SCARCE COLLATERAL, THE TERM PREMIUM, AND QUANTITATIVE EASING

Williamson (2016, *JET*)

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Main features of the model:

- Private financial intermediaries perform a liquidity transformation role. [Diamond and Dybvig (1983), Williamson (2012)]
- Intermediary liabilities are subject to limited commitment, and the assets of the financial intermediary must serve as collateral.
- Different assets have different degrees of pledgeability

A term premium will arise in equilibrium under two conditions:

(I) short-maturity government debt has a greater degree of pledgeability than long-maturity government debt;

(II) collateral is collectively scarce, in that the total value of collateralizable wealth is too low to support efficient exchange.
## Repo Haircuts

*(percent)*

<table>
<thead>
<tr>
<th>Repo haircuts (%)</th>
<th>Spring 2007</th>
<th>Spring 2008</th>
<th>Fall 2008</th>
<th>Spring 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Treasuries (short-term)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>U.S. Treasuries (long-term)</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
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<tr>
<td>Agency mortgage-backed securities</td>
<td>2.5</td>
<td>6</td>
<td>8.5</td>
<td>6.5</td>
</tr>
<tr>
<td>Corporate bonds, A–/A3 or above</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Collateralized mortgage obligations, AAA</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Asset-backed securities, AA/Aa2 and above</td>
<td>10</td>
<td>25</td>
<td>30</td>
<td>35</td>
</tr>
</tbody>
</table>

*Source:* The data in the first three columns is from the Depository Trust and Clearing Corporation (provided by Tobias Adrian of the New York Fed), with the column for fall of 2008 filled out from reports of investment banks.
Main results

- Purchases of long-maturity government debt at the zero lower bound reduce the nominal yield on long-maturity government bonds and flatten the yield curve.

- Real bond yields increase because QE, involving swaps of better collateral for worse collateral, increases the value of collateralizable wealth, making collateral less scarce, and the liquidity premium is reduced.

- Inflation falls because one of the effects of QE is to increase the real stock of currency held by the private sector, and agents require an increase in currency’s rate of return (a fall in the inflation rate) to induce them to hold more currency.
**Model: agents and assets**

- A continuum of buyers (sellers) each with unit mass.
  - Buyer scan produce in the CM, but not in the DM.
  - Sellers can produce in the DM, but not in the CM.
  - One unit of labor input produces one unit of the perishable consumption good, in either the CM or the DM.

- Assets:
  - government-issued currency, $\phi_t$ in the CM goods
  - reserves (issued by the central bank), account balance at the central bank
  - short-maturity and long-maturity government bonds (issued by the fiscal authority)
Model: CM

In the CM, debts are first paid off, then a Walrasian market opens.

- reserves: \( Z^m_t \) units of money \( \rightarrow \) one unit of money
- short-maturity: \( Z^s_t \) units of money \( \rightarrow \) one unit of money
- long-maturity: \( Z^l_t \) units of money \( \rightarrow \) one unit of money

in every future CM
Model: DM

- Limited commitment - no one can be forced to work - and so lack of memory implies that there can be no unsecured credit.
- $\rho$ of DM transactions: the seller can only verify the buyer’s currency holdings.
  $1 - \rho$ of DM transactions: the seller can verify the entire portfolio held by the buyer.
- Reserves and government debt cannot be transferred until the next CM.
- Buyers make take-it-or-leave-it offers.
Credit

- Credit arrangements in this model will involve promised payments at the beginning of the CM that must be collateralized, given limited commitment and lack of memory. (Banks have the same commitment problem.)

- Assume that a buyer (bank) can abscond with fraction $\theta_s$ of short-maturity debt, reserves or currency and with $\theta_l$ of long-maturity government debt.
Banks

- Any agent can operate a bank.
- A bank issues deposits in the CM, but before buyers learn what their type will be.
- A deposit-holder can withdraw currency, or the deposit turns into a tradeable claim to be redeemed by the bank in the next CM in goods. ⇒ A bank deposit is essentially an option.
- A banking arrangement essentially permits currency to be allocated only to currency transactions, and government debt and reserves to non-currency transactions.
- The bank’s deposit claims must be backed with collateral, and the only available collateral in the model is government debt and reserves.
Bank’s problem

All quantities are expressed in units of the CM good in period $t$ ($d_t$ denotes claims to CM goods of period $t + 1$).

