CHAPTER 22
OPTIONS AND CORPORATE FINANCE: *BASIC CONCEPTS*

Answers to Concept Questions

1. A call option confers the right, without the obligation, to buy an asset at a given price on or before a given date. A put option confers the right, without the obligation, to sell an asset at a given price on or before a given date. You would buy a call option if you expect the price of the asset to increase. You would buy a put option if you expect the price of the asset to decrease. A call option has unlimited potential profit, while a put option has limited potential profit; the underlying asset's price cannot be less than zero.

2. *a.* The buyer of a call option pays money for the right to buy....
b. The buyer of a put option pays money for the right to sell....
c. The seller of a call option receives money for the obligation to sell....
d. The seller of a put option receives money for the obligation to buy....

3. An American option can be exercised on any date up to and including the expiration date. A European option can only be exercised on the expiration date. Since an American option gives its owner the right to exercise on any date up to and including the expiration date, it must be worth at least as much as a European option, if not more.

4. The intrinsic value of a call is Max[S – E, 0]. The intrinsic value of a put is Max[E – S, 0]. The intrinsic value of an option is the value at expiration.

5. The call is selling for less than its intrinsic value; an arbitrage opportunity exists. Buy the call for $10, exercise the call by paying $35 in return for a share of stock, and sell the stock for $50. You've made a riskless $5 profit.

6. The prices of both the call and the put option should increase. The higher level of downside risk still results in an option price of zero, but the upside potential is greater since there is a higher probability that the asset will finish in the money.

7. False. The value of a call option depends on the total variance of the underlying asset, not just the systematic variance.

8. The call option will sell for more since it provides an unlimited profit opportunity, while the potential profit from the put is limited (the stock price cannot fall below zero).

9. The value of a call option will increase, and the value of a put option will decrease.

10. The reason they don't show up is that the U.S. government uses cash accounting; i.e., only actual cash inflows and outflows are counted, not contingent cash flows. From a political perspective, they would make the deficit larger, so that is another reason not to count them! Whether they should be included depends on whether we feel cash accounting is appropriate or not, but these contingent liabilities should be measured and reported. They currently are not, at least not in a systematic fashion.

11. Increasing the time to expiration increases the value of an option. The reason is that the option gives the holder the right to buy or sell. The longer the holder has that right, the more time there is for the option to increase (or decrease in the case of a put) in value. For example, imagine an out-of-the-money option that is about to expire. Because the option is essentially worthless, increasing the time to expiration would obviously increase its value.

12. An increase in volatility acts to increase both call and put values because the greater volatility increases the possibility of favorable in-the-money payoffs.

13. A put option is insurance since it guarantees the policyholder will be able to sell the asset for a specific price. Consider homeowners insurance. If a house burns down, it is essentially worthless. In essence, the homeowner is selling the worthless house to the insurance company for the amount of insurance.

14. The equityholders of a firm financed partially with debt can be thought as holding a call option on the assets of the firm with a strike price equal to the debt's face value and a time to expiration equal to the debt's time to maturity. If the value of the firm exceeds the face value of the debt when it matures, the firm will pay off the debtholders in full, leaving the equityholders with the firm's remaining assets. However, if the value of the firm is less than the face value of debt when it matures, the firm must liquidate all of its assets in order to pay off the debtholders, and the equityholders receive nothing. Consider the following:

Let VL = the value of a firm financed with both debt and equity
FV(debt) = the face value of the firm's outstanding debt at maturity

	If V_L < FV(debt)	If V_L > FV(debt)
Payoff to debtholders	V_L	FV(debt)
Payoff to equityholders	0	V_L – FV(debt)
	V_L	V_L

Notice that the payoff to equityholders is identical to a call option of the form Max(0, S_T – K), where the stock price at expiration (S_T) is equal to the value of the firm at the time of the debt's maturity and the strike price (K) is equal to the face value of outstanding debt.

15. Since you have a large number of stock options in the company, you have an incentive to accept the second project, which will increase the overall risk of the company and reduce the value of the firm's debt. However, accepting the risky project will increase your wealth, as the options are more valuable when the risk of the firm increases.

16. Rearranging the put-call parity formula, we get: S – PV(E) = C – P. Since we know that the stock price and exercise price are the same, assuming a positive interest rate, the left hand side of the equation must be greater than zero. This implies the price of the call must be higher than the price of the put in this situation.

17. Rearranging the put-call parity formula, we get: S – PV(E) = C – P. If the call and the put have the same price, we know C – P = 0. This must mean the stock price is equal to the present value of the exercise price, so the put is in-the-money.

18. A stock can be replicated using a long call (to capture the upside gains), a short put (to reflect the downside losses) and a T-bill (to reflect the time value component – the "wait" factor).

