Online Supplementary Appendix to "Liquidity and the Threat of Fraudulent Assets"

This supplementary appendix establishes results to complement and extend the main analysis of the paper. Each section is self-contained and can be read separately.

- Appendix B formulates and solves the model of retention and haircuts outlined in the main body of the paper.
- Appendix C offers extensions of our benchmark model delivering fraud in equilibrium.
- Appendix D provides a formal proof of the outcome equivalence between the original game and the reverse-ordered game and it establishes that a milder refinement generates the same outcome as the one in the text.

B Asset retention and haircuts

Our benchmark model assumes that asset portfolios are private information, and that the production of fraudulent assets involves only a fixed cost. With this cost structure, private information is with no loss in generality as asset retention would have no signaling value. In the following we generalize the model to allow buyer to make their portfolio observable, assuming that fixed and proportional costs of producing fraudulent assets. Because of the proportional cost, a buyer who retains an observable amount of some asset in its portfolio, can credibly signal its value. As we will discuss, retention is a standard incentive mechanism for asset backed securities and, in the context of collateralized loans, can be interpreted in terms of haircuts.[34]

B.1 The bargaining game

We consider a variant of our model where, in the DM, agents can issue asset backed securities, or ABS. The set of assets that can back securities is denoted by $\{1, ..., S\}$. An asset can be a commodity, like gold, or it can be a financial asset such as a mortgage loan. Each asset comes in two versions: a genuine version, with time $t = 2$ payoff normalized to one, and a fraudulent version with zero payoff. At the end of $t = 0$ a buyer can create ABSs of type s. Each unit of ABS is backed by one unit of asset s. If s is gold, one can think of the ABS as a banknote. If s is a mortgage loan, then the ABS is a mortgage-backed security. The extent to which a security is backed by genuine or fraudulent versions of the asset is not observed by a seller in a match in the OTC market.

As before, fraud involves a fixed cost, $k_f(s)$, which is specific to the underlying asset and has to be incurred irrespective of the extent of fraud on an ABS of type s. Differently from before, there is also a proportional cost, $k_v(s)$, to produce a fraudulent asset of type s. Precisely, the variable cost to generate one unit of type s ABS backed by $\eta \in [0, 1]$ units of genuine assets and $1 - \eta$ units of fraudulent assets is $\eta\phi(s) + (1 - \eta)k_v(s)$. Such an ABS pays off η at the end of the period.

The reverse-ordered game has the following timing. First, the buyer posts an offer to be executed in the OTC market at time $t = 1$ provided a match is formed. This offer specifies, for all possible

[34] According to Krishnamurthy (2010) haircuts in the repo market have two main justifications: (i) the probability of a borrower defaulting on the repo loan; and (ii) the recovery value when liquidating the collateral if default occurs. Our model captures the second consideration. According to Krishnamurthy (2010) the first of these considerations is usually quite small while the second consideration has played a dramatic role in the crisis.

ABS backed by s, whose names are indexed by $j \in \mathcal{J}(s)$, a quantity $a(j) \geq 0$ of securities created by the buyer, and a security transfer $d(j) \leq a(j)$ to the seller.[35] Hence, $a(j) - d(j)$ is the observable quantity of security j retained by the buyer.[36]

Second, for each security, $j \in \mathcal{J}(s)$, the buyer chooses $\eta(j)$, the extent to which the security is backed by genuine assets. Third, if the buyer is matched with a seller in the OTC market, with probability σ, the seller observes the posted offer and chooses whether to accept it or not.

The expected payoff of the buyer is:

$$\begin{aligned} U^b \quad \equiv \quad & - \sum_{s \in S, j \in \mathcal{J}(s)} \left\{ [\phi(s) - 1]\, \eta(j) a(j) + k_v(s) \, [1 - \eta(j)] \, a(j) \right\} - k_f(s) \mathbb{I}_{\left\{ \sum_{j \in \mathcal{J}(s)} [1 - \eta(j)] a(j) > 0 \right\}} \\ & + \sigma \pi \left[u(q) - \sum_{s \in S, j \in \mathcal{J}(s)} \eta(j) d(j) \right]. \end{aligned} \tag{32}$$

The payoff, in (32), has the following interpretation. In the first term, for each possible security, $j \in \mathcal{J}(s)$, the buyer chooses the issue size $a(j)$ and the extent to which the security is backed by the asset, $\eta(j) \in [0, 1]$. The quantity of genuine asset s purchased by the buyer to back security j is then $\eta(j) a(j)$ and the cost of holding those assets is $\phi(s) - 1$; the quantity of fraudulent assets produced is $[1 - \eta(j)]\, a(j)$ at a unit cost of $k_v(s)$. In the second term, the fixed cost of fraud for asset s, $k_f(s)$, is incurred if some asset-s backed securities are not fully backed, $\sum_{j \in \mathcal{J}(s)} [1 - \eta(j)]\, a(j) > 0$. Finally, in the third term, the buyer is matched with a seller with probability σ and the seller accepts the offer with probability π. In that event, the buyer enjoys the utility of consumption, $u(q)$, but he gives up $d(j)$ units of security j, where each units pays off $\eta(j)$.

Given an offer, $o \equiv \langle \{a(j), d(j); s \in S, j \in \mathcal{J}(s)\}, q \rangle$, the buyer will choose optimally the extent

[35] We view $\mathcal{J}(s)$ as an exogenously given, finite, set of names for ABS backed by s. This is without much loss of generality: for example, $\mathcal{J}(s)$ could be the set of all N-letter words, for some large N. If $a(j) = 0$, the buyer chooses to issue no security with name j.

[36] In principle the asset demand of the buyer could include an unobservable part. That is, the buyer could show $a(j)$ to the seller and hide some extra amount $\Delta a(j)$, with a total asset demand equal to $a(j) + \Delta a(j)$. As will become clear later, in the case of interest of a liquidity shortage, $\phi(s) > 1$ for all s, and so a buyer will find it optimal to hold $\Delta a(j) = 0$. When liquidity is abundant, $\phi(s) = 1$ for all s, and buyers have no strict incentive to include an unobservable part in his asset demand. See the discussion at the beginning of Section B.2.

of fraud for each category of assets, given rational belief about the seller's acceptance probability:

$$\{\eta(j)\}_{j\in\mathcal{J}(s)} \in \arg\min_{\hat{\eta}(j)} \sum_{s\in S, j\in\mathcal{J}(s)} \left\{ [\phi(s)-1]\,\hat{\eta}(j)a(j) + k_v(s)\,[1-\hat{\eta}(j)]\,a(j) + \sigma\pi\hat{\eta}(j)d(j) \right\} + k_f(s)\mathbb{I}_{\left\{\sum_{j\in\mathcal{J}(s)}[1-\hat{\eta}(j)]a(j)>0\right\}} \tag{33}$$

Similarly, the seller chooses an acceptance probability given rational beliefs about asset quality:

$$\pi \in \arg\max_{\hat{\pi}} \hat{\pi} \left[-q + \sum_{s\in S, j\in\mathcal{J}(s)} \eta(j)d(j) \right]. \tag{34}$$

Definition 1 *A subgame perfect equilibrium of the reverse-ordered game is a collection of strategies, $\langle o, \eta, \pi\rangle$, such that:*

(i) The offer, o, maximizes U^b taking as given π and η.

