

Private Money, Bank Operations, and Government Regulations

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Abstract

We study competition between inside and outside money in economies with trading frictions and financial intermediation. We show that, claims on banks circulate if the redemption rate is low. When the quantity of fiat money is scarce, coexistence of inside and outside money dominates equilibria with a unique medium of exchange. If outside money is ample, banks choose to redeem claims in outside money, which increases welfare. Under binding reserve requirements, tightening monetary policy leads to credit rationing. Our results support recent trends toward lower reserve requirements. However, we also identify situations where restrictions on note issue are beneficial.

Key words: Private money; Reserve requirement policy; Random matching models

JEL classification: E40; E43; E50; E52

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1 Introduction

An important issue in monetary economics is how private liabilities become a generally accepted medium of exchange, or inside money. This is of considerable historical interest, because inside money, and in particular privately-issued bank notes, have been a common medium of exchange throughout history. Moreover, due to recent regulatory changes where many countries have removed legal impediments to the creation of private money, and technological improvements in communication and information, it has become easier to issue various forms of currency substitutes, including electronic money. This leads to potential competition between inside and outside money, and focuses the attention of monetary authorities on the nature of private money and on the effectiveness and appropriateness of government intervention.

Historically it is not uncommon for governments to require note-issuing banks to hold government liabilities (bonds or fiat money) as a portion of assets, presumably with the objective of securing seigniorage or enhancing the circulation of outside money.¹ This backing requirement affects the cost of intermediation, and influences banks' ability to extend credit and issue financial claims. The goal of this paper is to investigate the effects of government regulations, including reserve requirements, on the circulation and the value of various forms of monies, and on the operation of financial intermediaries. To this end, we construct a model with an endogenous role of a medium of exchange and with an explicit role for financial intermediation.

As is now fairly standard, the model uses random matching to generate a role for a medium of exchange, as in Trejos and Wright (1995) or Shi (1995). In addition, financial intermediaries mitigate a mismatch between the timing of investment payoffs and agents' desires for consumption, as in Diamond and Dybvig (1983) or Williamson (1999).² A bank may issue and redeem claims (or bank notes) backed by its portfolio. Claims may be used as a medium of exchange in the decentralized random-matching market. We study equilibrium under two distinct re-

¹Ricardo (1816) proposed requiring currency suppliers to hold government liabilities, of which the value should be proportional to the value of the currency issued. As described in Schreft (1997), under the free banking laws, once a bank opened it had to deposit certain assets such as state bonds with the state. In 1833 Bank of England notes gained widespread circulation by their becoming legal reserve for the country banks (White 1995). Also, during the 19th century banks in the United States were required to keep a certain reserve proportion of their notes plus deposits in the form of legal tender currency, which then was composed of specie and greenbacks, to achieve a larger market for government bonds and unified currency (Smith 1990).

²We consider two versions of the banking sector: one with a monopoly bank, and the other with competitive banking industry. For simplicity, the main results are derived in the former version in the text, while the latter is sketched and the differences are discussed in the Appendix.

demption rules: one where the bank uses outside money, and one where it does not. We also distinguish equilibria by whether bank notes circulate, and by whether fiat money circulates.

In terms of results, we find that, regardless of the redemption rules, circulation of bank notes requires the redemption rate be low. This result is consistent with historical observations that low redemption rates were very often necessary for generating enough profits for the issuing banks. Then we show that if bank notes are redeemed solely in goods, fiat money may or may not be valued in equilibrium. If the quantity of fiat money is scarce, equilibria with private money as the unique medium of exchange dominates the economy with fiat money as the unique means of payment, but welfare is highest when private money and fiat money circulate concurrently. Fiat money can still play a welfare-improving role in an economy with privately-issued money. Also, we show that, an increase in the quantity of fiat money reduces bank profits, and the value of both fiat money and bank notes, but it raises the redemption value of notes and consumers' expected utility.

If the bank redeems notes in outside money, fiat money is valued in any equilibrium. Existence of an equilibrium requires a sufficient quantity of fiat money to facilitate the redemption process. Compared to the other redemption rule, redemption in outside money earns higher profits for the bank and higher welfare for the public. If the bank chooses the redemption rule that brings it higher profits, it would choose to redeem in outside money. Hence, as long as outside money is ample, a profit-maximizing bank will coordinate the economy on a better equilibrium.

A particular policy considered here is reserve requirements. This effectively makes the bank take deposits in fiat money – so fiat money is necessarily valued. We also investigate the effects of reserve requirements on a bank's ability to extend credit and issue claims. If the required reserve ratio is higher than what a bank would choose without intervention, a tightening monetary policy forces it to fund fewer investment projects. This creates a credit-rationing phenomenon. Under binding reserve requirements, raising the supply of fiat money or lowering the required reserve ratio will increase the number of investment projects, reduce the value of money and notes, but enhance welfare.

Our results in general support the recent trend toward lower reserve requirements. However, in an economy with abundant investment opportunities it is possible that lower reserve requirements would drive the value of notes down sufficiently to lower welfare. In this situation, higher reserve requirements, which in effect restrict issuance of notes, are good. Thus, to determine whether lowering reserve requirements is beneficial, one must weigh the benefit from increasing

productive intermediation against the cost of reducing the value of the media of exchange.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 discusses the existence and properties of various types of equilibria. Section 4 discusses welfare issues. In section 5, we introduce the reserve requirement policy and study its effect on economic activity. Section 6 concludes with suggestions for possible extensions.

2 The Basic Model

2.1 The Environment

Time is discrete and the horizon is infinite. There is a $[0, 1]$ continuum of infinitely-lived agents. Consuming q units of consumption goods at period t yields utility $\varphi_t u(q_t)$, where φ_t is an i.i.d. preference shock with $\varphi_t \in \{0, 1\}$ and $\Pr[\varphi_t = 1] = \theta$. The utility function $u(q)$ is defined on $[0, \infty)$, is strictly increasing and twice differentiable and $u(0) = 0$, $u'(0) = \infty$, and $u''(q) < 0$ for all $q > 0$. Each agent is endowed with a production technology that requires cost q (in terms of disutility) to yield q units of consumption good instantaneously. Goods are not storable. Assume that agents do not eat what they produce and so trade is desirable. There is a $\hat{q} > 0$ such that $u(\hat{q}) = \hat{q}$. Each agent maximizes expected discounted utility with a discount rate r .

At each period of time a random investment shock, which yields an indivisible investment project, arrives to an agent according to a Poisson process at a constant rate of β . The investment project requires production cost (in terms of disutility) γ at period t to yield R units of consumption good at period $t + k$ with probability α , conditional on not having paid off in any period $t + 1, t + 2, \dots, t + k - 1$. Agents who receive the investment shocks decide whether they wish to fund the project.

There are two sectors in the economy: a *search sector* and a *banking sector*. The search sector is characterized by bilateral random matching of agents. To abstract from the absence-of-double-coincidence problem, we assume the matching technology is such that producers never meet with each other and so barter is ruled out (this can be endogenized as in Burdett et al. 1995). Individual's trading history is private information and there is no enforcement technology for agents to commit future actions. Thus, credit arrangements in the search sector are not feasible.

In the banking sector there is a monopoly bank. The bank is assumed to have expertise in intermediating investment projects so that agents who fund the investment must deposit the

projects at the banking sector.³ In return the bank gives the investor one unit of indivisible financial claim (bank note). Bank financial claims are backed by future returns paid off from the investment projects deposited in the bank. We assume that the bank enjoys a commitment technology that private individuals do not because, for example, there is a public record of bank's transactions (see Calvacanti and Wallace 1999).⁴ Hence, bank notes can be redeemed on demand at any time in the future.

