Payments and liquidity under adverse selection
Guillaume Rocheteau a,b,*

a University of California, Irvine, United States
b Federal Reserve Bank of Cleveland, United States

ABSTRACT

Informational asymmetries regarding the future value of assets affect their role in exchange. I construct a random-matching economy composed of two assets: a risk-free bond and a Lucas tree whose terminal value is privately known to its holder. No restrictions are imposed on payment arrangements. The main finding supports a pecking-order theory of payments: Agents use their risk-free bonds first in order to finance their spending shocks, and they use their information-sensitive assets only if their holdings of bonds are depleted. The theory has implications for the optimal provision of risk-free bonds, the structure of asset returns, and liquidity.

1. Introduction

Liquidity considerations matter for macroeconomics. They help explain asset pricing anomalies, the codetermination of asset prices and macroeconomic conditions, and the transmission mechanism of monetary policy. Liquid assets have an essential role in economies where unsecured credit arrangements that would allow households and firms to finance spending shocks (e.g., consumption or investment opportunities) are not feasible. A critical observation from Kiyotaki and Moore (2005) and Lagos (2010a) is that not all assets are equally suitable for helping agents handle these shocks: some assets are more liquid than others. In Kiyotaki and Moore, agents who hold land and capital can use only a fraction of their capital stock to finance investment opportunities. In Lagos, agents hold risk-free bonds and equity, but equity shares can only be used to finance a fraction of their consumption opportunities. While these liquidity differences among assets help to explain several macroeconomic phenomena, the differences in liquidity themselves are left unexplained by the proposed theories.

The objective of this paper is to investigate how informational asymmetries regarding the future value of assets affect their role in exchange, and the implications those asymmetries have for liquidity and the structure of asset yields. History is marked by episodes where the imperfect recognizability of some assets has impaired their ability to serve as media of exchange. In the Antebellum United States thousands of different notes issued by hundreds of banks circulated, and a fraction of these notes were worthless, since they were either counterfeits or the floating issue of insolvent banks. More recently, during the financial market turmoil of 2007–2008, the recognizability problem with asset-backed securities arising from their complexity and heterogeneity has contributed to a major liquidity crunch by reducing investors’ ability to use them as collateral for loans.

I construct an environment that features pairwise meetings, a meaningful role for means of payment (or collateral), and two types of assets that differ in terms of their exposure to private-information problems: risk-free bonds and risky

* Correspondence address: University of California, Irvine, United States.
E-mail address: grochete@uci.edu

Lucas (1978) trees. Agents holding a Lucas tree are better informed about its future performance than agents who receive it. The Lucas tree, which is information sensitive, can be thought of as private equity, a corporate bond, or an asset-backed security. 2 Beside being a realistic description of the way many assets are traded, having pairwise meetings allows the model to provide explicit game-theoretic foundations for the exchange of goods and assets, as well as for the transfer of information that takes place in a match. The intricate part is determining the terms of trade in matches where the two parties are asymmetrically informed. Most of the recent literature in monetary theory neglects this issue. (See Section 1.1 for a review of the relevant literature.) In contrast, the transfer of goods and assets in this model will be the outcome of an explicit bargaining game, and it will be shown that asset holders can select offers that signal the quality of their assets. The possibility of such signaling generates new insights for payment arrangements in pairwise meetings.

The main insight is that payment arrangements exhibit a pecking-order property. Agents have a strict preference for risk-free bonds as means of payment (or, equivalently, as collateral): they spend their risk-free bonds first in order to finance their consumption, and they use their information-sensitive assets as means of payment only if their holdings of risk-free bonds are depleted. Moreover, information-sensitive assets are (partially) illiquid in the sense that agents sell only a fraction of their asset holdings even when their consumption is inefficiently low. Intuitively, by retaining a fraction of their information-sensitive assets, agents are able to credibly reveal their private information regarding the future value of their assets. In contrast, if an agent attempts to sell too large a quantity of information-sensitive assets, then his offer will get rejected because it will be attributed to someone holding low-value assets. This result captures the notion that large trades of information-sensitive assets involve sizeable transaction costs—a standard notion of asset illiquidity.

A major insight of Kiyotaki and Wright (1989) was to show that the acceptability of a good depends on its storage cost as well as other fundamentals (e.g., the pattern of specialization) and beliefs. In the same vein, the liquidity of the information-sensitive asset depends on the stochastic process that drives its fundamental value. The asset becomes more illiquid as the dispersion of its future values across states increases. The methodology in this paper could be applied to Kiyotaki and Moore (2005) or Lagos (2010a) models in order to relate the partial illiquidity of some assets to their risk characteristics. In the limiting case where the asset has no value in some states, it becomes fully illiquid and, in the absence of risk-free assets, trade shuts down. While it is well understood that adverse-selection problems can cause markets to cease to function, here it occurs in the context of a market with bilateral trades and bargaining — the two main characteristics of an over-the-counter market — despite agents’ private information being revealed in equilibrium through signaling.

The model has implications for asset prices and the liquidity structure of asset yields. Risk-free bonds exhibit a liquidity premium – the difference between the market price and the discounted terminal value of the asset — if there is a shortage of information-insensitive assets. 3 Moreover, the rate of return of the risk-free bonds is less than the rate of return of the information-sensitive assets even though agents are endowed with quasilinear preferences. This rate-of-return difference occurs because risk-free bonds are preferred means of payment. This finding is consistent with the observed convenience yield of Treasury securities relative to corporate bonds (Krishnamurthy and Vissing-Jorgensen, 2008).

Finally, the model provides a channel through which policy — described as a change in the supply of the risk-free bonds — affects asset prices, the structure of asset returns, and output. If the quantity of bonds is below a threshold, an increase in the supply of bonds raises the risk-free rate, output, and welfare. A policy that would consist of issuing risk-free bonds in order to substitute them for information-sensitive assets would be welfare improving. The optimal policy is such that the demand for risk-free bonds is satiated. In that case, asset prices are driven down to their fundamental values, and the information-sensitive asset is illiquid, i.e., its transaction velocity (in some states) is zero.

The paper is organized as follows. Section 1.1 provides a review of the relevant literature. The environment is described in Section 2. Section 3 analyzes the bargaining game under incomplete information. Section 4 embeds the bargaining game into a general equilibrium structure and studies the effects of policy and fundamentals on asset liquidity. The proofs of lemmas and propositions are in an online appendix.

1.1. Related literature

There is a related literature that studies adverse selection in decentralized asset markets with pairwise meetings. 4 This includes Cuadras-Morató (1994) on the emergence of a commodity money, Velde et al. (1999), and Burdett et al. (2001) on Gresham’s law, and Hopenhayn and Werner (1996) on the liquidity structure of asset returns. These papers differ from this one in that they avoid the thorny issue of the determination of the terms of trade under asymmetric information by restricting asset holdings to [0,1] and by assuming that agents cannot hold multiple assets. Even when consumption goods are divisible, such restrictions reduce agents’ ability to signal their private information, and they prevent the emergence of separating equilibria. Aiyagari (1989) circumvents the difficulty of analyzing strategic interactions within

---

2 An asset is information sensitive if its future value is random and if it is subject to a private-information problem. According to this definition, a risk-free asset is information insensitive. If agents are symmetrically informed about the future value of a risky asset, the asset is also information insensitive. The notion of information sensitivity is related to Jevons’s (1875) notion of cognizability defined as the capability of a substance for being easily recognized and distinguished from all other substances.

3 The idea that the available supply of liquid assets is relatively scarce is empirically relevant. See, e.g., Caballero (2006).

4 There is a recent literature on liquidity in decentralized asset markets. See, e.g., Duffie et al. (2005) and Lagos and Rocheteau (2009). These papers, however, do not incorporate informational asymmetries.
bilateral matches, and the possibility of information transmission through signaling or screening, by assuming competitive trades in overlapping generations economies with privately informed agents. Huggett and Krasa (1996) adopt a mechanism design approach – instead of an equilibrium approach – to show the essentiality of fiat money in a model with alternating endowments, a storage technology with positive returns, and differential information.

