Scarce Collateral, the Term Premium, and Quantitative Easing

Williamson (2016, JET)

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Introduction

- Main features of the model:
  - Private financial intermediaries perform a liquidity transformation role. (Diamond and Dybvig, 1983)
  - Intermediary liabilities are subject to limited commitment, and the assets of the financial intermediary must serve as collateral.
  - Different assets have different degrees of pledgeability

- A term premium will arise in equilibrium if
  (i) short-maturity government debt has a greater degree of pledgeability than long-maturity government debt;
  (ii) collateral is collectively scarce, in that the total value of collateralizable wealth is too low to support efficient exchange.
## Repo Haircuts

(percenbt)

<table>
<thead>
<tr>
<th>Repo Haircuts (%)</th>
<th>Spring 2007</th>
<th>Spring 2008</th>
<th>Fall 2008</th>
<th>Spring 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Treasuries (short-term)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>U.S. Treasuries (long-term)</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Agency mortgage-backed securities</td>
<td>2.5</td>
<td>6</td>
<td>8.5</td>
<td>6.5</td>
</tr>
<tr>
<td>Corporate bonds, A-/A3 or above</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Collateralized mortgage obligations, AAA</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Asset-backed securities, AA/Aa2 and above</td>
<td>10</td>
<td>25</td>
<td>30</td>
<td>35</td>
</tr>
</tbody>
</table>

*Source:* The data in the first three columns is from the Depository Trust and Clearing Corporation (provided by Tobias Adrian of the New York Fed), with the column for fall of 2008 filled out from reports of investment banks.
Main results

• Purchases of long-maturity government debt at the zero lower bound reduce the nominal yield on long-maturity government bonds and flatten the yield curve.

• Real bond yields increase because QE, involving swaps of better collateral for worse collateral, increases the value of collateralizable wealth, making collateral less scarce, and the liquidity premium is reduced.

• Inflation falls because one of the effects of QE is to increase the real stock of currency held by the private sector, and agents require an increase in currency’s rate of return (a fall in the inflation rate) to induce them to hold more currency.
Model: agents and assets

• A continuum of buyers (sellers) each with unit mass.
  • Buyer scan produce in the CM, but not in the DM.
  • Sellers can produce in the DM, but not in the CM.
  • One unit of labor input produces one unit of the perishable consumption good, in either the CM or the DM.

• Assets:
  • government-issued currency, $\phi_t$ in the CM goods
  • reserves (issued by the central bank), account balance at the central bank
  • short-maturity and long-maturity government bonds (issued by the fiscal authority)
Model: CM

In the CM, debts are first paid off, then a Walrasian market opens.

reserves: \( Z^m_t \) units of money \( \rightarrow \) one unit of money
short-maturity: \( Z^s_t \) units of money \( \rightarrow \) one unit of money
long-maturity: \( Z^l_t \) units of money \( \rightarrow \) one unit of money in every future CM
Model: DM

- Limited commitment - no one can be forced to work - and so lack of memory implies that there can be no unsecured credit.

- $\rho$ of DM transactions: the seller can only verify the buyer’s currency holdings.
  $1 - \rho$ of DM transactions: the seller can verify the entire portfolio held by the buyer.

- Reserves and government debt cannot be transfered until the next CM.

- Buyers make take-it-or-leave-it offers.
Credit

- Credit arrangements in this model will involve promised payments at the beginning of the CM that must be collateralized, given limited commitment and lack of memory. (Banks have the same commitment problem.)

- Assume that a buyer (bank) can abscond with fraction $\theta_s$ of short-maturity debt, reserves or currency and with $\theta_l$ of long-maturity government debt.
Banking

- Any agent can operate a bank.

- A bank issues deposits in the CM, but before buyers learn what their type will be.

- A deposit-holder can withdraw currency, or the deposit turns into a tradeable claim to be redeemed by the bank in the next CM in goods. ⇒ A bank deposit is essentially an option.

- A banking arrangement essentially permits currency to be allocated only to currency transactions, and government debt and reserves to non-currency transactions.