In the initial period, a fraction M of agents is endowed with one unit of indivisible fiat money. Agents can freely dispose of money if it is not valued. Assume that the storage capacity is limited so that an agent can hold only one unit of fiat money or bank note.

The sequence of events within a period occurs as follows. Each agent begins a period holding one unit of asset or nothing. At the beginning of a period, each agent receives an investment shock and a preference shock. Agents who receive investment projects and wish to fund the projects go to the banking sector. The rest of the agents enter the search sector. If a pairwise random matching results in a trade, production and consumption occur. After trading, they leave the meeting and wait for the beginning of the next period. Agents who do not successfully trade in the search sector can go to the banking sector and trade with the bank.⁵ After trading, they leave the banking sector, and a new period begins.

2.2 Bank operations, exchange and prices

In this economy the bank intermediates investment projects, issues and redeems financial claims, and takes fiat money as deposits.

An agent who wishes to fund an investment project must incur production cost γ (in terms of

³This is a simplifying assumption so that we do not need to consider whether agents prefer investing on their own to depositing the projects in the bank. One rationale for "expertise" of banks in this paper is that agents do not know what to do with investment projects which arrive at their doors and need a banker to start it up.

⁴We describe briefly the banker commitment technology as follows. Assume that the technology keeps record of notes issued and redeemed by the bank. Defection by the bank (i.e., consumes all the returns from the portfolio in a period and refuses to redeem any of its notes) is punished by having the bank get the payoff from autarky, which is zero. Comparing the utility from defection to the continuation value of staying in business, one finds that the no-defection condition requires the discount rate be small.

⁵This setup implies that the bank has a superior matching technology so that people can always trade with it if they wish to. It may not always be the case in reality especially when issuing banks were located in far away cities. Substantial transportation and information costs gave a role to middlemen in the bank notes business. If one wants to take into account the fact that banks may not have such superior matching technology, he can consider a probability less than one that people can trade with banks (see Temzelides and Williamson 2001).

disutility) and pay d units of good as a fee to the bank for intermediating the projects. In return the bank gives the investor one unit of bank note. The payoff to one of these notes, if redeemed, is determined by the total returns on the portfolio in that period and the redemption rule. If the bank does not take money deposits, redemption of notes yields q_r units of consumption good. If it takes money deposits, it gives depositors q_b units of good in return for one unit of fiat money,⁶ and redemption of bank notes yields one unit of fiat money plus q_s units of good as side payment. This setup implies that the upper bound on money holdings is not limited for the bank, though it is restricted to unity for nonbank individuals. This asymmetric treatment allows us to consider redemption of bank notes in outside money while maintaining tractability of the model. Note that whether redemption of bank notes involves outside money depends on whether the bank takes fiat money deposits.

To facilitate discussion, we first study equilibrium under exogenous redemption rules and then endogenize it as bank's choice. The bank determines the fee for intermediating the investment projects, d , price of taking money deposits, q_b , and redemption value of notes, q_r and q_s . We consider that the bank provides just enough incentives for agents to participate in certain types of trade, and focus on the determination of redemption prices. In particular, we consider a simple version of bilateral bargaining approach used in Shi (1995) and Trejos and Wright (1995) by assuming that the bank makes take-it-or-leave-it offers to agents who deposit investment projects and fiat money. This implies that the bank extracts the entire trade surplus from these types of exchange.

In the search sector, trade must be intermediated by fiat money or bank notes. A producer produces q_m units of good to money holders in exchange for one unit of fiat money, and produces q_n units of good to noteholders in exchange for one unit of bank notes. To simplify the analysis we assume that in the search sector agents who consume goods in a trade (money holders and noteholders) make take-it-or-leave-it offers to producers. This implies that consumers extract the entire trade surplus.

Let $V_i, i = m, n, o$, denote the life-time expected value to an agent holding one unit of money, bank note and nothing at the end of a period. Take-it-or-leave-it offers by money holders and noteholders in the search sector make the producers indifferent from accepting and rejecting the

⁶That the operation is called money deposits can be interpreted in the following way. Agents deposit fiat money at the bank and get claims or checks in return. Since they have a consumption need in that period, they spend the claims immediately at the banking sector.

trade, which implies

$$\begin{aligned} q_m &= V_m - V_o \\ q_n &= V_n - V_o. \end{aligned}$$

Trade is acceptable to the money holder and noteholder if and only if

$$u(q_m) + V_o - V_m \geq 0$$

and

$$u(q_n) + V_o - V_n \geq 0,$$

respectively, which are equivalent to $0 \leq q_m \leq \hat{q}$, and $0 \leq q_n \leq \hat{q}$, respectively. In this paper we look for equilibria where the above participation conditions are satisfied.

Bank's take-it-or-leave-it offer to producers who deposit investment projects in the banking sector implies

$$d = V_n - V_o - \gamma,$$

since agents who have no assets in hand must pay a cost γ and fee d to fund the investment projects and get bank notes in return.

Before proceeding to study the existence of equilibrium, we discuss some distinctions between the present model and Williamson (1999). In the economy considered here, agents can redeem bank notes if they wish to, and a profit-maximizing bank sets the redemption price of notes. Thus, the redemption rate and quantity of outside money would affect the redemption price, which then affects the feasibility of a private monetary system. We will see below that the different setups also lead to different welfare implications of fiat money. Moreover, we consider in more detail bank operations such as taking fiat money deposits and redeeming notes in outside money. This enables us to study the effects of different redemption rules and reserve requirements on the circulation and the value of inside and outside money.

3 Equilibria

We confine our analysis to steady states where strategies and distributions are time invariant. Agents choose trading strategies to maximize their expected lifetime utility, taking as given others' strategies. Equilibria satisfy each agent maximizing expected discounted utility, rational expectations and maximization of bank profits. We study equilibrium under two distinct

redemption rules: one where the bank uses outside money, and one where it does not. We also distinguish equilibria by whether bank notes circulate, and by whether fiat money is valued. Potential equilibria are summarized in Table 1.

3.1 The bank does not take fiat money deposits

In this case bank notes are redeemed solely in goods. The assumption that the bank and consumers make take-it-or-leave-it offers to producers implies zero expected utility to a producer, i.e., $V_o = 0$. We look for the equilibria in which producers choose not to forgo the opportunities of funding investment projects and trading with consumers, though the take-it-or-leave-it offers make them indifferent to accepting and rejecting the opportunities.

Let $p_i, i = m, n, o$, denote the proportion of agents who are money holders, noteholders and producers, respectively. Then,

$$p_m + p_n + p_o = 1.$$

The expected value to a money holder satisfies the following Bellman's equation:

$$rV_m = \theta p_s \max[u(q_m) + V_o - V_m, 0], \quad (1)$$

where

$$p_s = (1 - \beta)p_o$$

is the fraction of producers who do not receive an investment opportunity and trade in the search sector. Equation (1) sets the flow return of holding money equal to the probability that the agent wants to consume this period and meets a producer in the search sector, θp_s , multiplied by the gains of trade.

From (1) one immediately sees that $V_m = q_m = 0$ is always one of the solutions. In this case, agents expect that money will not be valued, so they never accept it, and this belief is justified. Those endowed with money at the initial period dispose of it and so $p_m = 0$. For fiat money to be valued in decentralized exchange, the condition

$$u(q_m) > q_m > 0 \quad (2)$$

must hold, and $p_m = M$. In this case, agents expect that money will be accepted, and so they always take it. Thus, whether or not fiat money is valued is a self-fulfilling phenomenon.