While my paper focuses on an alternative-selection problem, there is a related literature that studies the role of money in the presence of moral hazard problems regarding the quality of goods. Williamson and Wright (1994), Trejos (1997, 1999), and Berentsen and Rocheteau (2004) consider an economy with pairwise trades, where consumption goods can be counterfeited and fiat money is the only asset. Li (1995) studies an environment with multiple commodity monies but where terms of trade are exogenous. In these models, the possibility of signaling, which underlies my pecking-order theory of payments, is shut down, because money holdings are restricted to {0,1} or allocations are restricted to those that are pooling. Banarjee and Maskin (1996) do not restrict asset holdings, but they study the emergence of commodity monies in an environment with Walrasian trading posts, where agents can allocate their labor to the production of low- or high-quality goods. The assumption of price-taking agents rules out the strategic considerations in the pairwise meetings that are the focus of this paper.

Lester et al. (2007) also extend the Lagos and Wright (2005) model to include multiple divisible assets, fiat money, and capital. The recognizability problem takes the form of claims on capital that can be costlessly counterfeited and can only be authenticated in an endogenous fraction of meetings. However, they simply assume that uninformed sellers do not accept claims on capital. Li and Rocheteau (2009) solve the bargaining game under incomplete information in the case where counterfeits are produced at a positive cost. Li and Rocheteau’s paper complements my analysis by showing the different implications of moral hazard and adverse selection for payment arrangements.

Papers in the search-theoretic literature concerned with liquidity and asset pricing include Lagos (2010a), Geromichalos et al. (2007), Lagos and Rocheteau (2008), and Ravikumar and Shao (2010).

Finally, my model provides a foundation for the trading restrictions that have been imposed in some recent models that have fiat money coexisting with other assets, e.g., Aruoba and Wright (2003), Aruoba et al. (2011), Kiyotaki and Moore (2005), Lagos (2010a), and Telyukova and Wright (2008).7

2. Environment

The environment is similar to the one in Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is discrete, starts at \( t = 0 \), and continues forever. Each period has two subperiods: a morning, where trades occur in a decentralized market (DM), followed by an afternoon, where trades take place in a competitive market (CM). There is a unit measure of infinitely lived households and a unit measure of firms. There are two perishable consumption goods, one produced in the DM and the other in the CM (Fig. 1).

The lifetime expected utility of a household from date 0 onward is

\[
E \sum_{t=0}^{\infty} \beta^t U(y_t, x_t, n_t),
\]

where \( x_t \) is the CM consumption of period \( t \), \( n_t \) is the hours of work in the CM, \( y_t \) is the DM consumption, and \( \beta \in (0, 1) \) is a discount factor. For tractability, the period utility is separable across stages and linear in the CM, i.e., \( U(y_t, x_t, n_t) = u(y_t) + x_t - n_t \). The utility function \( u(y) \) is twice continuously differentiable, \( u(0) = 0 \), \( u'(0) = \infty \), \( u'(y) > 0 \), and \( u''(y) < 0 \). The production technology in the CM is linear, with labor as the only input, \( c(0) = 0 \), and \( c'(0) = 0 \).

The DM output is produced by firms. Each firm invests \( k > 0 \) units of the CM good at \( t = 1 \). This allows it to generate at \( t \) any amount \( y \in [0, Y] \) of the DM good, plus \( x = f(Y - y) \) of the CM good, where \( Y > 0 \). \( f \) is twice continuously differentiable, strictly increasing and concave, \( f'(0) = \infty \), and \( f'(Y) = 0 \). This makes \( c(y) = f(y) - f(Y - y) \) the opportunity cost of selling \( y \) in the CM, where \( c(0) = 0 \), \( c'(0) = 0 \), and \( c'(Y) = \infty \). Let \( y^* = \arg \max_y [u(y) - c(y)] \). Assume \( k < \beta f(Y) \), so that it is always profitable to participate in the market. The profits of the firms (defined as income net of investment cost) are redistributed in a lump-sum fashion to the households.

At the beginning of the CM, each household is endowed with \( A > 0 \) units of one-period-lived, divisible Lucas trees that can be interpreted as private equity, corporate bonds, or asset-backed securities. Trees are subject to an idiosyncratic shock, \( K \in \{k_i, k_f\} \), at the beginning of the DM, which is privately known to the holder of the trees. With probability \( P_h \in (0,1) \), the terminal output of a tree is \( K = k_h \), and with complement probability \( P_i, K = k_i \), where \( 0 < k_i < k_h \) and

---

5 Smith (1989), in an overlapping generations model, and Jafarey and Rupert (2001) in a model with alternating endowments explain the usefulness of fiat money when credit is available by a private-information friction regarding individuals’ abilities to repay their debts.

6 There is a related literature on counterfeiting, which includes Green and Weber (1996), Williamson (2002), and Nosal and Wallace (2007).

7 Aruoba and Wright (2003) and Aruoba et al. (2011) also refer to the lack of portability of capital goods to justify the assumption that capital cannot be used as a means of payment in decentralized markets. Telyukova and Wright (2008, Section 4) lay down an extension of their model with Lucas trees, in which agents pay a fixed cost if they use their real assets as means of payment.

8 The assumption of an infinite time horizon is not necessary but it makes the model comparable to standard monetary models. It can be readily extended to have long-lived assets or fiat money. See Rocheteau (2008, 2009).

9 The description of firms is borrowed from Berentsen et al. (2011).
πK1 + πK2 = 1.10 (With no loss in generality, the expected income of each asset is normalized to one.) The realizations of the productivity shocks are common to all the trees held by a household, but they are independent across households.

The government supplies a constant quantity, Z, of one-period-lived, risk-free bonds. The safety of bonds is backed by the unrestricted ability of the government to tax households in the CM. Bonds are perfectly divisible, and each unit pays one unit of output in the CM. The interest payments are financed by lump-sum taxes in the CM.

In the CM, households can trade consumption goods, bonds, and trees competitively. In the DM, each firm is matched bilaterally and at random with a household. The household makes an offer that the firm accepts or rejects. If the offer is accepted, the trade is implemented.11 Unsecured credit arrangements are not incentive-feasible since households are anonymous and cannot commit. In this case, DM trade is either quid pro quo, or equivalently, for the purpose of this paper, collateralized debt.12 Matched agents can transfer any nonnegative quantity of DM output and any quantity of their asset holdings.

3. Payments under private information

Consider the bargaining game between a household holding a portfolio composed of a trees and z bonds, and a firm. The analysis of the bargaining game is simplified by assuming that the household’s portfolio is common knowledge in the match.13

3.1. Description of the bargaining game

In order to define the payoffs in the bargaining game, it is useful to derive first some properties of the value functions in the CM. Let \( W(z,a,K) \) denote the value function of a household holding z bonds and a trees before the CM opens, when the terminal value of a tree is \( K \in \{ K_1, K_2 \} \).

\[
W(z,a,K) = \max_{x \in \mathbb{R}, z \in \mathbb{R}} \{ x - n + \beta E V(z', a', K') \}
\]  

subject to the budget constraint (3).

\[
x + qz' + qa' + T = n + ka + z + qaA + II,
\]

where \( V(z,a,K) \) is the value function of the household at the beginning of the DM, \( q_z \) is the price of bonds (expressed in CM output), \( q_a \) is the price of trees, \( T \) is the lump-sum tax by the government, and \( II \) are the profits paid by the firms (revenue net of the investment cost). The expectation is taken with respect to the terminal value \( K' \). According to (2), each household chooses its net consumption, \( x-n \), and its portfolio, \( z' \) and \( a' \), in order to maximize its expected lifetime utility subject to the budget constraint (3). According to (3), the value of the household’s initial portfolio in terms of CM output is \( ka + z \). In order to hold a portfolio \( (z',a') \) in the next CM, the household must invest \( q_z z' \) of current output in bonds and \( qa' \) in trees. It must also pay some lump-sum taxes \( T \), and it receives an endowment of trees worth \( qaA \) and the profits of the firms. Substitute \( x-n = ka + z - qz' + qa(A-a') - T + II \) from (3) into (2) to obtain

\[
W^b(z,a,K) = ka + z + qaA - T + II + \max_{z',a'} \{ -qz' - qaA' + \beta(\pi_K V^b(z', a', K) + \pi_r V^b(z', a', K')) \}.
\]

The household’s value function in the CM is linear in its wealth. Moreover, a household’s portfolio choice is independent of its initial portfolio when it entered the period. Both properties greatly simplify the model.

