d \) in the first subperiod, he receives \( (1 + i_d) d \) units of money, and if he has borrowed \( \ell \), he should repay \( (1 + i) \ell \). Substituting \( h \) from the budget constraint into the objective function we obtain

\[
W(m, a, \ell, d) = \phi (m + T) + (\psi + \rho) a + \phi (1 + i_d) d - \phi (1 + i) \ell
\]

\[
+ \max_{x, m+1, a+1} \{ U(x) - x - \phi m+1 - \psi a+1 + \beta V_1(m+1, a+1) \}.
\]

The first order conditions are \( U'(x) = 1 \) and

\[
\phi \geq \beta V_m(m+1, a+1), \quad \text{“=” if } m+1 > 0, \quad (1)
\]

\[
\psi \geq \beta V_a(m+1, a+1), \quad \text{“=” if } a+1 > 0, \quad (2)
\]

where \( V_m(m+1, a+1) \) and \( V_a(m+1, a+1) \) are the marginal values of an additional unit of money and the real asset, respectively, taken into the first subperiod of \( t + 1 \). The optimal choice of \( x^* \) satisfies \( U'(x^*) = 1 \) for all agents. Conditions (1) and (2) determine the portfolio \( (m+1, a+1) \), independent of the initial holdings of \( m \) and \( a \). Therefore, the distribution of holdings of money and the real asset is degenerate at the beginning of period \( t + 1 \). The envelope conditions are

\[
W_m = \phi, \quad (3)
\]

\[
W_a = \psi + \rho, \quad (4)
\]

\[
W_\ell = -\phi (1 + i), \quad (5)
\]

\[
W_d = \phi (1 + i_d). \quad (6)
\]

**The first subperiod**

Let \( q_b \) and \( q_s \) denote the quantities consumed by a buyer and produced by a seller, respectively, and \( p \) denote the nominal price of the good. Because sellers do not take loans and buyers do not
make deposits, an agent holding a portfolio of \((m, a)\) entering the first subperiod has the expected lifetime utility

\[
V(m, a) = (1 - n)[u(q_b) + W(m + \ell - pq_b, a, \ell)] + n[-c(q_s) + W(m - d + pq_s, a, d)].
\] (7)

An agent may be a buyer with probability \(1 - n\), and spends \(pq_b\) units of money to get \(q_b\) consumption, or he may be a seller with probability \(n\), and receives \(pq_s\) units of money from \(q_s\) production.

As agents trade in a centralized market, they take the price \(p\) as given. A seller solves

\[
\max_{q_s, d} -c(q_s) + W(m - d + pq_s, a, d) \quad \text{s.t.} \quad d \leq m.
\]

Let \(\lambda_d\) denote the multiplier on the deposit constraint. The first order conditions are

\[
-c'(q_s) + pW = 0, \quad \text{and} \quad -W_m + W_d - \lambda_d = 0.
\]

Using (3) and (6), the first order conditions become

\[
p = \frac{-c'(q_s)}{\phi}, \quad \lambda_d = \phi i_d.
\] (8)

Equation (8) implies that a seller’s production is such that the marginal cost of production, \(c'(q_s)\), equals the marginal revenue, \(p\). The production \(q_s\) is independent of the seller’s initial portfolio brought to the first subperiod. Moreover, for any \(i^d > 0\), the deposit constraint binds and sellers deposit all money balances.

A buyer’s problem is

\[
\max_{q_b, \ell} u(q_b) + W(m + \ell - pq_b, a, \ell) \quad \text{s.t.} \quad pq_b \leq m + \ell \quad \ell \leq \bar{\ell}.
\]
The buyer faces the budget constraint that his spending cannot exceed his money holdings, \( m \), plus borrowing, \( \ell \). He also faces the credit constraint that his borrowing is bounded above by the credit limit, \( \bar{\ell} \). Let \( \lambda \) and \( \lambda_\ell \) be the multipliers on the buyer’s budget constraint and borrowing constraint, respectively. From (3), (5) and (8), we rewrite the first order conditions of the buyer’s problem as

\[
\begin{aligned}
    u'(q_b) &= c'(q_s)(1 + \frac{\lambda}{\phi}), \\
    \phi i &= \lambda - \lambda_\ell.
\end{aligned}
\]

If \( \lambda = 0 \), (9) reduces to \( u'(q_b) = c'(q_s) \), which implies trades are efficient. If \( \lambda > 0 \), the budget constraint binds, and buyers spend all money. Combining (9) and (10) we obtain

\[
\frac{u'(q_b)}{c'(q_s)} = 1 + i + \frac{\lambda_\ell}{\phi}.
\]

If the borrowing constraint does not bind, \( \lambda_\ell = 0 \) and

\[
\frac{u'(q_b)}{c'(q_s)} = 1 + i,
\]

which implies that buyers borrow up to the point at which the marginal benefit of an additional unit of borrowed money, \( \frac{u'(q_b)}{c'(q_s)} \), equals the marginal cost, \( 1 + i \). If \( \lambda_\ell > 0 \), the borrowing constraint binds, \( \ell = \bar{\ell} \), and

\[
\frac{u'(q_b)}{c'(q_s)} > 1 + i.
\]

Buyers wish to borrow more money, but banks may not be willing to lend because of the concern about default. The buyer thus borrows \( \bar{\ell} \) and spends all his money to consume, so

\[
q_b = (m + \bar{\ell})/p.
\]

In the first subperiod, banks accept deposits from the sellers and make loans to the buyers. If banks have full enforcement on repayment, the borrowing constraint does not bind. When enforcement is limited, banks choose the credit limit \( \bar{\ell} \) to ensure voluntary repayment. The zero-profit condition for competitive banks is (see Appendix B for the details to derive solutions to the bank’s problem)

\[
i = i_d.
\]

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13 Given the market price \( p \), one unit of borrowed money buys \( \frac{1}{p} \) units of the good, which generates utility \( \frac{u'(q_b)}{p} \). To compare the utility from consumption with the nominal cost of borrowing, one needs to convert the utility in terms of money; i.e., dividing \( \frac{u'(q_b)}{c'(q_s)} \) by \( \phi \), which becomes \( \frac{u'(q_b)}{c'(q_s)} \) by using (8).
In a symmetric equilibrium, the loan market clearing condition is

$$(1 - n)\ell = nd.$$  

The goods market clearing condition in subperiod 1 is

$$nq_s = (1 - n)q_b.$$  

The market clearing conditions for money and the real asset are $m = M_{-1}$ and $a = A$, respectively.

### 3.1 Optimal portfolio choices

To find an agent’s optimal portfolio and determine the asset’s price, we first derive the marginal values of money and the real asset in the first subperiod (see Appendix A for the derivation). The marginal value of money is

$$V_m(m, a) = \phi[(1 - n)\frac{u'(q_b)}{c'(q_s)} + n(1 + i_d)].$$  

The benefits of holding an additional unit of money include the expected gains from spending the money on goods as a buyer, and the interest payments from making deposits as a seller. The marginal value of holding the real asset is

$$V_a(m, a) = (1 - n)\phi\left[\frac{u'(q_b)}{c'(q_s)} - (1 + i)\right]\frac{\partial \ell}{\partial a} + (\psi + \rho).$$

The benefits of carrying an additional unit of the real asset include two terms: the net gains of consumption accrued from the loans secured by the asset minus the debt repayment, and the resale price and the dividend that the holder of the asset is entitled to when entering the second subperiod.

Using (1) lagged one period to eliminate $V_m(m, a)$ from (13), an agent’s optimal money holdings satisfy

$$\phi_{-1} \geq \beta\phi[(1 - n)\frac{u'(q_b)}{c'(q_s)} + n(1 + i_d)], \quad \text{“=” if } m > 0.$$  

Similarly, an agent’s optimal asset holdings satisfy

$$\psi_{-1} \geq \beta\{\psi + \rho + (1 - n)\phi[\frac{u'(q_b)}{c'(q_s)} - (1 + i)]\frac{\partial \ell}{\partial a}\}, \quad \text{“=” if } a > 0.$$  

11
We focus on the equilibrium in which agents hold money and the real asset. Hence, (15) and (16) hold at equality:

\[
\frac{\gamma - \beta}{\beta} = (1 - n)[u'(q_b) - 1] + ni_d; \tag{17}
\]

\[
\frac{1 - \beta}{\beta} \psi = \rho + (1 - n)\phi\left[\frac{u'(q_b)}{c'(q_s)} - (1 + i)\right] \frac{\partial \ell}{\partial a}. \tag{18}
\]

Equation (18) illustrates a key feature of our model: the extra loan amount secured by a marginal unit of the real asset provides a link between the asset price and its role in generating liquidity. To see this, recall that if the credit constraint does not bind, the marginal benefit of receiving an additional dollar of a loan equals the cost, \( \frac{u'(q_b)}{c'(q_s)} = 1 + i \). The second term in (18) vanishes, and the asset price \( \psi \) is determined only by the dividend flows. If agents are credit constrained, \( \frac{u'(q_b)}{c'(q_s)} > 1 + i \), the asset price is influenced not only by dividends but also by the extent to which the real asset relaxes credit constraints. We will elaborate more in Section 4.

### 3.2 Equilibrium with full enforcement

When banks can force borrowers to repay loans at no cost, the borrowing constraint does not bind. This immediately leads to the following proposition:

**Proposition 1** With full enforcement, the asset price is the present value of dividends; i.e., \( \psi = \psi_u \) where

\[
\psi_u = \frac{\beta \rho}{1 - \beta}. \tag{19}
\]

To find an equilibrium, substituting \( i = i_d \) into (17) we obtain

\[
i = \frac{\gamma - \beta}{\beta}. \tag{20}
\]

From (17) and (20),

\[
\frac{\gamma - \beta}{\beta} = \frac{u'(q_b)}{c'(\frac{1 - n}{n}q_b)} - 1. \tag{21}
\]

Under full enforcement, a monetary equilibrium with credit is a list \((\psi, i, q_b)\) satisfying (19) – (21).
4 Limited enforcement and record keeping

When the bank’s ability to force repayment is limited, borrowers have an incentive to renege on debts. If default occurs, banks seize collateral, and they may exclude defaulters from the banking sector. Whether exclusion is feasible relies heavily on the collection and sharing of repayment information—the record-keeping technology, among other institutions. We capture the efficiency of record keeping by the probability with which agents’ default records are updated. Specifically, at the end of each period if an agent repudiates debt obligations, his default record is updated with probability $\zeta$. The records of default are kept within the banking system, and defaulters listed in the records are excluded from the banking sector forever. With probability $1 - \zeta$ the updating does not occur, and a defaulter starts the next period as a nondefaulter. One can also interpret $\zeta$ as a measure of financial development as it affects the ability of a financial system to control for the moral hazard problem.$^{14}$

4.1 Extremely limited record keeping

In the economy with extremely limited record keeping, the update on defaults never occurs ($\zeta = 0$), and the only punishment levied on defaulters is seizing their collateral. Consider a buyer entering the second subperiod who repays his loan and holds no money. His expected discounted utility in a stationary equilibrium is

$$W(m, a) = U(x^*) - h_b + \beta V_{+1}(m+1, a+1),$$

where $h_b$ is a buyer’s production in the second subperiod if he repays the loan. If a borrower defaults, the benefit is enjoying more leisure since he doesn’t have to produce to repay the debt, while the cost is the loss of his collateral. He then starts the next period as a nondefaulter. This implies that a defaulter would choose the same portfolio as nondefaulters, and his expected discounted utility from the next period is also $V_{+1}(m+1, a+1)$. The expected discounted utility of a deviating buyer entering the second subperiod is

$$\hat{W}(m, a) = U(\hat{x}) - \hat{h}_b + \beta V_{+1}(m+1, a+1),$$

$^{14}$One may justify the exclusion of defaulters in an environment such as that considered in Gomis-Porqueras and Sanches (2010), where a costly technology allows agents to report their trades and identities to the credit center. This creates public information regarding private trades, which can enforce the repayment of private liabilities.
where the hat indicates a deviator’s optimal choice.