3.1.1 Bank notes do not circulate

If spending bank notes in the search sector is not favorable enough, noteholders will simply hold on to the notes and make redemption when they need to consume. For agents to forgo the opportunity of spending bank notes in the decentralized markets, the net gains of redemption must be higher than that of trading in the search sector:

$$q_r \geq q_n. \quad (3)$$

The expected returns of holding notes is simply through redemption:

$$rV_n = \theta \max[u(q_r) + V_o - V_n, 0]. \quad (4)$$

For redemption to be incentive compatible, we need

$$u(q_r) + V_o - V_n \geq 0, \quad (5)$$

which implies $u(q_r) - q_n \geq 0$. We can interpret (5) as a feasibility condition for the existence of a private monetary system.

Given perfect diversification by the bank, in steady state the number of maturing investment projects must equal the flow of new projects into the bank's portfolio in each period, βp_o , which also equals the number of notes issued.⁷ The steady state also requires that the number of outstanding bank notes be constant; i.e., the number of notes issued equals the number of notes redeemed. Therefore, we have

$$\beta p_o = \theta p_n. \quad (6)$$

Bank profits are calculated in terms of output acquired from its operation.⁸ To check whether a steady-state equilibrium exists, we look for bank's maximizing strategy, given other agents' strategies and distributions in steady states. The expected returns to the bank in period t is represented by

$$V_{B,t} = \max_{q_{r,t}} \beta p_o R + \beta p_o d - p_n q_{r,t} + \left(\frac{1}{1+r}\right) V_{B,t+1}.$$

In period t , total output that the bank acquires equals the return R multiplied by the number of matured projects, βp_o , plus the fee it takes from intermediating investment. The amount

⁷Suppose x is the quantity of projects held by the bank in the steady state. The number of investment projects paid off is αx , and the quantity of investment in new projects is βp_o . Hence, in steady state $\alpha x = \beta p_o$. In the following discussions we will use βp_o instead of αx to represent the number of matured projects.

⁸Using quantity of output as a measure of profit simplifies the algebra. It may be interpreted as implicitly assuming bankers are risk neutral.

of output it pays out equals the number of notes redeemed multiplied by the redemption price $q_{r,t}$. The bank maximizes the life-time sum of discounted period-wise profit flows by choosing a sequence of $q_{r,t}$. Given other agents' steady-state maximizing strategies, and that the measures p_o , p_n , and the number of notes redeemed, θp_n , are time-invariant, bank's pricing strategy in a period would not affect profits in the following periods. Therefore, the problem amounts to maximizing profit flows in each period. Also, given a profile of other agents' steady-state strategies and steady-state distributions, bank profits in each period are identical, so a sequence of time-invariant $q_{r,t}$ is optimal for the bank. Therefore, to solve for $q_{r,t}$, we consider a representative period's profits (subscript t is omitted):

$$\pi = \beta p_o R + \beta p_o d - \theta p_n q_r. \quad (7)$$

A steady state equilibrium where the bank does not take fiat money deposits and bank notes do not circulate consists of $(V_o, V_m, V_n, q_m, q_n, q_r, p_o, p_n, p_m)$ satisfying $V_o = 0$, (1) and (4), take-it-or-leave-it offers, bank maximizing π defined in (7) by choosing q_r , constraints (3) and (5), and steady-state condition (6). If fiat money is not valued, $V_m = q_m = 0$ and $p_m = 0$; otherwise, $u(q_m) > q_m > 0$ and $p_m = M$.

We establish the non-existence result in the following proposition and proof in the Appendix.

Proposition 1 *When bank notes are redeemed solely in goods, there exists no equilibrium where bank notes do not circulate.*

The profit-maximizing bank sets a redemption price q_r , under which agents find it more appealing to use bank notes as a medium of exchange than hold on to them until redemption; namely, the constraint (3) is violated. Unlike intrinsically worthless fiat currency, bank notes in this economy are backed by real output and the “intrinsic value” makes it hard not to be accepted in exchange.

3.1.2 Bank notes circulate

In this type of equilibrium people spend bank notes in the decentralized markets, and if they do not have successful trade they redeem notes at the banking sector. The flow return to holding notes now satisfies (we incorporate equilibrium conditions in the value functions)

$$rV_n = \theta p_s [u(q_n) + V_o - V_n] + \theta(1 - p_s) [u(q_r) + V_o - V_n]. \quad (8)$$

The first term in (8) describes the expected gains of spending bank notes in decentralized trade. The second term represents the expected gains of redemption, which equals the probability that an agent wishes to consume but does not meet a producer in the search sector, $\theta(1 - p_s)$, multiplied by the gains of redeeming notes.

Since agents can freely trade in both sectors, circulation of bank notes requires that the gains of using notes in decentralized exchange be high enough so that agents would not forgo the trading opportunity. Thus, the circulating value is greater than the fundamental value ($q_n \geq q_r$) when bank notes circulate. Positive gains for redeeming notes requires (5) to hold. The steady-state condition that the quantity of notes issued equals the quantity of notes redeemed satisfies

$$\beta p_o = \theta(1 - p_s)p_n. \quad (9)$$

With an argument similar to the previous case, the bank maximizes profits represented by

$$\pi = \beta p_o R + \beta p_o d - \theta(1 - p_s)p_n q_r. \quad (10)$$

There are two types of equilibria where bank notes circulate: fiat money may or may not be valued.

Equilibrium G1. Bank notes circulate and fiat money is not valued

In this equilibrium, bank notes are the unique medium of exchange – a pure private monetary system. The following proposition summarizes the results.

Proposition 2 *When fiat money is not valued, bank notes circulate as the medium of exchange if $\theta \leq \theta_1$, where θ_1 is a function of parameters.*

Given other parameters, larger θ means a higher tendency for consumption, which makes agents value higher the media of exchange (i.e., the circulating price of bank notes is higher). A larger θ also implies a higher redemption rate, other things being equal. When θ is sufficiently big, a profit-maximizing bank may set a redemption value, q_r , that would not provide noteholders with enough incentives to redeem the notes. That is, the feasibility condition for the existence of a private monetary system, (5), is violated. This result is consistent with some historical observations that low redemption rates were very often necessary for circulation of notes in order to generate enough profits for the issuing banks.⁹

⁹Martin, Monnet and Weber (2000) argue that some bank notes were expected to be redeemed very quickly and so such notes would bear high marginal costs.

Equilibrium G2. Bank notes circulate and fiat money is valued

In this equilibrium there are two competing media of exchange – fiat money and bank notes. The following proposition establishes the existence result and demonstrates some properties of the coexistence of public and private monetary system.¹⁰

Proposition 3 *For $r > 0$, when fiat money is valued, there exists an equilibrium with circulating bank notes if $\theta \leq \theta_2$. The threshold θ_2 increases in the quantity of fiat money. Moreover, an increase in the quantity of fiat money raises the redemption value of bank notes, but lowers the circulating value of fiat money, notes, and bank profits.*

The coexistence of public and private monetary systems requires that the redemption rate be low and, preferably, the quantity of outside money is not too scarce. Fiat money is a competitor to bank notes as a medium of exchange but, on the other hand, is complementary in facilitating trade in the decentralized markets. When more fiat money circulates in the market, there are fewer outstanding bank notes, which results in fewer notes for redemption for a given θ . It is thus more plausible that a profit-maximizing bank sets a redemption price acceptable to noteholders. Outside money thus plays a role in the existence of a private monetary system. Proposition 3 also shows that, as the quantity of outside money increases, the value of both media of exchange is lower; however, redemption of bank notes commands higher returns. This also explains why the quantity of outside money should not be too scarce for the redemption of notes to be incentive compatible.