(ii) Conditional on any offer o, the decision to commit fraud, η, solves (33) given π.

(iii) Conditional on any offer o, the decision to accept, π, solves (34) given η.

As in the main model, the auxiliary problem is to maximize U^b with respect to an outcome $\langle\{a(j), d(j), \eta(j); s\in S, j\in\mathcal{J}(s)\}, \pi, q\rangle$, subject to the incentive constraints of the buyer, (33), and that of the seller, (34).

Lemma 1 *The solution to the auxiliary problem is such that $\eta(j) = 1$ for all $j \in \mathcal{J}(s)$ and all $s \in S$.*

Proof. For any feasible outcome, consider the subset of $\eta(j)$ such that $\eta(j) \in (0,1)$. Because the outcome satisfies the incentive constraint, (33), the buyer must be indifferent between any $\eta(j) \in (0,1)$. Thus, for all such j, one can simultaneously set $\eta(j) = 1$ and increase q correspondingly so that π remains a solution of (34). Because q increases strictly, the payoff of the buyer increases strictly as well. Now consider the subset of $\eta(j)$ such that $\eta(j) = 0$. For all such j, one can simultaneously set $a(j) = d(j) = 0$ and $\eta(j) = 1$. The incentive constraints of the buyer and the seller are satisfied, and the payoff of the buyer increases strictly because this saves on the fixed cost. Therefore, any feasible offer in which $\eta(j) < 1$ for some j, is strictly dominated by an offer such that $\eta(j) = 1$ for all j, establishing the claim. ■

Denote $d(s)$ the transfer of all ABS of type s and $a(s)$ the quantity of all ABS of type s publicly held by the buyer. Then,

$$\begin{aligned} d(s) &= \sum_{j \in \mathcal{J}(s)} d(j) \\ a(s) &= \sum_{j \in \mathcal{J}(s)} a(j). \end{aligned}$$

Substituting $\eta(j) = 1$ for all j, the seller's best response, (34), can be reexpressed as a function of $d(s)$ only:

$$\pi \in \arg\max \hat{\pi} \left[-q + \sum_{s \in S} d(s) \right]. \tag{35}$$

Similarly, the buyer's objective can also be expressed as a function of $a(s)$ and $d(s)$ only:

$$U^b \equiv - \sum_{s \in S} [\phi(s) - 1] \, a(s) + \sigma \pi \left(u(q) - \sum_{s \in S} d(s) \right). \tag{36}$$

Equations (35) and (36) show that only $a(s)$ and $d(s)$, the aggregate quantities of ABS of type s, matter for the buyer's expected payoff and the seller's accept/reject decision. Still, the composition of the offer matters in principle for the buyer's fraud decision. To see this, note that with Lemma 1, the incentive constraints of the buyer writes:

$$\begin{aligned} \eta = \{1, \ldots, 1\} \in \arg\min_{\hat{\eta}(j)} \sum_{j \in \mathcal{J}(s)} \quad & \left\{ [\phi(s) - 1] \, \hat{\eta}(j) a(j) + k_v(s) \left[1 - \hat{\eta}(j) \right] a(j) + \sigma \pi \hat{\eta}(j) d(j) \right\} \\ & + k_f(s) \mathbb{I}_{\left\{ \sum_{j \in \mathcal{J}(s)} [1 - \hat{\eta}(j)] a(j) > 0 \right\}}, \end{aligned} \tag{37}$$

for all $s \in S$, and so it depends on the composition of the offer. We now show that there are no gain from varying the composition of an offer. Namely, a buyer always find it optimal to issue at most one ABS for each type of asset, i.e. $a(j) \neq 0$ for at most one $j \in \mathcal{J}(s)$.

Lemma 2 *Consider any feasible outcome such that $\eta(j) = 1$ for all j. Then there is a payoff equivalent feasible outcome in which the buyer issues at most one ABS for each type of asset. The corresponding incentive constraint can be written:*

$$[\phi(s) - 1 - k_v(s)] \, a(s) + \sigma \pi d(s) \leq k_f(s) \quad \textit{for all } s \in S. \tag{38}$$

Proof. Given any feasible outcome $\omega = \langle a(j), d(j), \eta(j), q, \pi \rangle$ such that $\eta(j) = 1$ for all j, consider the alternative outcome in which the buyer issues at most one security for each type s of asset: $\omega' = \langle a(s), d(s), \eta, \pi, q \rangle$, with $\eta = 1$ and where $a(s) = \sum_{j \in \mathcal{J}}(s) a(j)$ and $d(s) = \sum_{j \in \mathcal{J}}(s) d(j)$. Clearly the alternative outcome ω' has the same payoff for the buyer as the outcome ω, and is incentive compatible for the seller. All we need to show is that it is incentive compatible for the buyer, i.e.,

$$\eta = 1 \in \arg\min_{\hat{\eta}} \left[\phi(s) - 1\right] \hat{\eta} a(s) + \big(1 - \hat{\eta}\big) k_v(s) a(s) + \pi \hat{\eta} d(s) + k_f(s) \mathbb{I}_{\{(1-\hat{\eta})a(s) > 0\}}. \tag{39}$$

Because of the fixed cost, the arg min is included in $\{0, 1\}$, and so the alternative outcome ω' is incentive compatible if and only if it satisfies (38). But (38) is implied by the incentive compatibility constraint (37) for the original outcome ω: indeed, (38) is equivalent to the observation that the right-hand side of (37) evaluated at $\hat{\eta} = \{1, \ldots, 1\}$, is smaller than the right-hand side evaluated at $\hat{\eta} = \{0, \ldots, 0\}$. ■

Lastly, it is clear that any solution of the auxiliary problem must satisfy $q = \sum_{s \in S} d(s)$. Therefore, the auxiliary problem associated with the reverse-ordered game reduces to

$$\max_{q, a(s), d(s), \pi} \; -\sum_{s \in S} a(s) \left[\phi(s) - 1\right] + \sigma \pi \left[u(q) - q\right] \tag{40}$$

$$\text{s.t.} \quad d(s) \leq a(s) \tag{41}$$

$$-q + \sum_{s \in S} d(s) = 0 \tag{42}$$

$$\left[\phi(s) - 1 - k_v(s)\right] a(s) + \sigma \pi d(s) \leq k_f(s) \quad \text{for all } s \in S. \tag{43}$$

Claim 3 *Any solution of the auxiliary problem, (40)-(43), has the property that* $u'(q) \geq 1$ *and* $\pi = 1$.