For the existence of equilibrium with credit, borrowers must voluntarily repay their loans, which requires \( W(m, a) \geq \hat{W}(m, a) \). Banks offer loan contracts such that borrowers will repay their debts, so the real borrowing constraint \( \phi \bar{\ell} \) satisfies

\[
W(m, a) = \hat{W}(m, a).
\]

**Lemma 1** When the only punishment on defaulters is seizing collateral, the real borrowing constraint \( \phi \bar{\ell} \) satisfies

\[
(1 + i)\phi \bar{\ell} = (\psi + \rho)a. \tag{22}
\]

The real credit limit \( \phi \bar{\ell} \) in (22) is obtained by equating the current-period benefit and cost of default.\(^{15}\) The left-hand side is the saving in the utility cost from not producing to repay loans, and the right-hand side is the cost of losing collateral, which includes the resale price and dividends accrued from possessing the asset.\(^{16}\)

For a given \( \bar{\ell} \), an agent’s demand for loans may be less than the credit limit imposed by banks, so that the borrowing constraint does not bind. Hence, \( \ell < \bar{\ell} \) in an unconstrained equilibrium; otherwise, \( \ell = \bar{\ell} \). Note that in any equilibrium with \( i > 0 \), banks lend out all deposits, so the real loan satisfies

\[
\phi \ell = \frac{n}{1 - n} \phi M_{-1}.
\]

**Definition 1** A monetary equilibrium with unconstrained credit is a list \((q_b, i, \psi)\) satisfying (19), (20) and (21), and \(0 < \phi \ell < \phi \bar{\ell} \), where \( \phi \bar{\ell} \) satisfies (22).

\(^{15}\)Because the market clearing conditions for money and loans are \( m = M_{-1} \) and \((1 - n)\ell = nd\), respectively, in this economy \( \ell = \frac{n}{1 - n} M_{-1} \); i.e., the nominal loan amount is determined by the money supply of last period. What is important for economic activity, such as consumption, is the real value of the loan, \( \phi \ell \). Hence, we focus on the loan amount and the credit limit in real terms.

\(^{16}\)Equation (22) looks similar to the credit constraint in Kiyotaki and Moore (1997). They assume that lenders have linear preferences and they are not credit constrained, and so in equilibrium the interest rate \( R \) equals their rate of time preference. Specifically, let \( k_t \) denote the borrower’s land that is used as collateral to borrow \( b_t \) at time \( t \), and \( q_{t+1} \) is the price of land at time \( t + 1 \). The binding credit constraint appears to be

\[
Rb_t = q_{t+1}k_t.
\]

However, in our model the loan rate and loan amount are jointly determined in the contract that make borrowers voluntarily repay their debts.
Even if enforcement is limited, the asset price is determined by the discounted dividend streams as long as credit constraints do not bind. Moreover, the asset price, loan rate, and allocations are identical to those in the economy with full enforcement. The equilibrium with unconstrained credit exists if $A > \bar{A}$, where

$$\bar{A} = \frac{n(1 - \beta)\phi M}{\rho(1 - n)\beta}.$$  

That is, if the supply of the asset is sufficiently abundant, it will be priced at the fundamental value.

If agents are credit constrained, banks charge a nominal loan rate, $i'$, and lend out $\ell = \bar{\ell}$ to induce voluntary repayment.

**Definition 2** A monetary equilibrium with constrained credit is $(q_b, i', \psi)$ satisfying (17), (18) and (22), and $0 < \phi \ell = \phi \bar{\ell}$, where $\phi \bar{\ell}$ satisfies (22).

From the borrowing constraint (22), the extra loan amount that the marginal unit of the asset can generate is

$$\frac{\partial \ell}{\partial a} = \frac{\psi + \rho}{\phi(1 + i)}.$$  

(23)

Substituting (23) into (18), we derive the asset price and the loan-to-value ratio (the ratio of the real loan amount to the real value of collateral, $\frac{\phi \ell}{\psi}$) in the following proposition.

**Proposition 2** When the only punishment on defaulters is seizing collateral, in a constrained equilibrium the asset price is

$$\psi_1 = \frac{\beta B \rho}{1 - \beta B},$$  

(24)

where

$$B = 1 + (1 - n)[\frac{u'(q_b)}{c'(q_s)} \frac{1}{1 + i} - 1].$$

The loan-to-value ratio is

$$\theta_1 = \frac{1 + r_p}{1 + i},$$  

(25)

where $r_p = \frac{p}{\psi}$ is the dividend-price ratio.
The asset pricing equation in (24) shows how asset prices, borrowing constraints, and credit market imperfections are intertwined. Denote $\beta B$ the “effective” discount factor, which takes into account credit market frictions. As agents are credit constrained, $\frac{u'(q_b)}{c'(q_s)} > 1 + i$ and $B > 1$; therefore, $\psi_1 > \psi_u$. Define the “liquidity premium” as the difference between the price of an asset and its fundamental value. The liquidity premium of the real asset arises from its role in relaxing the borrowing constraint, and the premium is increased by the discrepancy between the marginal benefit of loan, $\frac{u'(q_b)}{c'(q_s)}$, and the marginal cost, $1 + i$. That is, the more severe the credit rationing, the higher the liquidity premium.

The loan-to-value ratio, which can be interpreted as the rate at which the asset generates liquidity to lubricate economic activity, depends positively on the dividend-price ratio of the asset pledged as collateral, and negatively on the loan rate. Although a lower price of the collateralized asset leads to a more stringent borrowing constraint (lower $\phi$), it increases the loan-to-value ratio. For a given asset price $\psi$, an increase in the loan rate makes agents more likely to default since the repayment cost is increased. Hence, banks should lend less by setting a lower loan-to-value ratio to deter default.\textsuperscript{17}

\textbf{Proposition 3} Monetary policy has similar effects on the loan rate, allocations, and prices in a constrained and unconstrained equilibrium: $\frac{\partial \psi}{\partial \gamma} > 0$, $\frac{\partial q_b}{\partial \gamma} < 0$, $\frac{\partial \phi}{\partial \gamma} < 0$, $\frac{\partial \psi}{\partial \gamma} > 0$. However, in a constrained equilibrium, $\frac{\partial \psi}{\partial \gamma} < 0$ if $u' + u''q_b > 0$, and $\frac{\partial \theta}{\partial \gamma} < 0$.

Higher inflation lowers the value of money, which reduces the incentives to produce, as is standard in monetary models. The binding credit constraints, however, constitute an additional transmission mechanism of monetary policy. An increase in the inflation rate may reduce the price of the asset, which has a positive effect on the loan-to-value ratio; it also results in a higher loan rate, which increases the incentives to default. It turns out that the interest rate effect dominates, so the loan-to-value ratio is lowered by inflation, and aggregate liquidity and output fall.\textsuperscript{18}

\textbf{Proposition 4} The effects of changes in the supply of the real asset and dividend flows are:

\textsuperscript{17}Note that $r_p < r$ (the rate of time preference), since $\psi_1 > \psi_u$. Hence, $\theta_1 < 1$ as long as $i \geq r$.

\textsuperscript{18}In an unconstrained equilibrium, the real credit limit is $\phi = \frac{\bar{\phi}}{1 - \phi}$. Even though higher inflation lowers the credit limit, the aggregate liquidity satisfies what agents need for consumption as long as the borrowing constraints do not bind. Hence, for an unconstrained equilibrium we emphasize the effects of monetary policy on allocations rather than the loan-to-value ratio.
1. A change in the asset supply does not affect the loan rate and allocations in an unconstrained equilibrium, but it has real effects in a constrained equilibrium: \( \frac{\partial \phi}{\partial A} > 0, \frac{\partial i}{\partial A} > 0, \frac{\partial \psi}{\partial A} < 0, \frac{\partial \psi}{\partial A} > 0, \frac{\partial q}{\partial A} < 0, \frac{\partial q}{\partial A} = 0 \).

2. A change in the asset’s dividend flows affects only the asset price in an unconstrained equilibrium: \( \frac{\partial \psi}{\partial p} > 0 \); however, it also affects the loan rate and allocations in a constrained equilibrium: \( \frac{\partial i}{\partial p} > 0, \frac{\partial q}{\partial p} > 0, \frac{\partial \psi}{\partial p} > 0, \frac{\partial q}{\partial p} < 0, \frac{\partial \psi}{\partial p} > 0 \) if \( \frac{\partial B/B}{\partial p/\rho} < 1 - \beta B, \frac{\partial q}{\partial p} > 0 \).

In an unconstrained equilibrium, the asset is priced at the fundamental value, and the quantity \( q_b \) and loan rate \( i \) are determined by \( \gamma \) and \( \beta \), and are independent of the asset supply or dividends. In contrast to frictionless asset-pricing models, the asset price depends negatively on the supply of the asset if credit rationing occurs. Although an increase in the asset supply does not affect the loan-to-value ratio, it exerts positive effects on the aggregate liquidity and output by increasing the collateralizable assets, which relaxes the borrowing constraints.

Higher dividends have a positive effect on the fundamental element of the asset price, and a negative general equilibrium effect on the asset price, by reducing the severity of credit rationing (because \( \frac{\partial B}{\partial p} < 0 \)). When the positive effect dominates, i.e., the elasticity of the severity of credit rationing with respect to the dividend flows is sufficiently small, higher dividends lead to a higher asset price. Moreover, higher dividend flows generate more liquidity to support consumption by raising the loan-to-value ratio. The predictions of Proposition 4 imply that the loan-to-value ratio can be positively correlated with the price of the collateralized asset and aggregate liquidity.

### 4.2 Superior record keeping

In the economy with superior record-keeping technology, banks take defaulters’ collateral and exclude them permanently from the banking system (\( \zeta = 1 \)). The technology, however, cannot exclude defaulters from the asset market so they can still trade money and the real asset. Moreover, banks can only take a defaulter’s assets that have been pledged as collateral; they are not able to seize a defaulter’s assets or income in the future.

---

\(^{19}\) Along the lines of Kiyotaki and Moore (1997) and Chen (2001), Chen and Wang (2007) use a panel transaction data set from Taiwan to investigate whether collateral’s leverage on bank credit exhibits procyclicality to asset price cycles. They find that the value of collateralized assets has positive and significant effects on loan amounts and that the leverage effect of collateral is procyclical to the asset price cycles.
Because defaulters cannot make deposits or receive any credit in all future periods, they need to bring enough money to execute trades in the first subperiod market. This implies that a deviator potentially would choose a different portfolio from non-deviators, and may trade at a different quantity, $q_b$. The expected discounted utility of a deviating buyer from the next period is denoted as $\tilde{V}_{+1}(\tilde{m}_{+1}, \tilde{a}_{+1})$, where the tilde indicates a deviator’s optimal choice. The expected discounted utility of a deviating buyer entering the second subperiod is

$$\tilde{W}(m, a) = U(\tilde{x}) - \tilde{h}_b + \beta \tilde{V}_{+1}(\tilde{m}_{+1}, \tilde{a}_{+1}).$$

The real borrowing constraint $\phi \tilde{l}$ solves $W(m, a) = \tilde{W}(m, a)$, from which we derive the following lemma.

**Lemma 2** When permanent exclusion of defaulters is feasible, the real borrowing constraint $\phi \tilde{l}$ satisfies

$$(1 + i)\phi \tilde{l} = \frac{\rho a}{1 - \beta} + \frac{\beta}{1 - \beta} \{(1 - n)\Psi(q_b, \tilde{q}_b) + \gamma(1 - \beta) c'(q_s)(\tilde{q}_b - (1 - n)q_b)\}.$$  \hspace{1cm} (26)

where

$$\Psi(q_b, \tilde{q}_b) = u(q_b) - u(\tilde{q}_b) - c'(q_s)(q_b - \tilde{q}_b) \geq 0.$$

The real borrowing constraint $\phi \tilde{l}$ shown in (26) comprises of two terms: the loss of collateral, and the long-term loss from being excluded from the banking sector, which is the difference of the expected discounted gains from trade between non-deviators and deviators. Note that the first term is not equal to the right-hand side of (22), because deviators choose not to hold any real assets in this economy. The reason is that the asset price incorporates extra benefits from backing loans, yet no such gains accrue to deviators because they can not borrow in the future. Therefore, in a constrained equilibrium if non-deviators choose to hold the real asset, it never pays for deviators to do so.