Rate of return dominance

We now show that bank's financial claims dominate fiat money in the rate of return. In equilibrium the gains to producers for selling goods for money or bank notes are identical. However, the circulating value of bank notes fully accounts for its fundamental value (see equation (8)). Let r_m and r_n denote the rate of return on fiat money and bank notes, respectively. Then $r_m = \theta p_s u(q_m)/q_m$ and $r_n = [\theta p_s u(q_n) + \theta(1 - p_s)u(q_r)]/q_n$. One can show that $r_n = r + \theta > r_m = r + \theta p_s$. Although fiat money and bank notes have identical acceptability, notes dominate money in the rate of return. The reason for this phenomenon lies in the dissimilar life-span of the two assets, and our model's ability to capture *explicitly* the medium-of-exchange role of private liabilities and frictions that preclude arbitrage. Money is infinitely-lived while

¹⁰For the proofs of Proposition 3 and 4, see the working paper version on the web site <http://ccms.ntu.edu.tw/~yitingli/>.

the purchasing power of bank notes would be lost when they are redeemed. Hence, bank notes command a “risk premium” over fiat money.

3.2 The bank takes fiat money deposits

If the bank takes fiat money deposits, fiat money must be valued, because individuals know that someone – the bank – is always willing to accept fiat money. This operation also enables the bank to adopt a different redemption rule: one note is redeemed for one unit of fiat money plus some amount of good q_s .

Equilibrium M1. The bank takes money deposits and bank notes do not circulate

In this equilibrium, fiat money is the unique medium of exchange and bank notes are held solely for redemption. The return of holding money and bank notes now satisfies, respectively, the following Bellman’s equations:

$$rV_m = \theta p_s[u(q_m) + V_o - V_m] + \theta(1 - p_s)[u(q_b) + V_o - V_m] \quad (11)$$

$$rV_n = \theta[u(q_s) + V_m - V_n]. \quad (12)$$

The second term of (11) describes the expected gains to a money holder from trading in the banking sector, which equals the probability that the agent does not successfully trade in the search sector, $\theta(1 - p_s)$, multiplied by the gain of making deposits. Note that the difference between (12) and (4) lies in the different redemption rules.

Given that notes are redeemed in fiat money, the incentive constraint for redemption now requires

$$u(q_s) + V_m - V_n \geq 0. \quad (13)$$

For notes not to circulate, redemption must yield higher gains of trade than what would have been obtained if spending it in the decentralized markets:

$$u(q_s) + V_m - V_n \geq u(q_n) + V_o - V_n,$$

which implies

$$u(q_s) + q_m \geq u(q_n). \quad (14)$$

To facilitate redemption of notes, the bank must keep some fiat money as reserves. Let $b_m \geq 0$ denote bank’s vault cash. The total amount of fiat money is held by private agents as well as by the bank:

$$p_m + b_m = M. \quad (15)$$

In steady state, the amount of fiat money deposited must equal the amount paid out through redemption process, i.e.,

$$\theta(1 - p_s)p_m = \theta p_n. \quad (16)$$

The take-it-or-leave-it offer by the bank to money depositors implies that the price q_b satisfies

$$u(q_b) + V_o - V_m = 0,$$

which implies $q_b = u^{-1}(q_m)$. Bank maximizes profits represented by

$$\pi = \beta p_o R + \beta p_o d - \theta p_n q_s - \theta(1 - p_s)p_m q_b. \quad (17)$$

The total amount of goods that the bank pays out equals the number of notes redeemed multiplied by the price q_s , plus the amount of money deposited in the bank, $\theta(1 - p_s)p_m$, multiplied by the price q_b .

A steady-state equilibrium where the bank takes money deposits and bank notes do not circulate consists of $(V_o, V_m, V_n, q_m, q_n, q_b, q_s, p_o, p_n, p_m, b_m)$ satisfying $V_o = 0$, (11) and (12), the bank maximizing π defined in (17) by choosing q_s , take-it-or-leave-it offers, constraints (2), (13) and (14), and steady-state conditions (6), (15), (16) and $b_m \geq 0$.

Proposition 4 *There exists an equilibrium where bank notes are redeemed in fiat money and do not circulate if $M \geq M_1$ and $\theta \geq \theta_3$.*

In this equilibrium the bank needs to keep some vault cash for redeeming notes, which is feasible if the quantity of fiat money is ample; i.e., $M \geq M_1$. The constraint for notes not to circulate, (14), is satisfied if θ is big. The intuitive reason is as follows. Fiat money is valued higher in exchange when agents have a higher consumption tendency. When θ is sufficiently big, redemption of notes in fiat money thus can generate high enough gains to noteholders so that people would rather hold on to them to realize the fundamental value.¹¹

Equilibrium M2. The bank takes money deposits and bank notes circulate

The value of holding bank notes now includes the expected gains of spending it in decentralized exchange:

$$rV_n = \theta p_s[u(q_n) + V_o - V_n] + \theta(1 - p_s)[u(q_s) + V_m - V_n].$$

¹¹This is analogous to the theoretical findings in Li (2002): Silver coins very often circulate as a medium of exchange due to the valuable metal contents, but when they are so intrinsically valuable that the market fails to generate acceptable terms of trade, they may be hoarded from circulation.

Circulation of bank notes requires $u(q_n) \geq u(q_s) + q_m$. The steady-state condition that equates the quantity of money deposited and the quantity paid out through the redemption process satisfies $\theta(1 - p_s)p_m = \theta(1 - p_s)p_n$, which implies $p_m = p_n$. Bank maximizes profits represented by

$$\pi = \beta p_o R + \beta p_o d - \theta(1 - p_s)p_n q_s - \theta(1 - p_s)p_m q_b.$$

For redemption in outside money to be feasible, fiat money must be ample, $M \geq M_2$.¹² For notes to circulate, θ must be small. We also find that given other parameters, $M_1 > M_2$, so this equilibrium can exist at a lower M when equilibrium M1 does not. The reason is that, when bank notes circulate, there are fewer notes redeemed, and so with less fiat money this redemption rule is still feasible.

When the bank redeems notes in outside money, all the prices, steady-state distributions and incentive constraints are not affected by changes in the quantity of fiat money. Its only effect is on the amount of bank reserves, b_m . That is, when the government injects more outside money into the economy, it does not affect the amount of investment projects intermediated by the bank, the number of outstanding bank notes, and the value of money and notes. If the government adopts a tightening monetary policy, it does not affect real activity either as long as the quantity of money is large enough for the bank to keep nonnegative reserves. Thus, *money is neutral* as long as it is ample enough for redeeming notes in fiat money.¹³ The reason for neutrality of money is that the monopoly bank absorbs extra liquidity from the market, and internalizes the externality effects caused by an increase in outside money.

3.3 Existence of equilibria when redemption rules are endogenously determined

The above analysis demonstrates the existence and properties of equilibria when the redemption rules are exogenously imposed. We now relax this assumption and let the bank choose the redemption rule, given the feasibility constraints. The existence of steady-state equilibrium now should incorporate the condition that, given other agents' maximizing strategies, the bank sets the redemption rule as well as redemption price to maximize profits.

¹²From the steady-state conditions one can solve for $M_1 = \frac{-\beta(2+\theta) + \sqrt{\beta(4\beta+4\theta+\beta\theta^2)}}{2\theta(1-\beta)}$ and $M_2 = \frac{-\beta(2+\theta) + \sqrt{\beta(4\beta+8\theta-4\beta\theta+\beta\theta^2)}}{4\theta(1-\beta)}$.

