Liquidity and the Threat of Fraudulent Assets*

Yiting Li Guillaume Rocheteau Pierre-Olivier Weill

September 24, 2012

Abstract

We study an over-the-counter (OTC) market with bilateral meetings and bargaining, where the usefulness of assets as a means of payment or collateral is limited by the threat of fraudulent practices. We assume that agents can produce fraudulent assets at a positive cost, which generates endogenous upper bounds on the quantity of each asset that can be sold or posted as collateral in the OTC market. Each endogenous, asset-specific, resalability constraint depends on the vulnerability of the asset to fraud, on the frequency of trade, and on the price of the asset. In equilibrium, the set of assets can be partitioned into three liquidity tiers, which differ in their resalability, prices, sensitivity to shocks, and responses to policy interventions. The dependence of an asset's resalability on its price creates a pecuniary externality, which leads to the result that some policies commonly thought to improve liquidity can be welfare reducing. Finally, an extension of our model with endogenous, asset-specific haircuts can explain a liquidity crisis caused by a heightened threat of fraud.

J.E.L. Classification: D82, D83, E40, E50, G10

Keywords: search, liquidity, fraud, money, collateral.

*A previous version of this paper was circulated under the title "Liquidity Constraints." We thank two anonymous referees and Rob Shimer for comments that greatly improved the paper. We also thank Veronica Guerrieri, Ben Lester and Neil Wallace for insightful discussions, Andy Atkeson, Bill Branch, Dean Corbae, Harold Demsetz, Manolis Galenianos, Hugo Hopenhayn, Thomas Philippon, Shouyong Shi, Amir Yaron, seminar participants at the University of California at Riverside and Santa Cruz, the Bank of Canada, the Federal Reserve Bank of Chicago, the Federal Reserve Bank of Cleveland, the Leeds School of Business at the University of Colorado, the Federal Reserve Bank of New-York, New-York University, Sciences-Po Paris, the National University of Seoul, the Singapore Management University, the Southern Methodist University, National Taiwan University, the University of Wisconsin, the Wharton school at the University of Pennsylvania, the University of Chicago Booth School of Business, the Federal Reserve Bank of Minneapolis and the Federal Reserve Bank of New York for useful comments. We thank Monica Crabtree-Reusser and Liz Galvin for editorial assistance. Yiting Li: National Taiwan University, yitingli@ntu.edu.tw; Guillaume Rocheteau: University of California, Irvine and Federal Reserve Bank of Cleveland, grochete@uci.edu; Pierre-Olivier Weill: University of California, Los Angeles, NBER, and CEPR, poweill@econ.ucla.edu.

1 Introduction

Fraud in monetary and financial affairs has been prevalent throughout history. Classical examples include the clipping of coins in ancient Rome and medieval Europe, and the counterfeiting of banknotes during the first half of the 19^{th} century in the United States (Sargent and Velde, 2002; Mihm, 2007). Modern financial assets are no less susceptible to fraud. According to Gorton and Metrick (2010), prior to the 2008 financial crisis large volumes of repurchase agreements ("repos") backed by securitized bonds were traded daily without extensive due diligence.[1] These securitized bonds were subject to moral hazard problems, fraudulent practices, and lax incentives (Keys, Mukherjee, Seru, and Vig, 2010; Barnett, 2012).[2]

In this paper we develop a theory of asset liquidity—the extent to which an asset can facilitate exchange as a means of payment or as collateral—that emphasizes the assets' vulnerability to fraudulent practices. We address questions related to the cross-sectional dispersion of liquidity premia, such as what fundamental characteristics cause some assets to have higher turnover and lower yields than others? We study the determinants of aggregate liquidity and ask whether open-market purchases and sales of assets are effective to mitigate liquidity shortages by altering the relative supplies of assets. We also investigate whether a shock that heightens the threat of fraud for a class of assets can trigger a liquidity crisis in which investors shift their portfolios towards the most liquid assets and demand larger haircuts on illiquid assets, causing aggregate liquidity and output to shrink.

We introduce the threat of fraud into a search-theoretic model of asset markets, building on recent work in monetary and financial economics (e.g., Lagos and Wright, 2005; Duffie, Gârleanu, and Pedersen, 2005). In the first period, agents choose a portfolio of assets that can be genuine or fraudulent. Fraudulent assets are worthless and indistinguishable from their genuine counterparts, and their production involves an asset-specific fixed cost. In the second period, agents trade services

[1] Repos are short-term transactions by which an institutional investor (e.g., a mutual fund) deposits money at a financial institution (a dealer of securities) in exchange for some bonds received as collateral. Gorton and Metrick (2010) describe repos as a form of money, similar to private bank notes before the U.S. Civil War.

[2] The 2010 "Performance and Activity Report" of the SEC details many cases of financial fraud related to mortgage-based securities. The U.S. Senate report (2011) on "Wall Street and the Financial Crisis: Anatomy of a Financial Collapse" provides examples of deficient lending and securitization practices, mortgage fraud, ratings deficiencies, and investment bank abuses leading to the recent financial crisis.

for assets in an over-the-counter (OTC) market, with bilateral meetings and bargaining. Because of lack of commitment, assets serve as a means of payment or as collateral in the OTC market. However, the extent to which an asset can play such a role is limited by the threat of fraud, which introduces a private information friction into the OTC bargaining problem.

A key insight of our analysis is that the threat of fraud generates asset-specific, endogenous resalability constraints. While there are no exogenous restrictions on the transfer of assets in bilateral matches in the OTC market, if the quantity of an asset offered is above some threshold, then the trade is rejected with positive probability because of the rational fear that the asset might be fraudulent. In equilibrium, agents never find it optimal to offer more of the asset than can be accepted with certainty, which prevents fraud from taking place.[3] The resulting resalability constraint has three determinants: the asset's vulnerability to fraud, the difference between the asset's price and the value of its cash flows (its holding cost), and the frequency of trades in OTC markets. We emphasize three main implications of these endogenous resalability constraints below.

First, as a result of asset-specific resalability constraints, prices and measures of liquidity vary across assets with identical cash flows—a generalization of the rate-of-return dominance puzzle or, equivalently, a violation of the Law of One Price. We obtain an endogenous three-tier categorization of assets: illiquid, partially liquid, and liquid assets, which differ in their resalability, prices, and sensitivity to shocks and policy interventions. While the price of an illiquid asset is equal to the value of its cash flows, the price of a partially liquid or liquid asset is strictly larger than the value of its cash flows; i.e., this asset enjoys a liquidity premium. This premium increases with the asset's specific cost of committing fraud, but it decreases with the asset's supply.[4]

The second main implication of our results concerns policies aimed at managing the aggregate supply of liquidity through open-market sales and purchases of assets. In our model, an open-market operation has a positive welfare effect if and only if it increases a simple measure of aggregate liquidity—a weighted sum of asset supplies. For instance, a purchase of illiquid assets with liquid ones raises aggregate liquidity and output. On the other hand, a purchase of partially liquid assets

[3] In Section 3.2 we show that our results are robust to an extension where fraud materialize in equilibrium because the cost of fraud is random.

[4] The negative relationship between liquidity premium and asset supply is consistent with empirical findings of Krishnamurthy and Vissing-Jorgensen (2012).

with liquid ones has unintended consequences: such a policy *reduces* aggregate liquidity, the yield of liquid assets, and output. The shrinkage of aggregate liquidity is a consequence of a pecuniary externality affecting the pricing of partially liquid assets. In order to ensure that fraud is not profitable, partially liquid assets are priced below liquid ones—via a binding incentive-compatibility constraint—even though both classes of assets have the same marginal contribution to aggregate liquidity. Due to this pecuniary externality, a balanced-budget, open-market purchase of illiquid assets with liquid ones syphons out more liquidity than it injects.

Third, we show that the same frictions that generate endogenous resalability constraints can also rationalize retention mechanisms in markets for asset-backed securities or haircuts in repo markets. To that end, we extend the main model by adding a variable component to the cost of fraud and by assuming that asset portfolios are (partially) verifiable. Our model offers a narrative for the malfunctioning of the repo market in the midst of the subprime financial crisis (e.g., Krishnamurthy, 2010): when the cost of fraud of an initially liquid asset (e.g., mortgage-backed securities) is reduced by a sufficient amount, then the asset becomes illiquid and aggregate liquidity shrinks, which raises haircuts on all illiquid assets. Moreover, the set of liquid assets endogenously shrinks, meaning that agents shift their demand to the assets that are the least susceptible to fraud, in accordance with a flight to liquidity or quality.

1.1 Literature review

Kiyotaki and Moore (2001, 2005) study limited resalability by assuming that in each period, agents cannot sell more than an exogenous proportion of their asset holdings. While such exogenous resalability constraints can be chosen to replicate our distribution of asset prices, they generate markedly different comparative statics and policy recommendations (see Appendix of Li, Rocheteau, and Weill, 2011). For instance, with proportional resalability constraints, an increase in the frequency of trading needs weakly increases the prices of all assets, while in our model it has asymmetric effects: it increases the prices of liquid assets, and decreases the prices of partially liquid assets, consistent with evidence on a flight to liquidity. As another example, with proportional liquidity constraints, an open-market purchase of partially liquid assets with liquid ones increases liquidity, asset yields, and welfare. In our model, because of a new pecuniary externality, we obtain the opposite effects, consistent with some evidence on quantitative easing (see Section 5.1).

In Holmstrom and Tirole's (2011, and references therein) corporate finance model, a moral hazard problem generates endogenous borrowing constraints, i.e., resalability constraints in the primary asset market. In the secondary market, corporate claims with identical cash flows enjoy the same liquidity premium. In our model, by contrast, we focus on moral hazard in secondary markets. We highlight the fact that agents' incentives to take hidden actions depend on contemporaneous asset prices and on OTC market frictions, and we generate cross-sectional differences in liquidity premia between assets with identical cash flows.

The search-theoretic literature on the liquidity structure of asset returns includes Wallace (1998, 2000), Weill (2008), and Lagos (2010), as well as related work on the rate-of-return-dominance puzzle. Our approach goes beyond this earlier search literature by showing how cross-sectional differences in liquidity arise from fraud-based endogenous resalability constraints.[5] Lester, Postlewaite, and Wright (2012) consider a private information problem where agents can recognize the quality of an asset at some cost, but to determine the terms of trade under asymmetric information they make the simplifying assumption that unrecognized assets are not accepted in a bilateral match.[6] They address this issue in an extension that follows our methodology closely.

There is literature that emphasizes adverse selection problems in asset markets with search frictions (e.g., Hopenhayn and Werner, 1996). The most closely related papers are Rocheteau (2011), who introduces an adverse selection problem in a monetary model to explain the illiquidity of risky assets, and Guerrieri, Shimer, and Wright (2010), who consider a competitive search environment to illustrate how trading delays emerge endogenously to screen high- and low-quality assets. Guerrieri and Shimer (2011) extend the previous paper to a general equilibrium framework and, among other results, provide an explanation for flights to liquidity based on a dynamic adverse selection problem. While the distinction between adverse selection and moral hazard in asset markets is often subtle, the methodologies for capturing the two frictions differ profoundly. We take the view

[5] Wallace (1998, 2000) emphasizes assets' indivisibilities, Weill (2008) assumes increasing returns in the matching technology, and Lagos (2010) introduces exogenous restrictions on the use of some assets as means of payment. Similarly, Shi (2008) studies the pricing of bonds in a search economy where exogenous legal restrictions prevent bonds from being used in payments in a fraction of trades.