**Definition 3** A monetary equilibrium with unconstrained credit is $(q_b, \tilde{q}_b, i, \psi)$ satisfying (19), (20), (21), and

$$\gamma - \frac{\beta}{\beta} = (1 - n)[\frac{u'(\tilde{q}_b)}{c'(q_s)} - 1],$$ \hspace{1cm} (27)

and $0 < \phi \ell < \phi \tilde{l}$, where $\phi \tilde{l}$ satisfies (26).
Definition 4 A monetary equilibrium with constrained credit is \((q_b, q_b, i', \psi)\) satisfying (17), (18), (26), (27), and \(0 < \phi \ell = \phi \bar{\ell}\), where \(\phi \bar{\ell}\) satisfies (26).

From (26), the extra loan amount generated by a marginal unit of the real asset is

\[
\frac{\partial \ell}{\partial a} = \frac{\rho}{\phi(1+i)(1-\beta)}.
\]

Substituting (28) into (18), we derive the asset pricing equation and determine the loan-to-value ratio in the following proposition.

Proposition 5 When permanent exclusion of defaulters is feasible, in a constrained equilibrium the asset price is

\[
\psi_2 = \frac{\beta B_2 \rho}{1 - \beta},
\]

where

\[
B_2 = 1 + \frac{1 - n}{1 - \beta} \left[ \frac{u'(q_b)}{c'(q_s)(1 + i)} - 1 \right].
\]

The loan-to-value ratio is

\[
\theta_2 = \frac{r_p}{(1 - \beta)(1 + i)} + \frac{\beta}{(1 - \beta)(1 + i)\psi A} \left\{ (1 - n)\Psi(q_b, q_b) \right. \\
+ \left. \frac{\gamma (1 - \beta)}{\beta} c'(q_s) [q_b - (1 - n)q_b] \right\}.
\]

The loan-to-value ratio, \(\theta_2\), is positively related to the asset’s dividend-price ratio and the extra gains of consumption from obtaining loans. The real asset commands a liquidity premium in a constrained equilibrium, again due to the role of the asset in relaxing the borrowing constraint.

The policy implications differ depending on whether the technology and institutions allow for the punishment of exclusion. From numerical examples we find that higher inflation raises the loan rate and reduces the asset price as in the previous economy without exclusion; however, it may raise the value of money, the loan-to-value ratio, aggregate liquidity, and output (see Table 1). The reason for the positive effects on allocations is this. Because deviators need to bring enough money to self-insure against consumption shocks, higher inflation raises the cost of being excluded from the banking sector and thus the incentives to repay debts. Interestingly, even though the asset price is reduced by policy, the real loan amount increases due to a higher loan-to-value ratio. This result
Table 1: The effects of changes in $\gamma$, $\rho$ and $A$ in an equilibrium with constrained credit

<table>
<thead>
<tr>
<th>Benchmark $\gamma = 1.015(\uparrow)$</th>
<th>$\gamma = 0.001(\downarrow)$</th>
<th>$A = 3(\uparrow)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>0.041</td>
<td>0.052</td>
</tr>
<tr>
<td>$q_b$</td>
<td>0.631</td>
<td>0.641</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.025</td>
<td>0.026</td>
</tr>
<tr>
<td>$\phi\ell$</td>
<td>0.378</td>
<td>0.384</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.014</td>
<td>0.012</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>13.938</td>
<td>15.394</td>
</tr>
</tbody>
</table>

Note: We set up the utility function, $u(q_b) = \frac{(q_b)^{0.8}}{0.8}$, and the cost function, $c(q_s) = q_s$, in the first subperiod. The utility function is $U(x) = C \log(x)$ in the second subperiod. The parameter values for the benchmark case are $n = 0.6$, $\beta = 0.95$, $\gamma = 1.01$, $C = 2.537$, $\rho = 0.0005$, and $A = 2$.

runs counter to previous theoretic models with exogenously given, constant loan-to-value ratios, which predict a positive correlation between the loan amount and the value of collateral. This distinction implies that whether to derive credit constraints explicitly under various technologies to deter default matters for policy implications.

Finally, changes in the asset supply and dividend flows have the same effects on prices and allocations as in the previous economy, while their effects on the loan-to-value ratios are different. Table 1 shows that, though an increase in $A$ or $\rho$ raises the real loan amount and output, it lowers the loan-to-value ratio, $\theta_2$. The expected cost of being excluded from the banking sector is raised by a higher $q_b$, which tends to raise $\theta_2$. An increase in $\rho$, which leads to a higher asset price, or an increase in $A$, which results in more collateralizable assets, has a negative effect on $\theta_2$. The loan-to-value ratio falls when the negative effect dominates.

4.3 A general case of record keeping $\zeta \in (0, 1)$

In this subsection we discuss the case in which the default records are updated with some probability. At the end of each period after banks have seized defaulters’ collateral, an agent’s default record is updated with probability $\zeta \in (0, 1)$, and the updating does not occur with probability $1 - \zeta$. Thus, with probability $\zeta$ a defaulter will be excluded permanently from the banking sector, and his expected utility is $\tilde{W}(m, a)$, while with probability $1 - \zeta$ the punishment is confined to seizing collateral, and the expected utility is $\hat{W}(m, a)$. 

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The expected utility of a deviating buyer in the second subperiod is

\[
W(m, a) = (1 - \zeta) \left[ U(\tilde{x}) - \tilde{h}_b + \beta V_{+1}(m_{+1}, a_{+1}) \right] + \zeta \left[ U(\tilde{x}) - \tilde{h}_b + \beta \tilde{V}_{+1}(\tilde{m}_{+1}, \tilde{a}_{+1}) \right],
\]

where the hat and tilde indicate a deviator’s optimal choices in the cases where the default record is not updated and updating occurs, respectively. Banks choose the real borrowing constraint \(\ell\) satisfying \(W(m, a) = \tilde{W}(m, a)\), from which one can derive the real borrowing constraint \(\ell\) and the loan-to-value ratio in a constrained equilibrium. In a constrained equilibrium, \(\ell = \tilde{\ell}\), and

\[
\frac{\partial \ell}{\partial a} = \frac{(1 - \beta + \beta \zeta) \rho + (1 - \beta)(1 - \zeta) \psi}{\phi(1 + i)(1 - \beta)}.
\]

Substituting (32) into (18), we have the following proposition.

**Proposition 6** Under the probability of updating default records, \(\zeta\), in a constrained equilibrium the asset price is

\[
\psi_3 = \frac{\beta B_3 \rho}{1 - \beta B_4},
\]

where

\[
B_3 = 1 + (1 - n)(1 + \frac{\beta \zeta}{1 - \beta}) \left[ \frac{u'(q_b)}{c'(q_s)} \frac{1}{1 + i} - 1 \right],
\]

\[
B_4 = 1 + (1 - n)(1 - \zeta) \left[ \frac{u'(q_b)}{c'(q_s)} \frac{1}{1 + i} - 1 \right].
\]

The loan-to-value ratio is

\[
\theta_3 = \frac{(1 - \beta + \beta \zeta) \rho + (1 - \beta)(1 - \zeta)}{(1 - \beta)(1 + i)} + \frac{\zeta \beta}{(1 - \beta)(1 + i) \psi a} \{(1 - n) \Psi(q_b, \tilde{q}_b) + \gamma(1 - \beta) \frac{c'(q_s)}{\beta} [\tilde{q}_b - (1 - n) q_b] \}.
\]

The loan-to-value ratio \(\theta_3\) captures the current-period loss of collateral and the expected long-term loss of being excluded from the banking sector. Note that \(\theta_3 = \theta_1\) if \(\zeta = 0\), the case with extremely limited record keeping, and \(\theta_3 = \theta_2\) if \(\zeta = 1\), the case with superior record keeping.

To see the effect of the improved technology on welfare, we use the expected lifetime utility of the representative agent at the beginning of a period as a measure of welfare:

\[
\Omega = \frac{1}{1 - \beta} \left[ (1 - n) u(q_b) - nc(q_s) + U(x^*) - x^* \right].
\]
We compare asset prices, allocations and welfare across constrained equilibria under various degrees of efficiency in record keeping. Figure 1 shows that advances in the technology that make exclusion more likely have non-monotonic effects on real loan amounts, loan rates, output, and welfare: all values decline and then rise over the range $\zeta \in [0, 1]$, with the highest levels at $\zeta = 1$.\footnote{In Figure 1 we use $u(q_x) = \frac{(q_x)^{0.8}}{q_x^{0.8}}$, $c(q_s) = q_s$, and $U(x) = C \log(x)$. The parameter values are $n = 0.6$, $\beta = 0.95$, $\gamma = 1.016$, $C = 2.537$, $p = 0.001$, and $A = 2.5$.} Improved efficiency in the technology exerts two opposite effects on the loan amount: the asset price is decreased, whereas the loan-to-value ratio is raised by $\zeta$. Because a more severe punishment of exclusion (caused by higher $\zeta$) makes collateral a less important commitment device for borrowing, asset prices fall. When financial development is in the primitive stage, an improvement in $\zeta$ results in a small increase in the loan-to-value ratio. If the effect is dominated by the decrease in the asset price, aggregate liquidity and output fall. When $\zeta$ is above some threshold, the punishment of exclusion is substantial enough to make the rise in the loan-to-value ratio a dominant effect, so aggregate liquidity and output rise. Finally, our numerical examples show that higher inflation may raise the real loan amounts, the loan-to-value ratios, aggregate liquidity, and output only when $\zeta$ is above a certain threshold.

5 The real asset is used as a means of payment and collateral

To concentrate on the role of the real asset as collateral, our basic model features fiat money as the unique medium of exchange. In this section we relax this assumption to explore the dual role of the real asset as a means of payment and as collateral. To do so, we assume in the first subperiod there are two locations where agents can trade the consumption good competitively. In location 1 sellers accept the real asset for payment (this can be justified by assuming a costless verification technology available to sellers in location 1, which enables them to fully ascertain the quality of the real asset). Location 2 is reminiscent of the basic model, in that money is the unique means of payment. At the beginning of a period an agent receives a location shock and a preference shock, both of which arrives independently. An agent goes to location 1 with probability $\alpha$, and to location 2 with the complementary probability $1 - \alpha$, where $0 < \alpha < 1$. For simplicity we assume
that buyers in location 1 cannot take loans, but sellers can make deposits if they wish to.\textsuperscript{21}

Spatial and informational frictions imply that arbitrage is limited across markets, so that the good may trade at different prices in different locations. Let $p_k$ denote the nominal price of the good in location $k$, and $q_{b,k}$ and $q_{s,k}$ denote the quantities consumed by a buyer and produced by a seller, respectively, in location $k = 1, 2$. Let $p_a = \frac{\psi + \varphi}{\phi}$ denote the value (including the resale price and dividend) of the real asset in monetary units. An agent with portfolio $(m, a)$ entering the first subperiod has the expected lifetime utility

$$V(m, a) = \alpha \{(1 - n)[u(q_{b,1}) + W(m + p_a a - p_1 q_{b,1})] + n[-c(q_{s,1}) + W(m - d + p_1 q_{s,1})]\}$$

$$+ (1 - \alpha) \{(1 - n)[u(q_{b,2}) + W(m + \ell - p_2 q_{b,2})] + n[-c(q_{s,2}) + W(m - d + p_2 q_{s,2})]\},$$

where we have dropped the last three elements in the value function $W$ to reduce notations. The interpretation of $V(m, a)$ is similar to (7) except that, with probability $\alpha$, a buyer can finance his consumption directly with fiat money and the real asset.