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. *a.* The value of the call is the stock price minus the present value of the exercise price, so:

C_0 = €55 – [€45/1.06] = €12.55

The intrinsic value is the amount by which the stock price exceeds the exercise price of the call, so the intrinsic value is €10.

b. The value of the call is the stock price minus the present value of the exercise price, so:

C_0 = €55 – [€35/1.06] = €21.98

The intrinsic value is the amount by which the stock price exceeds the exercise price of the call, so the intrinsic value is €20.

c. The value of the put option is €0 since there is no possibility that the put will finish in the money. The intrinsic value is also €0.

2. *a.* The calls are in the money. The intrinsic value of the calls is $3.

b. The puts are out of the money. The intrinsic value of the puts is $0.

c. The Mar call and the Oct put are mispriced. The call is mispriced because it is selling for less than its intrinsic value. If the option expired today, the arbitrage strategy would be to buy the call for $2.80, exercise it and pay $80 for a share of stock, and sell the stock for $83. A riskless profit of $0.20 results. The October put is mispriced because it sells for less than the July put. To take advantage of this, sell the July put for $3.90 and buy the October put for $3.65, for a cash inflow of $0.25. The exposure of the short position is completely covered by the long position in the October put, with a positive cash inflow today.

3. *a.* Each contract is for 100 shares, so the total cost is:

Cost = 10(100 shares/contract)(Rs.7.60)
Cost = Rs.7,600

b. If the stock price at expiration is Rs.140, the payoff is:

Payoff = 10(100)(Rs.140 – 110)
Payoff = Rs.30,000

If the stock price at expiration is Rs.125, the payoff is:

Payoff = 10(100)(Rs.125 – 110)
Payoff = Rs.15,000

c. Remembering that each contract is for 100 shares of stock, the cost is:

Cost = 10(100)(Rs.4.70)
Cost = Rs.4,700

The maximum gain on the put option would occur if the stock price goes to Rs.0. We also need to subtract the initial cost, so:

Maximum gain = 10(100)(Rs.110) – Rs.4,700
Maximum gain = Rs.105,300

If the stock price at expiration is Rs.104, the position will have a profit of:

Profit = 10(100)(Rs.110 – 104) – Rs.4,700
Profit = Rs.1,300

d. At a stock price of Rs.103 the put is in the money. As the writer, you will make:

Net loss = Rs.4,700 – 10(100)(Rs.110 – 103)
Net loss = –Rs.2,300

At a stock price of Rs.132 the put is out of the money, so the writer will make the initial cost:

Net gain = Rs.4,700

At the breakeven, you would recover the initial cost of Rs.4,700, so:

Rs.4,700 = 10(100)(Rs.110 – S_T)
S_T = Rs.105.30

For terminal stock prices above Rs.105.30, the writer of the put option makes a net profit (ignoring transaction costs and the effects of the time value of money).

4. *a.* The value of the call is the stock price minus the present value of the exercise price, so:

C_0 = 元 80 – 70/1.06

C_0 = 元 13.96

b. Using the equation presented in the text to prevent arbitrage, we find the value of the call is:

元 80 = [(元 95 – 75)/(元 95 – 90)]C_0 + 元 75/1.06
C_0 = 元 2.31

5. *a.* The value of the call is the stock price minus the present value of the exercise price, so:

C_0 = \$70 – \$45/1.05
C_0 = \$27.14

b. Using the equation presented in the text to prevent arbitrage, we find the value of the call is:

\$70 = 2C_0 + \$60/1.05
C_0 = \$6.43

6. Using put-call parity and solving for the put price, we get:

¥7,000 + P = ¥7,500$e^{-(.026)(3/12)}$ + ¥450
P = ¥901.41

7. Using put-call parity and solving for the call price we get:

¥5,300 + ¥489 = ¥5,000$e^{-(.036)(.5)}$ + C
C = ¥878.19

8. Using put-call parity and solving for the stock price we get:

S + €2.87 = €70$e^{-(.048)(3/12)}$ + €4.68
S = €70.98

9. Using put-call parity, we can solve for the risk-free rate as follows:

฿65.80 + ฿2.86 = ฿65$e^{-R(2/12)}$ + ฿4.08
฿64.58 = ฿65$e^{-R(2/12)}$
$0.9935 = e^{-R(2/12)}$
$\ln(0.9935) = \ln(e^{-R(2/12)})$
$-0.0065 = -R(2/12)$
$R_f = 3.89\%$

10. Using the Black-Scholes option pricing model to find the price of the call option, we find:

$d_1 = [\ln(\text{Ca\$38}/\text{Ca\$35}) + (.06 + .54^2/2) \times (3/12)] / (.54 \times \sqrt{3/12}) = .4951$