- The bank’s deposit claims must be backed with collateral, and the only available collateral in the model is government debt and reserves.
Bank’s problem

All quantities are expressed in units of the CM good in period $t$ ($d_t$ denotes claims to CM goods of period $t+1$).

$$\max_{d_t, c_t, k_t, m_t, b_t^s, b_t^l} \left[ -k_t + \rho u \left( \frac{\beta \phi_{t+1} c_t}{\phi_t} \right) + (1 - \rho) u(\beta d_t) \right]$$  \hspace{1cm} (1)

s.t

$$k_t - \rho c_t - z_t^m m_t - z_t^s b_t^s - z_t^l b_t^l + \left[ -(1 - \rho) \beta d_t + \beta b_t^s \frac{\phi_{t+1}}{\phi_t} \\
+ \beta b_t^l \frac{\phi_{t+1}}{\phi_t} (1 + z_{t+1}^l) + \beta \frac{\phi_{t+1}}{\phi_t} m_t \right] \geq 0$$  \hspace{1cm} (2)

$$- (1 - \rho) d_t + \left[ b_t^s \frac{\phi_{t+1}}{\phi_t} + b_t^l \frac{\phi_{t+1}}{\phi_t} (1 + z_{t+1}^l) + \frac{\phi_{t+1}}{\phi_t} m_t \right] \geq \frac{\phi_{t+1}}{\phi_t} \left[ \theta_s (m_t + b_t^s) + \theta_l b_t^l (1 + z_{t+1}^l) \right]$$  \hspace{1cm} (3)

$$k_t, c_t, m_t, d_t, b_t^s, b_t^l \geq 0$$  \hspace{1cm} (4)

$$(3) \Rightarrow (1 - \rho) d_t + (b_t^s + m_t) \frac{\phi_{t+1}}{\phi_t} (1 - \theta_s) + b_t^l \frac{\phi_{t+1}}{\phi_t} (1 + z_{t+1}^l) (1 - \theta_l) \geq 0$$  \hspace{1cm} (5)
Consolidated balance sheet

- $C_t, M_t$: nominal quantities of currency and reserves
- $B^s_t, B^l_t$: nominal quantities of gov’t debt held in the private sector
- $\tau_t$: real value of the transfer to each buyer in the CM

\[
\phi_0\left(C_0 + z^m_0 M_0 + z^s_0 B^s_0 + z^l_0 B^l_0\right) - \tau_0 = 0
\]

\[
\phi_t\left\{\left(C_t - C_{t-1} + z^m_t M_t - M_{t-1}\right.ight.
+ z^s_t B^s_t - B^s_{t-1} + z^l_t B^l_t - (z^l_t + 1) B^l_{t-1}\left.\right) - \tau_t = 0, \quad t = 1, 2, 3, \ldots
\]

- market clearing conditions for assets
Channel system and floor system

Channel system

• CB targets a short-term nominal interest rate, and pays interest on reserves at a rate below that target rate.

• In US before 2008.10 it was essentially a channel system, with $z_t^m = 1$, i.e. no interest paid on reserves.

Floor system

• Interest is paid on reserves, and a positive stock of reserves is held overnight.

• $z_t^m = z_t^s$ must hold in eqm, as the role played by reserves in the private sector is identical to that of short-term gov’t debt.

• Difference: Away from the zero lower bound on the short rate, CB can issue short-term debt (reserves), and swap that short-term debt for long-term gov’t debt.
A Channel System: Equilibrium

- If \( z_t^m > z_t^s \), then it is optimal for banks to hold no reserves \( (m_t = 0) \).
- Constraint (2) must bind, as the objective function is strictly increasing in both \( c_t \) and \( d_t \), and the LHS of (2) is strictly decreasing in \( c_t \) and \( d_t \).
- We restrict attention to the case where (5) binds \( \Rightarrow \) the multiplier \( \lambda_t \) on (5).
Equilibrium: FOC

\[ [c_t]: \quad \frac{\beta \phi_{t+1} u'}{\phi_t} \left( \frac{\beta \phi_{t+1} c_t}{\phi_t} \right) - 1 = 0 \quad (16) \]

\[ [d_t]: \quad \beta u' (\beta d_t) - \beta - \lambda_t = 0 \quad (17) \]

