Money, Credit, and Monetary Policy

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Abstract

We study liquidity effects and short-term monetary policies in a model with fully flexible prices, and with an explicit role for money and financial intermediation. Banks may hold some deposits and money injections as reserves. If banks hold liquidity buffers, liquidity effects exist when the fraction of money injections used to finance spending is larger than that of the initial money stock. The lower the substitutability between newly issued money and the initial money stock, the larger the liquidity effect. We determine the coefficients of the interest rate rules in response to shocks from the first-order conditions for a loss function. One implication is that, to minimize the loss caused by fluctuations in inflation and output is equivalent to setting the money growth rate at the target inflation rate. If the central bank targets inflation only, the optimal coefficient may depend on the magnitude of the liquidity effect.

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1 Introduction

As governments across the globe struggle to deal with the aftermath of recent financial crisis, another major round of financial regulations has been in the works. A recent Basel III proposal on global banking regulation would require banks to hold liquidity buffers sizeable enough to enable them to withstand a severe short-term shock.\footnote{The short-term liquidity buffers (mostly comprising cash, central bank reserves, and domestic sovereign bonds), known as the liquidity coverage ratio, require a bank to have enough highly liquid assets on the balance sheet to cover its net cash outflows over a 30-day period following a shock event, such as a three-notch downgrade to its public credit rating. This will come into effect in 2015. See Basel III: International framework for liquidity risk measurement, standards and monitoring, December 2010.} Although Basel III rules will not be fully effective for several years, we can project what the requirement on liquidity buffers might imply for the monetary transmission mechanism, and how it will affect the conduct and effectiveness of monetary policy. In this paper, we offer a general equilibrium framework to tackle these issues, in line with the macroprudential approach.\footnote{As Hanson, Kashyap, and Stein (2011) argue, a framework with microprudential regulations, which is partial equilibrium in its conception and aimed at preventing the costly failure of individual financial institutions, is limited. Instead, we need a macroprudential approach to recognize the importance of general equilibrium effects, and safeguard the financial system as a whole.}

Our framework features flexible prices and frictions, which give rise to the roles of money and financial intermediation (see, e.g., Lagos and Wright 2005, and Berentsen, Camera, and Waller 2007). Banks channel funds from people with idle cash to those who need liquidity to finance unanticipated consumption. The central bank injects money through financial intermediaries. Agents make decisions on money holdings before they learn the shocks of preference and money injections. An unexpected money injection increases the nominal amount of loans (the loanable funds effect); nonetheless, it raises the expected inflation and lowers the real value of money and loans (the Fisher effect). The existence of a liquidity effect, where output rises and nominal interest rates fall in response to money supply shocks, depends on whether the loanable funds effect outweighs inflation expectations.

Due to limitations on record keeping, enforcement, and commitment, fiat money is used as the medium of exchange in the decentralized market. Agents, however, are not subject to the standard cash-in-advance constraint because, before trading, they can borrow cash from banks to supplement their money holdings. The amount that agents can borrow is affected by the banks'
holdings of liquidity buffers—the fraction of deposits and money injections that may be held as reserves, due to regulations or liquidity management considerations.\textsuperscript{3} If the fractions of the initial money stock and money injections that are used to finance spending are identical, then the loanable funds effect offsets the Fisher effect. Agents make portfolio decisions \textit{as if} they knew the future money injections. Thus, in contrast to the previous literature, the informational friction in our model does not necessarily generate liquidity effects.\textsuperscript{4} On the other hand, if the fraction of money injections used to finance spending is larger than that of the initial money stock, the loanable funds effect dominates the Fisher effect. Consequently, output increases and interest rates fall. In this case, the higher the fraction of money injections used to finance spending, the larger the liquidity effect.

Given that our model identifies conditions for the existence of liquidity effects, we use it to study a class of short-term monetary policies. Much discussion on monetary policies nowadays is centered on “Taylor rules,” which specify the nominal interest rate set by the central bank as a reaction function to changes in inflation and output, among other variables. While the coefficients of Taylor rules in conventional New Keynesian models are usually constant (see Woodford 2003), Board staff at the FOMC meeting of November 1995 suggested that equal weights on inflation and the output gap in Taylor rules may not always be appropriate.\textsuperscript{5} One may like to know what the optimal reaction coefficients are when an economy faces different shocks. We attempt to ask: What are the optimal reaction coefficients when an economy faces different shocks? How should the coefficients be determined in an economy with flexible prices and frictions as the one we consider here?

Following the approach of choosing the reaction coefficients for a target rule as proposed by Svensson (2003), we determine the coefficients of Taylor rules from the first-order conditions for a specific loss function. Our model suggests that the central bank should identify the source of inflation when determining the coefficients. If a negative supply shock occurs, the central bank faces

\textsuperscript{3}Requiring a bank to hold sufficiently high liquid assets is a cost on intermediation. Because of less available information on banks’ net worth, however, developing countries have to rely on the quantitative liquidity regulation (Freedman and Click, 2006; Ratnovski, 2009).

\textsuperscript{4}For instance, Lucas (1990), Fuerst (1992) and Christiano (1991) attribute the reason for the liquidity effect to agents’ inability to adjust their portfolios at the time of the money injections.

a dilemma: trying to control inflation will dampen output, whereas stimulating aggregate demand will escalate inflation. The best strategy is to set a sufficiently small coefficient on inflation, which amounts to setting the growth rate of money at the target inflation rate. On the other hand, when a demand shock occurs and pushes up the interest rate, raising the money growth rate to lower the interest rate will cause inflation to rise. To minimize the fluctuations in inflation and output, the central bank should choose a sufficiently large coefficient on inflation, which is equivalent to setting a sufficiently small money growth rate. In the limiting case, the monetary growth rate should be set at the target inflation rate. These two examples thus deliver the same message: the central bank can accomplish more (i.e., minimize the loss) by doing less. Finally, if the central bank targets on inflation only, the optimal coefficient may depend on the magnitude of the liquidity effect.

The current paper is related to two strands of literature on liquidity effects and Taylor rules. Using a model with segmented markets, Williamson (2004) shows in a model with segmented markets that, if households can issue private money after they learn the shocks to money injections, the liquidity effect is eliminated. The mechanism in the current paper is similar to Williamson’s: bank’s lending of money injections, like the private money in Williamson’s model, effectively removes the buyer’s cash-in-advance constraint. The distinction is that our paper identify how bank’s holding reserves affects the magnitude of the liquidity effect, and we discuss short-term monetary policy. In Berentsen and Waller (2011) a stabilization policy generates real effects because the central bank can commit itself to a price-level path and undo the current money injection at a future date. Our paper differs in that we do not assume the central bank can undo money injections; moreover, we discuss a class of interest rate rules with which the central bank can minimize the variation of inflation and output. As for studies on the interest rate rule, Alvarez, Lucas, and Weber (2001) use models of segmented markets to show that increasing the interest rate to reduce inflation can be rationalized with quantity-theoretic models of monetary equilibrium. While delivering a similar implication in a model with financial intermediation and endogenous cash-in-advance constraints,

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6Previous literature using limited participation models to study liquidity effects includes, for example, Grossman and Weiss (1983), Rotemberg (1984), and Williamson (2006). They identify the distributional effect of money injections as the underlying mechanism, but often the models are not analytically tractable, except Williamson (2006).

7The result is similar to Lagos (2011), who shows in a framework of multiple assets that real money balances are reduced by a higher nominal interest rate. He also finds that the optimal monetary policy is a zero interest rate policy—the Friedman rule.
we further determine the coefficients of Taylor rules.\textsuperscript{8}

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 derives the equilibrium conditions. In Section 4 we study the liquidity effect. We discuss issues related to Taylor rules in section 5, and conclude in section 6.

2 The Environment

The basic environment is based on Lagos and Wright (2005) and Berentsen, Camera, and Waller (2007). There is a $[0,1]$ continuum of infinitely lived agents. Time is discrete and continues forever. Each period is divided into two subperiods, and in each subperiod trades occur in competitive markets. There are perishable and perfectly divisible goods, one produced in the first subperiod, and the other (called the \textit{general good}) in the second subperiod. The discount factor across periods is $\beta = \frac{1}{1 + \rho} \in (0, 1)$, where $\rho$ is the rate of time preference.

In the beginning of the first subperiod, an agent receives a preference shock that determines whether he consumes or produces. With probability $\theta$ an agent can consume but cannot produce; with probability $1 - \theta$ the agent can produce but cannot consume. We refer to consumers as buyers and producers as sellers. This is a simple way to capture the uncertainty of the opportunity to trade. Consumers get utility $u(q)$ from $q$ consumption, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$ and $u'(-\infty) = 0$. Producers incur disutility $c(q)$ from producing $q$ units of output, where $c'(q) > 0$, $c''(q) \geq 0$. To motivate a role for fiat money, we assume that all goods trades are anonymous, and there is no public record of individuals’ trading histories.