Proof. The first claim holds because otherwise one could reduce the quantity produced, increase the expected utility of the buyer, and satisfy all the constraints. To prove the second claim suppose, towards a contradiction, that $\pi < 1$. Note first that the value of the auxiliary problem must be positive: a small offer $q' = d'(s_0) > 0$ where $d'(s_0) < k_f(s_0)$, $d'(s) = 0$ for $s \neq s_0$, and $\pi' = 1$ yields a positive payoff. This implies that both $q > 0$ and $\pi > 0$. Consider a deviation according to which the probability of the offer being accepted is increased by $\Delta\pi > 0$. At the same time, whenever

$d(s) > 0$, the transfer is reduced so that the incentive-compatibility constraint (43) continues to hold. To a first order, this implies a change $\Delta d(s) = -\frac{d(s)}{\pi}\Delta\pi$, for all $s \in S$. That is, the transfer is reduced if $d(s) > 0$, and remains equal to zero otherwise. Thus the output in any bilateral match is changed by $\Delta q = -\sum_{s\in S}\Delta d(s) = -\frac{\Delta\pi}{\pi}q$. The expected surplus of the buyer increases by

$$\Delta U = \sigma\left\{[u(q)-q] - \left[u'(q)-1\right]q\right\}\Delta\pi > 0.$$

The equality is strict because of two facts: first, $u(q)$ is strictly concave and second, $q > 0$, since the value of the auxiliary problem is positive. ■

Lastly we have:

Claim 4 *Any equilibrium outcome must solve the auxiliary problem, (40)-(43).*

The proof is the same as in the paper and is thus omitted. Using that $\pi = 1$, the resalability constraint, (43), can be reexpressed as

$$d(s) \leq \frac{\frac{k_f(s)}{a(s)} + k_v(s) + 1 - \phi(s)}{\sigma}a(s). \tag{44}$$

To mitigate fraud incentives, the buyer only spend a fraction of his holdings in a match, and retains the rest in his portfolio. This fraction decreases with the quantity of assets held, it increases with the fixed and proportional costs of producing fraudulent assets, it decreases with the price of the asset and the frequency of trading opportunities in the OTC market. As argued in the text, when interpreting the payment as a collateralized loan, asset retention can be interpreted as a haircut that the borrower concedes to the lender to signal the quality of his collateral.

B.2 Liquidity structure of asset returns

The demand for assets. Taking stock of the above result, the asset demands an offer solve, when liquidity is scarce and $\phi(s) > 1$ for all s:

$$\max_{q,a(s),d(s)} -\sum_{s\in S}a(s)\left[\phi(s)-1\right] + \sigma\left[u(q)-q\right] \tag{45}$$

$$\text{s.t.}\quad d(s) \leq a(s) \tag{46}$$

$$-q + \sum_{s\in S}d(s) = 0 \tag{47}$$

$$\left[\phi(s) - 1 - k_v(s)\right]a(s) + \sigma d(s) \leq k_f(s) \quad \text{for all } s \in S. \tag{48}$$

When liquidity is abundant and $\phi(s) = 1$ for some s, asset demands continue to solve the above program. To see this, note first that, in case $\phi(s) = 1$, one needs in principle to distinguish explicitly the amount of asset demanded, $a(s)$, and the observable amount of asset shown by the buyer to the seller, $c(s) \leq a(s)$. With this distinction in mind, one can show that the auxiliary problem is the same as before, after replacing the feasibility constraint (41) by $d(s) \leq c(s) \leq a(s)$ and the incentive compatibility constraint (44) by

$$[\phi(s) - 1 - k_v(s)]\, c(s) + \sigma\pi d(s) \leq k_f(s).$$

Next, observe that, when $\phi(s) = 1$, showing $c(s) = a(s)$ only relaxes the incentive compatibility constraint. Therefore, to determine asset prices, we can always assume that $c(s) = a(s)$ and that optimal asset demands and offers are determined by the problem (45)-(48).

The Lagrangian associated with the auxiliary problem is

$$\mathcal{L} = -\sum_{s\in S} [\phi(s) - 1]\, a(s) + \sigma [u(q) - q] + \xi \left[\sum_{s\in S} d(s) - q\right]$$
$$+ \sum_{s\in S} \lambda(s) \Big\{ k_f(s) - [\phi(s) - 1 - k_v(s)]\, a(s) - \sigma d(s) \Big\} + \sum_{s\in S} \nu(s) [a(s) - d(s)]\,.$$

The first-order conditions are

$$\xi = \sigma \left[u'(q) - 1\right] \tag{49}$$

$$\phi(s) = 1 + \nu(s) - \lambda(s) [\phi(s) - 1] + \lambda(s) k_v(s) \tag{50}$$

$$\xi = \sigma\lambda(s) + \nu(s). \tag{51}$$

The first equation, (49) is the derivative of the Lagrangian with respect to q and is the same as in the main model. The second equation, (50), is the derivatives with respect to $a(s)$ and differs from its counterpart in the main model. The left-hand side is the price. The right-hand side is the buyer's marginal value of increasing her observable asset holdings: it reflects the fundamental value of the asset, 1, a tightening of the no-fraud constraint because the buyer is making a larger purchase of genuine assets, $-\lambda(s) [\phi(s) - 1]$, and a loosening of the no-fraud constraint, $\lambda(s) k_v(s)$, because the cost of producing a corresponding quantity of fraudulent assets increases.

Equilibrium asset pricing. To derive the three-tier categorization of assets, we impose market clearing, i.e., $a(s) = A(s)$ for all $s \in S$, and we consider the case of a liquidity shortage, $q < q^*$.

Liquid assets ($\lambda(s) = 0$). Liquid assets are such that only the feasibility constraint, $d(s) \leq a(s)$, binds. It follows from (51) that $\nu(s) = \xi$ and from (50)

$$\phi(s) = 1 + \xi. \tag{52}$$

Moreover, $\nu(s) > 0$ implies $d(s) = A(s)$. The no-fraud constraint is not binding if

$$\frac{k_f(s)}{A(s)} + k_v(s) \geq \xi + \sigma.$$

Partially liquid assets ($\lambda(s) > 0$ and $\nu(s) > 0$). Partially-liquid assets are such that both the incentive-compatibility and the feasibility constraints bind. From $\nu(s) > 0$, $d(s) = A(s)$ and from the no-fraud constraint at equality, (44),

$$\phi(s) = (1 - \sigma) + k_v(s) + \frac{k_f(s)}{A(s)}. \tag{53}$$

From (50) and (51), the conditions $\lambda(s) > 0$ and $\nu(s) > 0$ can be rewritten as

$$\begin{aligned}
\lambda(s) &= \frac{\sigma + \xi - k_v(s) - \frac{k_f(s)}{A(s)}}{\frac{k_f(s)}{A(s)}} > 0 \\
\nu(s) &= \frac{\xi \frac{k_f(s)}{A(s)} - \sigma \left[\sigma + \xi - k_v(s) - \frac{k_f(s)}{A(s)}\right]}{\frac{k_f(s)}{A(s)}} > 0,
\end{aligned}$$

or, equivalently after some algebra,

$$\frac{k_f(s)}{A(s)} + k_v(s) < \sigma + \xi < \frac{k_f(s)}{A(s)} \frac{\sigma + \xi}{\sigma} + k_v(s).$$

The left-hand-side inequality is equivalent to $\lambda(s) > 0$, and the right-hand-side inequality to $\nu(s) > 0$.