¹³Similar results are also found in Burdett et al. (2001) where the total money supply (commodity money and fiat money) is endogenous. They find that as long as small amount of fiat money is introduced, it does not affect the exchange process because each unit of fiat money crowds out a unit of commodity money.

We show existence of equilibria in Fig. 1, where the redemption rule is bank's choice. First note that, when $M \geq M_2$, bank profits are higher if redemption is in fiat money rather than solely in goods; the bank thus chooses to redeem notes in outside money. Since we have shown that fiat money is always valued if the bank redeems notes in fiat money, the above result implies that bank's choice of redemption rule gives rise to valued fiat money. Second, when $M < M_2$, redemption in outside money is not feasible and the bank redeems notes solely in goods. In this case, equilibria G1 and G2 coexist when $\theta \leq \theta_1$. Whether fiat money is valued is thus determined by agents' beliefs, not by the bank's choice.

4 Welfare

We discuss welfare issues such as whether fiat money plays a welfare-improving role when it coexists with circulating bank notes, and which redemption rule yields higher expected utility to individuals. We use a representative individual's long-term expected utility, not conditional on the current status, as the welfare criterion:

$$W = p_m V_m + p_n V_n.$$

Our discussion in this section mainly draws from the observations of numerical examples.

We have shown that, when bank notes are redeemed solely in goods, fiat money may or may not be valued. Both types of equilibria coexist for some parameter values, and we want to do welfare comparisons. We find that, the equilibrium with valued fiat money entails lower bank profits but higher welfare than otherwise, when the quantity of fiat money is small (Ex. 4 of Table 2). Thus, fiat money can still play a welfare-improving role in an economy where privately-issued financial claims are generally accepted in exchange. If the quantity of fiat money is scarce, equilibria with private money as the unique medium of exchange dominates the economy with fiat money as the unique means of payment, but welfare is highest when private money and fiat money circulate concurrently. Hence, the coexistence of privately-issued money and fiat money may yield the most desirable outcome (a similar result is also found in Azariadis et al. 2001).

Our result that fiat money can improve welfare when it coexists with circulating bank notes is contrary to the findings in Williamson (1999). Note that in Williamson (1999) fiat money decreases welfare because it crowds out productive intermediation. Although this crowding out effect also appears here (as we have shown that an increase in the quantity of fiat money lowers the number of outstanding notes), it is not as severe for the following reason. In Williamson

(1999) the zero-profit condition pins down the redemption price to be equal to the return of a maturing project, R , while in the present model the bank chooses the redemption value to maximize profits. Proposition 3 shows that an increase in the quantity of fiat money reduces the amount of notes redeemed and raises the redemption value. When money supply is small, the benefit from improving the terms of trade in redeeming notes may outweigh the loss in production due to the crowding-out effect, and thus, welfare may be improved.¹⁴ This result also implies the following distributional effect: an increase in the quantity of fiat money may redistribute the profits from a monopoly bank to private individuals so that they enjoy a higher level of utility.

We now do welfare comparisons between the equilibria where bank notes are redeemed in outside money (Ex. 1 and 2 in Table 2). When θ and r are relatively large, circulation of bank notes yields higher welfare than otherwise. Note that the redemption rule is composed of two steps of exchange and represents a way to smooth consumption. Agents consume on the spot of redeeming notes, and then use fiat money acquired from redemption to buy goods later. If agents spend notes in the search sector, they consume in a single step of exchange. Circulation of bank notes is thus particularly valuable when agents are less patient and the consumption tendency is relatively high.

Comparing equilibria with different redemption rules, we find that, redemption in outside money earns higher profits for the bank and higher welfare for the public (Ex. 2 and 3 in Table 2).¹⁵ Fiat money is valued in decentralized trade as well as in the banking business. More trading opportunities for fiat money is beneficial to private individuals. It also enables the bank to adopt an operation that increases the value of both inside and outside money and, thus, entails higher welfare to the society. We have shown that, if the bank chooses the redemption rule that brings it higher profits, it would choose to redeem in outside money. The resulting implication is that, as long as outside money is ample, a profit-maximizing bank can coordinate the economy on a better equilibrium.

¹⁴One may wonder whether bank's profits plus private individuals' welfare (denoted *total benefit*) is also higher in the equilibrium with valued fiat money. We find from numerical examples that, the total benefit is lower in the equilibrium with valued fiat money than in the one where fiat money is not valued. This is because bank profits are lower when fiat money is used as a medium of exchange.

¹⁵Freeman (1996) shows in a model where agents are spatially separated, banks perform a role as a clearinghouse of private debt, and private debt creation and settlement in outside money is efficient. He argues that to prevent the over-issue of bank notes, they must be fully backed by reserves of outside money.

5 The Reserve Requirement Policy

We examine the effects of the reserve requirement policy on economic activity when inside money is redeemed in outside money. Let the reserve ratio be measured by the ratio of bank's vault cash to the amount of outstanding notes. The reserve requirement policy specifies the minimum reserve ratio that must be complied with in bank's reserve management.

Given the required reserve ratio and existing quantity of outside money, the bank may not be able to issue as many notes as it wishes were there no government intervention. The reserve requirement policy in effect restricts the maximum amount of notes that banks can issue, which then affects the liquidity available to the economy. This policy also sets a limit on the quantity of investment projects that banks can fund and therefore affects the intermediation function. To account for these effects, in this section we consider one more dimension of bank operations – determining the number of investment projects that the bank wishes to fund.

Suppose that government imposes a required reserve ratio r_b . We study policy r_b in equilibrium M1, where bank notes are redeemed in fiat money and do not circulate (analysis is similar for equilibrium M2 and is omitted). Let η denote the fraction of investment projects intermediated by the bank in each period. The number of projects paid off equals the flow of new investment projects into the bank's portfolio, $\eta\beta p_o$. Bank profits now become

$$\pi = \eta\beta p_o R + \eta\beta p_o d - \theta p_n q_s - \theta(1 - p_s)p_m q_b.$$

The steady-state condition that the number of notes issued equals that redeemed satisfies

$$\eta\beta p_o = \theta p_n.$$

The value functions, incentive constraints and other steady-state conditions are the same as described in the previous sections.

The reserve requirement policy implies the following constraint facing the bank:

$$b_m/p_n \geq r_b.$$

Note that in the present model bank profits would be lower as it funds fewer investment projects. A profit-maximizing bank thus will choose to fund as many investment projects as it can without violating the reserve requirement policy. Hence, when the quantity of fiat money and required reserve ratio permit, the bank will fund all investment projects that are deposited in. If outside money is relatively scarce, the bank may be able to fund only a fraction of investment projects turned in; that is, the bank adopts a credit-rationing policy.

We find that, if r_b is lower than the reserve ratio that the bank would have chosen without government intervention (i.e., the reserve requirement policy is not binding), changes in r_b or quantity of fiat money have no real effects. That is, the number of investment projects intermediated by the bank, quantity of bank notes issued, and the value of fiat money and bank notes remain unchanged. The bank funds all investment projects and $\eta = 1$.

When r_b policy is binding, the bank is unable to fund as many investment projects as it wishes and $\eta < 1$. This implies that only a fraction of investment projects turned in get funding. Under a binding r_b policy, when government reduces money supply or raises the required reserve ratio, the bank is forced to fund fewer investment projects, print fewer notes and earn lower profits. A tightening monetary policy thus creates a credit-rationing phenomenon. On the other hand, an expansionary policy enables the bank to fund more investment projects and issue more notes. The ease of liquidity also lowers the value of fiat money and bank notes.