In the second subperiod, all agents can produce and consume the general good, getting utility $U(x)$ from $x$ consumption, where $U'(x) > 0$, $U''(x) \leq 0$, $U'(0) = \infty$, and $U'(\infty) = 0$. Agents can produce one unit of the general good with one unit of labor, which generates one unit of disutility. This setup allows us to introduce an idiosyncratic preference shock while keeping the distribution of money holdings analytically tractable.

A government is the sole issuer of fiat money. The evolution of the money stock is $M_t = (1 + z_t)M_{t-1}$, where $M_t$ denotes the per capita currency stock, and $z_t$ is the money growth rate, in period $t$. Assume $z_t = \mu + \varepsilon_t$, where $\mu$ is the long-run money growth rate, and $\varepsilon_t$ is a random variable.

\textsuperscript{8}For more studies on Taylor rules, see, e.g., Woodford (2003) for further references.
with density \( f \) on \([\xi, \tau]\). The random variable, \( \varepsilon_t \), generates a monetary shock, which becomes known at the beginning of period \( t \). In the first subperiod, the central bank injects money, \( \tau_t = z_t M_{t-1} \), through the banking system, which extends funds to borrowers. This transfer scheme is merely an analytical device to mimic open-market operations. We assume full enforcement, so the central bank can levy nominal taxes to extract cash from the economy, which implies \( \tau_t < 0 \) and \( z_t < 0 \).

Competitive banks accept nominal deposits and make nominal loans. Sellers in the first subperiod can deposit their money holdings in banks at the nominal interest rate, \( i_{d,t} \), and are entitled to withdraw funds in the second subperiod. Buyers can borrow money from banks at the nominal loan rate, \( i_{b,t} \), and repay their loans in the second subperiod. We assume that loans and deposits are not rolled over, and so all financial contracts are one-period contracts.\(^9\) Moreover, banks have zero net worth, and there are no operating costs.

Banks keep records on financial histories but not on trading histories in the goods market. The record-keeping technology is not available to individuals, so credit between private agents is not feasible. Under the full enforcement of debt repayment, default is not possible.\(^{10}\) In equilibrium, the loan rate \( i_{b,t} \) clears the loan market. Assume that banks are owned by private agents. Because the central bank injects money through financial intermediaries, banks may have nonzero profits. A bank’s profits are distributed to private agents as dividends, or are withdrawn from agents’ bank accounts in the case of \( z_t < 0 \). Moreover, we assume that the central bank injects money equally among banks, and leaves no room for an individual bank to use injections to compete away customers.

We consider an economy in which banks may keep a buffer stock of reserves, due to regulations (such as those in the Basel III proposal) or liquidity risk management considerations. A rationale for liquidity risk management is proposed by, for instance, Kashyap, Rajan, and Stein (2002): Banks provide customers with liquidity on demand to satisfy their unexpected needs. Liquidity is provided by offering demand deposits and loan commitments, which give a borrower the option to take the loans on demand over a certain specified period of time. Both of these products require

\(^9\)With the assumption on the linear utility costs of production in the second subperiod, agents do not gain by spreading the repayment of loans or redemption of deposits across periods.

\(^{10}\)But see, e.g., Berentsen, Camera, and Waller (2007) and Li and Li (2010), for considering the possibility of default to study the effects of inflation on credit arrangements, output, and asset prices.
explicit liquidity risk management. Specifically, we assume that banks lend out a constant fraction, \( \nu \in (0, 1] \), of deposits, and a fraction, \( \chi_m \in (0, 1] \), of money that the central bank injects into banks. Though we use regulations and liquidity management considerations to motivate banks’ holding liquidity buffers, in Appendix we offer a model with random withdrawal shocks as in the Diamond-Dybvig model to justify banks’ holding reserves.\(^{11}\)

The timing of events is summarized as follows. At the beginning of the first subperiod, each agent receives a preference shock. Money injections take place after the realization of preference shocks. Then sellers make deposits and buyers take loans. In the second subperiod, agents settle financial claims, receive dividends from banks, and adjust money holdings. In section 5, we add demand shocks or supply shocks in the first subperiod, which occur after the preference shocks but before the money injections.

3 Equilibrium

We study symmetric stationary equilibria in which end-of-period real balances are time-invariant; i.e., \( \phi_{t-1} M_{t-1} = \phi_t M_t \), where \( \phi_t \) is the value of money in terms of the good produced in the second subperiod. Thus, \( \frac{\phi_{t-1}}{\phi_t} = \frac{M_t}{M_{t-1}} = 1 + z_t \). As such, the money growth rate, \( z_t \), also represents the inflation rate in the second subperiod of period \( t \).

Let \( V(m_t) \) denote the expected value from trading in the first subperiod with \( m_t \) units of money. Let \( W(m_t, b_t, d_t) \) denote the expected value from entering the second subperiod with \( m_t \) units of money, \( b_t \) debt, \( d_t \) deposits, where loans and deposits are in the units of fiat money. We study a representative period \( t \) and work backwards from the second to the first subperiod, using a similar approach as in Berentsen, Camera, and Waller (2007) to characterize equilibria.

The second subperiod

In the second subperiod an agent consumes \( x_t \), produces \( h_t \) goods, redeems deposits, repays

\(^{11}\)Bencivenga and Camera (2011) consider banks and capital in the Lagos-Wright model where depositors make heterogeneous withdrawals, in the spirit of Diamond and Dybvig (1983). Banks, therefore, always hold some positive amount of reserves to satisfy the heterogeneous liquidity needs of buyers. In contrast, banks in our model make loans to satisfy the heterogeneous liquidity needs of agents.
loans, receives dividends, $F_t$, and adjusts his money holdings. He solves the following problem:

$$ W(m_t, b_t, d_t) = \max_{x_t, h_t, m_{t+1}} U(x_t) - h_t + \beta E_t V(m_{t+1}), $$

s.t. $x_t + \phi t m_{t+1} = h_t + \phi t (m_t + F_t) + \phi_t (1 + i_{d,t}) d_t - \phi_t (1 + i_{b,t}) b_t,$

where $E_t$ is the expectations operator based on the information of the current period. If an agent has deposited $d_t$ in the first subperiod, he receives $(1 + i_{d,t}) d_t$ units of money, and if he has borrowed $b_t$, he should repay $(1 + i_{b,t}) b_t$ units of money. Substituting $h_t$ from the budget constraint into the objective function, we obtain

$$ W(m_t, b_t, d_t) = \phi t (m_t + F_t) + \phi_t (1 + i_{d,t}) d_t - \phi_t (1 + i_{b,t}) b_t $$

$$ + \max_{x_t, m_{t+1}} \{ U(x_t) - x_t - \phi t m_{t+1} + \beta E_t V(m_{t+1}) \}. $$

The first order conditions are:

$$ U'(x_t) = 1, $$

$$ \beta E_t V_m(m_{t+1}) \leq \phi_t, \quad " = " \text{ if } m_{t+1} > 0, $$

where $V_m(m_{t+1})$ is the marginal value of an additional unit of money taken into the first subperiod of $t + 1$. Equation (2) implies $x_t = x^*$ for all agents and for all $t$. The intertemporal equation (3) determines $m_{t+1}$, independent of the initial holdings of $m_t$ when entering the second subperiod. Therefore, the distribution of money holdings is degenerate at the beginning of a period. The envelope conditions are

$$ W_m = \phi_t, $$

$$ W_b = -\phi_t (1 + i_{b,t}), $$

$$ W_d = \phi_t (1 + i_{d,t}). $$

The first subperiod

Let $q_{b,t}$ and $q_{s,t}$ denote the quantities consumed by a buyer and produced by a seller, respectively, and $p_t$ denote the nominal price of the good, in period $t$. An agent may be a buyer with probability $\theta$, spending $p_t q_{b,t}$ units of money to get $q_{b,t}$ consumption, or he may be a seller with probability $1 - \theta$.
1 − θ, receiving \( p_t q_{s,t} \) units of money from \( q_{s,t} \) production. Because buyers do not make deposits and sellers do not take out loans, in what follows we let \( b_t \) denote loans taken out by buyers and \( d_t \) deposits of sellers, and drop these arguments in \( W(m_t, b_t, d_t) \) where relevant for notational simplicity. The expected utility of an agent entering the first subperiod of period \( t \) with money holdings \( m_t \) is

\[
V(m_t) = \theta[u(q_{b,t}) + W(m_t + b_t - p_t q_{b,t}, b_t)] + (1 - \theta)[-c(q_{s,t}) + W(m_t - d_t + p_t q_{s,t}, d_t)].
\] (7)

Agents trade in a centralized market, so they take the price \( p_t \) as given. A seller solves

\[
\max_{q_{s,t}, d_t} -c(q_{s,t}) + W(m_t - d_t + p_t q_{s,t}, d_t)
\]

s.t. \( d_t \leq m_t \).