Illiquid assets ($\lambda(s) > 0$ and $\nu(s) = 0$). Illiquid assets are such that the feasibility constraint does not bind, i.e., buyers spend only a fraction of their assets. From (50) and (51),

$$\phi(s) = 1 + \frac{k_v(s)\xi}{\sigma + \xi}. \tag{54}$$

Provided that $k_v(s) > 0$ illiquid assets are priced above their fundamental value. The reason why the asset price exhibits a liquidity premium is because an increase in asset holdings allows the buyer to raise the quantity of asset he spends. The intuition is that, when the buyer acquires one more unit of a genuine asset and makes it observable to the seller, he increases his cost of committing fraud. Because fraud incentives are reduced, he can make a larger transfer $d(s)$ to the seller. Finally, the condition $d(s) \leq A(s)$ is equivalent to

$$\frac{k_f(s)}{A(s)} + \frac{\sigma k_v(s)}{\sigma + \xi} \leq \sigma.$$

Solving for the equilibrium ξ. Adding up the payments $d(s)$ over all categories of assets we define aggregate liquidity is defined as before by $L = \sum_{s \in S} \theta(s)A(s)$, where

$$\theta(s) = \min\left[\frac{k_f(s)}{\sigma A(s)} + \frac{k_v(s)}{\sigma + \xi}, 1\right] . \tag{55}$$

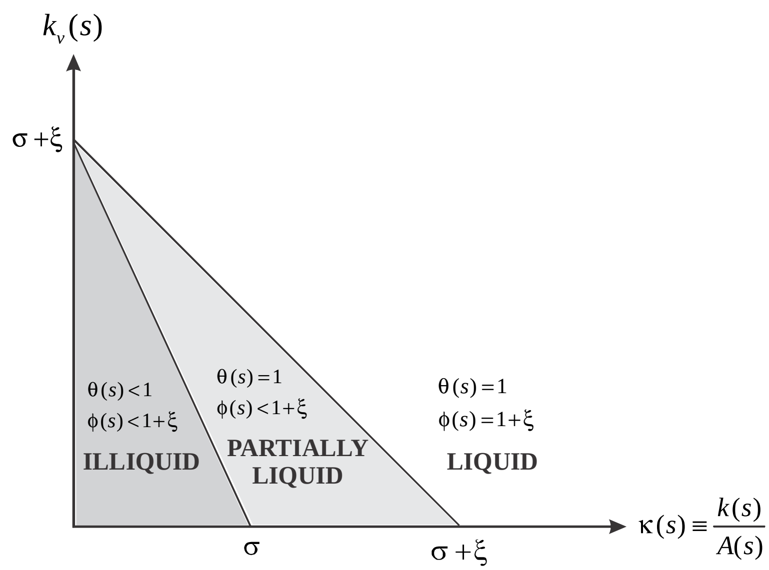

Figure 4: Liquidity structure

In Figure 4 we represent the three-tier categorization of assets as a function of the fixed and proportional costs of producing fraudulent assets. The liquid assets are the ones with the largest costs. For those assets, velocity is maximum, $\theta(s) = 1$, and the price is maximum and independent of the costs of fraud. Partially-liquid assets are such that the variable cost of fraud is lower than a threshold, $\sigma + \xi$, and the fixed cost is between two thresholds. Velocity is maximum but the asset price is less than the price of liquid assets. As $k_v(s)$ or $k_f(s)$ increases, the price of the asset increases. Finally, illiquid assets are such that both $k_v(s)$ and $k_f(s)$ are smaller than a threshold. Only a fraction of those assets are used for payments, $\theta < 1$, and their prices are less than the ones of liquid or partially liquid assets. The asset price increases with $k_v(s)$ but it is independent of $k_f(s)$. The liquidity of the asset increases with both $k_f(s)$ and $k_v(s)$.

Finally, to complete the characterization of the equilibrium we turn to the determination of q. Consider first equilibria where $q = q^*$. Then, from (49), $\xi = 0$. The first best can be implemented if $\sum_{s\in S} \theta(s)A(s) \geq q^*$, which can be rexpressed as

$$\sum_{s\in S} \min\left[A(s), \frac{k_f(s) + A(s)k_v(s)}{\sigma}\right] \geq q^*. \tag{56}$$

Consider next equilibria where $q < q^*$. From the definition of aggregate liquidity, $q = L = \sum_{s\in S} \theta(s)A(s)$, which gives

$$q = \sum_{s\in S} \min\left[A(s), \frac{k_f(s)}{\sigma} + \frac{A(s)k_v(s)}{\sigma u'(q)}\right].$$

When (56) does not hold then the intermediate value theorem implies there exists some $q < q^*$ solving the above equation.

B.2.1 Self-fulfilling liquidity shortages

A new feature of the present model is the possibility of multiple equilibria. Some preliminary intuition can be gained by thinking of an equilibrium in terms of supply and demand of liquidity.

Liquidity demand. Given a shadow price of liquidity, ξ, the demand is determined by the familiar first-order condition:

$$\xi = \sigma\left[u'(q^d) - q\right]$$

and is naturally downward sloping.

Liquidity supply. Now consider the problem of maximizing (45)-(48) with respect to $a(s)$ and $d(s)$ only, given shadow price ξ. One can view it as the problem of maximizing the value of liquidity services using assets as input, subject to feasibility and incentive compatibility. After imposing equilibrium in the asset market, one obtains that the aggregate amount of liquidity supplied is:

$$q^s = \sum_{s \in S} \min \left\{ A(s); \frac{k_f(s)}{\sigma} + \frac{A(s) k_v(s)}{\sigma + \xi} \right\}.$$

Its key novel feature is that, due to the new incentive compatibility constraint, this supply curve can be downward slopping.

Precisely, for liquid and partially liquid assets, the supply of liquidity is equal to $A(s)$ and so is inelastic in $\xi > 0$. For liquid assets this is because the incentive compatibility constraint does not bind and so in an asset market equilibrium all the supply of liquid assets is used to produce liquidity services. For partially liquid assets this is because the asset price adjusts until all the asset supply is used to produce liquidity services.

Thus, the downward slopping liquidity supply curve arises because of illiquid assets. Indeed, when ξ is high even the prices of illiquid assets go up. The incentives to commit fraud go up, the resalability constraint tightens, and the aggregate supply of liquidity goes down.