\[ [b_s^t]: \quad - z_s^t + \beta \frac{\phi_{t+1}}{\phi_t} + \lambda_t (1 - \theta_s) \frac{\phi_{t+1}}{\phi_t} = 0 \quad (18) \]

\[ [b_l^t]: \quad - z_l^t + \beta \frac{\phi_{t+1}}{\phi_t} (1 + z_{t+1}^l) + \lambda_t (1 - \theta_l)(1 + z_{t+1}^l) \frac{\phi_{t+1}}{\phi_t} = 0 \quad (19) \]

\[ [\lambda_t]: \quad - (1 - \rho) d_t + b_s^t \frac{\phi_{t+1}}{\phi_t} (1 - \theta_s) + b_l^t \frac{\phi_{t+1}}{\phi_t} (1 + z_{t+1}^l)(1 - \theta_l) = 0 \quad (20) \]
Binding incentive constraint

\[-(1 - \rho)d_t + b_t^s \frac{\phi_{t+1}}{\phi_t} (1 - \theta_s) + b_t^l \frac{\phi_{t+1}}{\phi_t} (1 + z_{t+1}^l)(1 - \theta_l) = 0\]

\[k_t - \rho c_t - z_t^m m_t - z_t^s b_t^s - z_t^l b_t^l + \left[ - (1 - \rho) \beta d_t + \beta b_t^s \frac{\phi_{t+1}}{\phi_t} \\
+ \beta b_t^l \frac{\phi_{t+1}}{\phi_t} (1 + z_{t+1}^l) + \beta \frac{\phi_{t+1}}{\phi_t} m_t \right] = 0\]

- If (20) binds, the bank must receive a payoff strictly greater than zero in the CM of \(t + 1\) to keep it from absconding.

- Binding (2) implies that what the bank receives from deposits in the CM of \(t\) is less than the value of assets it acquires. The difference is bank capital, i.e. the bank must acquire capital to keep itself honest.

- Eq. (20) binds in eqm because of the aggregate scarcity of collateral.
Stationary equilibria: \[\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu} \quad \forall t\]

(16) - (20) \Rightarrow

\[\frac{\beta}{\mu} u' \left( \frac{\beta c}{\mu} \right) - 1 = 0 \quad (21)\]

\[z^s = \frac{\beta}{\mu} [u'(\beta d)(1 - \theta_s) + \theta_s] \quad (22)\]

\[z^l = \frac{\beta}{\mu} \frac{[u'(\beta d)(1 - \theta_l) + \theta_l]}{1 - \frac{\beta}{\mu} [u'(\beta d)(1 - \theta_l) + \theta_l]} \quad (23)\]

\[- (1 - \rho)d + \frac{b^s(1 - \theta_s)}{\mu} + \frac{b^l(1 - \theta_l)}{\mu - \beta [u'(\beta d)(1 - \theta_l) + \theta_l]} = 0 \quad (24)\]
Bond yields and the term premium

- nominal yields on short-maturity and long-maturity:

\[ R^s = \frac{1 - z^s}{z^s}, \quad R^l = \frac{1 - z^l}{z^l} \]

- nominal term premium:

\[ R^l - R^s = \frac{\mu[u'(\beta d_t) - 1](\theta_l - \theta_s)}{\beta[u'(\beta d_t)(1 - \theta_l) + \theta_l][u'(\beta d_t)(1 - \theta_s) + \theta_s]} \]

- \( R^l - R^s > 0 \) requires

  (i) \( \theta_l > \theta_s \) (long-maturity debt is relatively poor collateral)

  (ii) \( u'(\beta d_t) > 1 \), i.e. (20) is binding (collective scarce of collateral).
Conclusion

• It is not the zero lower bound on the short-term nominal interest rate that makes QE matter.

• In this model, QE matters for two reasons.
  • Collateral is scarce in the aggregate.
  • QE acts to relax incentive constraints by increasing the average quality of the stock of collateral in the economy, thus increasing the effective quantity of collateral.

• A floor system allows the central bank to swap reserves for long-maturity gov’t debt, whereas under a channel system the central bank must rely on the willingness of the private sector to hold currency, in order to purchase long-maturity debt.