$$\max_{d_t, c_t, k_t, m_t, b^s_t, b^l_t} \left[ -k_t + \rho u \left( \frac{\beta \phi_{t+1} c_t}{\phi_t} \right) + (1 - \rho) u(\beta d_t) \right]$$

(1)

$$s.t \quad k_t - \rho c_t - z^m_t m_t - z^s_t b^s_t - z^l_t b^l_t +$$

$$\begin{pmatrix}
- (1 - \rho) \beta d_t + \beta b^s_t \frac{\phi_{t+1}}{\phi_t} \\
+ \beta b^l_t \frac{\phi_{t+1}}{\phi_t} (1 + z^l_{t+1}) + \beta \frac{\phi_{t+1}}{\phi_t} m_t
\end{pmatrix} \geq 0$$

(2)

$$- (1 - \rho) d_t + \left[ b^s_t \frac{\phi_{t+1}}{\phi_t} + b^l_t \frac{\phi_{t+1}}{\phi_t} (1 + z^l_{t+1}) + \frac{\phi_{t+1}}{\phi_t} m_t \right]$$

(3)

$$\geq \frac{\phi_{t+1}}{\phi_t} \left[ \theta_s (m_t + b^s_t) + \theta_l b^l_t (1 + z^l_{t+1}) \right]$$

(4)

$$k_t, c_t, m_t, d_t, b^s_t, b^l_t \geq 0$$

(5)

$$(3) \Rightarrow (1 - \rho) d_t + (b^s_t + m_t) \frac{\phi_{t+1}}{\phi_t} (1 - \theta_s) + b^l_t \frac{\phi_{t+1}}{\phi_t} (1 + z^l_{t+1})(1 - \theta_l) \geq 0$$
CONSOLIDATED BALANCE SHEET

- $C_t, M_t$: nominal quantities of currency and reserves
- $B^{s}_t, B^{l}_t$: nominal quantities of gov’t debt held in the private sector
- $\tau_t$: real value of the transfer to each buyer in the CM

\begin{equation}
\phi_0(C_0 + z^m_0 M_0 + z^s_0 B^s_0 + z^l_0 B^l_0) - \tau_0 = 0
\end{equation} 

\begin{equation}
\phi_t\{(C_t - C_{t-1} + z^m_t M_t - M_{t-1} \\
+ z^s_t B^s_t - B^s_{t-1} + z^l_t B^l_t - (z^l_t + 1) B^l_{t-1}\} - \tau_t = 0, t = 1, 2, 3, \ldots
\end{equation}

- Market clearing conditions for assets (12) - (15)
A Channel System vs Floor System

- **Channel system**
  - CB targets a short-term nominal interest rate, and pays interest on reserves at a rate below that target rate. Overnight reserves are essentially zero.
  - In US before 2008.10 it was essentially a channel system, with $z^m_t = 1$, i.e. no interest paid on reserves.

- **Floor system**
  - Interest is paid on reserves, and a positive stock of reserves is held overnight.
  - $z^m_t = z^s_t$ must hold in eqm, as the role played by reserves in the private sector is identical to that of short-term gov’t debt.
  - Difference: Away from the zero lower bound on the short rate, CB can effectively issue short-term debt (reserves), and swap that short-term debt for long-term gov’t debt.
A Channel System: Equilibrium

- If $z_t^m > z_t^s$, then it is optimal for banks to hold no reserves ($m_t = 0$).
- Constraint (2) must bind, as the objective function is strictly increasing in both $c_t$ and $d_t$, and the LHS of (2) is strictly decreasing in $c_t$ and $d_t$.
- We restrict attention to the case where (5) binds $\Rightarrow$ the multiplier $\lambda_t$ on (5).
**EQUILIBRIUM: FOC**

\[ [c_t] : \quad \frac{\beta \phi_{t+1}}{\phi_t} u' \left( \frac{\beta \phi_{t+1} c_t}{\phi_t} \right) - 1 = 0 \quad (16) \]

\[ [d_t] : \quad \beta u' (\beta d_t) - \beta - \lambda_t = 0 \quad (17) \]

\[ [b^s_t] : \quad - z^s_t + \beta \frac{\phi_{t+1}}{\phi_t} + \lambda_t (1 - \theta_s) \frac{\phi_{t+1}}{\phi_t} = 0 \quad (18) \]

\[ [b^l_t] : \quad - z^l_t + \beta \frac{\phi_{t+1}}{\phi_t} (1 + z^l_{t+1}) + \lambda_t (1 - \theta_l) (1 + z^l_{t+1}) \frac{\phi_{t+1}}{\phi_t} = 0 \quad (19) \]