A one asset example. For simplicity, we illustrate the possibility of multiple equilibria in the single-asset case, $S = 1$. The output in a bilateral match in the DM is determined by:

$$q = \min \left[q^*, A, F(q) \right], \tag{57}$$

where

$$F(q) \equiv \frac{k_f}{\sigma} + \frac{A k_v}{\sigma u'(q)}.$$

If the resalability constraint is binding, then output is determined by $F(q) = q$. Otherwise it is the minimum between the socially-efficient quantity, q^*, and the supply of assets, A.

Suppose that $F(0) = \frac{k_f}{\sigma} < \min[q^*, A]$. It is a necessary condition for the asset to be illiquid. Moreover, $F(\infty) = \infty$ and $F'(q) > 0$ since $u''(q) < 0$. We distinguish the following cases.

1. Suppose $F''(q) < 0$. This condition is satisfied for $u(q) = \frac{q^{1-\gamma}}{1-\gamma}$ with $\gamma < 1$. Then, the right

side of (57) is concave. Provided that $k_f > 0$ there is a unique $q \in (0, q^*]$ solution to (57). See left panel of Figure 5.

2. Suppose next that $F''(q) > 0$ and $A < q^*$. This condition is satisfied for $u(q) = \frac{(q+b)^{1-\gamma} - b^{1-\gamma}}{1-\gamma}$ with $\gamma > 1$. Assume $F(A) > A$, i.e., $\frac{k_f}{A} + \frac{k_v}{u'(A)} > \sigma$. If there is a solution to $F(q) = q$ with $q \in (0, A)$, then there is a second solution in $(0, A)$. Moreover, $q = A$ is also a solution to (57). In summary, there are two solutions where the asset is illiquid, and one solution where the asset is partially liquid or liquid. See right panel of Figure 5. Equilibria with a higher q are associated with a lower asset price and a higher welfare.

3. Suppose next that $F''(q) > 0$ and $q^* < A$. This case is similar as the case above. There can be multiple equilibria, one of them being such that $q = q^*$ and $\phi = 1$.

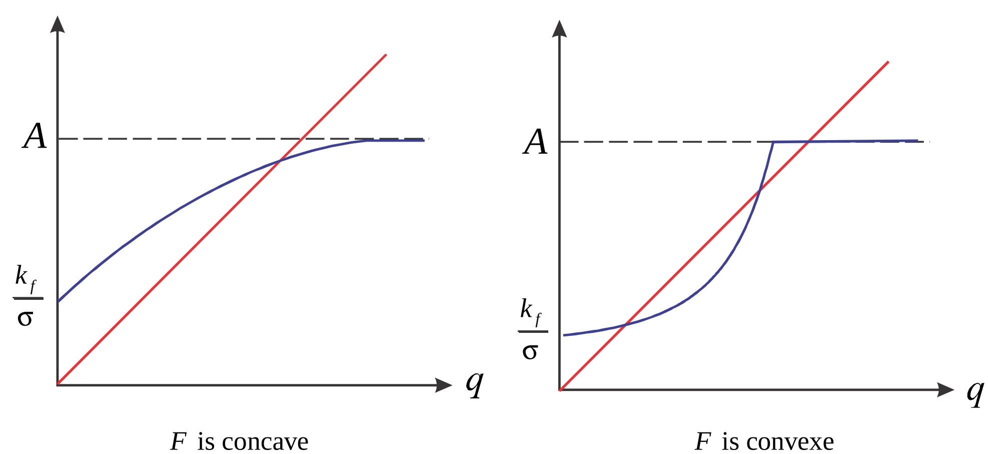

Figure 5: Left panel: Unique equilibrium; Right panel: Three equilibria including two with a binding resalability constraint

In the presence of multiple equilibria the high equilibrium corresponds to times of confidence: output is high, there are no haircut, and liquidity premia are low. The low equilibrium corresponds to times of fear: output is low, haircuts are large, and liquidity premia are large.

It is also worth noticing that retention mechanisms also emerge in models with adverse selection (e.g., DeMarzo and Duffie, 1999; Rocheteau, 2011) but typically they do not generate multiple equilibria. In our model the fact that agents can choose between genuine and fraudulent assets, and the price of genuine assets is endogenous, is crucial to obtain the multiplicity of equilibria.

C More on fraud in equilibrium

In this Appendix we review extensions of our benchmark model that deliver fraud in equilibrium. First, we provide detailed derivations for the model of Section 3.2. Second, we allow the cost of fraud to be drawn from a continuous distribution. Third, we introduce a proportional component to the cost of fraud and we assume observable portfolios.

C.1 Detailed derivations for the model of Section 3.2

We assume that there is uncertainty about the cost of fraud and that offers in the DM are set before the cost of fraud is realized. We take the reverse-ordered game as the primitive game and the sequence of moves is as follows. At the beginning of $t = 0$, buyers set the terms of a contract, $(q, \{d(s)\})$, to be executed in a bilateral match in the DM. For simplicity there are only two possible realizations for the cost of fraud: $k(s) > 0$ with probability $\omega(s) \in (0, 1]$ and zero with complement probability, $1 - \omega(s)$. The realizations for $k(s)$ are independent across assets and across buyers. So a fraction $1 - \omega(s)$ of buyers can produce fraudulent assets of type s at no cost. Next, buyers make their portfolio choices of both genuine and fraudulent assets. At $t = 1$ a fraction σ of buyers and sellers are matched and they trade according to the posted offers.

In the state where the fraud on asset s is costless the buyer will always find it profitable to execute his offer with fraudulent assets irrespective of the seller's probability of accepting. In the state where the fraud on asset s is costly a buyer will acquire fraudulent assets of type s with probability $\eta(s)$, where $\eta(s)$ minimize the cost of finance, i.e.

$$\{\eta(s)\} \in \arg\min_{\{\hat{\eta}(s)\}} \sum_{s\in S} \left\{ k(s)\left[1 - \hat{\eta}(s)\right] + \left[\phi(s) - 1\right]\hat{\eta}(s)d(s) + \sigma\pi\hat{\eta}(s)d(s) \right\}. \tag{58}$$

Multiplying the buyer's payoff by $\omega(s) > 0$ the condition (58) can be reexpressed as

$$\{\eta(s)\} \in \arg\min_{\{\hat{\eta}(s)\}} \sum_{s\in S} \left\{ \omega(s)k(s)\left[1 - \hat{\eta}(s)\right] + \left[\phi(s) - 1\right]\hat{\eta}(s)\omega(s)d(s) + \sigma\pi\hat{\eta}(s)\omega(s)d(s) \right\}. \tag{59}$$

Following an offer (q, d) the seller's strategy is:

$$-q + \sum_{s\in S} \eta(s)\omega(s)d(s) \begin{array}{c} > \\ < \\ = \end{array} 0 \Rightarrow \pi \begin{array}{l} = 1 \\ = 0 \\ \in [0,1] \end{array}. \tag{60}$$