[6] There is also a related literature on counterfeiting, e.g., Green and Weber (1996), Williamson and Wright (1994), and Nosal and Wallace (2007). In those studies, there is a single asset, asset holdings are restricted to $\{0, 1\}$, and assets are indivisible, while those restrictions are all relaxed in our model.

that informational asymmetries in asset markets often result from strategic behavior, which allows us to focus the model more squarely on the effects of the threat of fraud on asset liquidity. At a more theoretical level, an important distinction between adverse selection and moral hazard is that the type distribution is exogenous with the former, but is endogenous with the latter. With an exogenous type distribution, under some conditions, agents can mitigate the asymmetric information friction by holding broadly diversified asset portfolios. As our model demonstrates, when the type distribution is endogenous, the asymmetric information friction remains relevant.

The next section presents the model. Section 3 solves the bargaining game under the threat of fraud. Section 4 solves for asset prices, and Section 5 presents three main implications. The appendix contains omitted proofs, and the online supplementary appendix presents additional results and extensions.

2 The model

The economy lasts for three periods, $t \in \{0, 1, 2\}$, and is populated by a continuum of agents who trade sequentially in two markets: in a centralized market (CM) at $t = 0$, and in a decentralized over-the-counter market (DM) at $t = 1$. There are two perfectly divisible goods. The first good, which we take to be the *numéraire*, is produced and consumed at $t = 0$ and at $t = 2$. The numéraire good is perishable and cannot be stored across periods. The second good, labeled the DM good, is produced and consumed in bilateral meetings at $t = 1$. There is a finite set of assets indexed by $s \in S$. Each asset pays off at $t = 2$ a dividend normalized to one unit of the numéraire.

Figure 1: Timing

Agents are divided evenly into two types, called *buyers* and *sellers*. Buyers wish to consume the DM good but cannot produce it, while sellers have the technology to produce the DM good but do not want to consume it. Together with the frictions described below, this preference structure creates a need for liquidity: buyers will acquire assets in the CM in order to finance the purchase of goods produced by sellers in the DM. The utility of a buyer is:

$$x_0 + u(q_1) + x_2, \tag{1}$$

where $x_t \in \mathbb{R}$ is the consumption of the numéraire good at time t, with $x_t < 0$ being interpreted as production, and $q_1 \in \mathbb{R}_+$ is the consumption of the DM good. The utility function, $u(q)$, over the DM good is twice continuously differentiable, with $u(0) = 0$, $u'(q) > 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $u''(q) < 0$. The utility of a seller is:

$$x_0 - q_1 + x_2, \tag{2}$$

where q_1 is the seller's production in a pairwise meeting in the DM. Let $q^* = \arg\max_q [u(q) - q] > 0$ denote the output level that maximizes the match surplus, so $u'(q^*) = 1$.

The CM is a perfectly competitive market, where agents trade the numéraire good and assets. The price of the asset, s, in terms of the numéraire good is denoted $\phi(s)$. Asset holdings are non-negative, i.e., agents cannot sell assets short (this constraint can be partially relaxed, see Footnote 22). In the first part of the paper we take asset prices as given. One interpretation is that assets are produced at the unit cost $\phi(s)$. Later we will endogenize $\phi(s)$ by assuming that there is a fixed supply, $A(s)$, for each asset, s. As will become clear, because of quasi-linear preference, the initial distribution of asset ownership is irrelevant.

The DM is an over-the-counter market, where a fraction $\sigma \in (0, 1]$ of buyers are matched bilaterally and at random with an equal fraction of sellers. Because of lack of commitment, unsecured credit is not incentive feasible in the DM since debtors would always renege on their obligations. Consequently, buyers purchase DM goods with assets or, equivalently, with loans collateralized by assets (see Footnote 11).

Terms of trade in pairwise meetings in the DM are determined according to a simple bargaining game, in which the buyer makes a take-it-or-leave-it offer.[7] The buyer, whose asset holdings are private information, asks for a given amount of the DM good in exchange for some specified portfolio

[7] In her discussion of our paper, Veronica Guerrieri investigated a version of the model with competitive search and showed that this alternative pricing mechanism generates the same resalability constraint as the one obtained under our simple bargaining game.

of assets.[8] The seller accepts or rejects the offer. If the seller accepts the offer, then the trade is implemented, provided that the asset transfer is feasible given the buyer's asset holdings. Matched agents split apart at the end of $t = 1$.

We introduce the possibility of asset fraud as follows. At the end of $t = 0$, after the CM has closed, a buyer can pay a fixed cost, $k(s) > 0$, to produce any quantity of fraudulent asset of type s. Fraudulent assets do not pay any dividend at $t = 2$. In the DM sellers are unable to verify assets' cash flows as they cannot distinguish genuine from fraudulent assets. Thus, we view the DM as a market where agents do not have the time or resources to exert extensive due diligence when trading assets, as in the examples below.

Counterfeiting of means of payment. A literal interpretation of the model concerns assets used as a means of payment, such as coins or banknotes, for which fraud consists of producing counterfeits.[9] During the first half of the 19th century, the fixed cost to produce fake banknotes included the cost to acquire plates and dies. Nowadays, this cost corresponds to the price of photo-editing software and copy machines.

Fraud on asset-backed securities (ABS). The first stage of our model describes the fraud taking place during the origination of ABS, such as deficient lending and securitization practices, mortgage fraud and ratings deficiencies. (See the online supplementary appendix for a reinterpretation of our model along these lines.) The cost of originating fraudulent securities is the cost of producing false documentation about the underlying asset and the cost of gaming the procedures used by rating agencies. The second stage of our model stands in for markets in which investors trade and collateralize ABS of uncertain qualities. This includes the primary and secondary markets for ABS and, importantly, the repo market.[10] Taken together, the two stages of our model offer

[8] In Section 5.2 we extend the model to consider the case of (partially) observable portfolios. This case generates new insights when the cost of fraud has a variable component.

[9] Our model can accommodate fraud on unsecured credit in bilateral matches. In this case, an agent has the option to produce a fake identity in the CM at a fixed cost (e.g., the cost incurred by a computer hacker to steal the identity of someone else) and he can issue an IOU in the DM if matched. The repayment of genuine IOUs can be enforced in the following CM. In contrast, IOUs based on fake identities are not repaid.

[10] In a supplementary appendix of our earlier working paper, Li, Rocheteau, and Weill (2011), we provide two extensions of our main model that capture separately both fraud in the securitization process and collateral fraud in OTC markets. First, in Appendix G, we develop a model of risk sharing in the OTC market, where assets are used as

a compact representation of the view that informational asymmetries throughout the securitization process affected the liquidity of ABS in repo markets.

3 The OTC bargaining game

In this section we solve for the equilibrium of the bargaining game between a buyer and a seller matched at random in the DM. We show that the threat of fraud gives rise to endogenous, asset-specific, resalability constraints. While fraudulent assets are never produced along the path of play of the benchmark game, in Section 3.2 we offer a simple extension of the model featuring fraud in equilibrium.

3.1 Bargaining under the threat of fraud

The game starts in the CM at $t = 0$ and ends at $t = 2$ when assets pay off. For now we take as given asset prices in the CM, $\phi(s)$, $s \in S$, and we anticipate that, in equilibrium, they will satisfy $\phi(s) \geq 1$, i.e., the price of asset s cannot be less than the final payoff, which would be the "fundamental price" of the asset in a frictionless economy.

The sequence of moves is as follows: (i) In the CM at $t = 0$, the buyer chooses a portfolio of $\{a(s)\}$ genuine and $\{\tilde{a}(s)\}$ fraudulent assets, subject to $a(s) \geq 0$ and $\tilde{a}(s) \geq 0$; (ii) In the DM at $t = 1$, the buyer is matched with a seller with probability σ, in which case he makes an offer, $(q, \{d(s)\})$, where q represents the output produced by the seller and $d(s)$ is the transfer of assets of type s (genuine or fraudulent) from the buyer to the seller; (iii) The seller decides whether to accept the offer; (iv) If the offer is accepted, the seller produces q units of the DM good for the buyer, and the buyer delivers $\tau(s) \in [0, a(s)]$ genuine and $\tilde{\tau}(s) \in [0, \tilde{a}(s)]$ fraudulent units of asset s to the seller, with $\tau(s) + \tilde{\tau}(s) = d(s)$.[11] The game tree on the left of Figure 2 summarizes the sequence of moves.

collateral for credit derivative contracts. Second, in Appendix H, we develop a model with risky assets and risk-averse agents, nesting the Capital Asset Pricing Model (CAPM). The implications of these two extensions, regarding asset prices and resalability constraints, are similar to the ones of our main model.

[11] We can reinterpret the offer, $(q, \{d(s)\})$, as the purchase of q units of DM output financed by a collection of loans. Each loan, $d(s)$, is fully collateralized with assets of type s and promises $d(s)$ units of CM output in period 2. If one asset is fraudulent, then the buyer will choose to default on his obligation, in which case the seller keeps the assets that serve as collateral. If assets are genuine, then the buyer is indifferent between repaying his debt or defaulting.

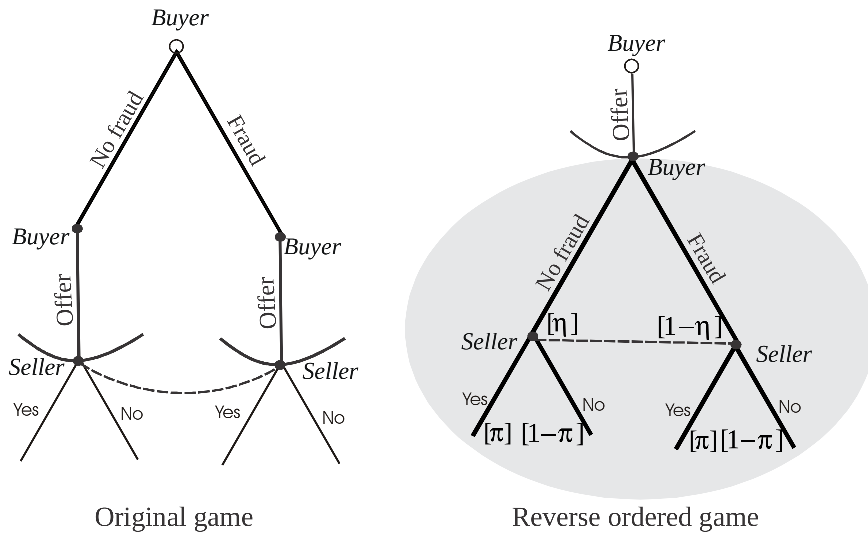

Figure 2: Game trees

Payoffs. The Bernoulli payoff of the buyer is:

$$-\sum_{s\in S}\left\{k(s)\mathbb{I}_{\{\tilde{a}(s)>0\}}+\phi(s)a(s)\right\}+\mu u(q)+\left\{\sum_{s\in S}[a(s)-\mu\tau(s)]\right\},$$

where $\mathbb{I}_{\{\tilde{a}(s)>0\}}=1$ if the buyer produces fraudulent assets of type s, $\tilde{a}(s)>0$, and zero otherwise. In the above, $\mu=1$ if the buyer meets a seller who accepts his offer, and $\mu=0$ otherwise. The first term is the payoff of the buyer at $t=0$. In order to accumulate $\tilde{a}(s)>0$ fraudulent units of asset s, the buyer must incur the fixed cost $k(s)$. In order to accumulate $a(s)$ units of genuine asset s, he must produce $\phi(s)a(s)$ units of the numéraire good in the CM. The second term is the utility of DM good consumption, $u(q)$, at $t=1$ if $\mu=1$, i.e., if the buyer meets a seller in the DM and his offer is accepted. The third term is the payoff at $t=2$ from his net holding of genuine assets, $a(s)-\mu\tau(s)$, the initial amount purchased net of the asset transfer to the seller, keeping in mind that each unit of genuine asset pays off one unit of the numéraire good. Collecting terms, we rewrite the payoff as

$$-\sum_{s\in S}\left\{k(s)\mathbb{I}_{\{\tilde{a}(s)>0\}}+[\phi(s)-1]\,a(s)\right\}+\mu\left\{u(q)-\sum_{s\in S}\tau(s)\right\}. \tag{3}$$

Similarly, the Bernoulli payoff of the seller is

$$\mu \left\{ -q + \sum_{s \in S} \tau(s) \right\}, \tag{4}$$

where we anticipate that, in equilibrium, sellers will not find it optimal to accumulate assets in the CM.[12] If the seller accepts the offer, $\mu = 1$, he suffers the disutility of producing, q, and receives $\tau(s)$ genuine units of asset s.