The bargaining game between the household and the firm has the structure of a signaling game.14 A strategy for the household specifies an offer \( (y,d,\tau) \in \mathcal{F} \equiv \mathbb{R}_+ \times [0,a] \times [0,z] \), where \( y \) is the output produced by the firm, \( d \) is the transfer of trees by the household, and \( \tau \) is the transfer of bonds, as a function of the household’s type (i.e., its private information about

---

10 Plantin (2009) justifies the “learning-by-holding” assumption for securitized pools of loans. Rocheteau (2008, Appendix D) shows that the model can be generalized to allow for more than two terminal values.

11 I chose a bargaining protocol in which the household makes a take-it-or-leave-it offer because it has been extensively used in monetary theory, and it remains tractable under private information. See Section 5.2 for more details.

12 The above-mentioned equivalence between the use of the asset as a means of payment or as collateral for a loan is described by Lagos (2010b).

13 This assumption is made in order to avoid having to specify firms’ beliefs regarding the portfolio held by households in the pairwise meetings. It will be shown in the following that the surplus functions in the DM are weakly monotonically increasing in the household’s asset holdings. Hence, if households had the possibility of showing their portfolios in a pre-stage of the bargaining game, there would be an equilibrium in which they would do so truthfully.

14 See Appendix B in Rocheteau (2009) for a more detailed presentation of signaling games.
the future value of the trees. The transfers of assets are constrained by the household’s portfolio. A strategy for the firm is an acceptance rule that specifies a set \( \mathcal{A} \subseteq \mathcal{F} \) of acceptable offers.

The household’s payoff in the state \( \kappa \) is

\[
[u(y) + W(z, y, a - d, \kappa)] \mathbb{1}_\mathcal{A}(y, d, \tau) + W(z, a, \kappa)[1 - \mathbb{1}_\mathcal{A}(y, d, \tau)],
\]

where \( \mathbb{1}_\mathcal{A}(y, d, \tau) \) is an indicator function that is equal to one if \((y, d, \tau) \in \mathcal{A}\). If an offer is accepted, then the household enjoys its utility of consumption in the DM, \( u(y) \), but it forgoes \( d \) trees and \( \tau \) bonds. Using the linearity of the household’s value function, and omitting the constant terms, the household’s payoff can be expressed as its surplus, \([u(y) - kd - \tau] \mathbb{1}_\mathcal{A}(y, d, \tau)\).

The firm’s payoff function is

\[
[f(Y - y) + \alpha + \gamma d] \mathbb{1}_\mathcal{A}(y, d, \tau) + f(Y)[1 - \mathbb{1}_\mathcal{A}(y, d, \tau)] = -c(y) + \alpha + \gamma d \mathbb{1}_\mathcal{A}(y, d, \tau) + f(Y).
\]

In order to accept or reject an offer, the firm will have to form expectations about the terminal value of the household’s trees. Let \((y, d, \tau) \in [0,1] \) represent the updated belief of a firm that the household holds high-value trees \((\kappa = \kappa_h)\), conditional on the offer \((y, d, \tau)\) being made. Then, \( E_z[\kappa] = \lambda(y, d, \tau)\kappa_h + [1 - \lambda(y, d, \tau)]\kappa_l \).

For a given belief system, the set of acceptable offers for a firm is

\[
\mathcal{A}(\lambda) = \{(y, d, \tau) \in \mathcal{F} : -c(y) + \lambda(y, d, \tau)\kappa_h + [1 - \lambda(y, d, \tau)]\kappa_l | d + \tau \geq 0\}.
\]

For an offer to be acceptable, the firm’s cost of production in the DM, \(-c(y)\), must be compensated for by its expected revenue in the next CM, \( E_z[\kappa]d + \tau\). Assuming a tie-breaking rule according to which a firm agrees to any offer that makes it indifferent between accepting or rejecting a trade, \(^{15}\) the problem of a household holding trees of quality \( \kappa \) is then

\[
\max_{y, d, \tau} u(y) - \gamma d - \tau \mathbb{1}_\mathcal{A}(y, d, \tau) \text{ s.t. } (y, d, \tau) \in \mathbb{R}_+ \times [0, a] \times [0, z].
\]

3.2. Equilibrium of the bargaining game

The equilibrium concept is perfect Bayesian equilibrium (PBE). An equilibrium of the bargaining game is a profile of strategies for the firm and the household, and a belief system, \( \lambda \). If \((y, d, \tau) \in \mathcal{A}\) is an offer made in equilibrium, then \(\lambda(y, d, \tau)\) is derived from the firm’s prior belief according to Bayes’ rule. Since there is no discipline for out-of-equilibrium beliefs, the equilibrium concept is refined by using the Intuitive Criterion of Cho and Kreps (1987).\(^{16}\) Denote \(U_Y^b\) the surplus of an \( h\)-type household and \(U_Y^l\) the surplus of an \( l\)-type household in a proposed equilibrium of the bargaining game. (The superscript \( b \) stands for buyer.) The proposed equilibrium fails the Intuitive Criterion if an offer \((y, d, \tau) \in \mathcal{F}\), and a household’s type \( \chi \in \{l, h\} \), such that the following is true:

\[
u(y) - \kappa_y d - \tau > U_Y^b,
\]

\[
u(y) - \kappa_y d - \tau < U_Y^b,
\]

\[-c(y) + \kappa_y d + \tau \geq 0,
\]

where \( (-\chi) = \{l, h\} \setminus \chi \). According to (9), the offer \((y, d, \tau)\) would make a \( \chi \)-type household strictly better off if it were accepted. According to (10), the offer \((y, d, \tau)\) would make the \(-\chi\)-type household strictly worse off. According to (11), the offer is acceptable provided that the firm believes it comes from a \( \chi \)-type.

**Definition 1.** An equilibrium of the bargaining game is a pair of strategies and a belief system, \( \langle [\kappa(\kappa), d(\kappa), \tau(\kappa)], \mathcal{A}, \lambda \rangle \), such that \([\kappa(\kappa), d(\kappa), \tau(\kappa)]\) is a solution to (8), with \( \kappa \in \{\kappa_l, \kappa_h\} \); \( \mathcal{A} \) is given by (7); \( \lambda : \mathcal{F} \to [0,1] \) satisfies Bayes’ rule whenever possible and the Intuitive Criterion.

Equilibria of the bargaining game are characterized in three steps. First, the Intuitive Criterion is used to eliminate all PBE with a pooling offer (Lemma 2). Second, it is shown that among separating PBE, all but the Pareto-efficient (or least-costly separating) one can be dismissed by the Intuitive Criterion (Lemma 3). Third, a system of beliefs is constructed that supports the Pareto-efficient separating PBE that complies with the Intuitive Criterion, and the firm’s acceptance rule is derived.

**Lemma 2.** In equilibrium, there is no pooling offer with \( d > 0 \).

\(^{15}\) A similar tie-breaking assumption is used in Rubinstein (1985, Assumption B-3). It is made so that the set of acceptable offers is closed, and the household’s problem has a solution.