The maximization problems and the optimal conditions of sellers and buyers in location 2 are identical to those in the basic model. We discuss briefly the optimal conditions in location 1. The first order condition to seller’s problem is

$$p_1 = \frac{c'(q_{s,1})}{\phi}. \quad (34)$$

Since the buyer can use fiat money and the real asset to make purchases, his problem is

$$\max_{q_{b,1}} u(q_{b,1}) + W(m + p_a a - p_1 q_{b,1})$$

s.t. $pq_{b,1} \leq m + p_a a$.

Let $\lambda_1$ denote the multiplier on the buyer’s budget constraint. The first order condition is

$$u'(q_{b,1}) = c'(q_{s,1})(1 + \frac{\lambda_1}{\phi})$$

If the budget constraint does not bind, $u'(q_{b,1}) = c'(q_{s,1})$, implying the quantity traded is efficient. If $\lambda_1 > 0$, the buyer spends all money and real assets, and

$$q_{b,1} = \frac{m + p_a a}{p_1}. \quad (35)$$

\textsuperscript{21}The assumption that buyers in location 1 cannot make loans is not restrictive, as we will show that in equilibrium agents prefer using the real asset as a means of payment rather than as collateral. Moreover, if we assume sellers in location 1 cannot make deposits, the main qualitative results still hold.
Finally, the goods market clearing condition at location $k$ is

$$n q_{s,k} = (1 - n) q_{b,k}.$$  

We focus on the equilibrium in which agents hold money and the real asset. An agent’s optimal holdings of money and assets satisfy

$$\phi_{-1} = \beta \phi\{(1 - n)\left[\alpha \frac{u'(q_{b,1})}{c'(q_{s,1})} + (1 - \alpha) \frac{u'(q_{b,2})}{c'(q_{s,2})}\right] + n(1 + i)\},$$

$$\psi_{-1} = \beta \{\alpha(\psi + \rho)[(1 - n)\frac{u'(q_{b,1})}{c'(q_{s,1})} + n] + (1 - \alpha)\{\psi + \rho + (1 - n)[\frac{u'(q_{b,2})}{c'(q_{s,2})} - (1 + i)]\} \frac{\partial \ell}{\partial a}\}.$$

The following two conditions must be satisfied in equilibrium:

$$\frac{\gamma - \beta}{\beta} = (1 - n)\{\alpha[\frac{u'(q_{b,1})}{c'(q_{s,1})} - 1] + (1 - \alpha)[\frac{u'(q_{b,2})}{c'(q_{s,2})} - 1]\} + ni, \tag{36}$$

$$\frac{1 - \beta B_m}{\beta} \psi = B_m \rho + (1 - \alpha)(1 - n)[\frac{u'(q_{b,2})}{c'(q_{s,2})} - (1 + i)] \frac{\partial \ell}{\partial a}, \tag{37}$$

where

$$B_m = 1 + \alpha(1 - n)[\frac{u'(q_{b,1})}{c'(q_{s,1})} - 1].$$

If the buyer’s budget constraint binds; i.e., the amounts of fiat money and real assets are not sufficient to buy the efficient quantity, then $B_m > 1$.

As in the basic model, under full enforcement the interest rate that clears the market is

$$i = \frac{u'(q_{b,2})}{c'(\frac{1}{n} q_{b,2})} - 1. \tag{38}$$

Substituting (38) into (36) and (37) we obtain

$$\frac{\gamma - \beta}{\beta} = \alpha(1 - n)[\frac{u'(q_{b,1})}{c'(\frac{1}{n} q_{b,1})} - 1] + (1 - \alpha + \alpha n)i, \tag{39}$$

and the asset price $\psi = \psi^m_u$, where

$$\psi^m_u = \frac{\beta B_m \rho}{1 - \beta B_m}. \tag{40}$$

The asset pricing equation (40) is the discounted sum of dividends, with $\beta B_m$ as the “effective” discount factor. The term $B_m$ depends on whether the amounts of fiat money and real assets are sufficient to achieve the efficiency, and it also reflects the liquidity return from facilitating trades.
When the buyer’s budget constraint binds, \( B_m > 1 \), and \( \psi_u^m > \psi_u \). The asset price under full enforcement is influenced by factors besides fundamentals, and the liquidity premium arises from the service provided by the asset to facilitate trades.

We discuss equilibria in the economy where seizing collateral is the only punishment; other cases may be analyzed in a similar way. Since buyers in location 1 do not take loans, in an equilibrium with \( i > 0 \) bank lending satisfies

\[
\phi \ell = \frac{n}{(1 - \alpha)(1 - n)} \phi M_{-1}.
\]

In an equilibrium with unconstrained credit, the asset price is \( \psi_u^m \), defined in (40). The loan rate and allocations are identical to those in the economy with full enforcement. Next, we study the constrained equilibrium.

**Definition 5** A monetary equilibrium with constrained credit is \((q_{b,1}, q_{b,2}, i', \psi)\) satisfying (22), (35), (36), and (37), and \( 0 < \phi \ell = \phi \bar{\ell} \), where \( \phi \bar{\ell} \) satisfies (22).

Substituting (23) into (37), we derive the asset price as follows.

**Proposition 7** When the real asset is used as a means of payment and collateral, in a constrained equilibrium the asset price is \( \psi = \psi_c \), where

\[
\psi_c = \frac{\beta B_c \rho}{1 - \beta B_c},
\]

and

\[
B_c = 1 + (1 - n) \left\{ \alpha \left[ \frac{u'(q_{b,1})}{c'(q_{s,1})} - 1 \right] + (1 - \alpha) \left[ \frac{u'(q_{b,2})}{c'(q_{s,2})} \left( \frac{1}{1 + i} - 1 \right) \right] \right\}.
\]

In the asset pricing equation (41), the “effective” discount factor \( \beta B_c \) takes into account the insufficiency of payment instruments and the frictions due to credit market imperfection: the first term in the big bracket of \( B_c \) reflects whether the amount of the means of payment is sufficient to achieve the efficiency, and the second term captures the severity of credit rationing. If any of the budget constraint or the credit constraint binds, \( B_c > 1 \) and \( \psi_c \) is higher than the fundamental value. The liquidity premium stems from the liquidity services provided by the asset to secure loans and to facilitate trades as a means of payment.
From the numerical examples we find that, in both constrained and unconstrained equilibrium, buyers in location 1 enjoy higher expected utility than those in location 2. This implies that, using the real asset as a payment instrument achieves higher consumption and welfare than using it as collateral. This result is obvious for a constrained equilibrium because agents cannot borrow sufficient funds to satisfy their consumption needs when credit rationing occurs. The reason for such a result in an unconstrained equilibrium is the loan interest payments. The payments reduce the amount that agents can borrow so that they cannot consume as much if they use the asset to borrow as if they use it as a means of payment.

6 Conclusion

We have explicitly derived credit constraints from rationality conditions in which lenders offer loan contracts that allow agents to borrow up to the amount that they have incentives to repay. Deriving the asset pricing equations from the role of the asset as collateral illustrates how the asset prices, liquidity, and credit constraints are intertwined. Credit constraints and loan-to-value ratios are shown to be an important transmission mechanism of monetary policy, which may result in some policy implications different from previous studies with exogenously imposed, constant loan-to-value ratios.

The current paper has depicted interactions between the endogenous credit constraints and the price of the collateralized asset under various technologies to deter default. We have also shown that imposing restrictions on the access to future credit may improve liquidity and allocations only when they constitute a substantial punishment on defaulters. This may shed some light on the question of the macroeconomic consequences of financial development and regulations.
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Appendix A.

Deriving the marginal values of holding money and the real asset in the first subperiod

Differentiating (7) with respect to $m$ and $a$, respectively, we obtain

$$V_m(m, a) = (1 - n)[u'(q_b)\frac{\partial q_b}{\partial m} + W_m(1 + \frac{\partial \ell}{\partial m} - p\frac{\partial q_b}{\partial m}) + W_a \frac{\partial a}{\partial m} + W_\ell \frac{\partial \ell}{\partial m}]$$

$$+ n[-c'(q_b)\frac{\partial q_b}{\partial a} + W_m(1 - \frac{\partial d}{\partial m} + p \frac{\partial q_b}{\partial m}) + W_a \frac{\partial a}{\partial m} + W_d \frac{\partial d}{\partial m}],$$

$$V_a(m, a) = (1 - n)[u'(q_b)\frac{\partial q_b}{\partial a} + W_m(\frac{\partial m}{\partial a} + \frac{\partial \ell}{\partial a} - p \frac{\partial q_b}{\partial a}) + W_a + W_\ell \frac{\partial \ell}{\partial a}]$$

$$+ n[-c'(q_b)\frac{\partial q_b}{\partial a} + W_m(\frac{\partial m}{\partial a} + \frac{\partial d}{\partial a} + p \frac{\partial q_b}{\partial a}) + W_a + W_d \frac{\partial d}{\partial a}].$$

Recall from (3) – (6) that $W_m = \frac{1}{n}$, $W_a = 1 + \frac{i}{n}$, $W_\ell = (1 + i_\ell)$, and $W_d = (1 + i_d)$. Moreover, $\frac{\partial d}{\partial a} = 0$, and $\frac{\partial d}{\partial m} = 1$ since a seller deposits all his cash when $i > 0$. Also, $\frac{\partial a}{\partial m} = 0$ and $\frac{\partial m}{\partial a} = 0$ because an agent’s portfolio $(m, a)$ is determined in the previous period. Since $i > 0$ implies $m + \ell - pq_b = 0$, we have $1 + \frac{\partial \ell}{\partial m} - p \frac{\partial q_b}{\partial m} = 0$ and $\frac{\partial m}{\partial a} + \frac{\partial \ell}{\partial a} - p \frac{\partial q_b}{\partial a} = 0$, from which one can derive the conditions $\frac{\partial q_b}{\partial m} = (1 + \frac{\partial \ell}{\partial m})/p$ and $\frac{\partial q_b}{\partial a} = (\frac{\partial \ell}{\partial a})/p$. Because the quantities produced by a seller are independent of his portfolio, $\frac{\partial q_b}{\partial m} = 0$ and $\frac{\partial q_b}{\partial a} = 0$. Hence,

$$V_m(m, a) = (1 - n)[u'(q_b)\frac{\partial q_b}{\partial m} - \phi(1 + i)\frac{\partial \ell}{\partial m}] + n\phi(1 + i_d),$$

$$V_a(m, a) = (\psi + \rho) + (1 - n)[\frac{u'(q_b)}{p} - \phi(1 + i)]\frac{\partial \ell}{\partial a}.$$  

Using $\frac{\partial \ell}{\partial m} = \frac{\partial q_b}{\partial m} - 1$, and after some manipulation we obtain equations (13) and (14).

Proof of Proposition 1  Substitute $\psi = \psi + 1$ and (12) into (18) to get the result.

Proof of Lemma 1  We derive the real borrowing constraint $\phi\ell$ under the extremely limited record-keeping technology. For a buyer entering the second subperiod who repays his loan and holds no money, the expected discounted utility in a stationary equilibrium is

$$W(m, a) = U(x^*) - h_b + \beta V_+(m+1, a+1),$$

where $h_b$ is a buyer’s production in the second subperiod if he repays the loan. Since banks’ punishment is confined to the current period, a defaulter will start a new period as nondefaulters,
and his continuation payoffs is denoted by $V_{m+1}(\hat{m}_{m+1}, \hat{a}_{m+1})$, where the hat indicates a deviator’s optimal choice. A deviating buyer’s expected discounted utility is

$$\hat{W}(m, a) = U(\bar{x}) - \hat{h}_b + \beta V_{m+1}(\hat{m}_{m+1}, \hat{a}_{m+1}).$$

In the second subperiod the deviating buyer’s problem is

$$\hat{W}(m, a) = \max_{\bar{x}, \hat{h}_b, \hat{m}_{m+1}, \hat{a}_{m+1}} U(\bar{x}) - \hat{h}_b + \beta V_{m+1}(\hat{m}_{m+1}, \hat{a}_{m+1})$$

s.t. $x + \phi m_{m+1} + \psi a_{m+1} = \hat{h}_b + \phi (m + T)$.