Finally, given equilibrium decision rules $\{\eta(s)\}$ and π, the optimal offer, $(q, \{d(s)\})$, maximizes the expected utility of consumption net of the expected cost of financing the offer:

$$-\sum_{s\in S}\left\{\omega(s)k(s)\left[1-\eta(s)\right]+\omega(s)\left[\phi(s)-1\right]\eta(s)d(s)\right\}$$
$$+\sigma\omega(s)\pi\left\{u(q)-\sum_{s\in S}\eta(s)d(s)\right\}+\sigma\left[1-\omega(s)\right]\pi u(q). \tag{61}$$

Recall that with probability $1-\omega(s)$ the buyer can produce fraudulent assets at no cost, in which case he enjoys the utility of consumption if the offer is accepted without incurring any cost. Eq. (61) can be simplified to

$$-\sum_{s\in S}\left\{\omega(s)k(s)\left[1-\eta(s)\right]+\omega(s)\left[\phi(s)-1\right]\eta(s)d(s)\right\}+\sigma\pi\left\{u(q)-\sum_{s\in S}\eta(s)\omega(s)d(s)\right\}. \tag{62}$$

It is clear from (59), (60), and (62) that the solution to the auxiliary problem is identical to the one in the main text $d(s)$ is replaced by $\omega(s)d(s)$ and $k(s)$ is replaced by $\omega(s)k(s)$. This leads to the proposition in Section 3.2.

C.2 Generalized model

In the previous Section we made the assumption that the cost of fraud was randomly drawn from a two-point distribution, which allowed us to immediately apply all the results of the paper, after rescaling variables. In this section we generalize the analysis by assuming that the cost of fraud is drawn from a continuous distribution. We show that fraud continues to occur in equilibrium. Namely, buyers find it optimal to commit fraud if the cost of fraud is lower than some endogenous threshold, and do not commit fraud otherwise.

Suppose for simplicity there is a single asset ($S = 1$) with price $\phi \geq 1$. As in Section 3.2, we take the reverse-ordered game as our primitive game: we assume that buyers commit to a contract before making portfolios choices and entering the DM. The cost of fraud, k, is random and it is only realized after terms of trade, (q, d), are set. It is drawn from some continuous distribution, with cumulative distribution function (cdf) $F(k)$ over the support $[0, \infty)$. We assume that $F(k)$ has thin enough tail in that $k\left[1-F(k)\right] \to 0$ as $k \to \infty$. The buyer chooses an offer and a decision to commit fraud conditional on the offer and on the realization of k, $\{\eta_k\}$. The seller chooses the probability of accepting, π, conditional on any offer. Following an offer, (q, d), the seller's strategy is:

$$-q + d \int \eta_k dF(k) \begin{array}{c} > \\ = \\ < \end{array} 0 \Rightarrow \pi \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array}. \tag{63}$$

Because the seller does not observe the cost of fraud, he forms a rational expectation about the buyer's probability of committing fraud, based on the buyer's strategy $\{\eta_k\}$ and on the cdf $F(k)$. Given this probability, the seller accepts an offer if the expected value of the asset proposed by the buyer is greater the cost of producing the output.

Conditional on k, the buyer's strategy is:

$$k \begin{array}{c} > \\ = \\ < \end{array} [\phi - 1 + \sigma\pi] \, d \Rightarrow \eta_k \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array}. \tag{64}$$

The buyer's strategy is identical to the one described in the text. The cost of fraud, k, is compared to the cost of holding and spending d units of genuine assets, $[\phi - 1 + \sigma\pi] \, d$. If the cost of fraud is greater than the holding cost, then the buyer acquires genuine assets; otherwise, he commits fraud.

The buyer's problem. Given the buyer's and seller's strategies in the subgame following an offer, (q, d), and a realization of k, the buyer chooses an offer that maximizes his expected payoff. The buyer's problem is then

$$\max_{q,d,\{\eta_k\},\pi} \int \Big\{ -(\phi - 1)\eta_k d - (1 - \eta_k)k + \sigma\pi \left[u(q) - \eta_k d\right] \Big\} dF(k) \tag{65}$$

subject to $\{\eta_k\}$ and π satisfying (63) and (64). The buyer's objective is the sum of his expected surplus from trade net of the cost of financing the trade for all possible realizations of k. Following the same reasoning as in the proof of Proposition 1 one can rule out outcomes such that $\eta_k \in (0, 1)$. Moreover, for each offer, (q, d), there is a threshold for k above which fraud is not optimal. This threshold is

$$\breve{k} = [\phi - 1 + \sigma\pi] \, d. \tag{66}$$

This threshold is equal to the cost of carrying d units of genuine asset. Finally, it is easy to check that the seller's participation constraint must hold at equality,

$$q = d \left[1 - F(\breve{k})\right]. \tag{67}$$

From the results above the buyer's objective can be reexpressed as:

$$-\int_0^{\breve{k}} kdF(k) - (\phi - 1 + \sigma\pi)\, d\left[1 - F(\breve{k})\right] + \sigma\pi u(q).$$

Using (67) to eliminate d from the objective and from the constraint, we obtain that the buyer solves:

$$\max_{q,\breve{k},\pi} - \int_0^{\breve{k}} kdF(k) - (\phi - 1 + \sigma\pi)\, q + \sigma\pi u(q)$$
$$\text{s.t.} \quad \breve{k}\left[1 - F(\breve{k})\right] = [\phi - 1 + \sigma\pi]\, q.$$

Using integration by parts the buyer's problem simplifies further to

$$\max_{q,\breve{k},\pi} - \int_0^{\breve{k}} [1 - F(k)]\, dk + \sigma\pi u(q) \tag{68}$$
$$\text{s.t.} \quad \breve{k}\left[1 - F(\breve{k})\right] = (\phi - 1 + \sigma\pi)\, q. \tag{69}$$

Now we establish that $\pi = 1$. Suppose $\pi < 1$. From (69) one can reduce q and raise πq so as to keep $\breve{k}$ constant, i.e., $\Delta(\pi q) = -\frac{(\phi-1)}{\sigma}\Delta q$. Equivalently, π increases by $\Delta\pi = -\left[\frac{\phi-1+\pi\sigma}{\sigma}\right]\frac{\Delta q}{q}$ (which is feasible since $\pi < 1$ and provided that Δq is small). Given that $\breve{k}$ is kept constant we can focus on the second term in the buyer's objective, which can be written as $\sigma\pi q\frac{u(q)}{q}$. This term is increasing in πq and decreasing in q. Therefore, such a deviation is profitable and $\pi < 1$ is not optimal. The equilibrium offer is chosen so that it is accepted with probability one. The final representation of the buyer's problem is then

$$\max_{q,\breve{k}} - \int_0^{\breve{k}} [1 - F(k)]\, dk + \sigma u(q) \tag{70}$$
$$\text{s.t.} \quad \breve{k}\left[1 - F(\breve{k})\right] = (\phi - 1 + \sigma)\, q. \tag{71}$$

The constraint, (71), defines q as a continuous function of $\breve{k}$, $Q(\breve{k})$, with $Q(0) = \lim_{k\to\infty} Q(k) = 0$. If the buyer chooses not to never commit fraud then $\breve{k} = 0$, $q = Q(0) = 0$, no trade can take place, and the buyer's expected payoff is zero. Alternatively, if chooses to always commit fraud, then $\breve{k} = \infty$, $Q(\infty) = 0$, no trade can take place either, and the buyer's expected payoff is $-\mathbb{E}[k]$. Moreover, given $u'(0) = \infty$, the buyer's expected payoff increases for low values of $\breve{k}$. Consequently, it is optimal for the buyer to choose $\breve{k} \in \left(0, \bar{k}\right)$. This shows that in equilibrium fraud will occur when the buyer draws a sufficiently low cost of fraud.