Let \( \lambda_{d,t} \) denote the multiplier on the deposit constraint. The first order conditions are

\[
-c'(q_{s,t}) + p_t W_m = 0,
\]
\[
-W_m + W_d - \lambda_{d,t} = 0.
\]

Using (4) and (6) the first order conditions become

\[
p_t = \frac{c'(q_{s,t})}{\phi_t},
\]
\[
\lambda_{d,t} = \phi_t i_{d,t}.
\] (8)

Equation (8) implies that a seller’s production is such that the marginal cost of production, \( \frac{c'(q_{s,t})}{\phi_t} \), equals the marginal revenue, \( p_t \). For \( i_{d,t} > 0 \), the deposit constraint binds and sellers deposit all money balances; i.e., \( d_t = M_{t-1} \). Moreover, the production \( q_{s,t} \) is independent of the seller’s initial portfolio brought to the first subperiod.

A buyer’s problem is

\[
\max_{q_{b,t}, b_t} u(q_{b,t}) + W(m_t + b_t - p_t q_{b,t}, b_t)
\]

s.t. \( p_t q_{b,t} \leq m_t + b_t \).
The buyer faces the cash constraint that his spending cannot exceed his money holdings, \( m_t \), plus borrowing, \( b_t \). He should have faced a constraint stating that his borrowing cannot exceed a certain credit limit. However, because banks can force borrowers to repay loans at no cost, the borrowing constraint does not bind; i.e., \( b_t \leq \infty \), and hence, we ignore this constraint. Let \( \lambda_t \) be the multiplier on the buyer’s cash constraint. Using (4), (5), and (8), the first order conditions are

\[
\frac{d(\bar{q}_b)}{c'(\bar{q}_s)} = \frac{\lambda_t}{\phi_t},
\]

(9)

\[
\phi_t i_{b,t} = \lambda_t.
\]

(10)

If \( \lambda_t = 0 \), (9) reduces to \( \frac{d(\bar{q}_b)}{c'(\bar{q}_s)} = \frac{\lambda_t}{\phi_t} \), implying \( i_{b,t} = 0 \). If \( \lambda_t > 0 \), the cash constraint binds, and the buyer spends all of his money; i.e.,

\[
\bar{q}_{b,t} = \frac{m_t + b_t}{p_t}.
\]

(11)

Combining (9) and (10), we obtain

\[
\frac{d(\bar{q}_b)}{c'(\bar{q}_s)} = 1 + i_{b,t},
\]

(12)

which implies that buyers borrow up to the point at which the marginal benefit of an additional unit of borrowed money, \( \frac{d(\bar{q}_b)}{c'(\bar{q}_s)} \), equals the marginal cost, \( 1 + i_{b,t} \).

To find an agent’s optimal money holdings, we take the derivative of (7) with respect to \( m \), and use (4) and (6) to get the marginal value of money:

\[
V_m(m_t, d_t) = \theta \frac{d(\bar{q}_b)}{p_t} + (1 - \theta) \phi_t (1 + i_{d,t}).
\]

(13)

An agent receives \( \frac{d(\bar{q}_b)}{p_t} \) from spending the marginal unit of money as a buyer, and if he is a seller, he deposits the idle cash in banks, which is valued \( \phi_t (1 + i_{d,t}) \) in the second subperiod. Using (3) lagged one period to eliminate \( V_m(m_t, d_t) \) from (13), an agent’s optimal money holdings satisfy

\[
\beta E_{t-1} [\theta \frac{d(\bar{q}_b)}{p_t} + (1 - \theta) \phi_t (1 + i_{d,t})] \leq \phi_{t-1}, \quad \text{“} = \text{” if } m_t > 0.
\]

(14)

Condition (14) states that the cost of acquiring an additional unit of money must be greater than the expected discounted benefit, with the equality holding if agents choose to hold money.
In a symmetric equilibrium, the market-clearing conditions for goods, money, and loan markets are

\[(1 - \theta)q_{s,t} = \theta q_{b,t}, \quad (15)\]
\[m_t = M_{t-1}, \quad (16)\]
\[\theta b_t = \nu(1 - \theta)d_t + \chi_m \tau_t, \quad (17)\]

respectively. In the loan market clearing condition, (17), the per capita funds available for banks to lend out include \(\nu\) fraction of deposits, \((1 - \theta)d_t\), plus \(\chi_m\) fraction of money injection, while per capita loans demanded is \(\theta b_t\).

Finally, banks are perfectly competitive with free entry, so they take as given the loan rate and the deposit rate. There is no strategic interaction among banks or between banks and agents, and no bargaining over the terms of the loan contract. Because the monetary authority injects money into all banks, competitive banks may earn positive profits.\(^{12}\) A bank’s per capita end-of-period profit is \(\omega = i_{b,t} \theta b_t - i_{d,t}(1 - \theta)d_t + \tau_t\). Substituting \((1 - \theta)d_t = \frac{\theta h - \chi_m \tau_t}{\nu}\) from (17), one can rewrite a bank’s profit as \(\omega = [1 + \frac{i_{d,t}}{\nu} \chi_m] \tau_t + [i_{b,t} - \frac{i_{d,t}}{\nu}] \theta b_t\). The zero marginal profit condition implies (see Appendix A2 for the details to derive solutions to the bank’s problem):

\[\nu i_{b,t} = i_{d,t}.\]

4 Liquidity Effects

In this section we identify the conditions for the existence of liquidity effects, and discuss whether the Friedman rule achieves the efficient allocation. An unexpected money injection results in two opposite effects: it increases the nominal amount of loans that buyers can borrow to spend (loanable funds effect), whereas it also raises inflation expectations and lowers the future value of money (Fisher effect). There are liquidity effects if the loanable funds effect outweighs the Fisher effect.

The total funds available for each buyer to finance consumption in the first subperiod include his money holdings, \(m_t\), and the money he borrows from banks, \(b_t\). From the loan-market-clearing

\(^{12}\)As argued by Fuerst (1994), there is nothing gained in explicitly modelling the open-market operations since the gains of the loanable reserves would be exactly offset by the loss of the interest-bearing securities.
condition, (17), we have
\[ b_t = \nu(1-\theta)d_t + \chi_m \tau_t. \]  
(18)

Substituting \( d_t = m_t \), \( \tau_t = z_t M_{t-1} \), and the market-clearing condition for money, \( m_t = M_{t-1} \), into (18), we obtain the total funds available per buyer in the first subperiod:
\[ m_t + b_t = \left( \frac{\chi + \chi_m z_t}{\theta} \right) M_{t-1}, \]
(19)

where \( \chi = \theta + (1-\theta)\nu \). Note that \( \chi \) is the fraction of the initial money stock, \( M_{t-1} \), that can be used to finance spending.

In equilibrium the cash constraint binds, i.e., \( q_{b,t} = \frac{m_t + b_t}{\rho_t} \), from which we derive the relationship between \( q_{b,t} \) and the money injection, \( z_t \):
\[ q_{b,t} c'(\frac{\theta q_{b,t}}{1-\theta}) = \left( \frac{\chi + \chi_m z_t}{\theta(1+z_t)} \right) \frac{\phi_{t-1} M_{t-1}}{1+z_t}. \]
(20)

by using (8), (19), and \( 1 + z_t = \frac{\phi_{t-1}}{\phi_t} \). Taking the derivative of (20) with respect to \( z_t \), we obtain
\[ \frac{\partial q_{b,t}}{\partial z_t} = \frac{\phi_{t-1} M_{t-1}(\chi_m - \chi)}{\theta(1+z_t)^2 c'(\frac{\theta q_{b,t}}{1-\theta}) + \frac{\theta q_{b,t}}{1-\theta} c''(\frac{\theta q_{b,t}}{1-\theta})} \left\{ \begin{array}{ll}
> 0 & \text{if } \chi_m - \chi > 0 \\
< 0 & \text{if } \chi_m - \chi < 0.
\end{array} \right. \]
(21)

Observe from (21) that the existence of liquidity effects (\( \frac{\partial q_{b,t}}{\partial z_t} > 0 \)) depends on whether the fraction of money injections used to finance spending, \( \chi_m \), is larger than that of the initial money stock, \( \chi \).

Note that the amount of money used to finance spending and thus, determine the inflation rate, in the first subperiod is \( (\chi + \chi_m z_t) M_{t-1} \), whereas the total end-of-period money stock, \( (1+z_t) M_{t-1} \), determines the inflation rate in the second subperiod. To see more clearly the underlying mechanism, rewrite (20) as
\[ \frac{\theta q_{b,t} c'(\frac{\theta q_{b,t}}{1-\theta})}{\chi \phi_t} = (1 + \frac{\chi_m}{\chi} z_t) M_{t-1}. \]
(22)

It is clear that the term, \( 1 + \frac{\chi_m}{\chi} z_t \), in (22) captures the money growth rate determining the inflation rate in the first subperiod, whereas \( 1 + z_t \) is the money growth rate determining the inflation rate in the second subperiod. If \( \chi_m > \chi \), an increase in \( z_t \) raises the total funds used, and inflation relatively higher than those in the second subperiod market. As a result, the price in the first subperiod rises more relative to the price in the second subperiod, causing higher incentives to
produce for sellers. The loanable funds effect dominates the Fisher effect, and consequently, output rises by an increase in the growth rate of money.