Note that the deviator’s constraint has taken into account the loss in collateral, and the benefit of not repaying the debt. The first-order conditions are $U'(\bar{x}) = 1, -\phi + \beta \frac{\partial V_{m+1}(\hat{m}_{m+1}, \hat{a}_{m+1})}{\partial m_{m+1}} = 0$ and $-\psi + \beta \frac{\partial V_{m+1}(\hat{m}_{m+1}, \hat{a}_{m+1})}{\partial a_{m+1}} = 0$, which imply $\bar{x} = x^*$ and $(\hat{m}_{m+1}, \hat{a}_{m+1}) = (m_{m+1}, a_{m+1})$. Therefore, a deviator would choose the same portfolio as non-deviators, and he has the expected discounted utility

$$\hat{W}(m, a) = U(\bar{x}) - \hat{h}_b + \beta V_{m+1}(m_{m+1}, a_{m+1}).$$

The real borrowing constraint $\phi \ell$ is the value such that $W(m, a) = \hat{W}(m, a)$, or, $\hat{h}_b = h_b$. For a non-deviator and a deviator, the labor used in production is, respectively,

$$h_b = x^* + \phi m_{m+1} + \psi a_{m+1} - \phi (m + \ell - pq_b) - \phi \tau m - (\psi + \rho) a + \phi (1 + i) \bar{\ell},$$

and

$$\hat{h}_b = x^* + \phi m_{m+1} + \psi a_{m+1} - \phi (m + \ell - pq_b) - \phi \tau m.$$

Hence, the real borrowing constraint $\phi \ell$ satisfies

$$(\psi + \rho) a = \phi (1 + i) \bar{\ell}.$$
by using $p = \frac{c'(q_s)}{\phi}$. In an unconstrained equilibrium, $\phi \ell < \phi \ell$. Recall that \( \phi \ell = \frac{(\psi + \rho)\alpha}{1+i} \) and in equilibrium $a = A$. Now define $\Delta = \frac{n\phi pq_b(1+i)}{\psi+\rho} - A$. Notice that $\Delta < 0$ in an unconstrained equilibrium. Substituting $\psi = \frac{\beta \rho}{1 - \beta}$ and $i = \frac{\gamma - \beta}{\beta}$ into $\Delta$, and using $(1-n)pq_b = M_{-1}$ and $M = \gamma M_{-1}$, we have

\[
\Delta = \frac{n\phi pq_b \gamma (1 - \beta)}{\rho \beta} - A
\]

\[
= \frac{n\phi M_{-1} \gamma (1 - \beta)}{\rho (1-n)\beta} - A
\]

\[
= \frac{n(1 - \beta)\phi M}{\rho (1-n)\beta} - A < 0.
\]

In an equilibrium with unconstrained credit,

$$ A > \frac{n(1 - \beta)\phi M}{\rho (1-n)\beta} = \bar{A}. $$

**Proof of Proposition 2** Substituting (23) into (18) and rearranging, one obtains the asset price $\psi = \psi_1$ where $\psi_1$ is defined in (24). The loan-to-value ratio is equal to the ratio of the real loan amount to the real value of collateral. From (22),

$$ \theta_1 = \frac{\phi \ell}{\psi \alpha} = \frac{1 + \rho/\psi}{1 + i}. $$

**Proof of proposition 3** In an unconstrained equilibrium, $(q_b, i, \psi)$ satisfy (19), (20) and (21). Define

$$ f^u(q_b, i, \psi; z) = \frac{u'(q_b)}{c'(q_s)} - 1 - \frac{\gamma - \beta}{\beta}, $$

$$ g^u(q_b, i, \psi; z) = i - \frac{\gamma - \beta}{\beta}, $$

$$ h^u(q_b, i, \psi; z) = \psi - \frac{\beta \rho}{1 - \beta}, $$

where $q_s = \frac{1-n}{n} q_b$. Let $k^u_x$ denote $\frac{\partial k^u}{\partial x}$, where $k = f^u, g^u, h^u$ and $x = q_b, i, \psi, z$. Then, $f^u_{q_b} = \frac{u''(q_b)}{c'(q_s)} - \frac{1-n}{n} \frac{u'(q_b)c''(q_s)}{[c'(q_s)]^2} < 0$, $f^u_i = f^u_{\psi} = g^u_{q_b} = g^u_{\psi} = h^u_{q_b} = h^u_{\psi} = h^u_i = h^u_{\psi} = h^u_{q_b} = h^u_{\psi} = 1$. Note that

$$ \begin{bmatrix} f^u_{q_b} & f^u_i & f^u_{\psi} \\ g^u_{q_b} & g^u_i & g^u_{\psi} \\ h^u_{q_b} & h^u_i & h^u_{\psi} \end{bmatrix} \begin{bmatrix} dq_b \\ di \\ d\psi \end{bmatrix} = - \begin{bmatrix} f^u_{q_b} dz \\ g^u_{q_b} dz \\ h^u_{q_b} dz \end{bmatrix}. $$

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Let $\Lambda^u$, $\Lambda^u_1$, $\Lambda^u_2$, $\Lambda^u_3$ denote the determinants of the following matrices, respectively:

\[
\Lambda^u = \begin{bmatrix}
f_{q_0} & f_i & f_{\psi} \\
g_{q_0} & g_i & g_{\psi} \\
h_{q_0} & h_i & h_{\psi}
\end{bmatrix}, \quad \Lambda^u_1 = \begin{bmatrix}
-f_{q_0} & f_i & f_{\psi} \\
-g_{q_0} & g_i & g_{\psi} \\
-h_{q_0} & h_i & h_{\psi}
\end{bmatrix},
\]
\[
\Lambda^u_2 = \begin{bmatrix}
f_{q_0} - f_{\psi} & f_i & f_{\psi} \\
g_{q_0} - g_{\psi} & g_i & g_{\psi} \\
h_{q_0} - h_{\psi} & h_i & h_{\psi}
\end{bmatrix}, \quad \Lambda^u_3 = \begin{bmatrix}
f_{q_0} - f_{\psi} & f_i & f_{\psi} \\
g_{q_0} - g_{\psi} & g_i & g_{\psi} \\
h_{q_0} - h_{\psi} & h_i & h_{\psi}
\end{bmatrix},
\]

and $\Lambda^u < 0$, $\Lambda^u_1 > 0$, $\Lambda^u_2 < 0$, $\Lambda^u_3 = 0$. Thus, $\frac{\partial q_0}{\partial \gamma} = \frac{\Lambda^u_1}{\Lambda^u} < 0$, $\frac{\partial i}{\partial \gamma} = \frac{\Lambda^u_2}{\Lambda^u} > 0$, $\frac{\partial \psi}{\partial \gamma} = \frac{\Lambda^u_3}{\Lambda^u} = 0$. Given $p = \frac{c'(q_0)}{\phi}$ and $\phi = \frac{(1-n)c'(q_0)q_0}{\Lambda^u}$, we have

\[
\frac{\partial \phi}{\partial \gamma} = 1 - n\frac{\left[1 - n \frac{c''(q_0)q_0 + c'(q_0)}{\phi} \right]}{\Lambda^u} < 0,
\]
\[
\frac{\partial p}{\partial \gamma} = -\frac{p^2(1 - n)}{\Lambda^u} \frac{\partial q_0}{\partial \gamma} > 0,
\]

because $\frac{\partial q_0}{\partial \gamma} < 0$.

In a constrained equilibrium, $(q_0, i', \psi)$ satisfy (17), (22), and (24). Define

\[
f(q_0, i', \psi; z) = (1 - n)[\frac{u'(q_0)}{c'(q_0)} - 1] + ni - \frac{\gamma - \beta}{\beta},
\]
\[
g(q_0, i', \psi; z) = (1 + i)nc'(q_0)q_0 - (\psi + \rho)A,
\]
\[
h(q_0, i', \psi; z) = (1 - \beta B)\psi - \beta B\rho,
\]

where we have substituted $\phi \ell = nc'(q_0)q_0$. Then, $f_{q_0} = (1 - n)[\frac{u''(q_0)}{c'(q_0)} - \frac{1 - n}{\left(\frac{c'(q_0)}{\phi}\right)^2}] < 0$, $g_{q_0} = (1 + i)[(1 - n)c'(q_0)q_0 + nc'(q_0)] > 0$, $f_i = n$, $g_i = nc'(q_0)q_0$, $g_{\psi} = -A$, $h_{q_0} = -\frac{\beta(\psi + \rho)(1 - n)u'(q_0)}{(1 + i)^2} < 0$, $h_i = \frac{\beta(\psi + \rho)(1 - n)u'(q_0)}{(1 + i)^2} > 0$, $f_{\psi} = g_{\psi} = h_{\psi} = 0$, $h_{\psi} = 1 - \beta B$, $f_{\psi} = -\frac{1}{\beta}$. Let $\Lambda$, $\Lambda_1$, $\Lambda_2$, $\Lambda_3$ denote the determinants of the following matrices, respectively:

\[
\Lambda = \begin{bmatrix}
f_{q_0} & f_i & f_{\psi} \\
g_{q_0} & g_i & g_{\psi} \\
h_{q_0} & h_i & h_{\psi}
\end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix}
-f_{q_0} & f_i & f_{\psi} \\
-g_{q_0} & g_i & g_{\psi} \\
-h_{q_0} & h_i & h_{\psi}
\end{bmatrix},
\]
\[
\Lambda_2 = \begin{bmatrix}
f_{q_0} - f_{\psi} & f_i & f_{\psi} \\
g_{q_0} - g_{\psi} & g_i & g_{\psi} \\
h_{q_0} - h_{\psi} & h_i & h_{\psi}
\end{bmatrix}, \quad \Lambda_3 = \begin{bmatrix}
f_{q_0} - f_{\psi} & f_i & f_{\psi} \\
g_{q_0} - g_{\psi} & g_i & g_{\psi} \\
h_{q_0} - h_{\psi} & h_i & h_{\psi}
\end{bmatrix}.
\]

We find that $\Lambda < 0$, $\Lambda_1 > 0$, $\Lambda_2 < 0$ and $\Lambda_3 > 0$. Thus, $\frac{\partial q_0}{\partial \gamma} = \frac{\Lambda_1}{\Lambda} < 0$ and $\frac{\partial i}{\partial \gamma} = \frac{\Lambda_2}{\Lambda} > 0$. Then, $\frac{\partial \phi}{\partial \gamma} < 0$ and $\frac{\partial p}{\partial \gamma} > 0$. Also, $\frac{\partial \psi}{\partial \gamma} = \frac{\Lambda_3}{\Lambda} = \frac{n(1 - n)\psi u'(q_0) + u''(q_0)q_0}{\Lambda} < 0$ if $u'(q_0) + u''(q_0)q_0 > 0$. Given

\[
\theta_1 = \frac{1}{\Lambda_1} \frac{\partial \phi}{\partial \gamma}, \quad \frac{\partial \theta_1}{\partial \gamma} = \frac{1}{\Lambda_1} \frac{1}{(1 + i)^2} \left[\frac{(1 - n)\psi u'(q_0) + u''(q_0)q_0}{\Lambda} \right] < 0 \text{ if } u'(q_0) + u''(q_0)q_0 > 0.
\]