Equilibrium concept. Our equilibrium concept is Perfect Bayesian Equilibrium: actions are sequentially rational, and beliefs accord with Bayes's rule whenever it is possible. The notion of Perfect Bayesian Equilibrium imposes little discipline on the seller's belief regarding the buyer's decision in the initial stage of the game to produce fraudulent assets, conditional on an out-of-equilibrium offer being made in the DM. In order to circumvent this difficulty, we adopt the equilibrium concept of In and Wright (2011) for signaling games with unobservable choices.[13] This refinement selects equilibrium outcomes of the original game that are also equilibrium outcomes of the reverse-ordered game, whose timing is as follows: (i) At the beginning of the CM, the buyer posts his DM offer, $(q, \{d(s)\})$; (ii) He chooses his portfolio composed of genuine and fraudulent assets; (iii) He is matched with a seller who chooses whether to accept or reject the offer. The game tree on the right of Figure 2 provides a graphical representation of the sequences of moves. The reverse-ordered game is strategically equivalent to the original game—payoffs, actions, and information available to players are the same—but the orders of the observable and unobservable moves of the buyer are reversed. While intuitively this order should not matter for equilibrium outcomes, assuming that players make their observable moves first puts greater discipline on players' beliefs off the equilibrium path.

The reordered game captures the idea that upon seeing the buyer's offer, the seller will infer that the buyer's unobservable actions (portfolio and production of fraudulent assets) were chosen

[12] Sellers have no incentives to accumulate assets if $\phi(s) > 1$, because they have no liquidity needs and assets are costly to hold.

[13] We cannot apply standard refinements of signaling games, such as the intuitive criterion, because "types" are chosen in the initial stage instead of being determined by Nature. We instead apply the *reordering invariance refinement* of In and Wright (2011), based on the invariance condition of strategic stability from Kohlberg and Mertens (1986), which requires that the solution of a game should also be the solution of any game with the same reduced normal form. This equilibrium notion has a strong decision-theoretic justification and desirable normative properties. A more detailed description of the merits of this approach is provided in In and Wright (2011).

optimally with the offer in mind. Formally, by virtue of reverse ordering, every out-of-equilibrium offer is followed by a proper subgame in which the buyer chooses whether to produce fraudulent assets or accumulate genuine ones, and the seller decides whether to accept or reject the offer. This proper subgame is represented by a gray area in Figure 2. By subgame perfection, agents' mixed strategies restricted to these subgames must form a Nash equilibrium. This means that for all out-of-equilibrium offers, a seller can correctly infer the buyer's randomized strategy over accumulating assets or faking them (η and $1-\eta$, respectively, in Figure 2), and a buyer can correctly infer the seller's randomized strategy over accepting or rejecting the offer (π and $1-\pi$, respectively, in Figure 2).[14] This key feature of the reverse-ordered game allows us to pin down beliefs following all out-of-equilibrium offers in a logically consistent way, and it improves tractability dramatically as subgame perfection becomes sufficient to solve the game. Finally, from a normative viewpoint, this refinement has the appealing property of selecting an equilibrium of the original game that yields the highest payoff to the buyer, the agent making the offer.

Solving for equilibrium. The analysis of the game can be simplified by making two observations. First, because of the fixed cost, the buyer will either produce the quantity of fraudulent assets that is necessary to execute the offer in a match or no fraudulent asset at all. Consequently, $\tilde{\tau}(s) = [1-\chi(s)]\, d(s)$ and $\tau(s) = \chi(s)d(s)$, where $\chi(s) = 0$ if the buyer produces fraudulent assets, and $\chi(s) = 1$ otherwise. Moreover, the buyer must be able to cover his intended transfer of genuine assets; i.e., $a(s) \geq \chi(s)d(s)$.

Second, we can solve for the buyer's optimal asset demand as a function of any candidate equilibrium offer. Indeed, if $\phi(s) = 1$, it follows from the buyer's payoff, (3), that any demand satisfying the constraint $a(s) \geq \chi(s)d(s)$ is optimal. If $\phi(s) > 1$, it is costly to hold assets, and so it is optimal to demand $a(s) = \chi(s)d(s)$. We then substitute these optimal asset demands into the buyer's objective. In both cases, this amounts to replacing $a(s)$ with $\chi(s)d(s)$.

[14] One may object to this level of sophistication because these randomized strategies are never played in equilibrium. If we take the reverse-ordered game as our primitive game, these randomized strategies can be played in a "trembling hand" equilibrium in which buyers are forced to put a small probability on all offers. Moreover, a milder refinement according to which sellers should never believe that buyers are playing a strictly dominated strategy would be sufficient to generate the same resalability constraints and the same outcomes as the ones under our equilibrium concept. See the online supplementary appendix.

With these observations in mind, a buyer's strategy specifies the following two objects: the offer, $(q, \{d(s)\})$, and conditional on any offer, a probability distribution over $\{\chi(s)\} \in \{0,1\}^S$, denoted by η. The seller's strategy specifies, conditional on any offer, $(q, \{d(s)\})$, the probability of accepting, denoted by π.

The game is solved by backward induction. Following an offer, $(q, \{d(s)\})$, the seller's decision to accept an offer must be optimal given the buyer's decision to produce fraudulent assets; i.e.,

$$\pi \in \arg\max_{\hat{\pi}\in[0,1]} \hat{\pi}\left\{-q + \sum_{s\in S} \eta(s)d(s)\right\}, \tag{5}$$

where $\eta(s)$ denotes the marginal probability of bringing genuine assets of type s.[15] The seller's value in accepting the offer depends on the disutility of producing q units of goods and on the expected quality of the asset transfer, determined by η.

Similarly, following an offer, $(q, \{d(s)\})$, the buyer minimizes the cost of financing his DM trade by choosing how many genuine or fraudulent assets to hold given the seller's probability of accepting; i.e.,

$$\{\eta(s)\} \in \arg\min_{\{\hat{\eta}(s)\}} \sum_{s\in S} \left\{k(s)\left[1-\hat{\eta}(s)\right] + \left[\phi(s)-1\right]\hat{\eta}(s)d(s) + \sigma\pi\hat{\eta}(s)d(s)\right\}. \tag{6}$$

From (6) the cost of financing the offer has three components: (i) the fixed cost of producing fraudulent assets, $k(s)$; (ii) the holding cost of genuine assets, $[\phi(s)-1]\,d(s)$; (iii) the expected cost of transferring genuine assets to a seller, $\sigma\pi d(s)$.

Finally, given equilibrium decision rules $\{\eta(s)\}$ and π, the optimal offer, $(q, \{d(s)\})$, maximizes the expected utility of consumption net of the cost of financing the offer, as given by (6) :

$$-\sum_{s\in S}\left\{k(s)\left[1-\eta(s)\right] + \left[\phi(s)-1\right]\eta(s)d(s)\right\} + \sigma\pi\left\{u(q) - \sum_{s\in S}\eta(s)d(s)\right\}. \tag{7}$$

A perfect Bayesian equilibrium that passes the reordering invariance refinement is a pair of buyer's and seller's strategies satisfying (5), (6), and (7). To solve for equilibrium, we first characterize the maximum equilibrium payoff attainable by a buyer: we maximize the buyer's objective

[15] Note that, after replacing $a(s)$ and $\tau(s)$ with $\chi(s)d(s)$ in (3) and (4), the payoffs of buyers and sellers become linear functions of the binary actions $\{\chi(s)\}$. Therefore, taking expectations with respect to η amounts to replacing $\chi(s)$ with the marginal probability $\eta(s)$.

with respect to $(q, \{d(s)\}, \{\eta(s)\}, \pi)$, subject to the incentive constraints that $(\{\eta(s)\}, \pi)$ must form a Nash equilibrium in the subgame following the offer $(q, \{d(s)\})$. The analysis yields two insights.

First, in an equilibrium attaining his maximum payoff, the buyer does not produce any fraudulent assets. Indeed, consider a candidate equilibrium in which the buyer brings genuine assets of type s_0 with a probability $\eta(s_0) \in (0, 1)$.[16] The buyer can deviate and accumulate genuine assets of type s_0 with higher probability, $\eta'(s_0) > \eta(s_0)$, while keeping $d(s_0)$ the same. This deviation does not change the expected cost of transferring assets, since the buyer is indifferent between genuine or fraudulent assets of type s_0, but it does raise the expected value that the seller assigns to the buyer's offer. Thus, the buyer can increase his payoff by demanding higher consumption in the DM, $q' > q$, while keeping the seller's expected surplus—and therefore π—unchanged.

Second, in an equilibrium attaining his maximum payoff, the buyer makes an offer that is accepted with probability one. The intuitive explanation for this finding is as follows. Consider a candidate equilibrium in which the offer is rejected with probability, $\pi \in (0, 1)$. In this case $-q + \sum_{s \in S} d(s) = 0$. Then, for at least one asset the buyer must be indifferent between accumulating genuine or fraudulent assets. (Otherwise, the buyer's payoff could be raised by increasing π without violating any incentive constraints). Suppose for simplicity that the buyer is indifferent only for one asset, say s_0. A profitable deviation for the buyer consists in reducing the quantity of asset s_0 offered, $\Delta d(s_0) = -\varepsilon q$ with $\varepsilon > 0$, and the quantity of output asked, $\Delta q = -\varepsilon q$, so that the seller's incentive compatibility constraint still holds. Given a fixed cost of producing fraudulent assets, smaller trade size induces lower incentive to commit fraud. As a result, the seller can accept the new offer with a higher probability $\pi + \Delta\pi$, where one can show that $\Delta\pi \geq \varepsilon\pi$, while keeping the buyer's incentive to commit fraud in check. The buyer's expected payoff increases by

$$\Delta\pi \left[u(q) - q\right] + \pi \left[u'(q) - 1\right] \Delta q > \varepsilon\pi \left\{\left[u(q) - q\right] - q \left[u'(q) - 1\right]\right\} > 0,$$

since $u(q) - q$ is strictly concave.