When $\chi = \chi_m$, output is independent of the inflation rate, a result which is different from some of the previous studies that also assume uncertainty in trading opportunities (e.g., Lagos and Wright, 2005). The reason is as follows. In this economy, money injections, borrowing, and the first subperiod goods trade take place within the same subperiod. Though agents make portfolio choices before money injections, buyers can borrow money and agents know the future value of money when they trade in the first subperiod. Thus, agents make portfolio decisions as if they knew what the future money injections would be. Or, equivalently, it is as if agents could choose money holding in the first subperiod, and thus, liquidity effects are eliminated. An immediate implication is that if banks hold no liquidity buffers ($\chi = \chi_m = 1$), there is no liquidity effect.

When $q_{b,t}$ rises in response to money injections, interest rates fall. To see this, note that $u'(q_{b,t}) = c'(\frac{\theta q_{b,t}}{1 - \theta})(1 + i_{b,t})$ from (12) and (15). The case with $\chi_m > \chi$ can happen only if banks hold liquidity buffers. This is so because given $\chi_m \leq 1$ and $\theta > 0$, $\chi < 1$ implies $\nu < 1$. In reality, banks do not lend out all deposits due to regulations or liquidity risk management, whereas often they have no such considerations for the money injected by the central bank. This implies $\chi_m = 1 > \chi$. As a result, the loanable funds effect dominates the effect of expected inflation, and money injections increase output and lower nominal interest rates.\(^{13}\)

Note that $\chi$ and $\chi_m$ affect not only the existence but also the magnitude of the liquidity effect. Equation (21) shows that the magnitude of the liquidity effect, measured by $\frac{\partial q_{b,t}}{\partial z_t}$, increases in $\chi_m$ and decreases in $\chi$. As the difference between $\chi_m$ and $\chi$ becomes larger, the loanable funds effect is much stronger than the Fisher effect, and consequently, the liquidity effect is larger. One implication is that, given $\chi_m = 1$, unexpected money injections will cause a larger effect on output and interest rates when banks keep a larger fraction of deposits as reserves.

The following proposition summarizes the main results.

**Proposition 1** If banks hold no liquidity buffers, liquidity effects are eliminated. If banks hold

\(^{13}\)In numerical examples, we set $u(q_b) = 2q_b^2$, $c(q_s) = \frac{q_s^2}{2}$, $\theta = .5$, $M_{t-1} = 100$, $\rho = .11$, $\chi_m = 1$ and $\nu = .8$. As the central bank injects money, e.g., $z_t$ increases from .05 to .051, consumption, $q_{b,t}$, increases from .0858866 to .0859009, and the loan rate, $i_{b,t}$, decreases from .256353 to .256258.
liquidity buffers, liquidity effects exist if and only if the fraction of money injections used to finance spending is larger than that of the initial money stock. In this case, the higher the fraction of money injections used to finance spending, the larger the liquidity effect.

**Financial Intermediation and the Friedman Rule.** We now show that the Friedman rule achieves the first-best allocation in our model, in which monetary policy works through financial intermediaries.\(^{14}\) When banks lend out all deposits and money injections, the loan rate equals the deposit rate, and from (8) and (12), (14) becomes

\[
\beta E_{t-1} [\phi_t (1 + i_{d,t})] \leq \phi_{t-1}. \tag{23}
\]

We define the average real return on money as \(\frac{1}{\gamma} + \beta\); i.e., \(E_{t-1} \frac{\phi_t}{\phi_{t-1}} = 1 + \gamma\). To implement the Friedman rule in this economy, the policy needs to set the expected return on money equal to the real interest rate; i.e., \(\frac{1}{\gamma} = \frac{1}{\beta}\). From (9) and (10), \(u'(q_{b,t}) = c'(q_{s,t})\) when \(i_{d,t} = 0\), and the Friedman rule achieves the efficient allocation. The Friedman rule ensures that agents can perfectly insure themselves against preference shocks because holding currency has zero costs.

The Friedman rule also achieves the efficient allocation in an economy where banks hold liquidity buffers. Given \(\nu i_{b,t} = i_{d,t}\), the loan rate is larger than the deposit rate. Equation (14) becomes

\[
\beta E_{t-1} \phi_t [\theta (1 + i_{b,t}) + (1 - \theta) (1 + i_{d,t})] \leq \phi_{t-1}. \tag{24}
\]

Under the policy that sets \(1 + \gamma = \beta\), we obtain \(u'(q_{b,t}) = c'(q_{s,t})\) from (24), as \(i_{d,t}\) and \(i_{b,t}\) approach to 0. The Friedman rule achieves the efficient allocation.

## 5 Inflation, Interest Rates, and Taylor Rules

It is widely accepted that a main goal of monetary policy is to stabilize the price level.\(^{15}\) There is also a consensus among practitioners that the monetary policy instrument used to achieve this goal should be the short-term interest rate. Economists have focused discussions on a class of strategies

\(^{14}\)Our discussion about the Friedman rule is similar to Berentsen, Camera, and Waller (2005), who consider the real effect of monetary injections without the banking system.

\(^{15}\)Friedman (1968) said that monetary policy can offset the major disturbances arising from other sources and provide a stable background for the economy.
known as the Taylor rule (Taylor, 1993), which suggests a simple instrumental rule whereby the monetary authority sets the instrument rate in response to inflation and output gap:

\[ i_t^s = \bar{i}^s + \delta_\pi (\pi_t - \bar{\pi}) + \delta_y (y_t - \bar{y}_t), \tag{25} \]

where \( i_t^s \) is the instrument rate in period \( t \), \( \bar{i}^s \) is the sum of the real interest rate and the target inflation rate, \( \pi_t \) is the actual inflation rate, \( \bar{\pi} \) is the inflation target, \( y \) is (log) output, and \( \bar{y}_t \) is (log) potential output. The coefficients in (25), \( \delta_\pi \) and \( \delta_y \), are positive, which govern the reaction of the central bank. While the coefficients of Taylor rules are assumed constant in many studies, one may wonder whether constant coefficients are optimal when the economy faces different shocks. In this section, we tackle this question by determining the coefficients of an instrument rule from the first-order conditions for a specific loss function, as suggested by Svensson (2003).

We adopt the following approach. First, we study the short-run connections among the money growth rate, inflation, and interest rates. In this economy, the central bank follows an instrument rule with exogenously specified coefficients, e.g., the Taylor rule (25). The variables that the central bank targets are the current inflation rate and the output gap, both of which are affected by shocks. We will show that raising the interest rate to reduce inflation is consistent with reducing the money growth rate, as implied by the quantity theory of money. Then, based on the short-run relationship explored, we will determine the coefficients of the Taylor rule for different shocks.

To address the issue of how the short-term policy responds to shocks while obtaining analytical results, we assume the following functional forms. The cost function is \( c(q_{s,t}) = \frac{q_{s,t}^2}{2(1+\alpha_t)} \), where \( \alpha_t \) is an i.i.d. shock with mean zero and variance \( \sigma_\alpha^2 < \infty \). The utility function is \( u(q_{b,t}) = \frac{(1+\eta_t)^{1-\psi}}{1-\psi} \), where \( \psi \) is the constant coefficient of relative risk aversion, and \( \eta_t \) is an i.i.d. shock with mean zero and variance \( \sigma_\eta^2 < \infty \). We call \( \alpha_t \) the supply shock, and \( \eta_t \) the real demand shock. In this section we consider only the commonly observed case in which banks keep some reserves but lend out all money injected by the central bank; i.e., \( \chi_m = 1 > \chi \), so liquidity effects exist.

5.1 The short-run equilibrium

We now explore the short-run connections among the money growth rate, inflation, and interest rates in the face of shocks. Suppose that the central bank uses the loan rate, \( i_{l,t}^s \), as the instrument
rate, and follows the Taylor rule (25). Then, \( i_{b,t}^s \) is set at
\[
\dot{i}_{b,t}^s = \rho + \bar{\pi} + \delta_\pi (\pi_t - \bar{\pi}) + \frac{\delta_\alpha}{2} (\alpha - \bar{\alpha}),
\]
(26)
where \( \bar{\alpha} \) is the potential output. Note that the output gap, \( \log y_t - \log \bar{y}_t \), is equal to \( \frac{1}{2} (\alpha_t - \bar{\alpha}) \) under our model specifications.\(^{16}\) If current inflation exceeds the targeted inflation, or there exists an output gap (\( \alpha_t - \bar{\alpha} > 0 \)), the central bank should raise the interest rate, \( i_{b,t}^s \), above its long-run level, \( \rho + \bar{\pi} \). Assume that \( 0 \leq \delta_\pi \leq \bar{\delta}_\pi < \infty \) and \( 0 \leq \delta_\alpha \leq \bar{\delta}_\alpha < \infty \), where \( \bar{\delta}_\pi \) and \( \bar{\delta}_\alpha \) are exogenous upper bounds of the coefficients. A policy is called “active” if an increase in inflation by one percentage point prompts the central bank to raise the instrument rate by more than one percentage point (\( \delta_\pi > 1 \)); and it is called “passive” if \( \delta_\pi < 1 \). Without loss of generality, we set \( \bar{\alpha} = 0 \) in the following discussion.