Given $\theta_1 = \frac{1}{\Lambda_1} \frac{\partial \phi}{\partial \gamma}$, $\frac{\partial \theta_1}{\partial \gamma} = \frac{1}{\Lambda_1} \frac{1}{(1 + i)^2} \left[\frac{(1 - n)\psi u'(q_0) + u''(q_0)q_0}{\Lambda} \right] < 0 \text{ if } u'(q_0) + u''(q_0)q_0 > 0$. Given

\[
\theta_1 = \frac{1}{\Lambda_1} \frac{\partial \phi}{\partial \gamma}, \quad \frac{\partial \theta_1}{\partial \gamma} = \frac{1}{\Lambda_1} \frac{1}{(1 + i)^2} \left[\frac{(1 - n)\psi u'(q_0) + u''(q_0)q_0}{\Lambda} \right] < 0 \text{ if } u'(q_0) + u''(q_0)q_0 > 0.
\]
Proof of proposition 4  Let $\Lambda^u_4$, $\Lambda^u_5$, $\Lambda^u_6$, $\Lambda_4$, $\Lambda_5$, $\Lambda_6$ denote the determinants of the following matrices, respectively:

$$
\begin{align*}
\Lambda^u_4 &= \begin{bmatrix} -f^u_A & f^u_i & f^u_\psi \\ -g^u_A & g^u_i & g^u_\psi \\ -h^u_A & h^u_i & h^u_\psi \end{bmatrix},
\Lambda^u_5 &= \begin{bmatrix} f^u_{q_b} & -f^u_A & f^u_\psi \\ g^u_{q_b} & -g^u_A & g^u_\psi \\ h^u_{q_b} & -h^u_A & h^u_\psi \end{bmatrix},
\Lambda^u_6 &= \begin{bmatrix} f^u_{q_b} & f^u_i & -f^u_\psi \\ g^u_{q_b} & g^u_i & -g^u_\psi \\ h^u_{q_b} & h^u_i & -h^u_\psi \end{bmatrix},
\Lambda_4 &= \begin{bmatrix} -f_A & f_i & f_\psi \\ -g_A & g_i & g_\psi \\ -h_A & h_i & h_\psi \end{bmatrix},
\Lambda_5 &= \begin{bmatrix} f_{q_b} & -f_A & f_\psi \\ g_{q_b} & -g_A & g_\psi \\ h_{q_b} & -h_A & h_\psi \end{bmatrix},
\Lambda_6 &= \begin{bmatrix} f_{q_b} & f_i & -f_\psi \\ g_{q_b} & g_i & -g_\psi \\ h_{q_b} & h_i & -h_\psi \end{bmatrix}.
\end{align*}
$$

Because $g_A = -\psi$, $f^u_A = g^u_A = h^u_A = f_A = h_A = 0$, we have $\Lambda_4 < 0$, $\Lambda_5 < 0$, $\Lambda_6 > 0$, $\Lambda^u_4 = \Lambda^u_5 = \Lambda^u_6 = 0$. Thus, in an unconstrained equilibrium, $\frac{\partial q_{b}}{\partial A} = \frac{\Lambda^u_4}{\Lambda^u_6} = 0$, $\frac{\partial i}{\partial A} = \frac{\Lambda^u_5}{\Lambda^u_6} = 0$, and $\frac{\partial \psi}{\partial A} = \frac{\Lambda^u_6}{\Lambda^u_6} = 0$. With the same argument in the proof of proposition 3, $\frac{\partial q_{b}}{\partial A} = 0$ and $\frac{\partial p}{\partial A} = 0$, because $\frac{\partial q_{b}}{\partial A} = 0$. In a constrained equilibrium, $\frac{\partial q_{b}}{\partial A} = \frac{\Lambda_4}{\Lambda_6} > 0$, $\frac{\partial i}{\partial A} = \frac{\Lambda_5}{\Lambda_6} > 0$, and $\frac{\partial \psi}{\partial A} = \frac{\Lambda_6}{\Lambda_6} < 0$. Also, $\frac{\partial \phi}{\partial A} > 0$ and $\frac{\partial p}{\partial A} < 0$, because $\frac{\partial q_{b}}{\partial A} < 0$. Finally, $\frac{\partial q_{b}}{\partial A} = \frac{\psi \partial^2 f_{q_{b}}}{\partial A} \left[ -\beta B + \frac{\psi}{\beta} (1 - \beta B) \right] = 0$ by substituting $\psi = \frac{\beta B \rho}{1 - \beta B}$.

Let $\Lambda^s_7$, $\Lambda^s_8$, $\Lambda^s_9$, $\Lambda_7$, $\Lambda_8$, $\Lambda_9$ denote the determinants of the following matrices:

$$
\begin{align*}
\Lambda^s_7 &= \begin{bmatrix} -f^u_\rho & f^u_i & f^u_\psi \\ -g^u_\rho & g^u_i & g^u_\psi \\ -h^u_\rho & h^u_i & h^u_\psi \end{bmatrix},
\Lambda^s_8 &= \begin{bmatrix} f^u_{q_b} & -f^u_\rho & f^u_\psi \\ g^u_{q_b} & -g^u_\rho & g^u_\psi \\ h^u_{q_b} & -h^u_\rho & h^u_\psi \end{bmatrix},
\Lambda^s_9 &= \begin{bmatrix} f^u_{q_b} & f^u_i & -f^u_\psi \\ g^u_{q_b} & g^u_i & -g^u_\psi \\ h^u_{q_b} & h^u_i & -h^u_\psi \end{bmatrix},
\Lambda_7 &= \begin{bmatrix} -f_\rho & f_i & f_\psi \\ -g_\rho & g_i & g_\psi \\ -h_\rho & h_i & h_\psi \end{bmatrix},
\Lambda_8 &= \begin{bmatrix} f_{q_b} & -f_\rho & f_\psi \\ g_{q_b} & -g_\rho & g_\psi \\ h_{q_b} & -h_\rho & h_\psi \end{bmatrix},
\Lambda_9 &= \begin{bmatrix} f_{q_b} & f_i & -f_\rho \\ g_{q_b} & g_i & -g_\rho \\ h_{q_b} & h_i & -h_\rho \end{bmatrix}.
\end{align*}
$$

One can show that $h^u_\rho = -\frac{\beta}{1 - \beta}, g_\rho = -\rho, h_\rho = -\beta B, f^u_\rho = g^u_\rho = f_\rho = 0$. Hence, $\Lambda^s_9 < 0$, $\Lambda_7 < 0$, $\Lambda_8 < 0$, $\Lambda^s_8 = \Lambda^s_9 = 0$. In an unconstrained equilibrium, $\frac{\partial q_{b}}{\partial \rho} = \frac{\Lambda^s_7}{\Lambda^s_9} = 0$, $\frac{\partial i}{\partial \rho} = \frac{\Lambda^s_8}{\Lambda^s_9} = 0$, and $\frac{\partial \psi}{\partial \rho} = \frac{\Lambda^s_9}{\Lambda^s_9} > 0$. Therefore, $\frac{\partial \phi}{\partial \rho} = 0$ and $\frac{\partial p}{\partial \rho} = 0$. In a constrained equilibrium, $\frac{\partial q_{b}}{\partial \rho} = \frac{\Lambda_7}{\Lambda_9} > 0$ and $\frac{\partial i}{\partial \rho} = \frac{\Lambda_8}{\Lambda_9} > 0$, and hence $\frac{\partial \phi}{\partial \rho} > 0$, $\frac{\partial p}{\partial \rho} < 0$, and $\frac{\partial B}{\partial \rho} = \frac{1}{1 + \beta} \left( \frac{\partial B}{\partial \rho} \frac{1}{1 + \beta} \frac{\partial \psi}{\partial \rho} \right) < 0$. Moreover, $\frac{\partial \psi}{\partial \rho} = \frac{\beta B}{1 - \beta B} \left[ 1 + \frac{\rho}{B(1 - \beta B)} \frac{\partial B}{\partial \rho} \right] > 0$ if $\frac{\partial B}{\partial \rho} / B(1 - \beta B) > 0$ because $f_{q_b} < 0$ and $A < 0$.

Proof of Lemma 2  We derive the real borrowing constraint $\phi \ell$ in the economy with superior record-keeping technology. For a buyer entering the second subperiod with no money, who repays his loan, the expected discounted utility in a stationary equilibrium is

$$W(m, a) = U(x^*) - h_b + \beta V_{m+1}(m+1, a+1),$$
where \( h_b \) is the production of a buyer who repays his loan. The punishment of exclusion adds a long-run cost to defaulters so they need to bring enough money to execute trades. This implies that a defaulter potentially will choose a different portfolio and have different continuation value from nondefaulters, \( \tilde{V}_{t+1}(\tilde{m}_{t+1},\tilde{a}_{t+1}) \), where the tilde indicates the optimal choice of a defiator. A deviating buyer’s expected discounted utility is

\[
\tilde{W}(m,a) = U(\tilde{x}) - \tilde{h}_b + \beta \tilde{V}_{t+1}(\tilde{m}_{t+1},\tilde{a}_{t+1}).
\]

Thus, \( \phi \tilde{z} \) is the value such that \( W(m,a) = \tilde{W}(m,a) \), which leads to

\[
U(x^*) - U(\tilde{x}) + \tilde{h}_b - h_b + \beta[V_{t+1}(m_{t+1},a_{t+1}) - \tilde{V}_{t+1}(\tilde{m}_{t+1},\tilde{a}_{t+1})] = 0. \tag{42}
\]

The continuation payoffs are

\[
V_{t+1}(m_{t+1},a_{t+1}) = (1 - \beta)^{-1}[(1 - n)u(q_b) - nc(q_s) + U(x^*) - h],
\]

\[
\tilde{V}_{t+1}(\tilde{m}_{t+1},\tilde{a}_{t+1}) = (1 - \beta)^{-1}[(1 - n)u(\tilde{q}_b) - nc(\tilde{q}_s) + U(\tilde{x}) - \tilde{h}]. \tag{43}
\]

Now we derive \( \tilde{x}, \tilde{q}_s, \tilde{q}_b, \tilde{h}_b, \tilde{m}, \) and \( \tilde{a} \). In the subperiod when the buyer defaults, his problem is

\[
\tilde{W}(m,a) = \max_{\tilde{x},\tilde{h}_b,\tilde{m}_{t+1},\tilde{a}_{t+1}} U(\tilde{x}) - \tilde{h}_b + \beta \tilde{V}_{t+1}(\tilde{m}_{t+1},\tilde{a}_{t+1})
\]

s.t. \( x + \phi \tilde{m}_{t+1} + \psi \tilde{a}_{t+1} = \tilde{h}_b + \phi (m + T) \).