\[ [\lambda_t] : \quad - (1 - \rho) d_t + b^s_t \frac{\phi_{t+1}}{\phi_t} (1 - \theta_s) + b^l_t \frac{\phi_{t+1}}{\phi_t} (1 + z^l_{t+1}) (1 - \theta_l) = 0 \quad (20) \]
Binding incentive constraint

\[-(1 - \rho) d_t + b^s_t \frac{\phi_{t+1}}{\phi_t} (1 - \theta_s) + b^l_t \frac{\phi_{t+1}}{\phi_t} (1 + z^l_{t+1})(1 - \theta_l) = 0\]

\[k_t - \rho c_t - z^m_t m_t - z^s_t b^s_t - z^l_t b^l_t + \left[ -(1 - \rho) \beta d_t + \beta b^s_t \frac{\phi_{t+1}}{\phi_t} \\
+ \beta b^l_t \frac{\phi_{t+1}}{\phi_t} (1 + z^l_{t+1}) + \beta \frac{\phi_{t+1}}{\phi_t} m_t \right] = 0\]

- If (20) binds, the bank must receive a payoff strictly greater than zero in the CM of \(t + 1\) to keep it from absconding.
- Binding (2) implies that what the bank receives from deposits in the CM of \(t\) is less than the value of assets it acquires. The difference is bank capital, i.e. the bank must acquire capital to keep itself honest.
- Eq. (20) binds in eqm because of the aggregate scarcity of collateral.
Stationary equilibria: \[ \frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu} \ \forall t \]

(16) - (20) \Rightarrow

\[
\frac{\beta}{\mu} u' \left( \frac{\beta c}{\mu} \right) - 1 = 0
\]

(21)

\[
z^s = \frac{\beta}{\mu} \left[ u' (\beta d) (1 - \theta_s) + \theta_s \right]
\]

(22)

\[
z^l = \frac{\beta}{\mu} \left[ u' (\beta d) (1 - \theta_l) + \theta_l \right]
\]

\[
1 - \frac{\beta}{\mu} \left[ u' (\beta d) (1 - \theta_l) + \theta_l \right]
\]

(23)

\[
- (1 - \rho) d + \frac{b_s (1 - \theta_s)}{\mu} + \frac{b_l (1 - \theta_l)}{\mu - \beta [u' (\beta d) (1 - \theta_l) + \theta_l]} = 0
\]

(24)
Assume under this fiscal policy regime, transfers respond passively after period 0 to central bank policy, holding constant the value of the consolidated gov’t debt outstanding.

We are interested in the case where $V$ is small, so that the quantity of collateralizable wealth is inefficiently low in eqm.

\[
\begin{align*}
\tau_0 &= V = \rho c + z^s b^s + z^l b^l, \\
\tau_t &= V\left(1 - \frac{1}{\mu}\right) + \frac{1}{\mu}\left[(z^m - 1)m + (z^s - 1)b^s - b^l]\right)
\end{align*}
\]
Bond Yields and the Term Premium

Bond yields and the term premium

- nominal yields on short-maturity and long-maturity:

\[ R^s = \frac{1 - z^s}{z^s}, \quad R^l = \frac{1 - z^l}{z^l} \]

- nominal term premium:

\[ R^l - R^s = \frac{\mu[u'(\beta d_t) - 1](\theta_l - \theta_s)}{\beta[u'(\beta d_t)(1 - \theta_l) + \theta_l][u'(\beta d_t)(1 - \theta_s) + \theta_s]} \] (30)

- \( R^l - R^s > 0 \) requires
  
  (I) \( \theta_l > \theta_s \) (long-maturity debt is relatively poor collateral)
  
  (II) \( u'(\beta d_t) > 1 \), i.e. (20) is binding (collective scarce of collateral).
BOND YIELDS AND THE TERM PREMIUM

DEFINITION OF EQUILIBRIUM

**Definition**

A stationary equilibrium under a channel system is quantities $c, d, b^s$ and $b^l$ prices $z^s$ and $z^d$, and gross inflation rate $\mu$ that solve (21)-(25) and

\begin{align}
0 &\leq z^s b^s \leq V_s \quad (26) \\
0 &\leq z^l b^l \leq V_l \quad (27)
\end{align}
Conventional Monetary Policy

- $z_t^m = 1$ (no interest paid on reserves)
- Assume
  \[ z^l b^l = a_l \]  \( (36) \)
  where $a_l$ is a constant, and $0 \leq a_l \leq V_l$.