In this section inflation is measured in terms of the nominal price in the first subperiod, \( p_t = c(q_s, t) \), instead of the nominal price in the second subperiod, \( \frac{1}{\phi_t} \). The reason is that our focus is on how the central bank reacts to shocks, which occur in the demand or supply of goods in the first subperiod and affect the market price \( p_t \); whereas \( \phi_t \) is determined by the total money stock in the second subperiod. Let \( \pi_t \equiv \frac{p_t}{p_{t-1}} - 1 \) denote the inflation rate in the first subperiod. Then,
\[
\pi_t - \bar{\pi} \approx \frac{-(\alpha_t - \alpha_{t-1})}{2} + \frac{1 + \chi}{2\chi} (z_t - \bar{\pi}) - \frac{1 - \chi}{2\chi} (z_{t-1} - \bar{\pi}),
\]
(27)
(See the Appendix for the derivations of equations (27) –(29)). In the long run, \( \alpha_t = \alpha_{t-1} = \bar{\alpha} = 0 \), \( \pi_t = \bar{\pi} \), and (27) implies that the money growth rate equals the target inflation rate, \( z_t = z_{t-1} = \bar{\pi} \).

That is, in the long run, inflation is a monetary phenomenon. In the short-run, however, a money injection or a negative supply shock can lift inflation.

To implement the policy, the central bank needs one more equation that describes the behavior of market interest rates. Applying the log-approximation (i.e., \( \log(1 + x) \approx x \)) on (12) and (20), we obtain
\[
i_{b,t} = \bar{i}_{b,t} + \eta_t + \frac{1 - \psi}{2} \alpha_t + \frac{(1 + \psi)(\chi - 1)}{2\chi} (z_t - \bar{\pi}),
\]
(28)
where \( \bar{i}_{b,t} \) is the long-run average loan rate (note that in the long run \( \alpha_t = 0 \), \( \eta_t = 0 \) and \( z_t = \bar{\pi} \)). Equation (28) implies that a positive shock from the demand or supply side will raise the interest

\(^{16}\)In the Appendix we show that \( q_{b,t} = \sqrt{1 - \frac{\phi_{t-1}(1 + \alpha_t)\chi}{\pi_t(1 + z_t)}} \), and \( q_{s,t} = \frac{\phi_{b,t}}{1 - \psi} = \sqrt{1 - \frac{\phi_{t-1}(1 + \alpha_t)\chi}{\phi_t(1 + z_t)}} \).

Therefore, the output gap, \( \log y_t - \log \bar{y}_t = \log q_{s,t} - \log \bar{q}_{s,t} = \frac{1}{2} (\alpha_t - \bar{\alpha}) \).
rate. A positive demand shock increases marginal utility, while a positive supply shock reduces the marginal cost and the price; both effects induce agents to borrow more, and hence, the interest rate rises.\footnote{From (28) we have the same observation as in Section 4: When banks keep some reserves ($\chi < 1$), higher money growth lowers interest rates, but if banks hold no liquidity buffers, the interest rate does not respond to money injections.}

Now we have three equations, (26), (27), and (28), to describe the relationship among three variables, $\{\pi_t, i_{b,t}, z_t\}$.\footnote{In the long run, the money growth rate and the inflation rate equal the target inflation rate, $\pi$. Substituting $z_t = \pi_t = \bar{\pi}$ into (26) and (28), one finds that $i_{b,t} = \bar{i}_b = \rho + \bar{\pi}$ in the long run.} To study the short-run equilibrium, we focus our attention on the behavior of $z_t$, from which one can infer the behavior of $\pi_t$ and $i_{b,t}$. Suppose that the market interest rate equals the target rate; i.e., $i_{b,t} = i_{b,t}^s$. Substituting $i_{b,t} = i_{b,t}^s$ into (26) and (28), we obtain $z_t$ as a first-order difference equation:

$$z_t - \bar{\pi} = \chi((\delta_{\pi} - \delta_{\alpha} + 1 - \psi)\alpha_t - \delta_{\pi}\alpha_{t-1} + 2\eta_t) + A_z(z_{t-1} - \bar{\pi}),$$

(29)

where $A_z = \frac{(1-\chi)\delta_{\pi}}{(1+\psi)(1-\chi) + (1+\chi)\delta_{\pi}} < 1$ is the parameter that captures the lingering effect of shocks. One can interpret (29) as the epitome of dynamic systems, and use it to understand the effects of demand or supply shocks on inflation and interest rates. Note that we have a stable solution of $z_t$: the speed of convergence, $A_z$, is always less than 1, due to the fact that we consider long-run stationary equilibria in which real balances are time invariant.

Two factors affect the lingering effect: a decrease in $\chi$ or an increase in $\delta_{\pi}$ raises $A_z$. Note that $\chi$ affects the magnitude of liquidity effects, and $\delta_{\pi}$ represents the activeness of Taylor rules. If liquidity effects are stronger (smaller $\chi$) or monetary policy is more active (larger $\delta_{\pi}$), the effect of shocks lasts longer (larger $A_z$), and so it takes more time to achieve the target inflation rate after shocks. The intuitive reason is this. A strong liquidity effect implies that, a given change in $z_t$ will cause a large deviation of inflation from its target level when approaching the long-run equilibrium. Hence, it takes longer for $z_t$ to approach its long-run value $\bar{\pi}$.

We use the following two examples to show that implementing the short-term interest rate rule to reduce inflation is consistent with the quantity theory of money. For the purpose of illustration, suppose that $z_{t-1} = \bar{\pi}$, $\alpha_{t-1} = 0$, and initially the central bank sets $z_t = \bar{\pi}$ if no shock occurs. We assume $\delta_{\pi} > \delta_{\alpha}$ as in Taylor (1993).
First, consider a negative supply shock, $\alpha_t < 0$, which lifts inflation and dampens output. The central bank faces a dilemma: it may decrease the instrument rate, $i_{bst}^t$, to stimulate output, or increase $i_{bst}^t$ to control inflation. Under the assumption $\delta_\pi > \delta_\alpha$, the central bank, following the Taylor rule, will increase the target rate.\footnote{A negative supply shock implies that $\pi_t - \bar{\pi} = -\frac{\alpha_t}{\delta_\alpha} > 0$ from (27). The instrument rate then becomes $i_{bst}^t - \rho - \bar{\pi} = \frac{\alpha_t}{-\delta_\alpha} + \frac{\delta_\pi}{\delta_\alpha} \alpha_t = \frac{\alpha_t (-\delta_\pi + \delta_\alpha)}{-\delta_\alpha} > 0$ because $\delta_\pi > \delta_\alpha$.} To raise the interest rate, the central bank has to decrease the amount of the money stock, as (28) shows. One can confirm this argument from observing that in (29) $z_t$ should fall below $\bar{\pi}$ when facing a negative supply shock; i.e., $z_t - \bar{\pi} = \frac{\chi(\delta_\pi - \delta_\alpha + 1 - \psi)\alpha_t}{(1 + \psi)(1 - \chi) + (1 + \chi)\delta_\pi} < 0$, because $\delta_\pi > \delta_\alpha$ and $\alpha_t < 0$. Therefore, raising the interest rate to control inflation is an indirect way of reducing the money supply.

When a positive demand shock occurs, $\eta_t > 0$, buyers are eager to consume and borrow from banks, so the loan rate rises, as (28) indicates. The interest rate is off the target, and the central bank increases money supply to drag it down, as implied by (28). While the central bank raises the money growth rate, inflation will climb. According to the interest rate rule implied by (26), the central bank has to raise the target rate in the face of higher inflation. That is, the interest rate will be higher than the original level.

### 5.2 The determination of the coefficients of Taylor rules

When deriving the short-run relationship among $z_t$, $i_{bst}^t$, and $\pi_t$ in Section 5.1, we imposed arbitrary coefficients in the Taylor rule. We now determine the coefficients of Taylor rules from the first-order conditions of a specific loss function. First consider a simple form of loss function, which is an equally weighted sum of the squared inflation gap and the squared output gap:

$$L = \frac{1}{2}E_t[(\pi - \bar{\pi})^2 + \hat{y}^2],$$

where $\hat{y}$ is the output gap, and is equal to $\frac{1}{2}(\alpha_t - \bar{\alpha})$ under our model specifications. The loss function can be interpreted as the sum of the variance of inflation and the output gap.