The investment technology in this economy is risk-free, implying that the backing of bank notes is safe. At the first glance it may seem that the more projects intermediated by the bank, the higher the output and, therefore, the higher expected utility enjoyed by private agents. Although this is true for equilibrium M1, it is not always the case for equilibrium M2. That is, when bank notes are held solely for redemption, welfare always increases as more investment is undertaken. In this situation, relaxing reserve requirements is socially optimal.

However, in the equilibrium with circulating bank notes, for some parameter values (e.g., the arrival rate of investment projects is relatively high) more investment projects intermediated may yield lower expected utility to private individuals. The reason is as follows. Bank notes in this equilibrium facilitate exchange in the decentralized markets and insure people against consumption shocks through redemption. More investment projects funded result in more outstanding notes, which drives lower the value of fiat money and notes, i.e., lower amount of output traded in every exchange in the decentralized markets. When the latter effect dominates, expected utility of private individuals is reduced by more issuance of notes, even though they are fully backed by risk-free asset. This implies that the bank funds too many investment projects or, equivalently, prints too many notes to grasp profits. In this situation, higher reserve requirements, which in effect restrict issuance of notes, are good. Hence, to determine whether lowering reserve requirements is beneficial, one must weigh the benefit from increasing productive intermediation against the cost of reducing the value of the media of exchange.

6 Conclusion

Removal of legal restrictions and development of technology have made it easier to create close currency substitutes. One example of such technological innovations is stored-value cards, that can be used for a single type of purchase or more widely at any merchant that has electronic equipment to read the cards. Although the use of electronic cash may involve multiple parties and a computer network, it shares many properties of bank notes described in this model: they are private liabilities that permit transactions at dispersed locations, and some may circulate (change hands many times without clearing and settlement occurring). Some may feel that the issuers of bank notes in the present model are not just banks; they are also centers of production and centers for retail trade. In fact, the issuers need not be banks; they can be any business firms. We think this formulation is relevant because in real world, any business could issue stored-value cards. Moreover, so far banks have been the leading issuers of the general-purpose stored-value cards; however, in the future, joint ventures between banks and nonbank firms may arise to issue stored-value cards that are widely accepted.

The present model can be analyzed by considering a monopoly bank or competitive banks. For simplicity, the main results are derived in the version with a monopoly bank, while the case with competitive banks is sketched and the differences are discussed in the Appendix. One distinction is worth mentioning here. Under competitive banks the zero-profit condition pins down the redemption price to be equal to the (exogenously given) return of a maturing investment project. However, under a monopoly bank the redemption price is set in response to changes in exogenous variables, and this would affect the value of bank notes and the feasibility of a private monetary system.

We have shown that fiat money is a competing medium of exchange with private money but is also complementary to private monetary systems. One may study other issues of currency competition in the present framework. By introducing costs associated with different monies, e.g., the default risk on privately-issued money and a sudden loss of purchasing power (a proxy for inflation) of fiat money, one can study how the relative cost and benefit affect the value and circulation of both currencies. If multiple banks issue distinguishable bank notes, and the value of notes issued by each bank is determined by the return and risk of the backing asset, then there may be strategic interactions among issuers in competing for the circulation of their notes.

For tractability we have assumed restrictions on individual's money holding and indivisibility of money and notes. If goods and money are divisible, we conjecture that bank notes

will circulate in any equilibrium because the market is always able to generate a price that is acceptable to the noteholders. Allowing for holding multiple units of money and notes may lead to a variety of additional issues. For example, one can study how agents' decisions to hold a portfolio of government and privately issued currencies and their spending strategies are affected by inflation rate and costs of private money. Nevertheless, the present model is dramatically simpler than a fully divisible asset version, and as we discussed, many of its predictions would seem to be robust.

Some related studies of the search monetary models concerning the effect of inside money on allocations include, for example, Shi (1996), Cavalcanti and Wallace (1999), Cavalcanti et al. (1999) and Li (2001). The major difference between this article and those studies is that we have explicitly model the institutional features of private money, which allows us to examine in more detail the effects of government regulations on intermediary activities. The model considered in this paper allows us to discuss private money circulation, credit rationing and banks' ability to extend credit all in a unified framework. One may also study many other questions, in this framework, such as how bank runs may occur and how such phenomena affect the stability and functions of private money, and the effects of a discount window policy.

Appendix

Proof of Proposition 1.

To find the equilibrium condition, we first solve from (4) for

$$q_n = \left(\frac{\theta}{r+\theta}\right)u(q_r). \quad (18)$$

Given the steady-state condition $\beta p_o = \theta p_n$ we rewrite bank's profits (7) as $\pi = \beta p_o(R + q_n - \gamma - q_r)$. After substituting (18) and solving for the profit-maximizing price q_r^* we get

$$\left(\frac{\theta}{r+\theta}\right)u'(q_r^*) = 1$$

Since constraint (3) implies (5) we need only check whether $q_r \geq q_n$ holds. This, given (18), amounts to checking $q_r \geq \left(\frac{\theta}{r+\theta}\right)u(q_r)$. Note that $\left(\frac{\theta}{r+\theta}\right)u'(q_r) > 1$ as $q_r \rightarrow 0$, $\left(\frac{\theta}{r+\theta}\right)u'(q_r) \rightarrow 0$ as $q_r \rightarrow \infty$ and $\left(\frac{\theta}{r+\theta}\right)u(\hat{q}) = \left(\frac{\theta}{r+\theta}\right)\hat{q} < \hat{q}$. Given $\left(\frac{\theta}{r+\theta}\right)u'(q_r^*) = 1$ and $\left(\frac{\theta}{r+\theta}\right)u'(\hat{q}) < \frac{\theta}{r+\theta} < 1$, and because $u'(q)$ is continuous and $u''(q) < 0$, we know $q_r^* < \hat{q}$. Hence, $\left(\frac{\theta}{r+\theta}\right)u(q_r^*) > q_r^*$, i.e., $q_n > q_r^*$, a contradiction. ■

Proof of Proposition 2.

From differentiating (8) we know that the profit-maximizing price q_r^* must satisfy

$$\frac{\theta(1-p_s)u'(q_r^*)}{(r+\theta) - \theta p_s u'(q_n)} = 1. \quad (19)$$

Also from (8) one can solve for

$$q_n = \frac{\theta p_s u(q_n)}{r+\theta} + \frac{\theta(1-p_s)u(q_r)}{r+\theta}.$$

We want to show that, if $q_r \in (0, q_n)$, there exists at least one $q_n \in (q_0, q_1)$ where q_0 solves $q_0 = \left(\frac{\theta p_s}{r+\theta}\right)u(q_0)$, when q_r takes the value of 0, and q_1 solves $q_1 = \left(\frac{\theta}{r+\theta}\right)u(q_1)$, when q_r takes the value of q_n , such that (i) $q_n \geq q_r^*$ and (ii) $u(q_r) \geq q_n$.