The last step, detailed in the proof, is to show that any equilibrium must achieve the maximum payoff attainable by a buyer. This result follows because the offer can be perturbed slightly such that all incentive constraints are slack and the Nash equilibrium following the offer is unique,

[16] Looking at $\eta(s_0) > 0$ is without loss. See the proof of Proposition 1 for details.

which guarantees the buyer a payoff arbitrarily close to the maximum. This leads to the following Proposition.

Proposition 1 *The equilibrium offers, $(q, \{d(s)\})$, and asset demands, $\{a(s)\}$, solve:*

$$\max_{q,\{d(s),a(s)\}} \left\{ -\sum_{s\in S} [\phi(s) - 1]\, a(s) + \sigma\, [u(q) - q] \right\} \tag{8}$$

$$s.t. \quad \sum_{s\in S} d(s) - q = 0 \tag{9}$$

$$d(s) \leq \frac{k(s)}{\phi(s) - 1 + \sigma}, \quad \textit{for all } s \in S \tag{10}$$

$$d(s) \in [0, a(s)]\,, \qquad \textit{for all } s \in S. \tag{11}$$

Moreover, the seller accepts any equilibrium offer with probability one, $\pi = 1$, and the buyer transfers genuine assets with probability one, $\eta(s) = 1$ for all s.

Proposition 1 shows that equilibrium asset demands and offers maximize the buyer's expected utility subject to three constraints. First is the individual rationality constraint, (9), which states that the seller's payoff must be zero, given that the buyer's assets are genuine. Second is the incentive compatibility constraint, (10), which states that the buyer must find it optimal to accumulate genuine assets with probability one, given that the seller accepts with probability one. Third is the feasibility constraint, (11), which states that the buyer must hold enough genuine assets to cover his transfer to the seller.

Endogenous resalability constraints. Perhaps the most important result of Proposition 1 is that the incentive-compatibility constraints, (10), take the form of resalability constraints, specifying upper bounds on the transfer of assets from buyers to sellers. The buyer could offer a quantity of asset greater than the upper bound in (10), but in equilibrium he chooses not to do so. Indeed, if a too large quantity of an asset was proposed, the seller would expect that the asset offered is fraudulent with positive probability and, as a consequence, would reject it with positive probability. To give a simple example, assume there is a single asset, $S = 1$, and the offer is such that $q < d$ with $d \in \left(\frac{k}{\phi-1+\sigma}, \frac{k}{\phi-1}\right)$: it is acceptable if the asset is believed to be genuine but it violates the resalability constraint. From (5) and (6) the unique Nash equilibrium of the subgame following any such offer is

$$\eta = \frac{q}{d} \in (0,1) \quad \text{and} \quad \pi = \frac{k - (\phi - 1)\, d}{\sigma d} \in (0,1).$$

Hence, there is now a positive probability that the buyer commits fraud, $1 - \eta$, and a positive probability that the seller rejects the offer, $1 - \pi$, with both probabilities increasing with the size of the transfer, d. This is the sense in which the (out of equilibrium) threat of fraud reduces the resalability of assets.

The resalability constraints depend on the cost of producing fraudulent assets, $k(s)$, the holding cost of an asset, $\phi(s) - 1$, and the frequency of trades in the DM, σ. From (10), an asset which is more susceptible to fraud is subject to a more stringent resalability constraint. To illustrate this point, suppose that there are no search frictions, $\sigma = 1$. Then, the resalability constraint of asset s is $\phi(s)d(s) \leq k(s)$. The real value of the asset that can be transferred in a bilateral match is simply the cost of producing fraudulent assets. In accordance with the Wallace (1998) dictum, the liquidity of an asset depends on its intrinsic properties, which here are captured by the ease of producing fraudulent assets.

The resalability constraints also depend on the frequency of trade in the DM. Increasing the frequency of trade exacerbates the threat of fraud because the trade surplus of a con artist, $u(q)$, is greater than the match surplus of an honest buyer, $u(q) - q$. Therefore, the upper bound must be lowered to keep incentives in line. To give a concrete example, if the process of securitization implies that an asset can be retraded more frequently, then an increase in securitization raises the threat of fraud and makes resalability constraints more likely to bind.[17]

Finally, the holding cost of the asset, $\phi(s) - 1$, enters the resalability constraint, because lack of commitment forces agents to accumulate assets before liquidity needs occur. An increase in the asset price raises the holding cost, which raises the buyer's incentives to produce fraudulent versions of the asset for a given size of the trade.

[17] Keys, Mukherjee, Seru, and Vig (2010) establish evidence that the securitization of subprime loans led to lax screening. Purnanandam (2011) finds that banks involved highly in the originate-to-distribute market, where the originator of loans sells them to third parties, originated excessively poor-quality mortgages.

3.2 Fraud in equilibrium

The result of Proposition 1 that fraud does not materialize in equilibrium arises because contracts in the DM are unrestricted, so it is optimal and feasible to always keep buyers' incentives to commit fraud in check. This property of the equilibrium can be at odds with some observations that fraud in monetary and financial instruments does occur. (See, e.g., Footnote 2.) The objective of this section is to offer a simple extension of our model where fraud emerges in equilibrium, but the DM trades are identical to the ones obtained in Proposition 1 up to some rescaling.

The key departure from the previous section is the assumption that there is uncertainty about the cost of fraud, which is only resolved after setting the terms of the contract, $(q, \{d(s)\})$. The assumption that the terms of the offer cannot be reoptimized to take into account the realized cost of fraud—thereby creating a form of contract incompleteness—requires that we adopt the timing of the reverse-ordered game as our primitive (i.e., we no longer use the original game). Namely, at the beginning of $t = 0$, buyers post the terms of a contract, $(q, \{d(s)\})$, to be executed in a bilateral match in the DM. Next, the cost of fraud is realized: $k(s) > 0$ with probability $\omega(s) \in [0, 1]$ and $k(s) = 0$ with complement probability, $1 - \omega(s)$.[18] The realizations of the cost of fraud are independent across assets and across buyers. Next, buyers make their portfolio choices of both genuine and fraudulent assets. At $t = 1$ a fraction σ of buyers and sellers are matched and they trade according to the posted offers.

When $k(s) = 0$ the buyer finds it optimal to execute his offer with fraudulent assets, irrespective of π. When the cost of fraud is equal to $k(s) > 0$ a buyer acquires genuine assets of type s with probability $\eta(s)$, where $\eta(s)$ is defined in (6). Following an offer, $(q, \{d(s)\})$, the seller's strategy solves (5) where $\eta(s)d(s)$ is replaced by $\omega(s)\eta(s)d(s)$. It is then straightforward to generalize Proposition 1 as follows.

Proposition 2 *Suppose the cost of fraud is random, equal to $k(s) > 0$ with probability $\omega(s) > 0$, and to zero with probability $1 - \omega(s)$. The equilibrium offers, $(q, \{d(s)\})$, and the asset demands*

[18] In Online Supplementary Appendix C we describe a one-asset version of the model where the cost of fraud is drawn from a continuous distribution, $F(k)$, with support $[0, \bar{k}]$. We establish that there is an endogenous threshold, $\check{k} \in (0, \bar{k})$, below which agents commit fraud. Moreover, the offer, (q, d), is always accepted.

conditional on $k(s) > 0$, $\{a(s)\}$, *solve:*

$$\max_{q,\{d(s),a(s)\}} \left\{ -\sum_{s\in S} [\phi(s) - 1]\,\omega(s)a(s) + \sigma\,[u(q) - q] \right\}$$

subject to $\sum_{s\in S} \omega(s)d(s) - q = 0$, *as well as constraints (10) and (11). Moreover, the seller accepts any equilibrium offer with probability one,* $\pi = 1$, *the buyer transfers genuine assets of type* s *with probability* $\eta(s) = 1$ *if* $k(s) > 0$, *and transfers fraudulent assets if* $k(s) = 0$.

The buyer's problem in Proposition 2 is identical to the one in Proposition 1 after rescaling by $\omega(s)$, i.e. after making the change of variable $\{\bar{k}(s), \bar{a}(s), \bar{d}(s)\} \equiv \omega(s) \times \{k(s), a(s), d(s)\}$. Moreover, for fraud to take place in equilibrium it is sufficient that $d(s) > 0$ for some asset s, which follows from $u'(0) = \infty$ and $k(s) > 0$.

4 The liquidity structure of asset returns

In this section we study the implications of our model for cross-sectional liquidity premia. We endogenize asset prices in the CM and show that the endogenous resalability constraints derived in Proposition 1 create liquidity and price differences across assets, even if they have the same cash flows. Our results help explain differences in asset prices that cannot be fully accounted for by risk, and shed light on a variety of evidence on the positive relationship between liquidity and asset prices.[19]

Assume that each asset $s \in S$ comes in fixed supply, denoted by $A(s)$. With no loss in generality, given quasi-linear preferences, we assume that assets are initially owned by buyers. We define a symmetric equilibrium to be a collection of prices, $\{\phi(s)\}$, asset demands, $\{a(s)\}$, and a DM offer, $(q, \{d(s)\})$, such that the asset demands and the offer solve the buyer's problem (8)-(11) given prices, and the asset market clears; i.e., $a(s) = A(s)$ for all $s \in S$.[20]

Guessing that $d(s) \geq 0$ does not bind, the first-order conditions of the buyer's problem are:

$$\xi = \sigma\left[u'(q) - 1\right] = \lambda(s) + \nu(s) \tag{12}$$

$$\phi(s) = 1 + \nu(s), \tag{13}$$

[19] Since Amihud and Mendelson (1986), liquidity (level and risk) has been shown to explain risk-adjusted asset return differentials. For recent studies, see, e.g., Chordia, Huh, and Subrahmanyam (2009).

[20] The symmetry restriction that all buyers have the same asset demands serves to pin down portfolios when some assets are priced at their fundamental values, $\phi(s) = 1$.

for all $s \in S$, where $\xi \geq 0$ is the Lagrange multiplier of the seller's participation constraint, (9), $\lambda(s) \geq 0$ is the multiplier of the resalability constraint, (10), and $\nu(s) \geq 0$ is the multiplier of the feasibility constraint, (11). The multiplier, ξ, measures the expected net utility of holding an additional unit of asset in the DM. A match with a seller occurs with probability σ, in which case the increased consumption yields marginal utility, $u'(q)$, to the buyer, and the asset transfer has an opportunity cost equal to one.

Taken together, (12) and (13) imply the following bounds on asset prices:

$$1 \leq \phi(s) \leq 1 + \xi. \tag{14}$$

The upper bound is the asset terminal payoff, 1, which we refer to as the "fundamental value"of the asset, augmented by the net utility of spending an additional unit of the asset in the DM, ξ. The lower bound is the "fundamental value" of the asset, 1, since a buyer can always hold on to any unit of the asset and consume its cash flow at the end of $t = 2$. Assuming for now that $q < q^*$, so that $\xi > 0$, these first-order conditions imply that there are three categories of assets.