\(^{16}\) The Intuitive Criterion is a refinement supported by much of the signaling literature. An equilibrium that fails the Intuitive Criterion gives an outcome that is not strategically stable in the sense of Kohlberg and Mertens (1986). See Riley (2001) for a survey of the applications of the Intuitive Criterion (and other refinements) in various contexts. It has been used in monetary theory by Nosal and Wallace (2007); in the corporate finance literature by DeMarzo and Duffie (1999); in bargaining theory by Rubinstein (1985, Assumption B-1). In the context of this paper, the Intuitive Criterion has the additional advantage of preserving the tractability of the model once the bargaining game is embodied in the general equilibrium structure in Section 4. For the sake of completeness, the model is also analyzed under the alternative refinement from Mailath et al. (1993) in Appendix C of Rocheteau (2009). See Section 5.
Any equilibrium in which there are transfers of trees between households and firms is separating. The logic of the argument goes as follows. Suppose there is a pooling offer such that \( d > 0 \). The ask price of the \( h \)-type household for a tree in terms of the DM good is defined as the minimum quantity of DM output the household would accept in exchange for an additional tree. It is equal to \( \kappa_h / u(y) \), which is larger than the ask price of the \( t \)-type household, \( \kappa_t / u(y) \). The household in the high state has, therefore, the possibility of signaling its state by reducing the transfer of its trees by a small amount, \( \varepsilon > 0 \), and its consumption by a quantity between \( \kappa_t \varepsilon / u(y) \) and \( \kappa_h \varepsilon / u(y) \). Such an offer would raise its payoff relative to the proposed equilibrium, but it would hurt households in the low state. Consequently, the firm should attribute this offer to an \( h \)-type household and should be willing to accept it given that it was willing to accept the pooling offer in the first place. (See Section 3.4 for a graphical illustration of this argument.)

The next lemma shows that among separating PBE, only the Pareto-efficient one survives the Intuitive Criterion.

**Lemma 3.** An optimal offer by a household in the low-value state is

\[
(y_l, d_l, \tau_l) \in \arg \max_{y_l, d_l} (u(y_l) - \kappa_l d_l - \tau_l),
\]

\[
\text{s.t.} \quad -c(y_l) + \kappa_l d_l + \tau_l \geq 0,
\]

\[
0 \leq \tau_l \leq z, \quad 0 \leq d_l \leq a.
\]

An optimal offer by a household in the high-value state is

\[
(y_h, d_h, \tau_h) \in \arg \max_{y_h, d_h} (u(y_h) - \kappa_h d_h - \tau_h),
\]

\[
\text{s.t.} \quad -c(y_h) + \kappa_h d_h + \tau_h \geq 0,
\]

\[
u(y_h) - \kappa_h d_h - \tau_h \leq u(y_h) - c(y_h),
\]

\[
0 \leq \tau_h \leq z, \quad 0 \leq d_h \leq a.
\]

The only way an \( t \)-type household can achieve a higher payoff than the one it would get in a game with complete-information is by making an offer with \( d_t > 0 \), which a firm would attribute to an \( h \)-type household with positive probability, but this has been ruled out by Lemma 2. Hence, households in the low-value state make their complete-information offer (which is always acceptable irrespective of firms' beliefs). The solution to (12)–(14) is

\[
y_l = y^*,
\]

\[
\kappa_l d_l + \tau_l = c(y^*),
\]

if \( \kappa_l a + z \geq c(y^*) \). If \( \kappa_l a + z < c(y^*) \), then

\[
\tau_t = z,
\]

\[
d_t = a,
\]

\[
y_t = c^{-1}(\kappa_l a + z).
\]

The only possible offer an \( h \)-type household can make in equilibrium is the one that maximizes its payoff in the class of all offers that are incentive-compatible and that satisfy the participation constraint of the firm, where the firm has the correct belief that it faces an \( h \)-type household. This offer is called the least-costly separating offer. The proof is by contradiction. Suppose there is a separating equilibrium where the \( h \)-type household makes an offer that is not the solution to (15)–(18). Then, the \( h \)-type household could make an offer arbitrarily close to the least-costly separating offer that would raise its payoff and lower the payoff of an \( t \)-type household. Hence, according to the Intuitive Criterion, such an offer should provide a profitable deviation to \( h \)-type households for reasonable systems of beliefs.

The last step in characterizing an equilibrium is to construct a belief system that generates an acceptance rule for firms that is consistent with the households’ offers in Lemma 3 and that satisfies the Intuitive Criterion. Beliefs regarding equilibrium offers are determined from Bayes’s rule. All out-of-equilibrium offers that would raise the payoff of households in the low state relative to their complete-information payoff are attributed to \( t \)-type households, and all other out-of-equilibrium offers are attributed to \( h \)-type households. By construction, this system of beliefs satisfies the Intuitive Criterion.\(^{17}\) Formally,

\[
\lambda(y, d, \tau) = 0, \quad \forall (y, d, \tau) \notin O \quad \text{s.t.} \quad u(y) - \kappa_t d - \tau > u(y_t) - c(y_t),
\]

\[
\lambda(y, d, \tau) = 1, \quad \forall (y, d, \tau) \notin O \quad \text{s.t.} \quad u(y) - \kappa_h d - \tau \leq u(y_t) - c(y_t),
\]

\(^{17}\) The belief system is not uniquely determined, but the output levels and agents’ payoffs are unique.
where \( O \) is the set of equilibrium offers. Under this belief system, the set of acceptable offers is

\[
A = \{(y,d,t) \in F : u(y) - \kappa_d d - t \leq u(y') - c(y') \quad \text{and} \quad -c(y') + \kappa_h d + t \geq 0\}.
\]

(26)

Feasible offers that violate the incentive-compatibility constraint (17) are attributed to \( \ell \)-type households. Consequently, they violate the firm's participation constraint (13), and they are rejected. All the offers that satisfy \( u(y) - \kappa_d d - t \leq u(y') - c(y') \) are attributed to \( h \)-type households, except \( (y,d,t) \). In order for such offers to be accepted, they must also satisfy (16).

**Proposition 4 (A pecking-order theory of payments).** Consider a match between a household holding a portfolio \((z,a)\) and a firm. There is a solution \((y_h,d_h,\tau_h)\) to (15)-(18), and it has the following properties:

If \( z \geq c(y^*) \), then

\[
y_h = y^*, \quad \tau_h = \kappa_d d_h = c(y^*), \quad d_h = 0.
\]

(27)

If \( z < c(y^*) \), then \( \tau_h = z \) and \((y_h,d_h) \in [0,y^*_1] \times [0,a] \) is the unique solution to:

\[
\kappa_d d_h = c(y_h) - z, \quad u(y_h) - c(y_h) + \left(1 - \frac{\kappa_d}{\kappa_h}\right) [c(y_h) - z] = u(y_t) - c(y_t),
\]

(30)

where \( y_t = \min\{y^*, c^{-1}(z+\kappa_d a)\} \). Moreover, if \( a > 0 \), then \( y_h < y_t \) and \( d_h \in (0,a) \).

Proposition 4 offers a pecking-order theory of payment choices: households with a consumption opportunity finance it with risk-free bonds first, and they use their risky, information-sensitive assets as a last resort. If households hold enough risk-free bonds to buy \( y^* \) \((z \geq c(y^*)\)), then they do not transfer any tree to the firms. In this sense, the risk-free bond is a preferred means of payment. Even when households do not have enough wealth to buy the surplus-maximizing level of output, they choose not to spend all their trees (i.e., they only use a fraction of their holdings of trees as collateral). By retaining a fraction of their trees, households signal the high future value of their assets, and hence they secure better terms of trade.

The partial illiquidity of trees can also be explained by using the more familiar notions of bid and ask prices. Consider a match between a household holding a portfolio \((z,a)\) and a firm. There is a solution \((y_h,d_h,\tau_h)\) to (15)-(18), and it has the following properties:

If \( z \geq c(y^*) \), then

\[
y_h = y^*, \quad \tau_h = \kappa_d d_h = c(y^*), \quad d_h = 0.
\]

(27)

If \( z < c(y^*) \), then \( \tau_h = z \) and \((y_h,d_h) \in [0,y^*_1] \times [0,a] \) is the unique solution to:

\[
\kappa_d d_h = c(y_h) - z, \quad u(y_h) - c(y_h) + \left(1 - \frac{\kappa_d}{\kappa_h}\right) [c(y_h) - z] = u(y_t) - c(y_t),
\]

(30)

where \( y_t = \min\{y^*, c^{-1}(z+\kappa_d a)\} \). Moreover, if \( a > 0 \), then \( y_h < y_t \) and \( d_h \in (0,a) \).