We now derive the explicit form of the loss function in this economy. From (27) and (29), the
variances of inflation and monetary shocks, \( \sigma_\pi^2 \) and \( \sigma_z^2 \), are

\[
\sigma_\pi^2 = \frac{\sigma_\alpha^2}{4} + \frac{(1 + \chi^2)\sigma_z^2}{4\chi^2},
\]

\[
\sigma_z^2 = \frac{[(\delta_\pi - \delta_\alpha + 1 - \psi)^2 + \delta_\pi^2] \chi^2 \sigma_\alpha^2 + 4\chi^2 \sigma_\eta^2 - 2(1 - \chi)\delta_\pi \sigma_{z\alpha}}{[(1 + \psi)(1 - \chi) + (1 + \chi)\delta_\pi]^2 - (1 - \chi)^2 \delta_\pi^2}.
\]

(30)

(31)

where \( \sigma_\alpha^2 \) is the variance of the supply shock, which also equals the variance of the output gap, and \( \sigma_{z\alpha} = \frac{\chi(\delta_\pi - \delta_\alpha + 1 - \psi)}{(1 + \psi)(1 - \chi) + (1 + \chi)\delta_\pi} \) is the covariance between the money growth and the supply shocks. From (30), \( \sigma_z^2 \) is a weighted average of the variances of the supply shock and the monetary shock, and the weight of the latter is related to the magnitude of the liquidity effects. Note that the variance of the output gap equals the variance of the supply shock, because \( \hat{y} = \frac{1}{2}(\alpha_t - \bar{\alpha}). \)

The loss function thus can be written as:

\[
L = \frac{1}{2} \left[ \frac{5\sigma_\alpha^2}{4} + \frac{(1 + \chi^2)\sigma_z^2}{4\chi^2} \right].
\]

Because the variance of the supply shock cannot be reduced by monetary policies, the only way the central bank can minimize the loss is to reduce the variance of money growth, \( \sigma_z^2 \). Therefore, we view \( \sigma_z^2 \) as the loss function that the central bank faces, denoted as \( \tilde{L} \); i.e., the loss function becomes

\[
\tilde{L} = \sigma_z^2.
\]

Next we determine the coefficients of Taylor rules in response to supply shocks or demand shocks. The policy maker’s problem is to choose the reaction coefficients, \( \delta_\alpha \) and \( \delta_\pi \), to minimize the loss function \( \tilde{L} = \sigma_z^2 \).

**Supply shocks.** Consider that a supply shock occurs. Substituting \( \sigma_\eta^2 = 0 \) into (31), we obtain

\[
\tilde{L} = \frac{[(\delta_\pi - \delta_\alpha + 1 - \psi)^2 + \delta_\pi^2] \chi^2 \sigma_\alpha^2 - 2(1 - \chi)\delta_\pi \sigma_{z\alpha}}{[(1 + \psi)(1 - \chi) + (1 + \chi)\delta_\pi]^2 - (1 - \chi)^2 \delta_\pi^2}.
\]

(32)

From the first-order condition, \( \frac{\partial \tilde{L}}{\partial \delta_\alpha} = 0 \), we have

\[
\delta_\pi - \delta_\alpha + 1 - \psi = \frac{2(1 - \chi)\chi \delta_z^2}{\chi^2 \sigma_\alpha^2(1 + \psi)(1 - \chi) + (1 + \chi)\delta_\pi} \equiv \delta_\pi f(\delta_\pi).
\]

(33)
Substituting (33) into (32), the central bank’s problem becomes:

$$\min_{\delta_\pi} \frac{[1 + \delta_\pi^2 f(\delta_\pi)^2]\chi^2\sigma^2_\alpha - 2(1 - \chi)\sigma z\alpha\delta_\pi^2}{[(1 + \psi)(1 - \chi) + (1 + \chi)\delta_\pi]^2 - (1 - \chi)^2\delta_\pi^2}.$$ 

The solution is $\delta_\pi = 0$, which, together with (33), implies that $\delta_\alpha = 1 - \psi$. When a negative supply shock occurs, the central bank faces a dilemma: trying to control inflation—decreasing the aggregate demand—will dampen output, whereas increasing the aggregate demand to stimulate the economy will escalate the inflation. Our result implies that, to minimize the volatility of inflation and output in this economy, the central bank should set $\delta_\pi$ as small as possible.\(^{20}\) This amounts to setting the money growth rate at the target inflation rate, as indicated by (29).

**Demand shocks.**

Now consider that a demand shock occurs. Substituting $\sigma^2_\alpha = \sigma z\alpha = 0$ into (31), we obtain

$$\bar{L} = \frac{4\chi^2\sigma^2_\eta}{[(1 + \psi)(1 - \chi)]^2 + 2(1 + \psi)(1 - \chi^2)\delta_\pi + 4\chi^2\delta_\pi^2}.$$ 

Observe that $\frac{\partial \bar{L}}{\partial \delta_\pi} < 0$, because $\chi < 1$. If the demand shock tends to fluctuate often and on a large scale, the central bank may intend to choose a sufficiently large $\delta_\pi$ to minimize the fluctuation of inflation. This result can be interpreted as follows. If the central bank sets $\delta_\pi = \bar{\delta}_\pi$, where $\bar{\delta}_\pi$ is the exogenously imposed upper bound of the weight, then from (29), $z_t = \bar{\pi} + \frac{2\chi\eta}{(1 + \psi)(1 - \chi) + (1 + \chi)\delta_\pi}$. The implication is that the central bank, in order to minimize the loss, chooses the smallest possible $z_t$ to control inflation. If there were no restrictions on coefficients, the central bank could set an arbitrarily large $\delta_\pi$; i.e., $\delta_\pi \to \infty$. This implies that, from (29), the central bank simply sets the money growth rate at the target inflation rate; i.e., $z_t = \bar{\pi}$.

We summarize the results in the following proposition.

**Proposition 2** Consider the cost function, $c(q_{s,t}) = \frac{q^2_{s,t}}{(1 + \alpha_t)}$, and the utility function, $u(q_{b,t}) = \frac{(1 + \eta)q^1_{b,t}}{1 - \psi}$, where $\alpha_t$ and $\eta$ are i.i.d. shocks. If the central bank follows Taylor rules specified in (25), the coefficients that would minimize the volatility of inflation and output gap are (i) $\delta_\pi = 0$ and $\delta_\alpha = 1 - \psi$ when there is a supply shock, and (ii) $\delta_\pi = \bar{\delta}_\pi$ when there is a demand shock.\(^{20}\)

\(^{20}\)In this case, if the central bank follows the policy rule as in Taylor (1993), with coefficients $\delta_\pi = 1.5$ and $\delta_\alpha = 0.5$, it will cause higher volatility of inflation.
As a final remark, the coefficients $\delta_{\pi}$ and $\delta_{\alpha}$ in the above examples do not depend on the magnitude of liquidity effects; however, it may not be so if we consider other types of Taylor rules. For instance, if the central bank targets inflation only, the optimal coefficient $\delta_{\pi}$ may depend on $\chi$. To illustrate this point, suppose that the central bank sets the interest rate as

$$i_{b,t}^s = \rho + \bar{\pi} + \delta_{\pi}(\pi - \bar{\pi}),$$

and a supply shock occurs. To minimize the loss function, the central bank would choose $\delta_{\pi} \to 0$ as $\chi \to 0$, and $\delta_{\pi} \to \delta_{\pi}$ as $\chi \to 1$.\(^{21}\) The intuitive reason is that when the liquidity effect is large (small $\chi$), a change in the money growth rate will induce a larger change in the interest rate. To minimize the volatility of inflation, therefore, the central bank should set a sufficiently small coefficient, $\delta_{\pi}$. When the liquidity effect is very small (large $\chi$), a large coefficient $\delta_{\pi}$ causes only a slight fluctuation of inflation. Actually, according to (29), the central bank should set the money growth rate at the target inflation rate as in previous scenarios. Our result supports Alan Blinder’s suggestion that the coefficients in Taylor rules will likely alter if the variables that the central bank targets are changed (FOMC, May, 1995).

6 Conclusion

This paper studies liquidity effects and short-term monetary policies in a model with fully flexible prices, and with a explicit role for money and financial intermediation. We have shown that, if banks hold no liquidity buuffers, money injections affect inflation but not interest rates or allocation. If banks hold liquidity buuffers, then liquidity effects exist if and only if the fraction of money injections used to finance spending is larger than that of initial the money stock. The lower the substitutability between newly issued money and the initial money stock, the larger the liquidity effect. We then apply our framework to determine the reaction coefficients for a class of Taylor rules. Our model suggests that the central bank should identify the source of inflation to choose the coefficients;

\(^{21}\) If $\chi = 1$, the loss function becomes $\bar{L} = \frac{[(\delta_{\pi} + 1 - \psi)^2 + \delta_{\pi}^2]^{2}\sigma_{\pi}^2}{2\sigma_{\pi}^2}$. To minimize the loss $\bar{L}$, the central bank sets $\delta_{\pi} = \delta_{\pi}$. Meanwhile, when $\delta_{\pi} = 0$, $\bar{L} = \frac{(1 - \psi)^2 \sigma_{\pi}^2}{(1 + \psi)^2 (1 - \psi)^2}$, when is minimized at $\psi = 1$ or $\chi = 0$. That is, if $\chi \to 0$, the central bank sets $\delta_{\pi} = 0$ to minimize the loss. In the numerical examples, we use the following function form: $c(q_{u,t}) = \frac{\tau}{2} \psi u(q_{b,t}) = \frac{\tau}{2} \psi b_{u}$, and we set $\psi = 0.5$. The optimal coefficient $\delta_{\pi} = 0.56$, as $\chi = 0.2$, and $\delta_{\pi} = \delta_{\pi}$ as $\chi = 0.8$. This implies that, when $\chi$ is relatively small or relatively large, the coefficient $\delta_{\pi}$ increases in $\chi$. 