Liquid assets. For this type of asset, the feasibility constraint is binding, $\nu(s) > 0$, but the resalability constraint is slack, $\lambda(s) = 0$. Therefore, the asset price is equal to the upper bound, $1 + \xi$. The asset is said to be perfectly liquid in the following sense: if the buyer holds an additional unit of the asset, he would spend it in the DM. Substituting the market clearing condition, $a(s) = A(s)$, and the price, $\phi(s) = 1 + \xi$, into the binding feasibility constraint and the slack resalability constraint, we obtain $d(s) = A(s) \leq \frac{k(s)}{\sigma+\xi}$. This last inequality can be equivalently written as $\kappa(s) \geq \sigma + \xi$, where $\kappa(s) \equiv k(s)/A(s)$ is the cost of fraud per unit of the asset.

Partially liquid assets. For this type of asset, both the resalability and feasibility constraints bind, $\lambda(s) > 0$ and $\nu(s) > 0$. In equilibrium, a buyer spends all his holdings of the asset. However, if he were to acquire and spend an additional unit, there would be a positive probability of the trade being rejected. The asset is thus said to be partially liquid and its price must be lower than the upper bound. From (10), $d(s) = A(s) = \frac{k(s)}{\phi(s)-1+\sigma}$, which leads to $\phi(s) = 1 + \kappa(s) - \sigma$, keeping in mind that $\kappa(s) = k(s)/A(s)$. The conditions $\lambda(s) = \xi + 1 - \phi(s) > 0$ and $\nu(s) = \phi(s) - 1 > 0$ can be written as $\sigma < \kappa(s) < \sigma + \xi$.

Illiquid assets. Lastly, there are assets for which the resalability constraint binds, $\lambda(s) > 0$, but the feasibility constraint is slack, $\nu(s) = 0$. In equilibrium the buyer does not spend a fraction of his asset holdings even though he is liquidity constrained. Therefore, the asset is said to be illiquid, and its price is equal to the lower bound, $\phi(s) = 1$. The binding resalability constraint implies that $d(s) = \frac{k(s)}{\sigma}$. Substituting this expression into the slack feasibility constraint, we obtain that $\kappa(s) \leq \sigma$.

The next step is to determine ξ and verify that $q < q^*$. From the above, we have:

$$d(s) = \min\left[A(s), \frac{k(s)}{\sigma}\right] = \theta(s)A(s), \quad \text{where} \quad \theta(s) = \min\left[1, \frac{\kappa(s)}{\sigma}\right].$$

That is, the buyer either transfers all his holdings of asset s, or the maximum holding consistent with the resalability constraint and the no-arbitrage restriction that $\phi(s) \geq 1$. Substituting the expression for $d(s)$ into the seller's binding participation constraint, (9), we obtain

$$q = L \equiv \sum_{s \in S} \theta(s)A(s). \tag{15}$$

The aggregate liquidity, L, is a weighted average of asset supplies, with endogenous weights depending on trading frictions and assets' characteristics.[21] Given q, the liquidity premium of liquid assets, ξ, is determined by (12). One can easily verify that, if $L < q^*$, the above asset prices, offer, and asset demands constitute a symmetric equilibrium. The condition $L < q^*$ means that the aggregate liquidity is not large enough to satiate buyers' liquidity needs, represented by q^*. If $L \geq q^*$, then the equilibrium has $q = q^*$ and $\phi(s) = 1$ for all $s \in S$. Summarizing:

Proposition 3 ***(The liquidity-return relationship)*** *There exists a unique symmetric equilibrium. If $L \geq q^*$, then $q = q^*$ and $\phi(s) = 1$ for all $s \in S$. If $L < q^*$, then $q < q^*$, $\xi \equiv \sigma\left[u'(q) - 1\right] > 0$. Letting $\underline{\kappa} \equiv \sigma$, and $\overline{\kappa} \equiv \sigma + \xi$, there are three categories of assets:*

[21] This approach is consistent with a definition of the quantity of money suggested by Friedman and Schwartz (1970) as "the weighted sum of the aggregate value of all assets, the weights varying with the degree of moneyness." Our definition of aggregate liquidity is also related to the Divisia monetary aggregates (e.g., Barnett, Fisher, and Serletis, 1992). A key difference is that in our approach the weight assigned to an asset in order to calculate liquidity changes is not equal to its holding cost, which has normative implications discussed in Section 5.

1. Liquid assets: for any $s \in S$, such that $\kappa(s) \geq \overline{\kappa}$,

$$\phi(s) = 1 + \xi \tag{16}$$

$$\theta(s) = 1. \tag{17}$$

2. Partially liquid assets: for any $s \in S$, such that $\kappa(s) \in (\underline{\kappa}, \overline{\kappa})$,

$$\phi(s) = 1 + \kappa(s) - \sigma \tag{18}$$

$$\theta(s) = 1. \tag{19}$$

3. Illiquid assets: for any $s \in S$, such that $\kappa(s) \leq \underline{\kappa}$,

$$\phi(s) = 1 \tag{20}$$

$$\theta(s) = \frac{\kappa(s)}{\sigma} < 1. \tag{21}$$

The central implication of Proposition 3 is that, whenever there is a liquidity shortage, $L < q^*$, assets with identical cash flows can have different prices. See Figure 3 for a graphical representation of these price differences. This departure from the Law of One Price is another formulation of the rate-of-return dominance puzzle, according to which monetary assets coexist with other assets with similar risk characteristics that generate a higher yield. In our model, price differentials across assets are attributed to differences in the cost of fraud. An asset which is less sensitive to fraudulent activities—as captured by a high cost of fraud—is used more intensively to finance random spending opportunities. Relative to assets that have a lower cost of fraud, this asset generates some non-pecuniary liquidity services, $\nu(s) = \phi(s) - 1$, also referred to as a convenience yield, and is sold at a higher price.[22]

Krishnamurthy and Vissing-Jorgensen (2011, 2012) document the existence of convenience yields for Treasury securities and, to a lesser extent, highly-rated bonds. They argue that a safety-premium (which they view as distinct from a standard risk premium) is an important component

[22] To see why the price differentials do not represent arbitrage opportunities, relax the short-selling constraint and assume that in order to sell an asset he does not own, an agent has to borrow it from someone else in exchange for a fee to be determined in equilibrium. The agent who borrows the asset can use it in the DM, but the agent who lends it cannot. The equilibrium remains unchanged, and the fee clearing the market for borrowing asset $s \in S$ is equal to its liquidity premium, $\phi(s) - 1$. Indeed, an agent who borrows a liquid or partially liquid asset must compensate the lender for his forgone liquidity services in the DM.

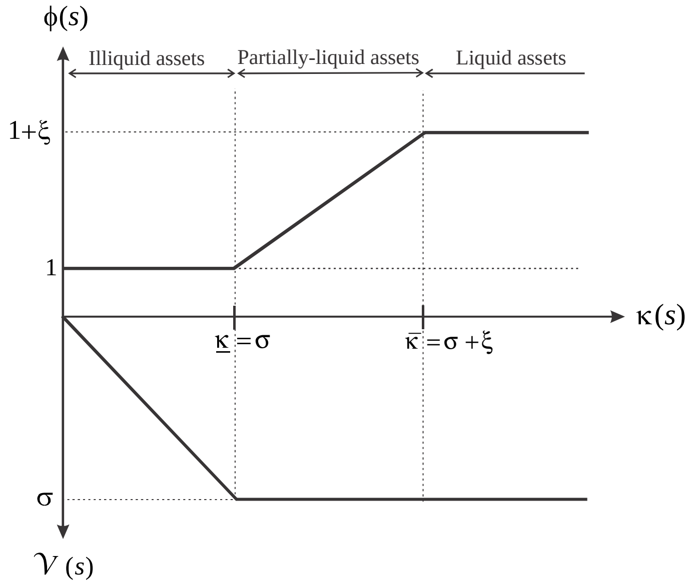

Figure 3: Liquidity structure of asset returns ($L < q^*$)

of asset prices. Through the lens of our model, we can interpret this safety premium as the premium offered by assets that are less sensitive to moral hazard considerations. Similarly, Vickery and Wright (2011) argue about the existence of a liquidity premium for agency mortgage-backed securities, which are better protected against the informational asymmetries that plague the securitization process.

Proposition 3 also has insights for cross-sectional differences in transaction velocity, a standard measure of liquidity in monetary economies. In our model, transaction velocity in the DM is $\mathcal{V}(s) \equiv \frac{\sigma d(s)}{A(s)} = \sigma\theta(s)$. Proposition 3 predicts a positive relationship between the price of an asset and its velocity. The most liquid assets (i.e., any asset s such that $\kappa(s) \geq \bar{\kappa}$) trade at the highest price, and their velocity is maximum and equal to the frequency of spending opportunities in the DM, σ. Illiquid assets (i.e., any asset s such that $\kappa(s) < \underline{\kappa}$) trade at the lowest price, equal to their fundamental value, and have lower velocities. This result is consistent with the view that bonds that are used more intensely as collateral in OTC markets tend to have higher prices (Duffie, 1996).

In reality, a myriad of assets are not used as a means of payment or collateral. This observation is consistent with our results if there is a mass of assets that do not circulate in the DM, $\theta(s) = 0$.

From (21) such assets must be characterized by $\kappa(s) = 0$: these are assets for which agents have so little knowledge about their mere existence or attributes that even simple, costless frauds can be deceptive.[23]

Finally, it is straightforward to generalize Proposition 3 to the case where fraud emerges in equilibrium. From Section 3.2, a measure $\omega(s)$ of buyers do not commit fraud on asset s and demand $a(s)$ genuine assets of type s while the remaining $1 - \omega(s)$ buyers produce fraudulent assets of type s. Thus, the market clearing condition becomes $\omega(s)a(s) = A(s)$ and Proposition 3 goes through with $\kappa(s) = \omega(s)k(s)/A(s)$.[24] This model offers predictions about the cross-sectional relationship between liquidity and the occurrence of fraud. Let the occurrence of fraud for an asset of type s be equal to the aggregate transfer of fraudulent assets per unit of genuine asset, $\frac{\sigma[1-\omega(s)]d(s)}{A(s)}$. The occurrence of fraud for liquid and partially liquid assets is equal to $\sigma\,[1 - \omega(s)]$. Among these assets, those with a high $\omega(s)$ are more liquid, have a higher price, and have a lower occurrence of fraud. The occurrence of fraud for illiquid assets is $\frac{[1-\omega(s)]k(s)}{A(s)}$. Note that a larger $k(s)$ translates into a larger transfer $d(s) = \frac{k(s)}{\sigma}$ and so, all else equal, into a larger fraud. Thus, among illiquid assets, those that are more resalable offer more opportunities for fraud.

5 Applications and Extensions

In the rest of the paper we consider three applications of our model: in Section 5.1 we assess the effectiveness of aggregate liquidity management policies; in Section 5.2 we study incentive mechanisms based on asset retention and over-collateralization; in Section 5.3 we identify a shock generating a dry-up of aggregate liquidity in the OTC market and a flight to liquidity.

5.1 Liquidity management

In this section we use our model to study the effectiveness of policies aimed at managing the supply of liquidity in the economy.