$d_2 = .4951 - (.54 \times \sqrt{3/12}) = .2251$

$N(d_1) = .6897$

$N(d_2) = .5891$

Putting these values into the Black-Scholes model, we find the call price is:

C = Ca\$38(.6897) – (Ca\$35$e^{-.06(.25)}$)(.5891) = Ca\$5.90

Using put-call parity, the put price is:

Put = Ca\$35$e^{-.06(.25)}$ + 5.90 – 38 = Ca\$2.38

11. Using the Black-Scholes option pricing model to find the price of the call option, we find:

d_1 = [ln(₪86/₪90) + (.04 + $.62^2/2$) × (8/12)] / (.62 × $\sqrt{8/12}$) = .2160

d_2 = .2160 – (.62 × $\sqrt{8/12}$) = –.2902

$N(d_1)$ = .5855

$N(d_2)$ = .3858

Putting these values into the Black-Scholes model, we find the call price is:

C = ₪86(.5855) – (₪90$e^{-.04(8/12)}$)(.3858) = ₪16.54

Using put-call parity, the put price is:

Put = ₪90$e^{-.04(8/12)}$ + 16.54 – 86 = ₪18.18

12. The delta of a call option is $N(d_1)$, so:

d_1 = [ln(₭87/₭85) + (.05 + $.56^2/2$) × .75] / (.56 × $\sqrt{.75}$) = .3678

$N(d_1)$ = .6435

For a call option the delta is .64. For a put option, the delta is:

Put delta = .64 – 1 = –.36

The delta tells us the change in the price of an option for a ₭1 change in the price of the underlying asset.

13. Using the Black-Scholes option pricing model, with a 'stock' price is Rs.20,000,000 and an exercise price is Rs.22,000,000, the price you should receive is:

d_1 = [ln(Rs.20,000,000/Rs.22,000,000) + (.05 + $.20^2/2$) × (12/12)] / (.20 × $\sqrt{12/12}$) = –.1266

d_2 = –.1266 – (.20 × $\sqrt{12/12}$) = –.3266

$N(d_1)$ = .4496

$N(d_2)$ = .3720

Putting these values into the Black-Scholes model, we find the call price is:

$C = \text{Rs.}20{,}000{,}000(.4496) - (\text{Rs.}22{,}000{,}000e^{-.05(1)})(.3720) = \text{Rs.}1{,}208{,}017.63$

14. Using the call price we found in the previous problem and put-call parity, you would need to pay:

$\text{Put} = \text{Rs.}22{,}000{,}000e^{-.05(1)} + 1{,}208{,}017.63 - 20{,}000{,}000 = \text{Rs.}2{,}135{,}064.96$

You would have to pay Rs.2,135,064.96 in order to guarantee the right to sell the land for Rs.22,000,000.

15. Using the Black-Scholes option pricing model to find the price of the call option, we find:

$d_1 = [\ln(\$86/\$90) + (.12 + .53^2/2) \times (6/12)] / (.53 \times \sqrt{(6/12)}) = .2262$

$d_2 = .2262 - (.53 \times \sqrt{6/12}) = -.1486$

$N(d_1) = .5895$

$N(d_2) = .4409$

Putting these values into the Black-Scholes model, we find the call price is:

$C = \$86(.5895) - (\$90e^{-.12(.50)})(.4409) = \$13.32$

Using put-call parity, we find the put price is:

$\text{Put} = \$90e^{-.12(.50)} + 13.32 - 86 = \$12.08$

a. The intrinsic value of each option is:

Call intrinsic value = Max[S – E, 0] = $0

Put intrinsic value = Max[E – S, 0] = $4

b. Option value consists of time value and intrinsic value, so:

Call option value = Intrinsic value + Time value
$13.32 = $0 + TV
TV = $13.32

Put option value = Intrinsic value + Time value
$12.08 = $4 + TV
TV = $8.08

c. The time premium (theta) is more important for a call option than a put option; therefore, the time premium is, in general, larger for a call option.

16. The stock price can either increase 15 percent, or decrease 15 percent. The stock price at expiration will either be:

Stock price increase = \$75(1 + .15) = \$86.25

Stock price decrease = \$75(1 – .15) = \$63.75

The payoff in either state will be the maximum stock price minus the exercise price, or zero, which is:

Payoff if stock price increases = Max[\$86.25 – 70, 0] = \$16.25

Payoff if stock price decreases = Max[\$53.75 – 70, 0] = \$0

The risk neutral probability of a stock price increase is:

Probability of stock price increase = (.15 + .12) / (.15 + .15) = .90

And the probability of a stock price decrease is:

Probability of stock price decrease = 1 – .90 = .10

So, the risk neutral value of a call option will be:

Call value = [(.90 × \$16.25) + (.10 × \$0)] / (1 + .08)
Call value = \$13.06

17. The stock price increase, decrease, and option payoffs will remain unchanged since the stock price change is the same. The new risk neutral probability of a stock price increase is:

Probability of stock price increase = (.08 + .15) / (.15 + .15) = .7667

And the probability of a stock price decrease is:

Probability of stock price decrease = 1 – .7667 = .2333

So, the risk neutral value of a call option will be:

Call value = [(.7667 × \$16.25) + (.2333 × \$0)] / (1 + .15)
Call value = \$11.54

Intermediate

18. If the exercise price is equal to zero, the call price will equal the stock price, which is £85.

19. If the standard deviation is zero, d_1 and d_2 go to $+\infty$, so $N(d_1)$ and $N(d_2)$ go to 1. This is the no risk call option formula, which is:

$C = S - Ee^{-rt}$
$C = €84 - €80e^{-.05(6/12)} = €5.98$

20. If the standard deviation is infinite, d_1 goes to positive infinity so $N(d_1)$ goes to 1, and d_2 goes to negative infinity so $N(d_2)$ goes to 0. In this case, the call price is equal to the stock price, which is Rs.35.

21. We can use the Black-Scholes model to value the equity of a firm. Using the asset value of ¥10,500,000 as the stock price, and the face value of debt of ¥10,000,000 as the exercise price, the value of the firm's equity is:

$d_1 = [\ln(¥10{,}500{,}000/¥10{,}000{,}000) + (.05 + .38^2/2) \times 1] / (.38 \times \sqrt{1}) = .4500$

$d_2 = .4500 - (.38 \times \sqrt{1}) = .0700$

$N(d_1) = .6736$

$N(d_2) = .5279$

Putting these values into the Black-Scholes model, we find the equity value is:

$\text{Equity} = ¥10{,}500{,}000(.6736) - (¥10{,}000{,}000e^{-.05(1)})(.5279) = ¥2{,}051{,}699.90$

The value of the debt is the firm value minus the value of the equity, so:

$D = ¥10{,}500{,}000 - 2{,}051{,}699.90 = ¥8{,}448{,}300.10$

22. *a.* We can use the Black-Scholes model to value the equity of a firm. Using the asset value of ¥11,200,000 as the stock price, and the face value of debt of ¥10,000,000 as the exercise price, the value of the firm if it accepts project A is:

$d_1 = [\ln(¥11{,}200{,}000/¥10{,}000{,}000) + (.05 + .55^2/2) \times 1] / (.55 \times \sqrt{1}) = .5720$

$d_2 = .5720 - (.55 \times \sqrt{1}) = .0220$

$N(d_1) = .7163$

$N(d_2) = .5088$

Putting these values into the Black-Scholes model, we find the equity value is:

$E_A = ¥11{,}200{,}000(.7163) - (¥10{,}000{,}000e^{-.05(1)})(.5088) = ¥3{,}183{,}369.72$

The value of the debt is the firm value minus the value of the equity, so:

$D_A = ¥11{,}200{,}000 - 3{,}183{,}369.72 = ¥8{,}016{,}630.28$

And the value of the firm if it accepts Project B is:

d_1 = [ln(¥11,500,000/¥10,000,000) + (.05 + $.34^2/2$) × 1] / (.34 × $\sqrt{1}$) = .7281

d_2 = .7281 – (.34 × $\sqrt{1}$) = .3881

$N(d_1)$ = .7667

$N(d_2)$ = .6510

Putting these values into the Black-Scholes model, we find the equity value is:

E_B = ¥11,500,000(.7667) – (¥10,000,000$e^{-.05(1)}$)(.6510) = ¥2,624,544.59

The value of the debt is the firm value minus the value of the equity, so:

D_B = ¥11,500,000 – 2,624,544.59 = ¥8,875,455.41

b. Although the NPV of project B is higher, the equity value with project A is higher. While NPV represents the increase in the value of the assets of the firm, in this case, the increase in the value of the firm's assets resulting from project B is mostly allocated to the debtholders, resulting in a smaller increase in the value of the equity. Stockholders would, therefore, prefer project A even though it has a lower NPV.

c. Yes. If the same group of investors have equal stakes in the firm as bondholders and stock-holders, then total firm value matters and project B should be chosen, since it increases the value of the firm to ¥11,500,000 instead of ¥11,200,000.

d. Stockholders may have an incentive to take on riskier, less profitable projects if the firm is leveraged; the higher the firm's debt load, all else the same, the greater is this incentive.