C.3 Fraud with proportional costs and observable portfolios

We now consider the version of the model with both fixed and proportional costs of fraud and observable portfolios. For simplicity there are only two possible realizations for the cost of fraud: $[k_f(s), k_v(s)] = \left[\bar{k}_f(s), \bar{k}_v(s)\right] > 0$ with probability $\omega(s) \in (0,1]$ and $[k_f(s), k_v(s)] = (0,0)$ with complement probability, $1-\omega(s)$. In the state where the fraud on asset s is costless the buyer will always find it profitable to execute his offer with fraudulent assets irrespective of the seller's probability of accepting the offer. In the state where the fraud on asset s is costly we can follow the reasoning in the text to show that buyers execute the offer with genuine assets only. Moreover, buyers choose offers that are accepted with probability one, $\pi = 1$. The equilibrium offers, $(q, \{d(s), a(s)\})$, solve:

$$\max_{q,\{d(s),a(s)\}} \left\{ -\sum_{s\in S} [\phi(s) - 1]\,\omega(s)a(s) + \sigma\,[u(q) - q] \right\} \tag{72}$$

$$\text{s.t.} \quad \sum_{s\in S} \omega(s)d(s) - q = 0 \tag{73}$$

$$\left[\phi(s) - 1 - \bar{k}_v(s)\right] a(s) + \sigma d(s) \le \bar{k}_f(s), \quad \text{for all } s \in S. \tag{74}$$

The problem (72)-(74) is identical to the one in the text where $d(s)$ is now replaced by $\omega(s)d(s)$, $a(s)$ is replaced by $\omega(s)a(s)$, and $k_f(s)$ is replaced by $\omega(s)\bar{k}_f(s)$. Notice that the proportional cost does not need to be rescaled since the expected cost of fraud is the expected fixed cost, $\omega(s)\bar{k}_f(s)$, plus the expected proportional cost, $\omega(s)\bar{k}_v(s)a(s)$, and asset holdings are already scaled up by a factor $\omega(s)$.

In order to endogenize asset prices we simply have to add the following market clearing condition, $\omega(s)a(s) = A(s)$. Therefore, it is clear that the asset pricing implications of the model are unaffected once the fixed cost of fraud is scaled up by $\omega(s)$. The amount of fraud in equilibrium is determined as follows. Buyers who have an opportunity to commit fraud on asset s at no cost, with probability $1-\omega(s)$, will produce $a(s) = A(s)/\omega(s)$ units of fraudulent assets. Note the difference with the case of unobservable portfolio: the buyer must "show" to the seller that he holds $a(s)$ units of assets and so he may need to fake more than the transfer $d(s)$. Therefore, the occurrence of fraud is $\frac{\sigma[1-\omega(s)]}{\omega(s)}A(s)$. The costs of fraud affect asset prices and resalability constraints but they do not affect the occurrence of fraud. Occurrence of fraud is a decreasing function of $\omega(s)$.

Our description for a flight to liquidity can now be reinterpreted as a decrease in $\omega(s)$ for a class of asset, $\hat{s}$. This means that buyers receive more opportunities to produce fraudulent versions of asset $\hat{s}$ at a very low cost. This effect to this shock for asset prices and aggregate liquidity are identical to the ones described in the main text. Moreover, in equilibrium the occurrence of fraud increases.

D Additional remarks on the bargaining game

In the following we provide a formal argument for the equivalence of the outcomes of the original game and the reverse-ordered game. We also show that a weaker refinement than the one used in the main text, requiring that offers satisfying the resalability constraints are considered as genuine, delivers the same outcome as the one in Proposition 1.

Sellers and buyers' strategies. The seller's strategy specifies a probability of acceptance $\pi(o)$ conditional on any offer $o \equiv (q, \{d(s)\})$. In both the original and the reversed ordered game, the buyer's behavioral strategy generates a joint probability distribution $\eta(p, o, t)$ over portfolios $p = \{a(s), \tilde{a}(s)\}$, offers $o = (q, \{d(s)\})$ and transfers $t = \{\tau(s), \tilde{\tau}(s)\}$ satisfying $\tau(s) + \tilde{\tau}(s) = d(s)$, $\tau(s) \leq a(s)$, and $\tilde{\tau}(s) \leq \tilde{a}(s)$. Conversely, any joint distribution over (p, o, t) is generated by some behavioral strategy of the original game. To see this, factor $\eta(p, o, t) = \eta(p) \times \eta(o, t \,|\, p)$, where $\eta(o, t \,|\, p)$ can be picked arbitrarily for p outside the support of $\eta(p)$. By construction, $\eta(p)$ and $\eta(o, t \,|\, p)$ form a behavioral strategy of the original game generating $\eta(p, o, t)$. The same argument, associated with the factorization $\eta(o) \times \eta(p, t \,|\, o)$, applies to the reverse-ordered game. Taken together, this discussion shows that, in both the original and the reverse-ordered game, one can think of the buyer as directly picking a joint probability distribution over p, o, and t. This makes it clear that, given the sellers' acceptance probability, $\pi(o)$, the sequence of moves does not matter for the buyer's problem.

Simplifying the buyer's expected payoff. The buyer's expected payoff is, then:

$$\mathbb{E}_\eta \left[-\sum_s \left\{ k(s)\mathbb{I}_{\{\tilde{a}(s)>0\}} + [\phi(s) - 1]\, a(s) \right\} + \sigma\pi\,(o) \left\{ u(q) - \sum_{s \in S} \tau(s) \right\} \right].$$

We can immediately make the following simplifications about the buyer's strategy. First, we can restrict our attention to portfolios that have the property that either $a(s) = 0$ or $\tilde{a}(s) = 0$. To see this, suppose η assigns a positive probability to a portfolio with $\tilde{a}(s) > 0$ and $a(s) > 0$. If the subsequent offers are never accepted, then the buyer can increase his payoff if he does not commit fraud, i.e. replacing the portfolio by $\tilde{a}(s) = a(s) = 0$ and making the offer $q = d(s) = 0$. If the subsequent offers are accepted with positive probability, the buyer can now increase his payoff by replacing the portfolio with $\tilde{a}'(s)=\tilde{a}(s)+a(s)$, $a'(s)=0$, while keeping the offer unchanged. Second, since $\phi(s)\geq 1$, we can assume without loss that all portfolios and offers in the support of η have the property that $a(s)=d(s)$ when $a(s)>0$, and $\tilde{a}(s)=d(s)$ when $\tilde{a}(s)>0$. With this in mind, we can represent the buyer's strategy as a joint probability distribution over $(\{\chi(s)\},\{d(s)\},q)$, where $\chi(s)=0$ if the buyer's chooses to commit fraud, and $\chi(s)=1$ otherwise.