[23] That assets, or claims on those assets, can be counterfeited at no cost has been the standard explanation in monetary theory for why capital goods are illiquid, since Freeman (1985), and more recently, Lester, Postlewaite, and Wright (2012).

[24] Recall from Proposition 2 that the problem that determines asset demands is identical to the one of our benchmark model after rescaling $\{d(s), a(s), k(s)\}$ by $\omega(s)$.

Measuring the social value of assets' liquidity services. Much of the analysis that follows is based on the following theoretical observation. In competitive models with reduced-form demand for liquidity (e.g., cash-in-advance or money-in-the-utility function), the liquidity premium of an asset not only measures the *marginal private value* of its liquidity services, but also its *marginal social value*.[25] In our model this property holds true for illiquid and liquid assets, but fails to hold for partially liquid assets. Formally, the marginal social value of the liquidity services provided by a unit of asset s is $\frac{\partial L}{\partial A(s)}\xi$, which is equal to ξ for liquid *and* partially liquid assets, and 0 for illiquid assets. Therefore, the liquidity premium of partially liquid assets, $\phi(s)-1<\xi$, underestimates the true marginal social value of their liquidity services.

To see this, consider first liquid assets, with $\kappa(s)>\sigma+\xi$, on the right side of Figure 3. For these assets, the cost of fraud is large, implying that the resalability constraint is slack when buyers hold and spend the entire supply, $A(s)$. As a result, these assets' liquidity premium, $\phi(s)-1$, is equal to ξ, the marginal social value of their liquidity services. Next, consider a partially liquid asset, with intermediate cost of fraud, $\kappa(s)\in(\sigma,\sigma+\xi)$. If its liquidity premium was as high as the one of a liquid asset, ξ, the market would not clear because an offer of $A(s)$ in the DM would violate the resalability constraint. Therefore, the liquidity premium has to fall along the upward-sloping line of Figure 3, until the resalability constraint binds exactly when buyers hold and spend the entire asset supply, $A(s)$. Hence, the binding resalability constraint depresses assets' liquidity premia below the marginal social value of their liquidity services, thereby creating a negative "pecuniary externality."[26] Finally, for an illiquid asset, $\kappa(s)<\sigma$, the resalability constraint binds, buyers hold $A(s)$ but spend less than $A(s)$ in the DM. Thus, the marginal social value of the asset's liquidity services is equal to the liquidity premium, $\phi(s)-1=0$.

Open-market purchases. Central banks routinely engage in aggregate liquidity management, by swapping assets with different degrees of liquidity, including reserves, treasuries, and recently

[25] This logic is underlying the calculation for the welfare cost of inflation in Lucas (2000), the measure of the liquidity services provided by Treasuries in Krishnamurthy and Vissing-Jorgensen (2012), and Barnett, Fisher, and Serletis's (1992) definition of Divisia monetary aggregates.

[26] By contrast, with exogenous proportional resalability constraints, there is no such pecuniary externality, and an asset's liquidity premium coincides with the marginal social value of its liquidity services. See the supplementary appendix of Li, Rocheteau, and Weill (2011).

mortgage-backed securities. Consider a policy maker in the CM, who sells a quantity, $\Delta A(s)$, of some liquid asset s from his portfolio, and simultaneously purchases a quantity, $\Delta A(s')$, of some other asset s'.[27] A small open-market operation has a small effect on prices, so that the budget constraint of the policy maker is, to a first-order approximation, $\phi(s)\Delta A(s) + \phi(s')\Delta A(s') = 0$. The welfare effect of such a policy is

$$\Delta L \times \xi = \left[\frac{\partial L}{\partial A(s)}\Delta A(s) + \frac{\partial L}{\partial A(s')}\Delta A(s')\right]\xi = \left[1 - \frac{\partial L}{\partial A(s')}\frac{\phi(s)}{\phi(s')}\right]\Delta A(s) \times \xi.$$

Suppose first that $\kappa(s') > \bar{\kappa}$, so both s and s' are liquid assets. Then, $\phi(s) = \phi(s')$, $\frac{\partial L}{\partial A(s')} = 1$, and $\Delta L = 0$. Such an open-market operation is irrelevant: it does not change aggregate liquidity and welfare, hence it has no effect on output and asset prices. Therefore, liquidity management by swapping assets has real effects only if it involves assets with different degrees of liquidity.

Suppose next that $\kappa(s') < \underline{\kappa}$, asset s' is illiquid. In this case aggregate liquidity does increase because the purchase of illiquid assets has no consequence on aggregate liquidity; i.e., $\Delta L \times \xi = \Delta A(s) \times \xi > 0$. Thus, welfare increases, the price of liquid assets decreases, and the price of illiquid assets is unaffected.

Finally, suppose that $\kappa(s') \in (\underline{\kappa}, \bar{\kappa})$; i.e., asset s' is partially liquid. Then, $\phi(s') < \phi(s)$ and $\Delta L \times \xi = \left[1 - \frac{\phi(s)}{\phi(s')}\right]\Delta A(s) \times \xi < 0$, implying that such a policy *reduces* aggregate liquidity and welfare. The intuition is in line with our earlier observation: while partially liquid and liquid assets have different prices, they contribute *equally* to aggregate liquidity. At the same time, because it has a higher price, one share of a liquid asset buys more than one share of a partially liquid one. Thus a balanced-budget open-market operation ends up syphoning out more liquidity than it is injecting; i.e., aggregate liquidity is reduced. The welfare effect of this open-market operation has the opposite sign than the yield difference between the asset that is withdrawn and the asset that is injected, and the prices of both assets s and s' increase.

The results above provide one interpretation of the effects of quantitative easing by the Federal Reserve that is consistent with the findings in Krishnamurthy and Vissing-Jorgensen (2011). They find that the purchases of Treasuries, agency bonds, and highly-rated corporate bonds in exchange

[27] While we assume that the policy maker has liquid assets in its portfolio, our analysis applies equally well to the case where the policy maker expands its balance sheet, i.e., it *issues* $\Delta A(s)$ liquid assets (backed by lump sum taxes) and sells them for some other assets.

for reserves led to a drop in interest rates but did not affect the yields on relatively illiquid assets (Baa corporate bonds). Suppose that Baa corporate bonds are illiquid assets, highly-rated bonds are partially liquid, and Treasuries and reserves are liquid. Our theory predicts that the purchase of Treasuries with reserves is neutral while the purchase of partially-liquid bonds reduces aggregate liquidity and leads to higher asset prices and lower returns. Moreover, in the context of our modelled economy the drop in interest rates is symptomatic of reduced liquidity, output, and welfare.

5.2 Asset retention and haircuts

Mechanisms have been designed to mitigate the informational asymmetries that affect the securitization process by aligning securitizers' incentives with the interest of investors, e.g., by requiring them to retain in their books some of the securities that they issue.[28] Similarly, concerns about the recovery value of collateral in the repo market has led to the use of overcollateralization and haircuts (Krishnamurthy, 2010; Gorton and Metrick, 2010). In the following we show that a simple extension of our model can generate a role for such incentive mechanisms.

A model of asset retention. To model the signaling role of retention or overcollateralization we make two assumptions. First, a buyer can credibly reveal his asset holdings, $a(s) \geq d(s)$, to the seller; in particular, if $a(s) > d(s)$, then the buyer reveals that he is retaining assets.[29] Second, fraud involves a variable cost, $k_v(s)$, per unit of fraudulent asset, in addition to the fixed cost $k_f(s)$.[30] Then, the resalability constraint of asset s becomes:

$$[\phi(s) - 1]\, a(s) + \sigma d(s) \leq k_f(s) + k_v(s) a(s). \tag{22}$$

The first term on the left side of (22) is the cost of acquiring $a(s)$ genuine type-s asset while the second term is the cost of transferring $d(s)$ genuine type-s assets, should a match occur. The right

[28]For an overview of such incentives mechanisms, see Board of Governors of the Federal Reserve System (2010). As an example, a leading regulatory measure of the Dodd-Frank Act, enacted in July 2010 in response to the 2007-08 financial crisis, is a requirement for securitizers to retain at least 5 percent of the credit risk they originate.

[29]In principle the buyer's portfolio could include an unobservable part, i.e., the buyer could hide some assets. As will become clear later, in the case of a liquidity shortage, a buyer will never find it optimal to hide assets. See the online supplementary appendix for a discussion.

[30]In Online Supplementary Appendix B we describe this model in detail. We assume that agents issue securities backed by assets in $\{1, ..., S\}$. The issuer of a type-s asset-backed security (ABS) chooses an underlying portfolio made of $\eta < 1$ units of genuine assets and $1 - \eta$ units of fraudulent assets. Such an ABS pays off η units of the numéraire good at $t = 2$.

side of (22) is the total cost of producing $a(s)$ fraudulent assets of type s.

This new resalability constraint, (22), implies that retention helps buyers signal the quality of the asset they offer. To see this in the simplest possible way, assume that $\phi(s) = 1$. In this case, it is not costly to retain a marginal unit of the genuine asset but, because of the variable cost, it is costly to retain a corresponding marginal unit of the fraudulent asset. Thus, from (22), increasing $a(s)$ by one marginal unit above $d(s)$ allows the buyer to raise his asset transfer and DM consumption by $\frac{k_v(s)}{\sigma} > 0$, which increases the buyer's utility if $q < q^*$.

If $q < q^*$ in equilibrium, then it must be costly to retain genuine assets, for otherwise buyers would have strict incentives to hold more than the actual asset supply. Thus, even illiquid assets must sell at a premium, $\phi(s) > 1$. This premium is equal to $\phi(s) - 1 = \frac{\partial d(s)}{\partial a(s)}\xi$, the marginal increase in the amount of asset offered, $\frac{\partial d(s)}{\partial a(s)}$, multiplied by the expected marginal surplus of an asset spent in the DM, $\xi = \sigma\left[u'(q) - 1\right]$. From (22) at equality,

$$\frac{\partial d(s)}{\partial a(s)} = \frac{k_v(s) - [\phi(s) - 1]}{\sigma}.$$

The marginal resalability of an illiquid asset is proportional to the difference between the variable cost of fraud and the holding cost of a genuine asset. Substituting $\frac{\partial d(s)}{\partial a(s)}$ by its expression into $\phi(s) - 1$ and solving for $\phi(s)$ leads to:

$$\phi(s) = 1 + \frac{\xi k_v(s)}{\sigma + \xi}. \tag{23}$$

The premium on illiquid assets is a fraction of the variable cost of fraud. Since buyers do not retain liquid or partially liquid assets, $d(s) = a(s) = A(s)$, their prices are determined as in Proposition 3 after redefining $\kappa(s)$ as $k_v(s) + \frac{k_f(s)}{A(s)}$.

Finally, aggregate liquidity is defined as

$$L = \sum_{s \in S} \theta(s) A(s), \quad \text{where } \theta(s) \equiv \min\left[\frac{k_f(s)}{\sigma A(s)} + \frac{k_v(s)}{\sigma + \xi}, 1\right]. \tag{24}$$

The weight, $\theta(s)$, depends on the asset-specific costs of fraud, $k_f(s)$ and $k_v(s)$, and aggregate liquidity. If aggregate liquidity is scarce, then ξ is high and, from (23), the prices of illiquid assets are also high. As a result, agents have more incentives to commit fraud, which tightens the resalability constraints of illiquid assets and makes aggregate liquidity scarcer. As shown in the online supplementary appendix, this mechanism can lead to multiple equilibria.