To check condition (i) is equivalent to verifying $u'(q_r^*) \geq u'(q_n)$. From (19) we know that $u'(q_r^*) \geq u'(q_n)$ iff $\left(\frac{\theta}{r+\theta}\right)u'(q_n) \leq 1$. Note that $\left(\frac{\theta}{r+\theta}\right)u(\hat{q}) = \left(\frac{\theta}{r+\theta}\right)\hat{q} < \hat{q}$, so $q_0 < \hat{q}$ and $q_1 < \hat{q}$. Also, $\left(\frac{\theta}{r+\theta}\right)u'(q_n) > 1$ as $q_n \rightarrow 0$, and $\left(\frac{\theta}{r+\theta}\right)u'(q_n) \rightarrow 0$ as $q_n \rightarrow \infty$. Given that $u'(q)$ is continuous and $u''(q) < 0$, we know $\left(\frac{\theta p_s}{r+\theta}\right)u'(q_0) < 1$ and $\left(\frac{\theta}{r+\theta}\right)u'(q_1) < 1$. Thus, there exists at least one $\tilde{q} \in (q_0, q_1)$ close enough to q_1 , such that $\left(\frac{\theta}{r+\theta}\right)u(\tilde{q}) > \tilde{q}$ and $\left(\frac{\theta}{r+\theta}\right)u'(\tilde{q}) \leq 1$. Let q_n take the value of \tilde{q} .

We next check condition (ii). From (8) we find that $u(q_r) - q_n$ takes the same sign as $(r + \theta p_s)q_n - \theta p_s u(q_n)$. That is, we need to show that

$$\left(1 + \frac{r}{\theta p_s}\right)\tilde{q} - u(\tilde{q}) \geq 0.$$

From the steady-state conditions we solve for p_o and $\frac{\partial p_o}{\partial \theta} > 0$, which implies $\frac{\partial p_s}{\partial \theta} > 0$. Thus, the above condition holds if $\theta \leq \theta_1$ where θ_1 solves $(1 + \frac{r}{\theta p_s})\tilde{q} = u(\tilde{q})$. ■

Proof of Proposition 3.

The proof for that bank notes circulate if $\theta \leq \theta_2$ is similar to proposition 2. We now show that there exists at least one solution to $q_m \in (0, \hat{q})$, so fiat money is valued. Given take-it-or-leave-it offers, we write (1) as $q_m = (\frac{\theta p_s}{r + \theta p_s})u(q_m)$. We know $u(0) = 0$ and $u'(q) > 1$ as $q \rightarrow 0$, it follows that $q < (\frac{\theta p_s}{r + \theta p_s})u(q)$ for q close to 0. Also, $(\frac{\theta p_s}{r + \theta p_s})u(\hat{q}) < \hat{q}$, and $(\frac{\theta p_s}{r + \theta p_s})u'(q) \rightarrow 0$ as $q \rightarrow \infty$. Since $u'(q)$ is continuous and $u''(q) < 0$, there exists at least one $q^* \in (0, \hat{q})$ such that $q^* = (\frac{\theta p_s}{r + \theta p_s})u(q^*)$ and $(\frac{\theta p_s}{r + \theta p_s})u'(q^*) < 1$.

We next show that θ_2 increases in M . From the proof of proposition 2 we know that $u(q_r) \geq q_n$ when $(1 + \frac{r}{\theta p_s})q_n - u(q_n) \geq 0$. This condition is more likely to hold if θp_s is smaller. From the steady-state condition we solve for p_o and $\frac{\partial p_o}{\partial M} < 0$, which implies $\frac{\partial p_s}{\partial M} < 0$. Thus, when M is bigger, $(1 + \frac{r}{\theta p_s})q_n - u(q_n) \geq 0$ may still hold even with a higher θ .

To show $\frac{dq_m}{dM} < 0$ we differentiate (1) to get

$$\frac{dq_m}{dM} = \frac{\theta[u(q_m) - q_m] \frac{\partial p_s}{\partial M}}{r + \theta p_s - \theta p_s u'(q_m)}.$$

Note that given $(\frac{\theta p_s}{r + \theta p_s})u'(q_m) < 1$, the denominator, $r + \theta p_s - \theta p_s u'(q_m)$, is positive. Thus, $\frac{dq_m}{dM} < 0$ because $\frac{\partial p_s}{\partial M} < 0$.

Similarly, we have

$$\frac{dq_n}{dM} = \frac{\theta[u(q_n) - u(q_r)] \frac{\partial p_s}{\partial M}}{r + \theta - \theta(1 - p_s)u'(q_r)}.$$

From (19), $u'(q_n) = \frac{(r + \theta) - \theta(1 - p_s)u'(q_r)}{\theta p_s}$. This implies that the denominator of the RHS in the above equation is positive. Thus, $\frac{dq_n}{dM} < 0$ given $\frac{\partial p_s}{\partial M} < 0$.

We now show $\frac{dq_r}{dM} > 0$. From differentiating (19) we get $\frac{dq_r}{dq_n} = \frac{-\theta p_s u''(q_n)}{\theta(1 - p_s)u''(q_r)} < 0$. Utilize the result $\frac{dq_n}{dM} < 0$ we have $\frac{dq_r}{dM} > 0$.

Finally, we show $\frac{\partial \pi}{\partial M} < 0$. Differentiating (10) we get

$$\frac{\partial \pi}{\partial M} = \beta(R + q_n - \gamma - q_r) \frac{\partial p_o}{\partial M} + \beta p_o \left(\frac{dq_n}{dM} - \frac{dq_r}{dM} \right) < 0$$

because $\frac{\partial p_o}{\partial M} < 0$, $\frac{dq_n}{dM} < 0$ and $\frac{dq_r}{dM} > 0$. ■

Proof of Proposition 4.

From the steady-state conditions we solve for the steady-state measures, each of which is between zero and one. From the steady-state conditions one can solve that $b_m \geq 0$ if $M \geq M_1$,

where

$$M_1 = \frac{-\beta(2 + \theta) + \sqrt{\beta(4\beta + 4\theta + \beta\theta^2)}}{2\theta(1 - \beta)}.$$

Note that $M_1 > 0$ always. The rest of proof is similar to Proposition 2 and is omitted. ■

Competitive Banks

Here we briefly describe existence results when banks operate under the zero-profit condition. Suppose that competition among banks for intermediating the investment projects drives down the fee to zero; $d = 0$. The zero-profit condition thus implies that the redemption price of notes equals the return on a maturing project; $q_r = R$. We maintain the take-it-or-leave-it assumption in determining terms of trade in the search sector. Since banks do not extract all the surplus in the trade with private individuals, $V_o \neq 0$.

I. Banks do not take fiat money deposits.

We consider the case where bank notes do not circulate (other cases can be analyzed in a similar way and are omitted). The flow return to a producer and noteholder is, respectively,

$$\begin{aligned} rV_o &= \beta(V_n - \gamma - V_o) \\ rV_n &= \theta[u(R) + V_o - V_n]. \end{aligned}$$

The expected value to holding fiat money, incentive constraints, and steady-state conditions are the same as in the model with a monopoly bank. Take-it-or-leave-it assumption implies $q_n = V_n - V_o = \frac{\theta u(R) + \beta\gamma}{r(\beta + r + \theta)}$.

Unlike the version with a monopoly bank, here the return R and cost γ affects individuals' value of being a producer and noteholder. The constraint for notes not to circulate is $R > q_n$. For redemption to be incentive compatible it requires $u(R) > q_n$. To find the existence condition, note that q_n is increasing in θ and $u(R) > R$. Therefore, we need only check $R > q_n$, which holds iff $\theta < \theta_0$ where θ_0 solves $q_n = \frac{\theta u(R) + \beta\gamma}{r(\beta + r + \theta)} = R$. Moreover, R must be big enough relative to γ for V_o and V_n to be non-negative.

If bank notes circulate, V_n includes the gains from using notes in the decentralized market. For notes to circulate it requires $q_n > R$. For the equilibrium to exist we thus need $u(R) > q_n > R$. Since q_n is increasing in θ , this equilibrium exist iff $\underline{\theta} < \theta < \bar{\theta}$, where $\underline{\theta}$ and $\bar{\theta}$ solves $q_n = u(R)$ and $q_n = R$, respectively, where $q_n = V_n - V_o$.