Proposition 4 offers a pecking-order theory of payment choices: households with a consumption opportunity finance it with risk-free bonds first, and they use their risky, information-sensitive assets as a last resort. If households hold enough risk-free bonds to buy \( y^* \) \((z \geq c(y^*)\)), then they do not transfer any tree to the firms. In this sense, the risk-free bond is a preferred means of payment. Even when households do not have enough wealth to buy the surplus-maximizing level of output, they choose not to spend all their trees (i.e., they only use a fraction of their holdings of trees as collateral). By retaining a fraction of their trees, households signal the high future value of their assets, and hence they secure better terms of trade.

3.3. Determinants of asset liquidity

The fraction \( \theta_h = d_h/a \) of its holdings of trees that a household spends in the DM is a function of its portfolio and the process that drives the terminal value of trees. For the functional forms \( u(y) = 2\sqrt{y} \) and \( c(y) = y \) the closed-form solution for \( \theta_h \) is

\[
\theta_h(\kappa_h,z,a) = \left(\frac{\kappa_h}{\kappa_d}\right)^2 \left[1 - \left(\frac{1 - \kappa_d}{\kappa_h}\right) \frac{2\sqrt{y_t} - y_t}{(1 - \kappa_d/a)}\right]^2 - z
\]

if \( z \leq y^* \),

(32)

\[
\theta_h(\kappa_h,z,a) = 0 \quad \text{otherwise},
\]

(33)

where \( y_t = \min(1,\kappa_d a + z) \) and \( \kappa_t = (1 - \pi_h \kappa_h)/\pi_t \). This expression points to the differences between the approach in this paper and the approaches of Kiyotaki and Moore (2005) and Lagos (2010a). In Kiyotaki and Moore (2005), agents can only sell a fraction, \( \theta \in (0,1) \), of their illiquid asset (capital) to raise funds; in Lagos (2010a), agents can use their illiquid asset (Lucas trees) in a fraction, \( \theta \), of the matches. In both cases, the parameter \( \theta \) is exogenous. In contrast, in this model households spend

---

18 The term “pecking order” was coined by Myers (1984, p. 581). It describes the predictions of models of capital structure choices under private information. According to the pecking-order theory, firms with an investment opportunity prefer internal finance (nondistributed dividends). If external finance is required, then they issue the safest security first, and they use equity as a last resort.

19 This result is reminiscent of some of the findings of the liquidity-based model of security design from DeMarzo and Duffie (1999). They consider the problem faced by a firm that needs to raise funds by issuing a security backed by real assets. The issuer has private information regarding the distribution of cash flows of the underlying assets. Using the Intuitive Criterion, they show that a signaling equilibrium exists in which the seller receives a high price for the security by retaining some fraction of the issue.
a fraction, \( \theta_h \), of their holdings of trees when their terminal value is high, where \( \theta_h \) is a function of the intrinsic characteristics of the asset \((\kappa_t, \kappa_h)\) and the composition of the portfolio held by the household \((z\) and \(a))\). Hence, the liquidity of the trees depends on their intrinsic characteristics, as well as policy, as captured by the supply of risk-free bonds.

**Proposition 5 (Asset liquidity and riskiness).** Assume \( z < c(y^*) \) and \( a > 0 \). Then:

\[
\frac{d\theta_h}{dk_h}\bigg|_{p_hK_h+\kappa_tK_t=1} < 0. 
\]  

(34)

The liquidity of trees decreases as the spread of the distribution of their terminal values increases. To understand this result, notice from (17) that, in order to separate themselves from \( t \)-type households, \( h \)-type households incur a signaling cost – the difference between the household’s surplus in the low state and the household’s surplus in the high state – equal to \((\kappa_h-\kappa_t)d_h > 0\). As \( \kappa_t \) gets closer to \( \kappa_h \), this signaling cost decreases, and the incentive-compatibility constraint is relaxed, which improves the liquidity of trees in the high state. Conversely, as \( \kappa_h-\kappa_t \) increases, the informational asymmetries become more severe, which makes the incentive-compatibility condition more binding.

**Proposition 6 (Market breakdowns).** Assume \( z < c(y^*) \) and \( a > 0 \). As \( \kappa_t \) tends to 0, then \( \theta_h \) approaches 0.

In the case where \( \kappa_t \) approaches 0, the adverse-selection problem is so severe that trees cease to be traded, and bonds become the only means of payment.\(^{20}\) There is a drying-up of liquidity in the market for trees when they become valueless in the worst state. The proof of this result goes as follows. From Lemma 2, an equilibrium of the bargaining game cannot be pooling, since, otherwise, \( h \)-type households would have a profitable deviation to signal the quality of their assets. If a tree in the low state is worthless, this immediately implies that \( t \)-type households cannot sell their trees at any positive price. But incentive-compatibility also implies that \( h \)-type households cannot sell their own trees.

**Proposition 7 (Payments and portfolio composition).** If \( z < c(y^*) \) and \( a > 0 \), then

\[
\frac{\partial (\kappa_h d_h)}{\partial z} = \frac{u'(y_h)/c'(y_h) - u'(y_h)/c'(y_h)}{u'(y_h)/c'(y_h) - \kappa_t/\kappa_h} < 0. 
\]  

(35)

If \( \kappa_t a + z < c(y^*) \), then

\[
\frac{\partial d_h}{\partial a} = \frac{\kappa_h u'(y_h)/c'(y_h) - 1}{\kappa_t u'(y_h)/c'(y_h) - 1} \in (0, 1). 
\]  

(36)

As the household accumulates a larger quantity of risk-free bonds, its use of trees (expressed in CM output) as means of payments (or collateral) decreases. An \( h \)-type household reduces its signaling cost by substituting information-insensitive bonds for information-sensitive trees, thereby relaxing the incentive-compatibility constraint (17). This dependence of \( \theta_h \) on \( z \) offers a channel through which the supply of bonds affects the liquidity of trees.

According to (36), the marginal propensity of a household to spend its trees in the high state is less than one. Provided that \( \kappa_t a + z < c(y^*) \), an additional tree raises the surplus that the household can obtain in the low state. As a consequence, the household in the high state that receives an additional tree can spend a fraction of it without giving \( t \)-type households incentives to imitate its offer. If \( \kappa_t a + z > c(y^*) \), then \( y_t = y^* \) and \( \partial d_h/\partial a = 0 \). In this case, the liquidity needs in the low state are satiated and, as a result, an additional tree does not affect the incentive-compatibility constraint, and hence the terms of trade, in the high state.

### 3.4. A benchmark

In the following, I describe an economy with no risk-free bonds, \( z=0 \). This special case lends itself to simple graphical representations of the results in Lemmas 2 and 3 and Proposition 4. In Fig. 2, the participation constraint of a firm that believes it is facing an \( h \)-type (respectively, \( t \)-type) household is represented by the frontier \( U^h_t \equiv (y,d): -c(y) + \kappa_h d = 0 \) (respectively, \( U^t_t \equiv (y,d): -c(y) + \kappa_t d = 0 \)). (The superscript \( s \) stands for seller.) The Intuitive Criterion selects a unique equilibrium among multiple PBE. As shown in Lemma 2, there is no equilibrium of the bargaining game with a pooling offer. The proof is illustrated in the left panel of Fig. 2. Consider an equilibrium with a pooling offer \((\bar{y},\bar{d})\) with \( \bar{d} > 0 \).