The first-order condition are \( U'(\tilde{x}) = 1 \), which implies \( \tilde{x} = x^* \), \( -\phi + \beta \frac{\partial \tilde{V}_{t+1}(\tilde{m}_{t+1},\tilde{a}_{t+1})}{\partial \tilde{m}_{t+1}} = 0 \) and \( -\psi + \beta \frac{\partial \tilde{V}_{t+1}(\tilde{m}_{t+1},\tilde{a}_{t+1})}{\partial \tilde{a}_{t+1}} = 0 \). Note that in the next period, if the deviator becomes a seller, the quantity that he sells is independent of his portfolio; i.e., \( q_s \) satisfies \( -c'(q_s) + p\phi = 0 \). So, \( \tilde{q}_s = q_s = \frac{1 - n}{n} q_b \); the deviator produces the same amount as non-deviating sellers. Because a defiator cannot borrow or make deposits, his expected utility in the future first subperiod is

\[
\tilde{V}(\tilde{m},\tilde{a}) = (1 - n)[u(\tilde{q}_b) + \tilde{W}(\tilde{m} - p\tilde{q}_b,\tilde{a})] + n[-c(q_s) + \tilde{W}(\tilde{m} + pq_s,\tilde{a})].
\]

The marginal value of holding money for a defiator is

\[
\tilde{V}_m(\tilde{m},\tilde{a}) = (1 - n)[u'(\tilde{q}_b) \frac{\partial \tilde{q}_b}{\partial \tilde{m}} + \tilde{W}_m(1 - p \frac{\partial \tilde{q}_b}{\partial \tilde{m}}) + \tilde{W}_a \frac{\partial \tilde{a}}{\partial \tilde{m}}] + n[-c'(q_s) \frac{\partial q_s}{\partial \tilde{m}} + \tilde{W}_m(1 + p \frac{\partial q_s}{\partial \tilde{m}}) + \tilde{W}_a \frac{\partial \tilde{a}}{\partial \tilde{m}}] = \phi[(1 - n) u'(\tilde{q}_b) c'(q_s) + n].
\]
A deviator’s choice of money holdings thus satisfies
\[
\frac{\gamma - \beta}{\beta} = (1 - n)[\frac{u'(\bar{q}_b)}{c'(q_b)} - 1],
\]
which is equation (27). Comparing (27) with (17), we find that when \( \gamma > \beta \) (which implies \( i > 0 \)),
\[
\frac{u'(\bar{q}_b)}{c'(q_b)} > \frac{u'(q_b)}{c'(q_b)},
\]
implying \( \bar{q}_b < q_b \). Moreover, because a deviator will be denied credit permanently, his marginal value of holding the real asset is
\[
\bar{V}_a(\tilde{m}, \tilde{a}) = (\psi + \rho).
\]
For a non-deviator the marginal value of holding the real asset is given by (14). Therefore, if non-deviators hold the real asset, the asset price \( \psi \) satisfies
\[
\psi_{-1} = \beta V_a(m, a) = \beta \{(\psi + \rho) + (1 - n)\phi[\frac{u'(q_b)}{c'(q_b)} - (1 + i)]\frac{\partial \ell}{\partial a}\}.
\]
Obviously, \( \beta V_a(m, a) > \beta \bar{V}_a(\tilde{m}, \tilde{a}) \) when \( \frac{u'(q_b)}{c'(q_b)} > (1 + i) \). That is, in a constrained equilibrium, \( \psi > \beta \bar{V}_a(\tilde{m}, \tilde{a}) \), so a deviator choose not to hold the real asset; i.e., \( \tilde{a} = 0 \).

Finally, using (42) and (43), we obtain
\[
h_b - \tilde{h}_b = \frac{\beta}{1 - \beta} \{(1 - n)[u(q_b) - u(\bar{q}_b)] + \tilde{h} - h\}. \tag{44}
\]
We now derive \( h_b - \tilde{h}_b \) and \( \tilde{h} - h \).

(i) Deriving \( h_b - \tilde{h}_b \), the difference in the production between a non-deviator and a deviator in the subperiod when default occurs:

If the buyer repays his loan, the labor used in production is
\[
h_b = x^* + \phi m_{+1} + \psi a_{+1} - \phi(m + \bar{\ell} - pq_b) - \phi \tau m - (\psi + \rho)a + \phi(1 + i)\bar{\ell}
\]
\[
= x^* + \phi \bar{\ell} + \phi pq_b - \rho a, \tag{45}
\]
where we have used \( m_{+1} = m + \tau m \) and \( a_{+1} = a = A \). If the buyer defaults on his loans, he works
\[
\tilde{h}_b = x^* + \phi \tilde{m}_{+1} + \psi \tilde{a}_{+1} - \phi(m + \bar{\ell} - pq_b) - \phi \tau m
\]
\[
= x^* + \phi \bar{\ell} + \phi pq_b, \tag{46}
\]

where we have used $\tilde{a}_{t+1} = 0$ and the equilibrium condition that a defaulter’s money holdings must grow at the rate $\gamma$, $\tilde{m}_{t+1} = (1 + \tau)\tilde{m} = \gamma\tilde{m}$. Thus,

$$h_b - \tilde{h}_b = \phi i\tilde{\ell} - \phi \gamma(\tilde{m} - m) + \phi \tilde{\ell} - \rho a$$

$$= \phi (1 + i)\tilde{\ell} - \phi \gamma(\tilde{m} - m) - \rho a. \quad (47)$$

(ii) Deriving $h - \tilde{h}$, the difference in the production between a non-deviator and a deviator in the next period following default:

If a seller never deviated in the past, in the second subperiod, he works

$$h_s = x^* + \phi m_{t+1} + \psi a_{t+1} - \phi (pq_s + \tau m) - (\psi + \rho)a - \phi (1 + i_d)d$$

$$= x^* + \phi (m_{t+1} - m - \tau m) + \psi (a_{t+1} - a) - \phi pq_s - \rho a - \phi im$$

$$= x^* - \frac{1 - n}{n} \phi i\tilde{\ell} - \frac{1 - n}{n} \phi pq_b - \rho a, \quad (48)$$

where we have used $i_d = i$, $d = m$ (since $i > 0$), $q_s = \frac{1 - n}{n} q_b$, $m = \frac{1 - n}{n} \tilde{\ell}$, and $m_{t+1} = (1 + \tau)m = \gamma m$.

From (45) and (48), a non-deviator’s expected hours worked are

$$h = (1 - n)h_b + nh_s = x^* - \rho a. \quad (49)$$

If an agent has defaulted in the previous period, he does not hold any real assets, and he cannot borrow nor make deposits in this period. If he is a buyer, he uses $\tilde{m}$ money to buy $\tilde{q}_b$ goods in the first subperiod, and in the second subperiod, he chooses money holdings brought to the next period, $\tilde{m}_{t+1}$, receives transfers $\tau \tilde{m}$, and works

$$\tilde{h}_b = x^* + \phi \tilde{m}_{t+1} - \phi (\tilde{m} - p\tilde{q}_b) - \phi \tau \tilde{m}$$

$$= x^* + \phi \tilde{q}_b.$$ 

where we have used $\tilde{m}_{t+1} = (1 + \tau)\tilde{m}$. If he is a seller, the hours worked is

$$\tilde{h}_s = x^* + \phi \tilde{m}_{t+1} - \phi (\tilde{m} + p\tilde{q}_s) - \phi \tau \tilde{m}$$

$$= x^* - \phi pq_s$$

$$= x^* - \frac{1 - n}{n} \phi pq_b.$$
where we have used $\bar{q}_s = q_s = \frac{1-n}{n}q_b$. Thus, a deviator’s expected hours worked are

$$\bar{h} = (1-n)\bar{h}_b + n\bar{h}_s = x^* + (1-n)\phi p(\bar{q}_b - q_b).$$  \hfill (50)

From (49) and (50),

$$\bar{h} - h = (1-n)\phi p(\bar{q}_b - q_b) + \rho a.$$ \hfill (51)

Substitute (47) and (51) into (44) to get

$$\phi \bar{\ell} = \frac{\rho a}{(1+i)(1-\beta)} + \frac{\beta}{(1-\beta)(1+i)} \{(1-n)\Psi(q_b, \bar{q}_b) + \frac{\gamma(1-\beta)}{\beta} c'(q_s)[\bar{q}_b - (1-n)q_b]\},$$

where

$$\Psi(q_b, \bar{q}_b) = u(q_b) - u(\bar{q}_b) - c'(q_s)(q_b - \bar{q}_b) \geq 0.$$

**Proof of Proposition 5** Substitute (28) into (18) to get $\psi = \psi_2$ where $\psi_2$ is defined in (29). Dividing the real loan amount from (26) by the real value of collateral one obtains $\theta_2$.

**Proof of Proposition 6** At the end of each period after banks have seized defaulters’ collateral, an agent’s default record is updated with probability $\zeta$, and the updating does not occur with probability $1-\zeta$. For a deviating buyer, his expected discounted utility in a stationary equilibrium is

$$W(m, a) = (1-\zeta) \left[ U(\bar{x}) - \bar{h}_b + \beta V_{+1}(m_{+1}, a_{+1}) \right]$$

$$+ \zeta \left[ U(\bar{x}) - \bar{h}_b + \beta \tilde{V}_{+1}(\tilde{m}_{+1}, \tilde{a}_{+1}) \right].$$

Thus, $\phi \bar{\ell}$ satisfies $W(m, a) = \bar{W}(m, a)$ or

$$U(x^*) - [(1-\zeta)U(\bar{x}) + \zeta U(\bar{x})] + [(1-\zeta)\bar{h}_b + \zeta \bar{h}_b] - h_b$$

$$+ \zeta \beta [V_{+1}(m_{+1}, a_{+1}) - \tilde{V}_{+1}(\tilde{m}_{+1}, \tilde{a}_{+1})] = 0.$$ \hfill (52)

From $\bar{x} = x^* = \bar{x}$, (43), and (52),

$$h_b - [(1-\zeta)\bar{h}_b + \zeta \bar{h}_b] = \frac{\beta \zeta}{1-\beta} [(1-n)[u(q_b) - u(\bar{q}_b)] + \bar{h} - h].$$ \hfill (53)
Deriving $h_b - [(1 - \zeta)\tilde{h}_b + \zeta\tilde{h}_b]$:

$$
\tilde{h}_b = x^* + \phi m_{+1} + \psi a_{+1} - \phi (m + \bar{\ell} - pq_b) - \phi \tau m \\
= x^* - \phi \bar{\ell} + \phi pq_b + \psi a,
$$

(54)

where we have used $m_{+1} = m + \tau m$. From (45), (46), and (54),

$$
h_b - [(1 - \zeta)\tilde{h}_b + \zeta\tilde{h}_b] = \phi(1 + i)\bar{\ell} - \rho a - (1 - \zeta)\psi a - \zeta \phi \gamma (\bar{m} - m).
$$

(55)

Substituting (51) and (55) into (53) and rearranging yields

$$
(1 + i)\phi \bar{\ell} = \frac{(1 - \beta + \zeta \beta)\rho a + (1 - \beta)(1 - \zeta)\psi a}{1 - \beta} \\
+ \frac{\zeta \beta}{1 - \beta} \{(1 - n)\Psi(q_b, \bar{q}_b) + \frac{\gamma (1 - \beta)}{\beta} c'(q_s)[\bar{q}_b - (1 - n)q_b]\},
$$

where

$$
\Psi(q_b, \bar{q}_b) = u(q_b) - u(\bar{q}_b) - c'(q_s)(q_b - \bar{q}_b) \geq 0.
$$

In a constrained equilibrium, $\phi \bar{\ell} = \phi \bar{\ell}$. The loan-to-value ratio $\theta_3$ is obtained by dividing $\phi \bar{\ell}$ of the above equation by $\psi a$. Substitute (32) into (18) to get the asset price $\psi_3$. 

Appendix B. Solving the bank’s problem

Since banks are perfectly competitive with free entry, they take as given the loan rate and the deposit rate. There is no strategic interaction among banks or between banks and agents, and no bargaining over the terms of the loan contract. The representative bank solves the following problem per borrower:

$$\max_{\ell} (i - i_d)\ell$$

s.t. $$\ell \leq \bar{\ell}, \quad u(q_b) + W(m, a, \ell, d) \geq \Gamma,$$

where $$\Gamma$$ is the reservation value of the borrower, which is the surplus from obtaining loans at another bank. If banks have full enforcement on repayment, the borrowing constraint is $$\bar{\ell} = \infty$$. When enforcement is limited, banks choose the credit limit $$\bar{\ell}$$ to ensure voluntary repayment. The first order condition to the bank’s problem is

$$i - i_d - \lambda_L + \lambda_{\Gamma}[u'(q_b)\frac{dq_b}{d\ell} + W] = 0,$$

where $$\lambda_L$$ and $$\lambda_{\Gamma}$$ are the Lagrangian multipliers on the lending constraint and borrower’s participation constraint, respectively. For $$i - i_d > 0$$, banks would like to make the largest loan possible to borrowers and, therefore, would choose a loan amount such that $$\lambda_{\Gamma} > 0$$.