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however, the policy implications are similar regardless of the source of inflation: the money growth rate should be set at the targeted inflation to minimize the fluctuation of inflation. Our paper offers insights on the implementation of short-term policy; it also illustrates the power and flexibility of models that include explicit roles for money and financial intermediation.
References


Appendix

A1. Derivation of Equations (27) to (29)

From (20), with \( \chi_m = 1 \) we get
\[
q_{b,t}c'(\theta q_{b,t}) = \frac{\phi_{t-1}M_{t-1}(\chi + z_t)}{\theta(1 + z_t)}.
\]

Since \( c(q_s) = \frac{q_{s,t}^2}{2(1+\alpha_t)} \),
\[
q_{b,t} = \sqrt{\frac{1 - \theta \phi_{t-1}M_{t-1}(1 + \alpha_t)(\chi + z_t)}{\theta(1 + z_t)}}. \tag{34}
\]

From \( c'(q_s) = p\phi_t \), and then \( p_t = \frac{\sqrt{(1 + z_t)\phi_{t-1}M_{t-1}}}{\phi_t} \). Take logarithms of both sides to get
\[
\log p_t = \frac{1}{2} \left[ -\log(1 - \theta) - \log(1 + \alpha_t) + \log \chi + \log(1 + z_t/t) + \log \phi_{t-1}M_{t-1} - \log \phi_t. \right. \tag{35}
\]

Define \( \pi_t = \frac{p_t}{p_{t-1}} - 1 \). Then take difference on both sides of (35) and use the approximation, \( \log(1 + x) \approx x \) to get (27).

From (9) and (10), we have
\[
1 + i_{b,t} = \frac{u'(q_{b,t})}{c'(q_{s,t})} = 1 - \frac{\theta}{\theta} (1 + \eta_t)(1 + \alpha_t)q_{b}^{-(1+\psi)}
\]

Use (34),
\[
1 + i_{b,t} = \frac{1 - \theta}{\theta} (1 + \eta_t)(1 + \alpha_t)\left[ \frac{\theta^2(1 + z_t)}{(\chi + z_t)\phi_{t-1}M_{t-1}(1 - \theta)(1 + \alpha_t)} \right]^{1+\psi}.
\]

Then, take logarithms of both sides to get (28). Note that \( \tilde{i}_{b,t} \) is a function of \( \theta \) and real balances that \( \tilde{i}_{b,t} = 1 - \frac{1+\psi}{2} \log (1 - \theta) - \psi \log \theta - \frac{1+\psi}{2} \log \phi_{t-1}M_{t-1} \).

Substitute \( \pi_t \) obtained in (27) into (26) to get
\[
\tilde{i}_{b,t} + \eta_t + \frac{1 - \psi}{2} \alpha_t + \frac{(1 + \psi)(\chi - 1)}{2\chi} (z_t - \bar{\pi}) = \rho + \bar{\pi} + \delta \pi \left[ \frac{-(\alpha_t - \alpha_{t-1})}{2} + \frac{1 + \chi}{2\chi} (z_t - \bar{\pi}) - \frac{1 - \chi}{2\chi} (z_{t-1} - \bar{\pi}) \right] + \frac{\delta \alpha(\alpha - \bar{\alpha})}{2}.
\]

Since \( \bar{\alpha} = 0 \), we get a first-order difference equation of \( z_t \) after rearranging:
\[
z_t - \bar{\pi} = \chi \left[ \frac{(\delta \pi - \delta \alpha + 1 - \psi)\alpha_t - \delta \pi \alpha_{t-1} + 2(\tilde{i}_{b,t} + \eta_t - \rho - \bar{\pi})}{(1 + \psi)(1 - \chi) + (1 + \chi)\delta \pi} \right] + A_t (z_{t-1} - \bar{\pi}),
\]
where \( A_t = \frac{(1-\chi)\delta \pi}{(1+\psi)(1-\chi)+(1+\chi)\delta \pi} \). Since \( \tilde{i}_{b,t} \) must equal \( \rho + \bar{\pi} \) in equilibrium, we get (29).
A2. Solving the bank’s problem

Since banks are perfectly competitive with free entry, they take as given the loan rate and the deposit rate. There is no strategic interaction among banks or between banks and agents, and no bargaining over the terms of the loan contract. The representative bank solves the following problem per borrower:

$$\max_{b_t} (i_{b,t} - \frac{i_{d,t}}{\nu}) i_{b,t}$$

s.t.  

$$u(q_{b,t}) + W(m_t, b_t, d_t) \geq \Gamma,$$

where $\Gamma$ is the reservation value of the borrower, which is the surplus from obtaining loans at another bank. The first order condition to the bank’s problem is

$$i_{b,t} - \frac{i_{d,t}}{\nu} + \lambda_\Gamma [u'(q_b) \frac{dq_{b,t}}{db_t} + W_b] = 0,$$

where $\lambda_\Gamma$ is the Lagrangian multiplier on the borrower’s participation constraint. For $i_{b,t} - \frac{i_{d,t}}{\nu} > 0$, banks would like to make the largest loan possible to borrowers and, therefore, would choose a loan amount such that $\lambda_\Gamma > 0$.

Banks compete for taking deposits and making loans, which results in a zero marginal profit condition:

$$\nu i_{b,t} = i_{d,t}.$$

From (8) and the buyer’s budget constraint, $\frac{dq_{b,t}}{db_t} = \frac{\phi_t}{c'(q_{b,t})}$. Therefore, from the bank’s first order condition the loan supplied by banks satisfies

$$\frac{u'(q_{b,t})}{c'(q_{b,t})} = 1 + i_{b,t}.$$
A3. A model with unexpected deposit withdrawals

In the basic model we assumed banks hold $1 - \nu$ fraction of deposits as reserves, where $\nu$ is an exogenous parameter. Here we provide an extension to the basic model to justify why banks hold reserves. In the spirit of Diamond and Dybvig (1983), we assume that some agents may face a liquidity shock and withdraw their deposits. As a result, banks have to hold reserves to meet unexpected withdrawals. We will show that the main results in the basic model still hold.

**Liquidity shocks and deposit withdrawals.** In the beginning of the first subperiod, an agent receives a preference shock that determines whether he is either a consumer (buyer), a producer (seller), or a non-trader. With probability $\theta_c$ an agent can consume but cannot produce; with probability $\theta_p$ the agent can produce but cannot consume; with probability $\theta_n$ he can neither consume nor produce, and is called a non-trader, where $\theta_c + \theta_p + \theta_n = 1$.

As in the basic model, sellers make deposits and buyers make loans. Non-traders also make deposits, with the reason that will become clear later. We assume that banks close after taking deposits and making loans, but those who need liquidity can withdraw deposits from automatic teller machines. After banks close, non-traders receive a consumption shock: with probability $\theta_{nc}$ a non-trader wants to consume (called the ‘late consumer’), and with probability $1 - \theta_{nc}$ he does not want to consume. We interpret $\theta_{nc}$ as a liquidity shock. Late consumers thus will withdraw their deposits to finance spending. Finally, the goods market in the first subperiod opens. Buyers and late consumers have the demand for goods, and sellers produce to supply goods.

For the purpose of exposition, we assume that $\theta_{nc}$ follows a discrete uniform distribution. In particular, assume that $\theta_{nc}$ can take the value from the set, $\{\theta_{nc}^1, \theta_{nc}^2, \ldots, \theta_{nc}^k\}$, with equal probability; that is, $\Pr(\theta_{nc} = \theta_{nc}^i) = \frac{1}{k}$, $i = 1, 2, \ldots, k$. The liquidity shock arrives in the following way. The nature draws from the set $\{\theta_{nc}^1, \theta_{nc}^2, \ldots, \theta_{nc}^k\}$ to determine the realized value of $\theta_{nc}, \widehat{\theta}_{nc}$. Afterwards, each non-trader wishes to consume with probability $\widehat{\theta}_{nc}$, or does not wish to consume with probability $1 - \widehat{\theta}_{nc}$.