II. Banks take fiat money deposits.

Competition among banks in issuing notes implies that banks would compete for fiat money deposits, and drives up q_b . But for fiat money to circulate, it must be valued in the search sector

no less than in the banking sector, $q_m \geq q_b$. Consider the case where bank notes do not circulate. The flow value to holding fiat money and bank notes satisfies, respectively,

$$\begin{aligned} rV_m &= \theta p_s[u(q_m) + V_o - V_m] + \theta(1 - p_s)[u(q_b) + V_o - V_m] \\ rV_n &= \theta[u(q_s) + V_m - V_n]. \end{aligned}$$

The zero-profit condition implies that any pair of $(q_s, q_b) \gg 0$ such that $q_s = R - q_b$ and satisfies the incentive constraints is a solution. From numerical experiments we find that this equilibrium exists when θ is small.

We now summarize some distinctions between this version and the version with a monopoly bank. First, under a competitive banking industry, there can exist equilibria where bank notes are redeemed solely in goods and do not circulate. The reason is as follows. Given the redemption value $q_r = R$, if R is big enough, it is possible that the fundamental value of notes is so high that people would rather hold on to them until redemption. Second, notes do not circulate when θ is small. When notes are redeemed solely in goods, the redemption price is pinned down by R , but the circulation value of notes, q_n , increases in θ . If the consumption tendency is low (θ is small), the circulating value would be low and so agents would rather hold on to them until redemption. However, a monopoly bank would adjust the redemption price in response to changes in θ . Consequently, it is not possible that the redemption value is high enough to prevent the notes from circulating. Third, the investment return R affects only the magnitude of profits of the monopoly bank in the basic model. Under competitive banks R determines the redemption price q_r and q_s and, thus, affects the existence results. Finally, neutrality of money may not hold under competitive banks. An increase in M increases p_m and reduces the fraction of producers and therefore, the total number of investment projects deposited in the banking sector. If a bank absorbs the extra liquidity by increasing vault cash, it incurs the cost but does not obtain the full benefit. This free-rider problem prevents the neutrality result from holding. A monopoly bank can internalize this externality effect.

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Table 1 Candidate Equilibria

| <u>The bank does not take fiat money deposits</u> | | |
|---|-----------------------------|----------------------|
| | Bank notes do not circulate | Bank notes circulate |
| Fiat money is not valued | N | $G1$ |
| Fiat money is valued | N | $G2$ |
| <u>The bank takes fiat money deposits</u> | | |
| | Bank notes do not circulate | Bank notes circulate |
| Fiat money is not valued | N | N |
| Fiat money is valued | $M1$ | $M2$ |

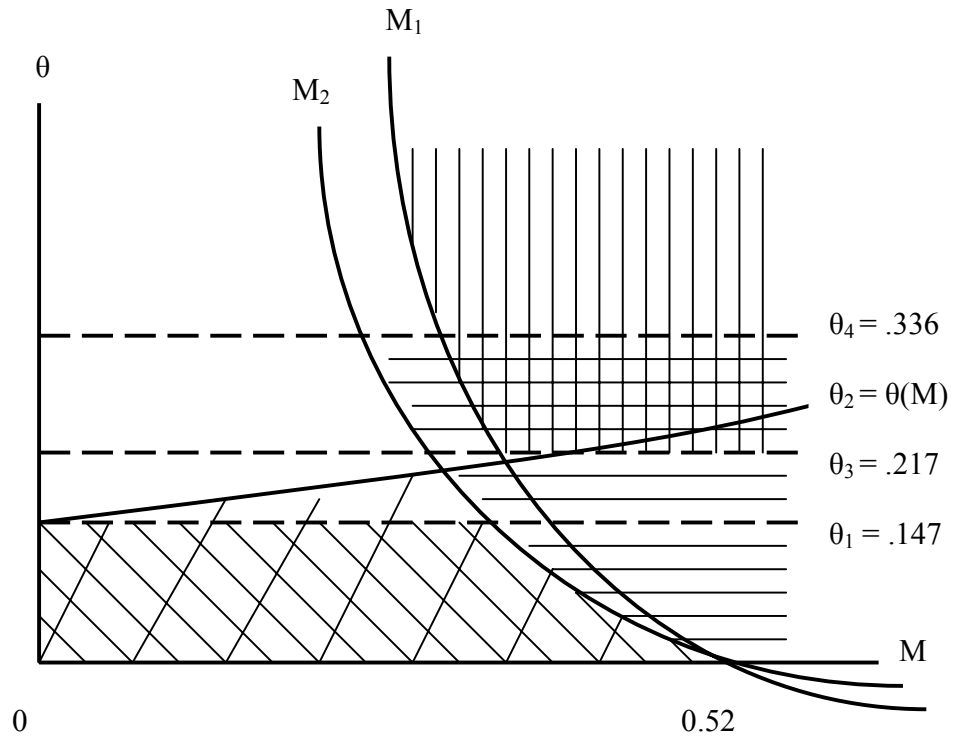
Key: N , equilibrium does not exist; $G1$, $G2$, bank notes are redeemed solely in goods; $M1$, $M2$, bank notes are redeemed in fiat money.




Table 2 Comparisons of Equilibria

| | | Equilibrium | p_o | p_n | p_m | V_m | V_n | $q_r(q_s)$ | π | W |
|-------|-------------------------|-------------|-------|-------|-------|-------|-------|------------|-------|------|
| Ex. 1 | $\theta = .25, M = .42$ | M2 | .44 | .28 | .28 | .437 | .645 | .102 | .054 | .307 |
| | | M1 | .48 | .19 | .33 | .465 | .735 | .174 | .059 | .295 |
| Ex. 2 | $\theta = .22, M = .42$ | M2 | .40 | .30 | .30 | .383 | .593 | .102 | .051 | .288 |
| | | M1 | .45 | .21 | .34 | .411 | .666 | .166 | .056 | .278 |
| | | G2 | .35 | .23 | .42 | .337 | .351 | .127 | .039 | .222 |
| Ex. 3 | $\theta = .13, M = .38$ | M2 | .32 | .34 | .34 | .181 | .381 | .096 | .037 | .192 |
| | | G2 | .30 | .32 | .38 | .171 | .273 | .105 | .032 | .152 |
| | | G1 | .44 | .56 | 0 | 0 | .282 | .089 | .048 | .158 |
| Ex. 4 | $\theta = .13, M = .25$ | G2 | .35 | .40 | .25 | .204 | .276 | .099 | .038 | .161 |
| | | G1 | .44 | .56 | 0 | 0 | .282 | .089 | .048 | .158 |

Note:

1. The example $u(q) = \sqrt{q}$ is used in Table 1. Parameter values are $r = .05, \gamma = .1, R = 1, \beta = .1$.
2. In an economy where fiat money is the unique medium of exchange, for the parameter values in Ex. 1 to Ex. 4, the welfare is .232, .217, .145 and .109, respectively, all of which are lower than the equilibria with private money.



-  Equilibrium G1 $\theta \leq \theta_1 ; M < M_2$
-  Equilibrium G2 $\theta \leq \theta_2 ; M < M_2$
-  Equilibrium M1 $\theta \geq \theta_3 ; M \geq M_1$
-  Equilibrium M2 $\theta \leq \theta_4 ; M \geq M_2$

Parameter Values: $r = 0.05, \gamma = 0.1, R = 1, \beta = 0.1$

Fig. 1. Existence of equilibria where the redemption rule is chosen by the bank