23. We can use the Black-Scholes model to value the equity of a firm. Using the asset value of ¥22,000,000 as the stock price, and the face value of debt of ¥20,000,000 as the exercise price, the value of the firm's equity is:

d_1 = [ln(¥22,000,000/¥20,000,000) + (.05 + $.53^2/2$) × 1] / (.53 × $\sqrt{1}$) = .5392

d_2 = .5392 – (.53 × $\sqrt{1}$) = .0092

$N(d_1)$ = .7051

$N(d_2)$ = .5037

Putting these values into the Black-Scholes model, we find the equity value is:

Equity = ¥22,000,000(.70512) – (¥20,000,000$e^{-.05(1)}$)(.5037) = ¥5,930,643.92

The value of the debt is the firm value minus the value of the equity, so:

D = ¥22,000,000 – 5,930,643.92 = ¥16,069,356.08

The return on the company's debt is:

¥16,069,356.08 = ¥20,000,000$e^{-R(1)}$
.803468 = e^{-R}
R_D = –ln(.803468) = 21.88%

24. *a.* The combined value of equity and debt of the two firms is:

Equity = ¥2,051,699.9 + 5,930,643.92 = ¥7,982,343.82

Debt = ¥8,448,300.10 + 16,069,356.08 = ¥24,517,656.18

b. For the new firm, the combined market value of assets is ¥32,500,000 and the combined face value of debt is ¥30,000,000. Using Black-Scholes to find the value of equity for the new firm, we find:

d_1 = [ln(¥32,500,000/¥30,000,000) + (.05 + $.31^2/2$) × 1] / (.31 × $\sqrt{1}$) = .5745

d_2 = .5745 – (.31 × $\sqrt{1}$) = .2645

$N(d_1)$ = .7172

$N(d_2)$ = .6043

Putting these values into the Black-Scholes model, we find the equity value is:

E = ¥32,500,000(.7172) – (¥30,000,000$e^{-.05(1)}$)(.6043) = ¥6,063,606.25

The value of the debt is the firm value minus the value of the equity, so:

D = ¥32,500,000 – 6,063,606.25 = ¥26,436,393.75

c. The change in the value of the firm's equity is:

Equity value change = ¥6,063,606.25 – 7,982,343.82 = –¥1,918,737.57

The change in the value of the firm's debt is:

Debt = ¥26,436,393.75 – 24,517,656.18 = ¥1,918,737.57

d. In a purely financial merger, when the standard deviation of the assets declines, the value of the equity declines as well. The shareholders will lose exactly the amount the bondholders gain. The bondholders gain as a result of the coinsurance effect. That is, it is less likely that the new company will default on the debt.

25. *a.* Using Black-Scholes model to value the equity, we get:

$d_1 = [\ln(\$22{,}000{,}000/\$30{,}000{,}000) + (.06 + .39^2/2) \times 10] / (.39 \times \sqrt{10}) = .8517$

$d_2 = .8517 - (.39 \times \sqrt{10}) = -.3816$

$N(d_1) = .8028$

$N(d_2) = .3514$

Putting these values into Black-Scholes:

$E = \$22{,}000{,}000(.8028) - (\$30{,}000{,}000e^{-.06(10)})(.3514) = \$11{,}876{,}514.69$

b. The value of the debt is the firm value minus the value of the equity, so:

$D = \$22{,}000{,}000 - 11{,}876{,}514.69 = \$10{,}123{,}485.31$

c. Using the equation for the PV of a continuously compounded lump sum, we get:

$\$10{,}123{,}485.31 = \$30{,}000{,}000e^{-R(10)}$
$.33745 = e^{-R10}$
$R_D = -(1/10)\ln(.33745) = 10.86\%$

d. Using Black-Scholes model to value the equity, we get:

$d_1 = [\ln(\$22{,}800{,}000/\$30{,}000{,}000) + (.06 + .39^2/2) \times 10] / (.39 \times \sqrt{10}) = .8806$

$d_2 = .8806 - (.39 \times \sqrt{10}) = -.3816$

$N(d_1) = .8107$

$N(d_2) = .3622$

Putting these values into Black-Scholes:

$E = \$22{,}800{,}000(.8107) - (\$30{,}000{,}000e^{-.06(10)})(.3622) = \$12{,}521{,}962.02$

e. The value of the debt is the firm value minus the value of the equity, so:

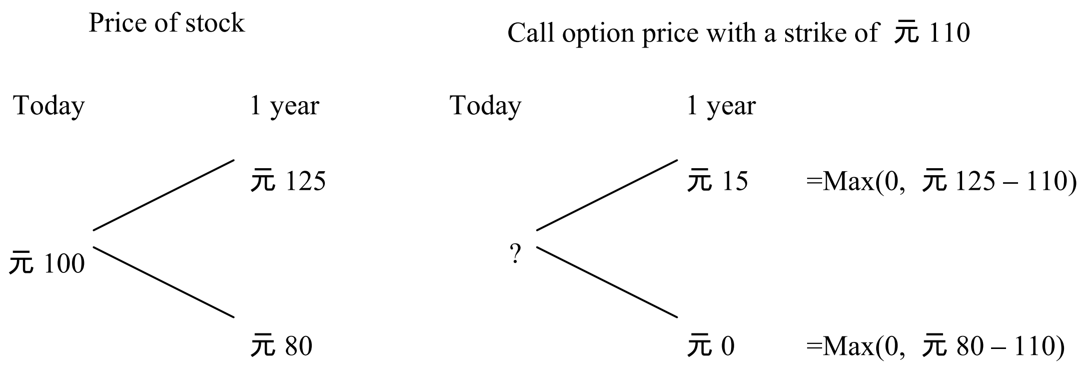
$D = \$22{,}800{,}000 - 12{,}521{,}962.02 = \$10{,}278{,}037.98$

Using the equation for the PV of a continuously compounded lump sum, we get:

$\$10{,}278{,}037.98 = \$30{,}000{,}000e^{-R(10)}$
$.34260 = e^{-R10}$
$R_D = -(1/10)\ln(.34260) = 10.71\%$

When the firm accepts the new project, part of the NPV accrues to bondholders. This increases the present value of the bond, thus reducing the return on the bond. Additionally, the new project makes the firm safer in the sense that it increases the value of assets, thus increasing the probability the call will end in-the-money and the bondholders will receive their payment.

26. *a.* In order to solve a problem using the two-state option model, we first need to draw a stock price tree containing both the current stock price and the stock's possible values at the time of the option's expiration. Next, we can draw a similar tree for the option, designating what its value will be at expiration given either of the 2 possible stock price movements.



The stock price today is 元 100. It will either increase to 元 125 or decrease to 元 80 in one year. If the stock price rises to 元 125, the call will be exercised for 元 110 and a payoff of 元 15 will be received at expiration. If the stock price falls to 元 80, the option will not be exercised, and the payoff at expiration will be zero.

If the stock price rises, its return over the period is 25 percent [= (元 125/元 100) – 1]. If the stock price falls, its return over the period is –20 percent [= (元 80/元 100) –1]. We can use the following expression to determine the risk-neutral probability of a rise in the price of the stock:

Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (\text{Probability}_{\text{Fall}})(\text{Return}_{\text{Fall}})$
Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (1 - \text{Probability}_{\text{Rise}})(\text{Return}_{\text{Fall}})$
.025 = $(\text{Probability}_{\text{Rise}})(0.25) + (1 - \text{Probability}_{\text{Rise}})(-0.20)$
$\text{Probability}_{\text{Rise}}$ = .50 or 50%

This means the risk neutral probability of a stock price decrease is:

$\text{Probability}_{\text{Fall}} = 1 - \text{Probability}_{\text{Rise}}$
$\text{Probability}_{\text{Fall}} = 1 - .50$
$\text{Probability}_{\text{Fall}}$ = .50 or 50%

Using these risk-neutral probabilities, we can now determine the expected payoff of the call option at expiration. The expected payoff at expiration is:

Expected payoff at expiration = (.50)(元 15) + (.50)(元 0)

Expected payoff at expiration = 元 7.50

Since this payoff occurs 1 year from now, we must discount it back to the value today. Since we are using risk-neutral probabilities, we can use the risk-free rate, so:

PV(Expected payoff at expiration) = 元 7.50 / 1.025

PV(Expected payoff at expiration) = 元 7.32

Therefore, given the information about the stock price movements over the next year, a European call option with a strike price of 元 110 and one year until expiration is worth 元 7.32 today.

b. Yes, there is a way to create a synthetic call option with identical payoffs to the call option described above. In order to do this, we will need to buy shares of stock and borrow at the risk-free rate. The number of shares to buy is based on the delta of the option, where delta is defined as:

Delta = (Swing of option) / (Swing of stock)

Since the call option will be worth 元 15 if the stock price rises and 元 0 if it falls, the delta of the option is 元 15 (= 15 – 0). Since the stock price will either be 元 125 or 元 80 at the time of the option's expiration, the swing of the stock is 元 45 (= 元 125 – 80). With this information, the delta of the option is:

Delta = 元 15 / 元 45

Delta = 1/3 or .3333

Therefore, the first step in creating a synthetic call option is to buy 1/3 of a share of the stock. Since the stock is currently trading at 元 100 per share, this will cost 元 33.33 [= (1/3)(元 100)]. In order to determine the amount that we should borrow, compare the payoff of the actual call option to the payoff of delta shares at expiration.