Recall banks make the decision of holding reserves before the realization of $\theta_{nc}$. Because banks need to meet the demand for withdrawals, they will hold $\theta_n\theta_{nc}^k$ fraction of deposits (per capita) as reserves. For simplicity, we assume that withdrawals incur no cost. Banks do not pay interests to
deposits withdrawn by late consumer, because deposits and withdrawals are made within the same subperiod. Now it is clear why non-traders would make deposits: they can withdraw money when they need liquidity, and can earn interest payments otherwise. All other setups are the same as in the basic model.

**Trade and bank operations in the first subperiod.** In the first subperiod, sellers, buyers and late consumers trade in the goods market. A seller’s problem and a buyer’s problem are the same as in the basic model. A late consumer uses the money withdrawn to finance his purchase, which is equal to the money holdings that he has deposited. A late consumer’s problem is thus

\[
\max_{q_{b,t}} u'(q_{b,t}) + W(m_t - p_t q_{b,t})
\]

s.t. \( p_t q_{b,t} \leq m_t. \)

We have dropped the last two arguments in \( W(\cdot) \), because a late consumer neither has deposit nor debt when he enters the second subperiod. The late consumer faces the cash constraint that his spending cannot exceed the money withdrawn, \( m_t \). Using (4) and (8), the first order condition is

\[
u'(q_{b,t}) = c'(q_{b,t})(1 + \frac{\lambda^t_{l}}{\phi_t}). \tag{36}\]

If \( \lambda^t_{l} = 0 \), (36) reduces to \( u'(q_{b,t}) = c'(q_{s,t}) \), implying the first-best outcome. If \( \lambda^t_{l} > 0 \), the cash constraint binds, and the late consumer spends all of his money; i.e.,

\[ q_{b,t} = \frac{m_t}{p_t}. \tag{37} \]

The marginal value of money in this economy is

\[ V_m(m_t) = \theta_c \frac{u'(q_{b,t})}{p_t} + \theta_p \phi_t (1 + i_d,t) + \theta_n (1 - \theta_{nc}) \phi_t (1 + i_d,t) + \theta_n \theta_{nc} \frac{u'(q_{b,t})}{p_t}. \tag{38} \]

From holding an additional unit of money to the first period, a buyer receives \( \frac{u'(q_{b,t})}{p_t} \) and a late consumer receives \( \frac{u'(q_{b,t})}{p_t} \), whereas a seller or a non-trader who does not want to consume has interest payments, \( \phi_t (1 + i_d,t) \), in the second subperiod. Using (3) lagged one period to eliminate \( V_m(m_t) \) from (38), an agent’s optimal money holdings satisfy

\[ \beta E_t-1 \{ \theta_c \frac{u'(q_{b,t})}{p_t} + [\theta_p + \theta_n (1 - \theta_{nc})] \phi_t (1 + i_d,t) + \theta_n \theta_{nc} \frac{u'(q_{b,t})}{p_t} \} \leq \phi_{t-1}, \quad \text{" if } m_t > 0. \tag{39} \]
Condition (39) states that the cost of acquiring an additional unit of money must be greater than the expected discounted benefit, with the equality holding if agents choose to hold money.

In a symmetric equilibrium, the market-clearing conditions for goods, money, and loan markets are

\[ \theta_p q_{s,t} = \theta_c q_{b,t} + \theta_n \theta_{nc} q_{b,t}, \]  
\[ m_t = M_{t-1}, \]  
\[ \theta_c b_t = [\theta_p + \theta_n (1 - \theta_{nc})] d_t + \chi_m \tau_t, \]

respectively. In the loan market clearing condition, (42), the per capita funds available for banks to lend out include \( \theta_p + \theta_n (1 - \theta_{nc}) \) fraction of deposits, plus \( \chi_m \) fraction of money injections, while per capita loans demanded is \( \theta_c b_t \).

A bank’s per capita end-of-period profit is \( \omega = i_{b,t} \theta_c b_t - i_{d,t} [\theta_p + \theta_n (1 - \theta_{nc})] d_t + \tau_t \). Substituting \( [\theta_p + \theta_n (1 - \theta_{nc})] d_t = \theta_c b_t - \chi_m \tau_t \) from (42), one can rewrite a bank’s profit as

\[ \omega = [1 + \frac{\chi_m}{\theta_p + \theta_n (1 - \theta_{nc})} i_{d,t}] \tau_t + [i_{b,t} - \frac{\theta_p + \theta_n (1 - \theta_{nc})}{\theta_p + \theta_n (1 - \theta_{nc})} i_{d,t}] \theta_c b_t. \]

As in the basic model, money injections create profits in the first subperiod for the competitive banks. These profits are distributed to agents as dividends in the second subperiod. The zero marginal profit condition implies

\[ \frac{\theta_p + \theta_n (1 - \theta_{nc})}{\theta_p + \theta_n (1 - \theta_{nc})} i_{b,t} = i_{d,t}. \]

Recall that in the basic model we have \( \nu i_{b,t} = i_{d,t} \), where \( \nu \) is the exogenous fraction of deposits that banks lend out. Here we have derived bank’s holding of reserves from a model with unexpected deposit withdrawals.

**Liquidity effects.** The total funds available for each buyer to finance consumption in the first subperiod include his money holdings, \( m_t \), and the money he borrows from banks, \( b_t \). From the loan-market-clearing condition, (42), we have

\[ b_t = \frac{[\theta_p + \theta_n (1 - \theta_{nc})] d_t + \chi_m \tau_t}{\theta_c}. \]
Substituting \( d_t = m_t, \tau_t = z_tM_{t-1}, \) and the market-clearing condition for money, \( m_t = M_{t-1}, \) into (43), we obtain the total funds available per buyer to finance consumption in the first subperiod:

\[
m_t + b_t = \frac{(\chi_l + \chi_m z_t)}{\theta_c} M_{t-1},
\]

(44)

where \( \chi_l = \theta_c + \theta_p + \theta_n(1 - \theta_n^k) = 1 - \theta_n \theta_n^k. \) Note that \( \chi_l \) is the fraction of the initial money stock, \( M_{t-1}, \) that can be used to finance spending.

In equilibrium the cash constraint binds, i.e., \( q_{b,t} = \frac{m_t + b_t}{p_t}, \) from which we derive the relationship between \( q_{b,t} \) and the money injection, \( z_t: \)

\[
q_{b,t} c'(q_{s,t}) = \frac{(\chi_l + \chi_m z_t) \phi_{t-1} M_{t-1}}{\theta_c (1 + z_t)},
\]

(45)

by using (8), (44), and \( 1 + z_t = \frac{\phi_{t-1}}{\phi_t}. \) Similarly,

\[
q_{l,b,t} c'(q_{s,t}) = \frac{\phi_{t-1} M_{t-1}}{(1 + z_t)},
\]

(46)

The aggregate demand \( q_b^A \) is thus defined as

\[
q_b^A = \theta_c q_{b,t} + \theta_n \theta_n^k q_{l,b,t} = \frac{(\chi_l + \chi_m z_t + \theta_n \theta_n^k) \phi_{t-1} M_{t-1}}{c'(q_{s,t})(1 + z_t)}.
\]

From (46), \( q_{b,t} \) is a decreasing function of \( z_t, \) and from (45), \( q_{b,t} \) is an increasing function of \( z_t \) if \( \chi_m > \chi_l. \) The aggregate demand, however, increases by money injections if \( \chi_m - \chi_l - \theta_n \theta_n^k > 0. \) Explicitly,

\[
\frac{\partial q_b^A}{\partial z_t} = \frac{\phi_{t-1} M_{t-1} (\chi_m - \chi_l - \theta_n \theta_n^k)}{(1 + z_t)^2 [c'(q_{b,t}^A) + \frac{q_{b,t}^A}{\sigma_p} c''(q_{b,t}^A)]} \begin{cases} > 0 \text{ if } \chi_m - \chi_l - \theta_n \theta_n^k > 0, \\ < 0 \text{ if } \chi_m - \chi_l - \theta_n \theta_n^k < 0. \end{cases}
\]

(47)

Note that if the realized value of \( \theta_n^k \) is \( \theta_n^k, \) we have \( \chi_l + \theta_n \theta_n^k = 1, \) and there is no liquidity effect even when \( \chi_m = 1. \) Because banks hold \( \theta_n \theta_n^k \) fraction of deposits as reserves, when the realized value is \( \theta_n^k, \) all reserves are withdrawn and used to purchase goods in the first subperiod. Consequently, it is as if agents could choose money holding in the first subperiod, and liquidity effects are eliminated. Moreover, because \( \theta_n = \theta_n^k \) occurs with probability \( \frac{1}{k}, \) the liquidity effect \( \frac{\partial q_b^A}{\partial z_t} > 0 \) is more likely to exist, when \( \chi_m \) is close to 1, and \( k \) is larger. Also, from \( u'(q_{b,t}) = c'(q_{s,t})(1 + i_{b,t}), \) the interest rate \( i_{b,t} \) decreases after money injections.