Haircuts. Suppose we interpret the offer in the DM as a multi-loan arrangement composed of asset-backed loan contracts, $\{d(s), a(s)\}$, that specify the size of the loan, $d(s)$, and the quantity of assets of a given type to serve as collateral for that loan.[31] The resalability constraint, (22), ensures that it is optimal to commit genuine collateral and to repay each loan. We define the aggregate haircut on type-s assets, denoted $h(s)$, as the percentage difference between the loan or payment value, $d(s)$, and the collateral value, $A(s)$, i.e., $h(s) \equiv 1 - \frac{d(s)}{A(s)}$. For liquid and partially liquid assets, $d(s) = A(s)$ and therefore $h(s) = 0$. For illiquid assets,

$$h(s) = 1 - \frac{k_f(s)}{\sigma A(s)} - \frac{k_v(s)}{\sigma + \xi}. \tag{25}$$

The size of the haircut increases with the asset supply and decreases with the costs of fraud. Moreover, haircuts tend to be larger when aggregate liquidity is scarce. Indeed, scarce liquidity inflates liquidity premia on illiquid assets, $\frac{\xi k_v(s)}{\sigma+\xi}$, which raises incentives to commit fraud. Haircuts also increase with the frequency of meetings in the DM, σ. Intuitively, buyers have more incentive to produce fraudulent assets if they don't have to hold onto them until they mature, which is the case if there are more opportunities to pass them to some sellers in the DM. Finally, assuming $k_f(s) \approx 0$, $\phi(s) = 1 + \xi - \xi h(s)$, asset prices are negatively correlated with haircut sizes.

5.3 Liquidity crisis

In what follows, we describe the effects of a shock raising the threat of fraud for a class of assets.[32] In accordance with the 2007-2008 subprime financial crisis, this shock triggers a flight to liquidity where market participants seek to reallocate their portfolios toward highly liquid assets, which leads to a widening yield spread between the most liquid and the less liquid assets. Moreover, liquidity dries up in the DM (repo) market, which generates larger haircuts on illiquid securities.[33]

[31] By assuming a multi-loan arrangement, the model can capture realistic asset-specific margin constraints (haircuts) as described, e.g., in Gârleanu and Pedersen (2011).

[32] If we endogenize fraud in equilibrium, as in Section 3.2, the decrease in the cost of fraud can be interpreted as an increase in the probability of costless fraud opportunities, $1-\omega(s)$. This would have the consequence of increasing the occurrence of fraud. For some prominent economists this type of shock is a central explanation for the financial crisis of 2008. In an interview to the Wall Street journal (09/24/2011), Robert Lucas argued that "the shock came because complex mortgage-related securities minted by Wall Street and certified as safe by rating agencies had become part of the effective liquidity supply of the system. All of a sudden, a whole bunch of this stuff turns out to be crap".

[33] Evidence shows that, during the subprime crisis, the flight-to-quality was confined to AAA-rated bonds, and the illiquidity component of the rate of return of bonds with lower grades rose sharply (Longstaff, 2010; Dick-Nielsen,

In the following analysis we assume that equilibrium is unique. Let a distribution of assets be a collection of triples $\langle k_f(s), k_v(s), A(s)\rangle$. Assume that, at one extreme of the distribution there is a type, s_0, such that $k_f(s_0) = k_v(s_0) = 0$. Type-s_0 assets are the most illiquid: they are not used as a means of payment or collateral, $\theta(s_0) = 0$, and they have zero liquidity premium, $\phi(s_0) - 1 = 0$. At the other extreme of the distribution there is a type, s_∞, such that $k_f(s_\infty) = k_v(s_\infty) = \infty$. Type s_∞ assets are the most liquid: assuming $q < q^*$, the entire supply is used as means of payment or collateral, $\theta(s_\infty) = 1$, and their liquidity premium is the maximum, $\phi(s_\infty) - 1 = \xi$.

We generate a flight to liquidity as follows. Take a type of assets, $\hat{s}$, that is initially liquid or partially liquid, i.e., $\theta(\hat{s}) = 1$. If the costs of fraud, $k_f(\hat{s})$ and $k_v(\hat{s})$, decrease by a sufficient amount, then asset $\hat{s}$ becomes illiquid, so that $\theta(\hat{s}) < 1$. For instance, agents realize that some assets (e.g., mortgage-backed securities) can be subject to a broader set of fraudulent practices than previously thought. Thus, the reduction in $\theta(\hat{s})$ causes aggregate liquidity, L, to fall. The set of liquid assets, $\frac{k_f(s)}{A(s)} + k_v(s) \geq \bar{\kappa} = \xi + \sigma$, shrinks. In accordance with a flight to liquidity, market demand for assets is concentrated on a smaller set of highly liquid assets. Moreover, the set of illiquid assets, requiring positive haircuts, $h(s) > 0$—or, equivalently from (25), assets such that $\frac{k_f(s)}{A(s)} + \frac{\sigma k_v(s)}{\sigma+\xi} \leq \sigma$—expands. Finally, haircuts for all illiquid assets, $h(s) = 1 - \frac{k_f(s)}{\sigma A(s)} - \frac{k_v(s)}{\sigma+\xi}$, increase.

The contraction in aggregate liquidity affects the whole distribution of asset prices. The liquidity premium on liquid assets, $\phi(s_\infty) - 1 = \xi$, increases while the price of the most illiquid assets are unchanged, $\phi(s_0) = 1$. Hence, the yield difference between the two extremes of the asset spectrum increases. The price of the newly illiquid assets, $\hat{s}$, decreases and, from (23), the price difference between liquid and illiquid assets, $\xi - \frac{k_v(s)}{\sigma+\xi}\xi$, increases. Prices of partially liquid assets stay unchanged, and therefore the price difference with liquid assets increases.

6 Conclusion

In this paper we have studied the pricing and liquidity implications of fraud in asset markets. We have shown that the threat of fraud creates resalability constraints, i.e., upper bounds on

Feldhutter, and Lando, 2012). During the crisis, haircuts on all classes of securities increased except U.S. Treasury bills. For details, see, e.g., Krishnamurthy (2010).

the quantity of assets agents can resell. These bounds depend on asset characteristics, such as their supplies and vulnerabilities to fraud, as well as market characteristics, such as the frequency of trading opportunities. In equilibrium the cross section of assets is partitioned endogenously into three liquidity tiers, which differ in resalability, prices, and sensitivity to policies and shocks. We have shown that the model was tractable and could be readily extended to explain fraud in equilibrium and to rationalize incentive mechanisms such as asset retention and haircuts. We used these extensions to provide a narrative for the experience of the repo market during the 2007-08 financial crisis.

References

Amihud, Yakov and Haim Mendelson. 1986. "Asset Pricing and the Bid Ask Spread." *Journal of Financial Economics* 17 (December): 223-49.

Barnett, Harold. 2012. "And Some with a Fountain Pen: Mortgage Fraud, Securitization and the Subprime Bubble." in *How They Got Away with it: White-Collar Crime and the Financial Meltdown*, edited by Susan Will, Stephen Handelman and David C. Brotherton. New York: Columbia University Press.

Barnett, William A., Douglas Fisher, and Apostolos Serletis. 1992. "Consumer Theory and the Demand for Money." *Journal of Economic Literature* 30 (December): 2086-2119.

Board of Governors of the Federal Reserve System. October 2010. "Report to the Congress on Risk Retention." Submitted to the Congress pursuant to section 941 of the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010.

Chordia, Tarun, Sahn-Wook Huh, and Avanidhar Subrahmanyam. 2009. "Theory-Based Illiquidity and Asset Pricing." *Review of Financial Studies* 22 (September): 3629-68.

Dick-Nielsen, Jens, Peter Feldhutter, and David Lando. 2012. "Corporate Bond Liquidity Before and After the Onset of the Subprime Crisis." *Journal of Financial Economics* 103 (March): 471-92.

Duffie, Darell. 1996. "Special Repo Rates." *Journal of Finance* 51 (June): 493-526.

Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen. 2005. "Over-the-counter Markets." *Econometrica* 73 (November): 1815-47.

Freeman, Scott. 1985. "Transactions Costs and the Optimal Quantity of Money." *Journal of Political Economy* 93 (February): 146-57.

Friedman, Milton and Anna J. Schwartz. 1970. *Monetary Statistics of the United States: Estimates, Sources, Methods*, NBER Studies in business cycles, no. 20, New York: National Bureau of Economic Research.

Garleanu, Nicolae and Heje Pedersen. 2011. "Margin-Based Asset Pricing and Deviations from the Law of One Price." *Review of Financial Studies* 24 (April): 1980-2022.

Green, Edward and Warren Weber. 1996. "Will the New $100 Bill Decrease Counterfeiting?" Federal Reserve Bank of Minneapolis *Quarterly Review* 20 (Summer): 3-10.

Gorton, Gary and Andrew Metrick. 2010. "Haircuts." Federal Reserve Bank of St. Louis *Review* 92 (November/December): 507-19.

Guerrieri, Veronica and Robert Shimer. 2011. "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality." Manuscript, Department of Economics, University of Chicago.

Guerrieri, Veronica, Robert Shimer, and Randall Wright. 2010. "Adverse Selection in Competitive Search Equilibrium." *Econometrica* 78 (November): 1823-62.

Holmstrom, Bengt and Jean Tirole. 2011. *Inside and Outside Liquidity*, Cambridge: MIT Press.

Hopenhayn, Hugo A. and Ingrid M. Werner. 1996. "Information, Liquidity, and Asset Trading in a Random Matching Game." *Journal of Economic Theory* 68 (February): 349-79.

In, Younghwan and Julian Wright. 2011. "Signaling Private Choices." Manuscript, KAIST Business School and National University of Singapore.

Keys, Benjamin J., Tanmoy K. Mukherjee, Amit Seru, and Vikrant Vig. 2010. "Did Securitization Lead to Lax Screening? Evidence from Subprime Loans." *Quarterly Journal of Economics* 125 (February): 307-62.

Kiyotaki, Nobuhiro and John Moore. 2001. "Liquidity, Business Cycles and Monetary Policy." Manuscript, Department of Economics, Princeton University.

Kiyotaki, Nobuhiro and John Moore. 2005. "Liquidity and Asset Prices." *International Economic Review* 46 (May): 317-49.

Kohlberg, Elon and Jean-Francois Mertens. 1986. "On the Strategic Stability of Equilibria." *Econometrica* 54 (September): 1003-37.

Krishnamurthy, Arvind. 2010. "How Debt Markets Have Malfunctioned in the Crisis." *Journal of Economic Perspectives* 24 (Winter): 3-28.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen. 2011. "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy." *Brookings Papers Economic Activity* (Fall): 215-65.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen. 2012. "The Aggregate Demand for Treasury Debt." *Journal of Political Economy* 120 (April): 233-67.

Lagos, Ricardo. 2010. "Asset Prices and Liquidity in an Exchange Economy." *Journal of Monetary Economics* 57 (November): 913-30.

Lagos, Ricardo and Randall Wright. 2005. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy* 113 (June): 463-84.