From (8) and the buyer’s budget constraint, $$\frac{dq_b}{d\ell} = \frac{\phi}{c'(q_s)}$$. We rewrite the first order condition of bank’s maximization problem as

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i + \frac{\lambda_L}{\phi\lambda_{\Gamma}}.$$

If banks can force repayment without any cost, the lending constraint does not bind, and $$\lambda_L = 0$$. The loan supplied by banks satisfies $$\frac{u'(q_b)}{c'(q_s)} = 1 + i$$. If $$\lambda_L > 0$$, the lending constraint binds and $$\frac{u'(q_b)}{c'(q_s)} > 1 + i$$. With limited enforcement, banks may have to conduct credit rationing.
Appendix C.

In this Appendix we describe a model in which banks buy assets from, instead of lending to, people who need liquidity to satisfy their consumption needs. We will show that it performs identically to the economy where seizing collateral is the only penalty for default. Suppose that in the first subperiod a competitive asset market opens after the consumption shocks are realized but before the trade of goods. We assume, as in the basic model, that sellers do not have the technology to verify the real asset so they do not participate in the asset market, whereas banks have the verification technology. Banks take deposits from the sellers and buy the real asset from the buyers.

We first look at the first-subperiod asset market. Let $p_A$ denote the nominal price (in monetary units) of the real asset in the first subperiod. We will show below that the zero-profit condition for banks implies that $p_A < \frac{\psi + \rho}{\delta}$. Hence, agents who do not need liquidity do not sell the real asset, because they can receive the dividends $\rho$ and the resale price $\psi$ in a frictionless market in the second subperiod. Let $a^s$ denote the amount of the real asset that a buyer wishes to sell in the first-subperiod asset market.

The expected lifetime utility of an agent with portfolio $(m,a)$ entering the first subperiod is

$$V(m,a) = (1-n)[u(q_b) + W(m + p_Aa^s - pq_b, a - a^s)] + n[-c(q_s) + W(m - d + pq_s, a, d)],$$

which is similar to (7), except that buyers sell the real asset at the price $p_A$ to acquire funds. A seller’s maximization problem is identical to that in the basic model, whereas a buyer’s problem becomes

$$\max_{q_b,a^s} u(q_b) + W(m + p_Aa^s - pq_b, a - a^s)$$

s.t. $pq_b \leq m + p_Aa^s$

$a^s \leq a$.

Let $\lambda_m$ and $\lambda_a$ be the multipliers on the buyer’s budget constraint and asset constraint that he
cannot sell more assets than what he holds, respectively. The first order conditions are

\[ u'(q_b) = c'(q_s)(1 + \frac{\lambda_m}{\phi}), \]
\[ \psi + \rho - p_A \phi = \lambda_m p_A - \lambda_a. \]

We have the following cases. Case (i): \( \lambda_m = 0 \) and \( \lambda_a = 0 \). In this case, \( u'(q_b) = c'(q_s) \); trade is efficient, and \( p_A = \frac{\psi + \rho}{\phi} \). Case (ii) \( \lambda_m > 0 \) and \( \lambda_a = 0 \). The buyer spends all funds available, \( q_b = \frac{m + p_A a_s}{p} \), but the asset constraint does not bind, \( a_s < a \). Combining the two first order conditions, we obtain

\[ \frac{p_A \phi u'(q_b)}{c'(q_s)} = \psi + \rho. \]

Equation (58) implies that the buyer sells the real asset up to the point at which the marginal benefit of selling an additional unit of assets, \( \frac{p_A \phi u'(q_b)}{c'(q_s)} \), equals the marginal cost (the asset’s value in the second subperiod), \( \psi + \rho \). Case (iii) \( \lambda_m > 0 \) and \( \lambda_a > 0 \). Both constraints bind, \( q_b = \frac{m + p_A a_s}{p} \) and \( a_s = a \), so we have

\[ \frac{p_A \phi u'(q_b)}{c'(q_s)} > \psi + \rho. \]

Buyers wish to acquire more funds to finance consumption, but they are constrained by their asset holdings. Finally, the case with \( \lambda_m = 0 \) and \( \lambda_a > 0 \) is not an equilibrium. If the budget constraint does not bind, buyers need not sell all assets, unless the sale price is higher than the asset’s value in the second subperiod; i.e., \( p_A > \frac{\psi + \rho}{\phi} \). Banks, however, will not buy any assets if \( p_A > \frac{\psi + \rho}{\phi} \) (see below).

Banks take deposits \( d \) per seller, and buy \( a^d \) units of the real asset per buyer, so they face the following resource constraint:

\[ (1 - n)p_A a^d \leq nd. \]

Given that \( i_d > 0 \), banks will use all deposits to buy assets, and so the equality holds. Notice that banks’ cost of funds is \( \phi(1 + i_d)nd \), and the revenue from selling assets in the second subperiod is \( (\psi + \rho)(1 - n)a^d \). The zero-profit condition for competitive banks thus implies \( (\psi + \rho)(1 - n)a^d - \phi(1 + i_d)nd = 0 \), from which and the resource constraint we derive

\[ p_A = \frac{\psi + \rho}{\phi(1 + i_d)}. \]
Note that $p_A < \frac{\psi + \rho}{\phi}$ if $i_d > 0$. The asset market clearing condition is $a^s = a^d = \frac{nd}{(1-n)p_A}$.

An agent’s optimal portfolio satisfies

$$\phi_1 \geq \beta \phi[(1 - n)\frac{u'(q_b)}{c'(q_s)} + n(1 + i_d)],$$

$$\psi_1 \geq \beta \{\psi + \rho + (1 - n)[\frac{p_A \phi u'(q_b)}{c'(q_s)} - \psi + \rho] \frac{\partial a^s}{\partial a}\}.$$  

In a stationary equilibrium, the following two conditions must be satisfied:

$$\gamma - \beta = (1 - n)[\frac{u'(q_b)}{c'(q_s)} - 1] + ni_d, \quad (61)$$

$$1 - \frac{\beta}{\psi} = \rho + (1 - n)[\frac{p_A \phi u'(q_b)}{c'(q_s)} - \psi + \rho] \frac{\partial a^s}{\partial a}. \quad (62)$$

If the asset constraint does not bind, the marginal benefit of selling an additional unit of asset equals the cost, $\frac{p_A \phi u'(q_b)}{c'(q_s)} = \psi + \rho$. The second term in (62) vanishes, and the asset price $\psi$ is determined by the dividend flows. If the asset constraint binds, $\frac{p_A \phi u'(q_b)}{c'(q_s)} > \psi + \rho$, then the asset price is determined not only by the fundamentals but also by the importance of the asset in financing people’s consumption needs.

We now show that the prices and allocations are identical to those in the economy where seizing collateral is the only punishment on defaulters. If the asset constraint does not bind, $a^s < a = A$. From (58), (60) and (61), $i_d = \frac{\gamma - \beta}{\beta}$, and $q_b$ satisfies (21). The asset price is the discounted sum of dividends, $\psi = \frac{\beta \psi}{1 - \beta}$. Thus, the asset price, deposit rate, and allocations are identical to those in the equilibrium with unconstrained credit.

When the asset constraint binds, $a^s = a$. In equilibrium, $\psi, p_A, i_d$ and $q_b$ satisfy (60), (61), (62), and the asset market clearing condition $a^s = \frac{nd}{(1-n)p_A} = A$. Substituting $d = m = M^{-1}$ into the asset market clearing condition, we get

$$p_A = \frac{nM^{-1}}{(1-n)A}. \quad (63)$$

From (60) and (63),

$$1 + i_d = \frac{\frac{(\psi + \rho)(1 - n)A}{\phi n M^{-1}}}{\phi n M^{-1}}, \quad (64)$$

which is identical to (22) by substituting $\phi \ell = \frac{n}{1-n} \phi M^{-1}$ and $a = A$. Moreover, comparing (61) and (17), we find that, because the interest rates in both economies are identical, so is $q_b$. Substituting
\( a^s = A, \frac{\partial a^s}{\partial \alpha} = 1 \) and (60) into (62), we obtain the asset pricing equation as described in (24). The asset price, deposit rate, and allocations are identical to those in the equilibrium with constrained credit.
Appendix D.

Here we consider an economy without banks, where buyers and sellers trade the real asset in a competitive asset market in the first subperiod. We will show that the asset prices and allocations are identical to the economy where seizing collateral is the only punishment. Assume that in the first subperiod a competitive asset market opens after the consumption shocks are realized but before the trade of goods. People who have consumption needs can liquidate the real asset, and those with idle cash may purchase the asset. We assume that there is no recognizability problem regarding the real asset.

Let $a^s$ and $a^d$ denote the amount of the real asset that a buyer wishes to sell and a seller wishes to buy, respectively. An agent’s expected lifetime utility in the first subperiod is

$$V(m,a) = (1 - n)[u(q_b) + W(m + p_Aa^s - pq_b, a - a^s)]$$

which is similar to (56), except that sellers may purchase the real asset instead of depositing their cash. A buyer’s maximization problem is as described in (57), whereas a seller’s problem is

$$\max_{q_s, a^d} -c(q_s) + W(m - p_Aa^d + pq_s, a + a^d)$$

s.t. $p_Aa^d \leq m$.

Let $\lambda_v$ denote the multiplier on the investment constraint. The first order conditions are

$$-c'(q_s) + pW_m = 0,$$

$$-p_AW_m + W_a - p_A\lambda_v = 0.$$ 

If the investment constraint does not bind, $a^d < \frac{m}{p_A}$, then $\lambda_v = 0$ and $p_A = \frac{\psi + \rho}{\phi}$. If $\lambda_v > 0$, then $a^d = \frac{m}{p_A}$ and $p_A < \frac{\psi + \rho}{\phi}$.

In equilibrium, $\psi$, $p_A$ and $q_b$ satisfy (62),

$$\frac{\gamma - \beta}{\beta} = (1 - n)[\frac{u'(q_b)}{c'(q_s)} - 1] + n(\frac{\psi + \rho}{\phi p_A} - 1),$$

and the asset market clearing condition $(1 - n)a^s = na^d$. 

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We have the following cases. Case (i) \( \lambda_v = 0, \lambda_m = 0 \) and \( \lambda_a = 0 \) (where \( \lambda_m \) and \( \lambda_a \) are defined in Appendix C). All of sellers’ investment constraint and buyers’ budget constraint and asset constraint are not binding. Then, \( p_A = \frac{\psi + \rho}{\phi} \) and \( u'(q_b) = c'(q_s) \). Case (ii) \( \lambda_v > 0, \lambda_m > 0 \) and \( \lambda_a = 0 \). From (58) we know that \( p_A = (\frac{\psi + \rho}{\phi} c'(q_s)) \). Substituting \( p_A = (\frac{\psi + \rho}{\phi} c'(q_s)) \) into (62) and (66), we find that the asset price and \( q_b \) are identical to those in the equilibrium with unconstrained credit.

Case (iii) \( \lambda_v > 0, \lambda_m > 0 \) and \( \lambda_a > 0 \). If buyers’ asset constraint binds, they liquidate all assets, \( a^s = a \). The asset market clearing condition is \( a^s = \frac{nM-1}{(1-n)\rho} = A \), from which we have \( p_A = \frac{nM-1}{(1-n)A} \). Substitute \( \frac{\partial a^s}{\partial a} = 1 \) and \( p_A = \frac{nM-1}{(1-n)A} \) into (62) to get the asset price \( \psi = \psi_A \), where

\[
\psi_A = \frac{\beta n[\rho + \frac{\phi M-1 u'(q_b)}{Ac'(q_s)}]}{1 - \beta n}.
\]

(67)

In this economy, there is no deposit interest rate. To make the comparison with the basic model, we substitute \( i_d = \frac{(\psi + \rho)nM-1}{\phi(1-n)A} - 1 \) from (64) into equations (17) and (24), which determine \( q_b \) and the asset price in the basic model. After rearranging, we obtain (66) and (67). Thus, the asset price and allocations are identical to those in the equilibrium with constrained credit.
Figure 1: The effects of advances in the record-keeping technology