Call Option

If the stock price rises to 元 125:	Payoff = 元 15
If the stock price falls to 元 80:	Payoff = 元 0

Delta Shares

If the stock price rises to 元 125:	Payoff = (1/3)(元 125) = 元 41.66
If the stock price falls to 元 80:	Payoff = (1/3)(元 80) = 元 26.66

The payoff of his synthetic call position should be identical to the payoff of an actual call option. However, owning 1/3 of a share leaves us exactly 元 26.66 above the payoff at expiration, regardless of whether the stock price rises or falls. In order to reduce the payoff at expiration by 元 26.66, we should borrow the present value of 元 26.66 now. In one year, the obligation to pay 元 26.66 will reduce the payoffs so that they exactly match those of an actual call option. So, purchase 1/3 of a share of stock and borrow 元 26.02 (= 元 26.66 / 1.025) in order to create a synthetic call option with a strike price of 元 110 and 1 year until expiration.

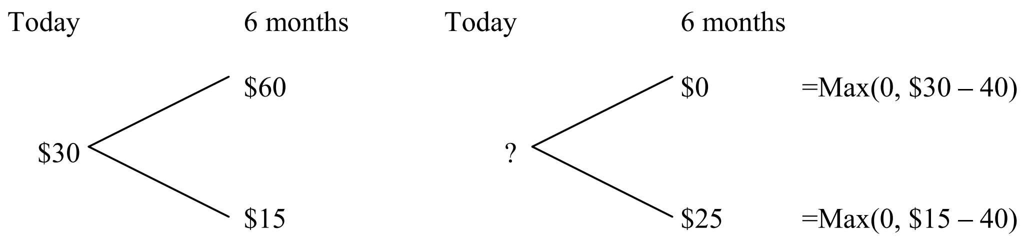
c. Since the cost of the stock purchase is 元 33.33 to purchase 1/3 of a share and 元 26.02 is borrowed, the total cost of the synthetic call option is:

Cost of synthetic option = 元 33.33 – 26.02
Cost of synthetic option = 元 7.32

This is exactly the same price as an actual call option. Since an actual call option and a synthetic call option provide identical payoff structures, we should not expect to pay more for one than for the other.

27. *a.* In order to solve a problem using the two-state option model, we first draw a stock price tree containing both the current stock price and the stock's possible values at the time of the option's expiration. Next, we can draw a similar tree for the option, designating what its value will be at expiration given either of the 2 possible stock price movements.

Price of stock | Put option price with a strike of $40



The stock price today is $30. It will either decrease to $15 or increase to $60 in six months. If the stock price falls to $15, the put will be exercised and the payoff will be $25. If the stock price rises to $60, the put will not be exercised, so the payoff will be zero.

If the stock price rises, its return over the period is 100% [= (60/30) – 1]. If the stock price falls, its return over the period is –50% [= (15/30) –1]. Use the following expression to determine the risk-neutral probability of a rise in the price of the stock:

Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (\text{Probability}_{\text{Fall}})(\text{Return}_{\text{Fall}})$
Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (1 - \text{Probability}_{\text{Rise}})(\text{Return}_{\text{Fall}})$

The risk-free rate over the next six months must be used in the order to match the timing of the expected stock price change. Since the risk-free rate per annum is 21 percent, the risk-free rate over the next six months is 10 percent [= $(1.21)^{1/2} - 1$], so.

$0.10 = (\text{Probability}_{\text{Rise}})(1) + (1 - \text{Probability}_{\text{Rise}})(-.50)$
$\text{Probability}_{\text{Rise}} = .40$ or 40%

Which means the risk-neutral probability of a decrease in the stock price is:

$Probability_{Fall}$ = 1 – $Probability_{Rise}$
$Probability_{Fall}$ = 1 – .40
$Probability_{Fall}$ = .60 or 60%

Using these risk-neutral probabilities, we can determine the expected payoff to put option at expiration as:

Expected payoff at expiration = (.40)($0) + (.60)($25)
Expected payoff at expiration = $15.00

Since this payoff occurs 6 months from now, we must discount it at the risk-free rate in order to find its present value, which is:

PV(Expected payoff at expiration) = $15.00 / $(1.21)^{1/2}$
PV(Expected payoff at expiration) = $13.64

Therefore, given the information about the stock price movements over the next six months, a European put option with a strike price of $40 and six months until expiration is worth $13.64 today.

c. Yes, there is a way to create a synthetic put option with identical payoffs to the put option described above. In order to do this, we need to short shares of the stock and lend at the risk-free rate. The number of shares that should be shorted sell is based on the delta of the option, where delta is defined as:

Delta = (Swing of option) / (Swing of stock)

Since the put option will be worth $0 if the stock price rises and $25 if it falls, the swing of the call option is –$25 (= $0 – 25). Since the stock price will either be $60 or $15 at the time of the option's expiration, the swing of the stock is $45 (= $60 – 15). Given this information, the delta of the put option is:

Delta = (Swing of option) / (Swing of stock)
Delta = (–$25 / $45)
Delta = –5/9

Therefore, the first step in creating a synthetic put option is to short 5/9 of a share of stock. Since the stock is currently trading at $30 per share, the amount received will be $16.67 (= 5/9 × $30) as a result of the short sale. In order to determine the amount to lend, compare the payoff of the actual put option to the payoff of delta shares at expiration.