---

\(^{20}\) Strictly speaking, the \( t \)-type households can still use trees in payments, but because \( \kappa_t \) tends to 0, the amount of output they buy with them approaches 0. Also, a well-known property of the equilibrium selected by the Intuitive Criterion is that the outcome is independent of the distribution of types \((n_a, n_t)\), which can make the adverse-selection problem look very severe when the occurrence of the low state is infrequent. Rocheteau (2009, Appendix C) checks the robustness of the result to the notion of undefeated equilibrium proposed by Mailath et al. (1993). If \( z \in [c(y^*), c(y^*)] \), where \( y \) is the solution to \( u'(y) = \kappa_h c'(y) \), then the unique undefeated equilibrium corresponds to the one selected by the Intuitive Criterion.
In order to separate itself, an (The superscript \(b\) firm, is at the intersection of only one satisfies the Intuitive Criterion, the one that maximizes the utility of \(h\) in the low state. Despite this inefficiently low consumption, households retain a fraction of their asset holdings, makes the least-costly separating offer. See the right panel.

Fig. 2. Pooling vs separating equilibria. The equilibrium of the bargaining game cannot be pooling since otherwise one could find offers that violate the Intuitive Criterion. See left panel. The equilibrium is separating. The \(l\)-type household makes the complete-information offer and the \(h\)-type household makes the least-costly separating offer. See the right panel.

The surpluses of the two types of households at the proposed equilibrium are denoted \(U^b_h \equiv u(y) - k_u d\) and \(U^b_l \equiv u(y) - k_l d\). (The superscript \(b\) stands for buyer.) The indifference curves \(U^b_h\) and \(U^b_l\) in Fig. 2 exhibit a single-crossing property, which is key to obtaining a separating equilibrium. The offer \((y, d)\) is located above \(U^b_h\) since it is accepted when \(\lambda < 1\). The shaded area indicates the set of offers that raise the utility of an \(h\)-type household (offers to the right of \(U^b_h\)), but reduce the utility of an \(l\)-type household (offers to the left of \(U^b_l\)), and are acceptable by firms provided that \(\lambda = 1\) (offers above \(U^b_h\)). These offers satisfy \((9)-(11)\) with \(\chi = h\), so that the proposed equilibrium with a pooling offer \((y, d)\) violates the Intuitive Criterion.

In order to separate itself, an \(h\)-type household reduces its DM consumption, \(\Delta y < 0\), as well as its transfer of assets to the firm, \(\Delta d < 0\). Provided that \(\Delta d/\Delta y\) is between the slopes of \(U^b_h\) and \(U^b_l\), \(u(y)/k_u\) and \(u(y)/k_h\), respectively, an \(h\)-type household would gain from such an offer while an \(l\)-type household would be made worse off. Among the separating PBE, only one satisfies the Intuitive Criterion, the one that maximizes the utility of \(h\)-type households, and the associated offer is at the intersection of \(U^b_h\) and \(U^b_l\). See the right panel of Fig. 2.

The last component of an equilibrium is a system of beliefs, \(\lambda\), and the associated acceptance rule for firms, \(A\). From \((24)\) to \((25)\) firms attribute all offers to the right of \(U^b_h\) to \(l\)-type households. See the shaded area in the right panel of Fig. 2. Such offers are also located to the right of \(U^b_l\), and therefore they are rejected. Offers to the left of \(U^b_l\) are attributed to \(h\)-type households. Among these offers, only the ones above \(U^b_h\) are acceptable. The acceptance rule, \(A\), is represented by a striped area in the right panel of Fig. 2. It shows clearly that there are offers with a larger transfer of trees than the one made in equilibrium, which are rejected even though they would raise the utility of \(h\)-type households and would be acceptable to firms under optimistic beliefs about the quality of the asset—such offers are located above \(d_h\) and between \(U^b_h\) and \(U^b_l\).

As shown in the figure, and proved in Proposition 4, \(y_h < y_r < y^*\). Households always consume less in the high state than in the low state. Despite this inefficiently low consumption, households retain a fraction of their asset holdings, \(d_h < d_r < a\) (Proposition 4).

4. Asset prices and liquidity

This section incorporates the bargaining game studied in Section 3 into the general equilibrium structure described in Section 2 in order to determine the conditions under which the price of each asset exhibits a liquidity premium, and the structure of asset returns.

The sequence of events is as follows. Households make a portfolio choice in the CM. At the beginning of the subsequent period, households receive a private and fully informative signal about the terminal value of their trees. Then, households get matched with firms. An implication of this timing is that the household’s portfolio does not convey any information about its private information. From Proposition 4, the terms of trade, \(\gamma(y, a, k), \tau(z, a, k), d(z, a, k)\), are functions of the household’s portfolio and its private signal.\(^{21}\)

4.1. Portfolio choices and asset prices

The missing element of the model so far is the determination of households’ portfolio choices. These choices depend on the benefits that a household expects to receive from holding assets in the DM. The expected lifetime utility of a household entering the DM with \(z\) units of bonds, \(a\) trees, and a private signal \(k\), is

\[
V(z, a, k) = u(y(z, a, k)) + W[z - \tau(z, a, k), a - d(z, a, k), k].
\]

\(^{21}\) The solutions to \((12)-(14)\) and \((15)-(18)\) might not be unique, e.g., if \(z > c(y^*)\), but agents’ surpluses are unique.
Using the linearity of $W$, (37) becomes

$$V(z,a,k_r) = S'(z,a) + z + k_r a + W(0,0,k_r), \quad \zeta \in \{l,h\},$$

where $S'(z,a)$ is the household’s surplus in the DM when the type of the trees is $\zeta$, i.e.,

$$S'(z,a) \equiv u(y(z,a,k_r)) - k_r d(z,a,k_r) - \tau(z,a,k_r) \quad \text{for} \quad \zeta \in \{l,h\}.$$

Substituting $V$ by its expression given by (38) into (4), the household’s portfolio problem reduces to

$$[z(j),a(j)] \in \arg \max_{(z,a) \in \mathbb{R}_+^2} \left\{ -\left( \frac{q_z - \beta}{\beta} \right) z - \left( \frac{q_a - \beta}{\beta} \right) a + \pi_b S_h(z,a) + \pi_t S'(z,a) \right\}, \quad \forall j \in \mathcal{H},$$

where $q_z/\beta - 1$ is the cost of holding bonds, $q_a/\beta - 1$ is the cost of investing in trees, and $\mathcal{H}$ is the set of households. The cost of holding an asset is approximately equal to the difference between the price of the asset and its fundamental value, $\beta$. According to (40), households choose their portfolios in order to maximize their expected surplus in the DM, net of the cost of holding the assets.

Finally, the clearing of the asset market implies

$$\int_{j \in \mathcal{H}} a(j) \, dj = A,$$

$$\int_{j \in \mathcal{H}} z(j) \, dj = Z.$$

**Definition 8.** An equilibrium is a list of portfolios, terms of trade in the DM, and the prices of trees and bonds, $(z(j),a(j),y(j),d(j),\tau(z,a,k_r))$, such that $[z(j),a(j)]$ is solution to (40) for all $j \in \mathcal{H}$; For all $(z,a) \in \mathbb{R}_+^2$, $[y(z,a,k_r),d(z,a,k_r),\tau(z,a,k_r)]$ is a solution to (12)–(14) if $k_r = k_l$ and to (15)–(18) if $k_r = k_h$; $(q_a,q_z)$ solves (41) and (42).

The characterization of the set of equilibria is done in two steps. The first step provides necessary and sufficient conditions for an optimal portfolio (Lemma 9). The second step uses the market-clearing conditions, (41) and (42), to determine asset prices and the DM allocations (Proposition 10). In order to characterize the household’s choice of asset holdings, let $S'_h$ and $S'_l$ denote the partial derivatives of the household’s surplus function for $\zeta \in \{l,h\}$. They represent the transactional benefits to a household that bonds and trees provide at the margin in the DM in the state $\zeta$.