The value of the long-term gov’t debt held by the private sector is fixed.

- We express eqm in terms of $(x_1, x_2)$, where $x_1 = \frac{\beta}{\mu} c$, $x_2 = \beta d$, denote consumption in currency transactions and in non-currency transactions in the DM, respectively.
Away from the zero lower bond $z_t^b < 1 = z_t^m$

Away from the zero lower bond ($z_t^b < 1$)

- $z_t^b < 1$ (no reserves are held in equilibrium)
- $(20) (22) (23) (25) \Rightarrow$

\[-(1 - \rho)\beta_d[u'(\beta_d)(1 - \theta_s) + \theta_s] - \frac{a_l(\theta_l - \theta_s)}{u'(\beta_d)(1 - \theta_l) + \theta_l} + (V - \rho c)(1 - \theta_s) = 0 \quad (37)\]

- $(21)$ and $(37) \Rightarrow$

\[-(1 - \rho)x_2[u'(x_2)(1 - \theta_s) + \theta_s] - \frac{a_l(\theta_l - \theta_s)}{u'(x_2)(1 - \theta_l) + \theta_l} + [V - \rho x_1 u'(x_1)](1 - \theta_s) = 0 \quad (38)\]
SOLVING FOR THE EQUILIBRIUM

From (21) - (25)

\[ z^s = \frac{[u'(x_2) (1 - \theta_s) + \theta_s]}{u'(x_1)} \] (39)

\[ z^l = \frac{u'(x_2) (1 - \theta_l) + \theta_l}{u'(x_1) - [u'(x_2) (1 - \theta_l) + \theta_l]} \] (40)

\[ \mu = \beta u'(x_1) \] (41)

Nominal and real bond yields for short-maturity and long-maturity bonds

\[ R^s = \frac{u'(x_1)}{u'(x_2) (1 - \theta_s) + \theta_s} - 1 \] (42)

\[ R^l = \frac{u'(x_1)}{u'(x_2) (1 - \theta_l) + \theta_l} - 1 \] (43)

\[ r^s = \frac{1}{\beta [u'(x_2) (1 - \theta_s) + \theta_s]} - 1 \] (44)

\[ r^l = \frac{1}{\beta [u'(x_2) (1 - \theta_l) + \theta_l]} - 1 \] (45)
Away from the zero lower bond $z_t^b < 1 = z_t^m$

**SOLVING FOR THE EQUILIBRIUM (CON’T)**

- In equilibrium, $z^s < 1$ or, from (39),
  \[
  \frac{[u'(x_2)(1 - \theta_s) + \theta_s]}{u'(x_1)} < 1
  \]  (46)

- (21), (25), (26), and (27) imply, in eqm,
  \[
  V_l - a_l \leq \rho x_1 u'(x_1) \leq V - a_l
  \]  (47)

- $\rho x_1 u'(x_1)$: real value of currency outstanding (CB’s liabilities)
- $V_l - a_l$: long-maturity gov’t debt on the CB’s balance sheet
- $V - a_l$: value of the total consolidated gov’t debt minus the long gov’t debt held by the private sector
Effects of a lower target rate

**Nominal short rate target**

- The Central bank chooses the price of short term nominal debt, \( z^s \), (or, equivalently, the nominal interest rate, \( \frac{1}{z^s} - 1 \)), and then (38) and (39) determine \( x_1 + x_2 \).

- Given \( z^s \), the consolidated government’s budget constraint:

\[
V = V_s + V_l = \rho c + z^s b^s + z^l b^l,
\]

- The value of short debt held by central bank:

\[
V_s - z^s b^s = a^l - V_l + \rho x_1 u'(x_1)
\]

- \( z^s \uparrow \) (short interest rate \( \downarrow \)) \( \rightarrow x_1 \uparrow \rightarrow V_s - z^s b^s \uparrow \) (supported by open market purchase of short-maturity govt't debt).
The one-time exchange of currency for short-term gov’t debt has increased one type of liquidity (currency), and reduced another (useful as collateral). $z^S \uparrow \rightarrow x_1 \uparrow, x_2 \downarrow$

- $(42)(43) \Rightarrow$ nominal bond yields $\downarrow$ induce a negative Fisher effect
- $(44)(45) \Rightarrow$ real bond yields $\downarrow$ because of less eligible collateral outstanding

Different from the conventional wisdom:

- inflation rate $\downarrow$
- collateral is more scarce (short-maturity debt is transferred from private sector to the central bank).