The buyer's expected payoff becomes:

$$\mathbb{E}_{\eta}\left[-\sum_{s}\left\{k(s)\left[1-\chi(s)\right]+\left[\phi(s)-1\right]\chi(s)d(s)\right\}+\sigma\pi\left(o\right)\left\{u(q)-\sum_{s\in S}\chi(s)d(s)\right\}\right]. \tag{75}$$

The buyer chooses a joint probability distributions η over fraud decisions and offers to maximizes the above objective given $\pi(o)$. Note that if we apply the law of iterated expectations and let $\eta(s\,|\,o)\equiv\mathbb{E}_{\eta}\left[\chi(s)\,|\,o\right]$, we can rewrite the the buyer's objective as:

$$\mathbb{E}_{\eta}\left[-\sum_{s}\left\{k(s)\left[1-\eta(s\,|\,o)\right]+\left[\phi(s)-1\right]\eta(s\,|\,o)d(s)\right\}+\sigma\pi\left(o\right)\left\{u(q)-\sum_{s\in S}\eta(s\,|\,o)d(s)\right\}\right], \tag{76}$$

where the outer expectation is taken with respect to the marginal distribution over offers, o.

The seller's payoff. After observing an offer $o=(q,\{d(s)\})$, the seller has beliefs $\gamma(s\,|\,o)$ about the probability that type-s assets are genuine, and chooses to accept with a probability $\pi(o)$ solving:

$$\pi(o)\in\arg\max_{\hat{\pi}}\hat{\pi}\left\{-q+\sum\gamma(s\,|\,o)d(s)\right\}. \tag{77}$$

A PBE of the original game is made up of a joint probability distribution η^{*} over fraud decisions, $\{\chi(s)\}$, and offers, $o\equiv(\{d(s)\},q)$, a probability of accepting conditional on any offer $\pi^{\star}(o)$, and a belief system $\gamma^{\star}(s\,|\,o)$ solving the buyer's and seller's problems, (75) and (77). Moreover, the seller's beliefs have to be consistent with Bayes' rule whenever applicable. Since we can only apply Bayes' rule on the equilibrium path, the restriction is:

$$\gamma^{\star}(s\,|\,o)=\eta^{\star}(s\,|\,o), \tag{78}$$

for all offers o in the support of $\eta^{\star}$. Beliefs are not pinned down for offers out of the support, because these do not materialize on the equilibrium path.

Outcome equivalence between the original and the reverse-ordered game. Take as the belief system of the original game the one specified by the reverse-ordered game. It follows immediately that the acceptance rule of the seller, $\pi(o)$, is the same for the two games. But in that case the solution to (75) is also the solution to the auxiliary problem of the reverse-ordered game, i.e., both games have the same portfolio choices and the same offer. This result corresponds to the outcome equivalence proposition of In and Wright (2011).

The auxiliary problem gives an upper bound to the buyer's payoff in the original game. Any candidate PBE must satisfy two necessary conditions. First, from (77) and (78), given any equilibrium offer, o in the support of $\eta^\star$, the seller's acceptance probability must solve

$$\pi(o) \in \arg\max_{\hat{\pi}} \hat{\pi}\left\{ -q + \sum_s \eta^\star(s \,|\, o) d(s) \right\}. \tag{79}$$

From (76), for all offers o in the support of $\eta^\star$, the conditional probability of genuine asset of type s, $\eta^\star(s \,|\, o)$, has to solve:

$$\eta^\star(s \,|\, o) \in \arg\min_{\hat{\eta}(s)} \quad k(s)\left[1 - \hat{\eta}(s)\right] + \left[\phi(s) - 1\right] \hat{\eta}(s) d(s) + \hat{\eta}(s)\sigma\pi(o)d(s). \tag{80}$$

Maximizing the buyer's objective over all possible equilibrium outcomes satisfying (79) and (80)—which corresponds to the auxiliary problem—we obtain an upper bound on the buyer's payoff in any PBE.

A weaker refinement. Consider an out-of-equilibrium offer, $o = \langle q, \{d(s)\}\rangle$, that has the property that, for some asset s_0, $d(s_0)$ satisfies the resalability constraint with a strict inequality i.e., $d(s_0) < \frac{k(s_0)}{\phi(s_0)-1+\sigma}$. Then, for any $\pi \in [0,1]$, $\left[\phi(s_0) - 1\right] d(s_0) + \sigma\pi d(s_0) < k(s_0)$. Therefore, any strategy with $\eta(s_0) < 1$ is strictly dominated by the same strategy with $\eta(s_0) = 1$. Therefore, a seller who receives the offer, o, should not believe that the buyer is playing a strictly dominated strategy and should view asset s_0 as genuine, $\gamma(s_0 | \, o) = 1$. To get more intuition for this requirement, suppose $\phi(s_0) = 1$. If the seller receives an offer such that the value of the asset is less than the fixed cost of fraud, $\phi(s_0)d(s_0) < k(s_0)$, then he should think that no buyer would have incentives to incur the fixed cost of fraud to make such an offer, irrespective of what the buyer thinks of the seller's strategy. We will adopt this minimum requirement as our refinement of γ.

Refinement: *The seller's belief system,* $\gamma^{\star}(s \mid o)$*, must be such that for any out-of-equilibrium offer,* o*, which is not in the support of* $\eta^{\star}$ *and such that* $d(s_0) < \frac{k(s_0)}{\phi(s_0)-1+\sigma}$ *for some asset,* s_0*, then* $\gamma^{\star}(s_0 \mid o) = 1$.

This argument implies that any offer that satisfies

$$\sum_{s \in S} d(s) - q > 0$$
$$d(s) < \frac{k(s)}{\phi(s) - 1 + \sigma}, \quad \text{for all } s \in S$$

should be accepted with probability one. The system of beliefs generated by the reverse-ordered game satisfies this condition.

Calculating the maximum buyer's payoff over all offers satisfying the above two constraints, we obtain a lower bound over the buyer's payoff in any PBE satisfying this refinement. Note that, by Proposition 1, this lower bound is equal to the value of the auxiliary problem. But we also know that the value of the auxiliary problem is an upper bound for the buyer's expected payoff across all PBE. Therefore, any PBE passing the refinement solves the auxiliary problem, with asset demands and offers given as in Proposition 1.

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