**Lemma 9 (Households’ portfolio choices).** If $q_z \geq \beta$ and $q_a \geq \beta$, then $(z,a)$ is a solution to the household’s portfolio problem, (40), if and only if

$$-\frac{q_z - \beta}{\beta} + \pi_h S'_h(z,a) + \pi_t S'_l(z,a) \leq 0 \quad " \quad \text{if} \quad z > 0,$$

$$-\frac{q_a - \beta}{\beta} + \pi_h S'_h(z,a) + \pi_t S'_l(z,a) \leq 0 \quad " \quad \text{if} \quad a > 0,$$

where

$$S'_h = \frac{S'_h}{k_l} = \frac{u'(y_l)}{c'(y_l)} - 1,$$

$$S'_l = \frac{u'(y_h)}{c'(y_h)} - 1 \left[ \frac{u'(y_l)/c'(y_l) - k_l/k_h}{u'(y_h)/c'(y_h) - k_l/k_h} \right],$$

$$S'_h = k_h \left[ \frac{u'(y_h)}{c'(y_h)} - 1 \right]^2 \left[ \frac{k_l}{k_h} \right].$$

If $q_z > \beta$ and $q_a > \beta$, then $(z,a)$ is unique.

If $q_z = \beta$, then $z \geq c(y^*)$.

If $q_a = \beta$, then $z + k_l a \geq c(y^*)$.

From (43) and (44), for an asset to be held, its cost must be equal to the expected marginal benefit that the asset confers in the DM. According to (45), a marginal unit of the risk-free bond allows an $l$-type household to purchase $1/c'(y_l)$ units of DM output; this additional output is valued according to the marginal surplus of the match, $u'(y_l) - c'(y_l)$. The first term in brackets on the right side of (46) has a similar interpretation for the high state. This term is multiplied by $1 + \delta(k_l a)/\partial z < 1$ because a household that accumulates one additional unit of the bond can cut down on its transfer of trees in order to reduce its signaling cost. Similarly, the first two terms on the right side of (47) correspond to the liquidity value of trees in the high state if households and firms are symmetrically informed. This liquidity component is multiplied by the marginal propensity to spend trees, $\delta d_h/\delta a \in [0,1)$, which is less than one in the private-information economy.
Proposition 10 (Equilibrium allocations and prices). An equilibrium exists, and it is such that \((q_a,q_z,y_i,y_h)\) is uniquely determined. Asset prices are

\[
q_z = \beta (1 + \mathcal{L}_z),
\]

\[
q_a = \beta (1 + \mathcal{L}_a),
\]

with

\[
\mathcal{L}_z = \pi_i \left[ \frac{u'(y_i)}{c'(y_i)} - 1 \right] + \pi_h \left[ \frac{u'(y_h)}{c'(y_h)} - 1 \right] \left[ \frac{k_h u'(y_i)}{c'(y_i)} - k_i \right],
\]

\[
\mathcal{L}_a = \pi_i k_i \left[ \frac{u'(y_i)}{c'(y_i)} - 1 \right] + \pi_h k_h \left[ \frac{u'(y_h)}{c'(y_h)} - 1 \right] \left[ k_i u'(y_i)/c'(y_i) - k_i \right],
\]

where \(y_i = \min\{y^a,c^{-1}(Z + \kappa_i A)\}\), and \(y_h\) solves \((31)\), with \(z = Z\).

An equilibrium exists, and it is essentially unique. The price of each asset is composed of its fundamental value, \(\beta\), times a liquidity premium.

4.2. Liquidity premia and asset returns

The next proposition determines the condition under which the liquidity premia, \(\mathcal{L}_z\) and \(\mathcal{L}_a\), are positive. Let \(r_z = 1/q_z\) denote the (gross) rate of return of risk-free bonds and \(r_a = 1/q_a\) the (gross) rate of return of information-sensitive trees.

Proposition 11 (Liquidity premia). \(\mathcal{L}_z > 0\) and \(r_z < \beta^{-1}\) if and only if \(Z < Z^* \equiv c(y^a)\).

\(\mathcal{L}_a > 0\) and \(r_a < \beta^{-1}\) if and only if \(Z < Z \equiv c(y^a) - \kappa_i A\).

The rate of return of risk-free bonds is below the rate of time preference whenever the supply of bonds, \(Z\), is too low relative to the liquidity needs of the economy, as measured by \(c(y^a)\). This low rate of return does not account for the liquidity services that bonds provide in the DM by reducing the signaling costs incurred by \(h\)-type households to reveal the terminal value of their trees. Information-sensitive trees can also provide liquidity services if households do not hold enough wealth to maximize the total surplus in \(l\)-type matches, \(Z + \kappa_l A < c(y^a)\). Finally, when \(Z \in (Z,Z^*)\), \(l\)-type households have enough wealth to purchase \(y^a\), while the \(h\)-type households consume \(y < y^a\). In this case trees do not pay a liquidity premium because \(l\)-type households do not wish to buy additional output in the DM, and \(h\)-type households cannot spend more than \(d_h < A\) because of a binding resalability constraint. Bonds pay a liquidity premium because they relax the liquidity constraint faced by \(h\)-type households.

Proposition 12 (Liquidity structure of asset returns). If \(Z < Z^*\), then \(\mathcal{L}_z > \mathcal{L}_a \geq 0\) and \(\beta^{-1} > r_a > r_z\).

The liquidity value of bonds is greater than the liquidity value of trees, so the rate-of-return differential between bonds and trees is positive. This finding addresses the rate-of-return dominance puzzle, according to which individuals hold monetary assets despite those assets being dominated in their rate of return by other assets. In my model, households hold some assets that are dominated in their rate of return because such assets are less sensitive to private-information problems and hence they provide greater liquidity services in the DM.

Fig. 3 represents the conditions on \(Z\) and \(A\) under which the liquidity factors \(\mathcal{L}_z\) and \(\mathcal{L}_a\) are positive for both the complete-information and the private-information economies. In the complete-information economy (right panel of Fig. 3) all asset prices exhibit a liquidity component, or none of them does. Provided that there is enough wealth in the economy, \(\kappa_i A + Z \geq c(y^a)\), assets are priced according to their fundamental values. In contrast, in the private-information economy (left panel of Fig. 3) bonds can be priced above their fundamental value even if the total wealth in the economy is very large. Consequently, a liquidity differential between trees and bonds (the gray areas in Fig. 3) is more likely to exist in the presence of an informational asymmetry.

The next proposition examines how the supply of bonds affects liquidity premia and assets’ rates of return.

Proposition 13 (Liquidity premia and the supply of risk-free bonds). If \(Z < Z^*\), then \(d\mathcal{L}_z/dZ < 0\) and \(dr_a/dZ > 0\).

As the supply of bonds increases, liquidity premia fall, while the rate-of-return differential between assets narrows. If households hold a larger quantity of bonds, then they can trade larger quantities in the DM, irrespective of their private information, and the marginal benefit from holding an additional unit of any asset is reduced. This result is consistent with the negative relationship found by Krishnamurthy and Vissing-Jorgensen (2008) between the convenience yield of Treasury securities and the supply of treasuries.

---

22 Any indeterminacy, such as the composition of the payments in terms of bonds and trees in the low state when \(Z + \kappa_i A > c(y^a)\), is payoff irrelevant.
Proposition 14 (Open-market purchases of information-sensitive assets). Assume $Z > z^*$. A policy that consists in issuing $A$ units of risk-free bonds in order to substitute them for the information-sensitive assets is welfare improving.

An obvious remedy to the private-information problem consists of replacing information-sensitive ones. Since households hold $A + Z$ units of risk-free bonds, the quantity of DM output they consume is at least as large as what they would consume in an $\ell$-type match in the presence of $Z$ units of risk-free bonds and $A$ units of information-sensitive assets (since $A + Z > \kappa_A Z$). Moreover, from Proposition 4, the output in $h$-type matches is less than the output in $\ell$-type matches if $Z < Z^*$. Therefore, by taking the information-sensitive assets out of circulation early enough that the private-information problem does not materialize, and by replacing them with risk-free bonds, the government raises the quantities traded in all matches, and hence social welfare. Moreover, from Proposition 12, $q_i \geq q_o$, so that the government raises enough revenue from the sale of its bonds to purchase all the information-sensitive assets, and the output generated by the trees in the following period covers the repayment of the bonds.

Proposition 15 (Optimal provision of risk-free bonds). For all $Z \geq Z^*$, $y^* = y^h = y^*$ and $r_Z = r_A = \beta^{-1}$.