Bank deposits $k \uparrow$ because of $\Delta x_1 > |\Delta x_2|$
Conventionally-studied channels for monetary easing typically work through temporary declines in real interest rates and increases in the inflation rate.

The effect is a permanent increase in the size of the central bank’s holdings of short-maturity govt debt - in real terms - which must be financed by an increase in the real quantity of currency held by the public.
**Zero Lower Bound**

**Zero Lower Bound**  \( z^b = z^m = 1 \)

- Banks are willing to hold reserves (39) ⇒

\[
z^s = 1 = \frac{u'(x_2)(1 - \theta_s) + \theta_s}{u'(x_1)}. \tag{49}
\]

- (38) and (49) solve \((x_1, x_2)\).
**Liquidty Trap**

- $m$ and $b^s$ (reserves and short-maturity gov’t debt held by the private sector) do not affect the eqm solution.
- From the consolidated gov’t budget constraint (25),
  
  \[ m + b^s = V - a_l - \rho x_1 u'(x_1). \]  
  (50)

where $m + b^s$, not the composition, is determined in eqm. (i.e. it does not matter whether the debt is issued by the central bank or the fiscal authority)

- The zero lower bound is a liquidity trap, in the sense that conventional swaps of outside money for short-term gov’t debt do not affect the inflation rate and nominal bond yields.
Away from the zero lower bound

**Purchase of long-maturity gov’t debt**

- Assume
  \[ z^s b^s + z^m m = a_s, \]
  where \( a_s \) is a constant.

- Away from the zero lower bound, the bank’s incentive constraint is
  \[
  - (1 - \rho) x_2 [u'(x_2)(1 - \theta_s) + \theta_s] + a_s (1 - \theta_s) + \\
  \left[ V - \rho x_1 u'(x_1) - a_s \right] (1 - \theta_l) [u'(x_2)(1 - \theta_s) + \theta_s] \\
  \frac{u'(x_2)(1 - \theta_l) + \theta_l}{u'(x_2)(1 - \theta_l) + \theta_l} = 0 \quad (51)
  \]


**Effects of buying long gov’t debt**

- Given $z^s$, in equilibrium, (51) and (39) determine $(x_1, x_2)$.
- The analogue of (47) is
  \[
  V_s - a_s \leq \rho x_1 u'(x_1) \leq V - a_s,
  \]
  and together with the consolidated gov’t budget constraint (25), the market value of long-maturity gov’t debt held by the public is
  \[
  z^l b^l = V - a_s - \rho x_1 u'(x_1).
  \]
- CB chooses $z^s$, and the OMO is buying long debt:
  A lower nominal interest rate implies that $x_1$ increases, and from (53) this implies a lower market value of long gov’t debt outstanding ($z^l b^l \downarrow$).
Banks hold reserves. The bank’s incentive constraint is

\[- (1 - \rho)x_2[u'(x_2)(1 - \theta_s) + \theta_s] + \frac{(m + b^s)(\theta_l - \theta_s)}{u'(x_2)(1 - \theta_l) + \theta_l} + \]

\[\frac{[V - \rho x_1 u'(x_1)](1 - \theta_l)[u'(x_2)(1 - \theta_s) + \theta_s]}{u'(x_2)(1 - \theta_l) + \theta_l} = 0 \quad (54)\]

In eqm, \((x_1, x_2)\) is determined by (54) and (49).

Liquidity trap

Quantitative easing (QE) matters, where QE here is a swap of reserves for long gov’t debt: from (54), \((x_1, x_2)\) increases
QE only matters in equation (54) if $\theta_l > \theta_s$.

QE acts to relax the incentive constraints of banks, and real bond yields rise because collateral is now less scarce in the aggregate.

In eqm, more currency is held ($x_1$ increases), and to induce buyers to hold more currency in real terms, the inflation rate must fall.
Conclusion

- It is not the zero lower bound on the short-term nominal interest rate that makes QE matter.
- In this model, QE matters for two reasons.
  - Collateral is scarce in the aggregate.
  - QE acts to relax incentive constraints by increasing the average quality of the stock of collateral in the economy, thus increasing the effective quantity of collateral.
Optimal Monetary Policy

- Any equilibrium allocation that is feasible for the central bank under a channel system is also feasible under a floor system.
- Why?
  A floor system allows the central bank to swap reserves for long-maturity gov’t debt, whereas under a channel system the central bank must rely on the willingness of the private sector to hold currency, in order to purchase long-maturity debt.