Lester, Benjamin, Andrew Postlewaite, and Randall Wright. 2012. "Information, Liquidity, Asset Prices, and Monetary Policy." *Review of Economic Studies* 79 (June): 1209-38.

Li, Yiting, Guillaume Rocheteau, and Pierre-Olivier Weill. 2011. "Liquidity and the Threat of Fraudulent Assets." Working Paper 11-24 (October), Federal Reserve Bank of Cleveland.

Longstaff, Francis A. 2010. "The Subprime Credit Crisis and Contagion in Financial Markets." *Journal of Financial Economics* 97 (January): 436-50.

Lucas, Robert E. 2000. "Inflation and Welfare." *Econometrica* 68 (March): 247-74.

Mihm, Stephen. 2007. *A Nation of Counterfeiters: Capitalists, Con men, and the Making of the United States.* Cambridge: Harvard University Press.

Nosal, Ed and Neil Wallace. 2007. "A Model of (the Threat of) Counterfeiting." *Journal of Monetary Economics* 54 (May): 229-46.

Purnanandam, Amiyatosh K. 2011. "Originate-to-Distribute Model and the Subprime Mortgage Crisis." *Review of Financial Studies* 24 (June): 1881-1915.

Rocheteau, Guillaume. 2011. "Payments and Liquidity under Adverse Selection." *Journal of Monetary Economics* 58 (April): 191-205.

Sargent, Thomas J. and Francois R. Velde. 2002. *The Big Problem of Small Change.* Princeton: Princeton University Press.

Shi, Shouyong. 2008. "Efficiency Improvement from Restricting the Liquidity of Nominal Bonds." *Journal of Monetary Economics* 55 (September): 1025-37.

U.S. Securities and Exchange Commission. 2010. "Performance and Accountability Report."

U.S. Senate. 2011. "Wall Street and the Financial Crisis: Anatomy of a Financial Collapse." U.S. Senate report.

Vickery, James and Joshua Wright. 2011. "TBA Trading and Liquidity in the Agency MBS Market." Federal Reserve Bank of New York *Economic Policy Review*, forthcoming.

Wallace, Neil. 1998. "A Dictum for Monetary Theory." Federal Reserve Bank of Minneapolis *Quarterly Review* 22 (Winter): 20-26.

Wallace, Neil. 2000. "A Model of the Liquidity Structure Based on Asset Indivisibility." *Journal of Monetary Economics* 45 (February): 55-68.

Weill, Pierre-Olivier. 2008. "Liquidity Premia in Dynamic Bargaining Markets." *Journal of Economic Theory* 140 (May): 66-96.

Williamson, Stephen and Randall Wright. 1994. "Barter and Monetary Exchange under Private Information." *American Economic Review* 84 (March): 104-23.

A Proof of Proposition 1

We define an outcome of the game as an offer $(q, \{d(s)\})$ made by the buyer, probabilities $\{\eta(s)\}$ of bringing genuine assets, and a probability $\pi \in [0,1]$ that the seller accepts the offer. Let us consider the auxiliary problem of choosing an outcome in order to maximize the expected utility of a buyer,

$$-\sum_{s\in S}\Big\{k(s)\left[1-\eta(s)\right]+\left[\phi(s)-1\right]\eta(s)d(s)\Big\}+\sigma\pi\left[u(q)-\sum_{s\in S}\eta(s)d(s)\right], \tag{26}$$

subject to the constraint that the probabilities π and $\{\eta(s)\}$ are the basis of an equilibrium in the sub-game following offer $(q, \{d(s)\})$; that is:

$$\pi \in \arg\max_{\hat{\pi}\in[0,1]} \hat{\pi}\Big\{-q+\sum_{s\in S}\eta(s)d(s)\Big\} \tag{27}$$

$$\eta(s) \in \arg\max_{\hat{\eta}\in[0,1]} \hat{\eta}\Big\{k(s)-\left[\phi(s)-1+\sigma\pi\right]d(s)\Big\},\ \text{for all } s\in S. \tag{28}$$

We start by showing that:

Claim 1 *Any solution of the auxiliary problem has the property that* $\eta(s) = 1$ *and* $q = \sum_{s\in S} d(s)$.

Proof. Consider first any feasible outcome $\langle q, d, \eta, \pi\rangle$ such that $\eta(s_0) < 1$ for some s_0. If $\eta(s_0) = 0$, then consider the alternative outcome, $\langle q', d', \eta', \pi'\rangle$, such that: (i) $q' = q$, $d'(s) = d(s)$ for all $s \neq s_0$, $d'(s_0) = 0$; (ii) $\eta'(s) = \eta(s)$ for all $s \neq s_0$ and $\eta'(s_0) = 1$; (iii) $\pi' = \pi$. The incentive constraint of the seller, (27), is satisfied since it only depends on the product $\eta(s)d(s)$. The incentive constraint of the buyer, (28), is obviously satisfied for $s \neq s_0$. For $s = s_0$ we have $k(s_0) > [\phi(s) - 1 + \sigma]\, d'(s_0) = 0$ and so $\eta'(s_0) = 1$ is optimal for the buyer. One can then verify that, with this alternative outcome, the expected utility of the buyer increases by $k(s_0) > 0$.

Next, consider any feasible outcome such that $\eta(s_0) \in (0,1)$: the buyer is indifferent between counterfeiting asset s_0 or not. We then increase $\eta(s_0)$ by $\varepsilon \in (0,1]$ and q by $\varepsilon d(s_0)$, which is positive since the incentive constraint of the buyer, (28), binds. The incentive constraint of the seller, (27) is satisfied because his payoff conditional on accepting the offer does not change. Because the buyer is indifferent between counterfeiting asset s_0 or not, an increase in $\eta(s_0)$ affects neither his payoff, (26), nor his incentive constraint, (28). The corresponding increase in q, however, increases his payoff strictly.

Next, consider any feasible outcome $\langle q, d, \eta, \pi \rangle$ such that $\eta(s) = 1$ for all s, but $q < \sum_{s \in S} d(s)$. Then the alternative outcome with $q' = \sum_{s \in S} d(s) > q$, $\eta'(s) = 1$, and $\pi' = \pi$, increases the expected payoff to the buyer by $\sigma\pi\,[u(q') - u(q)] > 0$ and satisfies all the constraints. ■

This claim implies that we can rewrite the auxiliary problem as

$$\max_{q,d,\pi} - \sum_{s \in S} [\phi(s) - 1]\, d(s) + \sigma\pi\,[u(q) - q] \tag{29}$$

$$\text{s.t.} \quad \sum_{s \in S} d(s) - q = 0 \tag{30}$$

$$k(s) \geq [\phi(s) - 1 + \sigma\pi]\, d(s), \;\; \text{for all } s \in S. \tag{31}$$

The second condition is the first-order necessary and sufficient condition for (28) evaluated at $\eta(s) = 1$. Next, we show:

Claim 2 *Any solution of the auxiliary problem, (26)-(28), has the property that* $u'(q) \geq 1$ *and* $\pi = 1$.

Proof. The first claim holds because otherwise one could reduce the quantity produced, increase the expected utility of the buyer, and satisfy all the constraints. To prove the second claim suppose, towards a contradiction, that $\pi < 1$. Note first that the value of the auxiliary problem must be positive: a small offer $q' = d'(s_0) > 0$, $d'(s) = 0$ for $s \neq s_0$, and $\pi' = 1$ yields a positive payoff. This implies that both $q > 0$ and $\pi > 0$. Moreover, at least one of the incentive constraints, (31), must be binding since otherwise one could increase π without violating any of the incentive constraints, and improve the objective. Let $S_B \subseteq S$ be the set of binding IC constraints. Since $[\phi(s) - 1 + \sigma\pi]\, d(s) = k(s)$ for all $s \in S_B$, it follows that $d(s) > 0$. Now consider the following variational experiment: increase π by some small ε and decrease the payments $d(s)$, for all $s \in S_B$, so that all the incentive constraints continue to bind. Up to second-order terms, the decrease in $d(s)$ is equal to $m(s)\varepsilon$, where

$$m(s) \equiv \frac{\sigma d(s)}{\phi(s) - 1 + \sigma\pi},$$

is the marginal rate of substitution between π and $d(s)$ in the IC constraint for asset $s \in S_B$. Lastly, to satisfy the participation constraint, the decrease in q must be, up to second order terms,

$\sum_{s\in S_B} m(s)\varepsilon$. The change in the buyer's expected utility is, up to second-order terms, equal to $\Delta U \times \varepsilon$, where

$$
\begin{aligned}
\Delta U &= \sum_{s\in S_B} [\phi(s)-1]\, m(s) - \sigma\pi \left[u'(q)-1\right] \sum_{s\in S_B} m(s) + 1\sigma \left[u(q)-q\right] \\
&> \sum_{s\in S_B} [\phi(s)-1]\, m(s) + \sigma \left[u'(q)-1\right] \left[q - \pi \sum_{s\in S_B} m(s)\right] \\
&\geq \sum_{s\in S_B} [\phi(s)-1]\, m(s) + \sigma \left[u'(q)-1\right] \sum_{s\in S_B} [d(s)-\pi m(s)] = \sum_{s\in S_B} [\phi(s)-1]\, m(s)u'(q) \geq 0,
\end{aligned}
$$

where we move from the first line to the second line using $u(q)-q > q\,[u'(q)-1] \geq 0$ (the equality is strict because of two facts: first, $u(q)$ is strictly concave and second, $q>0$, since the value of the auxiliary problem is positive); from the second line to the third line using $q \geq \sum_{s\in S_B} d(s)$; and from the third to the fourth line by noting that $d(s)-\pi m(s) = [\phi(s)-1]\, m(s)/\sigma$. ■

From Claims 1-2 and the result according to which $a(s) \geq \chi(s)d(s)$ if $\phi(s)=1$, and $a(s) = \chi(s)d(s)$ if $\phi(s)>1$, it follows that the auxiliary problem, (26)-(28), reduces to the maximization problem of Proposition 1, (8)-(11). Now we note that the solution to the auxiliary problem is an upper bound on the value of the buyer in any equilibrium of the game. Let $(\tilde{q}, \{\tilde{d}(s)\})$ be one solution of the auxiliary problem. Because, as argued above, the value of the auxiliary problem is positive, it must satisfy $\tilde{q}>0$ and $\tilde{d}(s)>0$ for some $s\in S$. Consider, for any $\varepsilon>0$ small enough, the offer $d^{\varepsilon}(s) = \max\{\tilde{d}(s)-\varepsilon, 0\}$ and $q^{\varepsilon} = \tilde{q}-(S+1)\varepsilon$. By construction, this offer is such that $[\phi(s)-1+\sigma]\, d^{\varepsilon}(s) < k(s)$, and $q^{\varepsilon} < \sum_{s\in S} d^{\varepsilon}(s)$. Thus, $\pi=1$ and $\eta(s)=1$ is the unique equilibrium in the subgame following $(q^{\varepsilon}, \{d^{\varepsilon}(s)\})$. By letting ε go to zero and making the above offer, the buyer can achieve a value arbitrarily close to that of the auxiliary problem. Therefore, in any equilibrium, the buyer's value must be equal to that of the auxiliary problem. Moreover, any equilibrium outcome satisfies (27) and (28). Therefore, any equilibrium outcome must solve the auxiliary problem.