The supply of risk-free bonds is optimal for all $Z \geq Z^*$. In this case, the quantities traded in the DM maximize the match surpluses, $y^* = y^h = y^*$. In the high state, households trade only bonds ($d_h = 0$), while in the low state households are indifferent between using bonds and trees to finance their consumption. The prices of the two assets are equal to their fundamental values ($q_i = q_o = \beta$). This condition for the optimal provision of bonds is reminiscent of the Friedman rule for the optimum quantity of money.\(^{23}\) The optimal quantity of monetary assets is such that the demand for liquidity services is satiated, i.e., the liquidity premia are zero, and all assets exhibit the same (risk-adjusted) rate of return.

5. Sensitivity analysis

The role played by two assumptions is succinctly reviewed: the use of the Cho-Kreps refinement; the assumption that the terms of trade are set by households.

5.1. An alternative refinement

Mailath et al. (1993) proposed an alternative to the Intuitive Criterion called the undefeated equilibrium. Rocheteau (2009) shows that the (separating) equilibrium of the bargaining game that satisfies the Intuitive Criterion is the only undefeated equilibrium if the household’s payoff at the Pareto-efficient separating equilibrium is greater than the one the household would enjoy at its preferred pooling equilibrium, i.e.,

$$U_b^h = u(q^p) - \kappa_A d^p - \tau^p < U_b^h = u(q_h) - \kappa_A d_h - \tau_h,$$

where $(q_h, d_h, \tau_h)$ is the solution to (15)–(18) and

$$(q^p, d^p, \tau^p) = \arg\max_{q, d, \tau} u(q) - \kappa_A d - \tau,$$

\(^{23}\) Rocheteau (2008) considers a version of the model with fiat money in which the optimum quantity of money requires the money growth rate to approach agents’ discount factor.
\begin{align}
&\text{s.t. } -c(q) + (\pi_h \kappa_h + \pi_r \kappa_r) d + \tau \geq 0, \\
&u(q) - \kappa_i d - \tau \geq u(q_i) - c(q_i), \\
&(d, \tau) \in [0, a] \times [0, z],
\end{align}

where \( q_i = \min[q^*, c^{-1}(\kappa_i a + z)] \). If \( U_h^b > U_h^p \) then there is an undefeated equilibrium and it is pooling. If \( U_h^b = U_h^p \) then there is both a pooling and a separating undefeated equilibrium. It can be shown that for all matches where \( a > 0 \) and \( z \in [c(\bar{q}), c(q^*)] \), where \( \bar{q} \) is the solution to \( u'(q) = (\kappa_h / \kappa_c) u''(q) \), the unique undefeated equilibrium of the bargaining game is separating. This result suggests that the separating outcome predicted by the Intuitive Criterion is consistent with other refinements for signaling games, provided that the supply of bonds is not too small.

### 5.2. Signaling vs screening

Instead of households making take-it-or-leave-it offers in pairwise meetings, suppose that firms are the ones to make offers.\(^{24}\) Firms choose a menu of contracts, \((y_h, d_h, \tau_h), (y_i, d_i, \tau_i)\), to maximize their expected profits subject to incentive-compatibility and individual rationality constraints. It is shown in an appendix that an optimal menu is such that the participation constraint of a household in the high state and the incentive-compatibility constraint of a household in the low state are binding, i.e.,

\begin{align}
&u(y_h) - \tau_h - \kappa_h d_h = 0, \\
&u(y_i) - \tau_i - \kappa_i d_i = u(y_h) - \tau_h - \kappa_h d_h - (\kappa_h - \kappa_i) d_h.
\end{align}

According to (57), firms leave no surplus to households in the high state, whereas, according to (58), households in the low state can extract a surplus proportional to the transfer of trees in the high state, \((\kappa_h - \kappa_i) d_h\). Moreover, households in the high state transfer fewer trees than households in the low state, \(d_h \leq d_i\). It can also be shown that the equilibrium of the screening game exhibits a pecking-order property: if the household holds enough bonds, then the firm asks to be paid with bonds only; if the quantity of bonds held by the household is small, then the firm will ask for all of the bonds of the household and some of its trees. The payment in trees decreases with the quantity of bonds held by the household. Since the household’s surplus in the DM is equal to \((\kappa_h - \kappa_i) d_h\), and \(d_h\) is decreasing with \(z\), households have no incentive to hold bonds. Suppose, for instance, that households do not have to bring their bonds in bilateral matches in the DM: they can keep them at home. Then, bonds are not used as means of payment in the DM, i.e., trees are the only means of payment, and therefore bonds have no liquidity value. This result is analogous to the nonexistence of a monetary equilibrium in economies where firms set the terms of trade unilaterally.\(^{25}\)

### 6. Conclusion

A model has been proposed where multiple assets are traded in pairwise meetings, no restrictions are placed on the transfer of goods, assets, or information, and assets differ in terms of their information sensitivity. This simple model has delivered new insights for payment arrangements, asset liquidity, and the distribution of asset returns.

In the model, an asset’s sensitivity to informational asymmetries reduces its usefulness as a means of payment or collateral. This description is consistent with several historical episodes, including the circulation of coins in medieval Europe or the coexistence of numerous banknotes in the 19th century United States. Some predictions of the model are also relevant to interpreting the drying-up of liquidity during the financial crisis of 2007–2008. For instance, the model predicts that the market for information-sensitive assets breaks down when some of these assets are valueless. The model also showed that there is an optimal provision of risk-free assets that can overcome the illiquidity of information-sensitive assets.

The findings in terms of liquidity premia are consistent with the evidence from Krishnamurthy and Vissing-Jorgensen (2008) regarding the convenience yield of Treasury securities relative to corporate bonds. Half of the convenience yield of Treasury securities relative to corporate bonds can be explained by a surety motive, where surety is the “value investors place on a sure cash-flow above and beyond what would be implied by the pricing kernel”. To get an idea of the quantitative importance of this convenience yield, Krishnamurthy and Vissing-Jorgensen value the liquidity services provided by the current level of treasuries at about 0.95% of GDP per year.

From a methodological viewpoint, my approach can be viewed as providing tractable microfoundations for some of the trading restrictions that have been imposed in recent monetary models. A natural next step is to construct a calibrated

---

\(^{24}\) Ennis (2008) studies a related trading mechanism in a model with fiat money where buyers have some private information about their marginal utility of consumption.

\(^{25}\) In order to allow households to capture some of the surplus that their recognizable assets generate, one could adopt the notion of competitive search, according to which firms post contracts and compete in order to attract households. See Guerrieri et al. (2010) for an application where agents trade a single indivisible asset available in two different qualities. As in this model, the contract is separating, and the low-quality asset has a higher velocity than the high-quality one.
version of the model incorporating more realistic features, such as risk aversion, infinitely lived assets, and a richer
information structure.

Acknowledgments

This paper has circulated under the title “A monetary approach to asset liquidity.” It has benefited from discussions
with Ricardo Lagos and Pierre-Olivier Weill. I also thank an anonymous referee for her/his comments and suggestions. I
also thank for their comments and suggestions of anonymous referees, Murali Agastya, Aditya Goenka, Steve LeRoy, Yiting
Li, Ed Nosal, Peter Rupert, Neil Wallace, Asher Wolinsky, Tao Zhu, and seminar participants at the Federal Reserve Bank of
Cleveland, Federal Reserve Bank of Chicago, Hong Kong University of Science and Technology, National Taiwan University,
National University of Singapore, Rice University, Singapore Management University, the Southern Workshop in
Macroeconomics (Auckland), the University of California at Irvine, the University of California at San Diego, the University
of Missouri, the University of Southern California, the University of Tokyo, the University of Wisconsin, the workshop on
“Networks and Payments” at the University of California at Santa Barbara, and the 2008 meeting of the Society of
Economic Dynamics in Cambridge. I thank Monica Crabtree-Reusser for editorial assistance.

Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jmoneco.2011.06.005.

References