<u>Put option</u>
If the stock price rises to $60: Payoff = $0
If the stock price falls to $15: Payoff = $25

<u>Delta shares</u>
If the stock price rises to $60: Payoff = (–5/9)($60) = –$33.33
If the stock price falls to $80: Payoff = (–5/9)($15) = –$8.33

The payoff of the synthetic put position should be identical to the payoff of an actual put option. However, shorting 5/9 of a share leaves us exactly \$33.33 below the payoff at expiration, whether the stock price rises or falls. In order to increase the payoff at expiration by \$33.33, we should lend the present value of \$33.33 now. In six months, we will receive \$33.33, which will increase the payoffs so that they exactly match those of an actual put option. So, the amount to lend is:

Amount to lend = \$33.33 / $1.21^{1/2}$
Amount to lend = \$30.30

c. Since the short sale results in a positive cash flow of \$16.67 and we will lend \$30.30, the total cost of the synthetic put option is:

Cost of synthetic put = \$30.30 – 16.67
Cost of synthetic put = \$13.64

This is exactly the same price as an actual put option. Since an actual put option and a synthetic put option provide identical payoff structures, we should not expect to pay more for one than for the other.

28. *a.* The company would be interested in purchasing a call option on the price of gold with a strike price of 元 3,750 per ounce and 3 months until expiration. This option will compensate the company for any increases in the price of gold above the strike price and places a cap on the amount the firm must pay for gold at 元 3,750 per ounce.

b. In order to solve a problem using the two-state option model, first draw a price tree containing both the current price of the underlying asset and the underlying asset's possible values at the time of the option's expiration. Next, draw a similar tree for the option, designating what its value will be at expiration given either of the 2 possible stock price movements.

Price of gold

Today	3 months
	元 4,000
元 3,500	
	元 3,250

Call option price with a strike of 元 3,750

Today	3 months	
	元 250	=Max(0, 元 4,000 – 3,750)
?		
	元 0	=Max(0, 元 3,250 – 3,750)

The price of gold is 元 3,500 per ounce today. If the price rises to 元 4,000, the company will exercise its call option for 元 3,750 and receive a payoff of 元 250 at expiration. If the price of gold falls to 元 3,250, the company will not exercise its call option, and the firm will receive no payoff at expiration. If the price of gold rises, its return over the period is 14.29 percent [= (元 4,000 / 元 3,500) – 1]. If the price of gold falls, its return over the period is –7.14 percent [= (元 3,250 / 元 3,500) –1]. Use the following expression to determine the risk-neutral probability of a rise in the price of gold:

Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (\text{Probability}_{\text{Fall}})(\text{Return}_{\text{Fall}})$
Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (1 - \text{Probability}_{\text{Rise}})(\text{Return}_{\text{Fall}})$

The risk-free rate over the next three months must be used in the order to match the timing of the expected price change. Since the risk-free rate per annum is 16.99 percent, the risk-free rate over the next three months is 4 percent [= $(1.1699)^{1/4} - 1$], so:

$.04 = (\text{Probability}_{\text{Rise}})(.1429) + (1 - \text{Probability}_{\text{Rise}})(-.0714)$
$\text{Probability}_{\text{Rise}} = .5200$ or 52.00%

And the risk-neutral probability of a price decline is:

$\text{Probability}_{\text{Fall}} = 1 - \text{Probability}_{\text{Rise}}$
$\text{Probability}_{\text{Fall}} = 1 - .5200$
$\text{Probability}_{\text{Fall}} = .4800$ or 48.00%

Using these risk-neutral probabilities, we can determine the expected payoff to of the call option at expiration, which will be.

Expected payoff at expiration = (.5200)(元 250) + (.4800)(元 0)

Expected payoff at expiration = 元 130.00

Since this payoff occurs 3 months from now, it must be discounted at the risk-free rate in order to find its present value. Doing so, we find:

PV(Expected payoff at expiration) = [元 130.00 / $(1.1699)^{1/4}$]

PV(Expected payoff at expiration) = 元 125.00

Therefore, given the information about gold's price movements over the next three months, a European call option with a strike price of 元 3,750 and three months until expiration is worth 元 125.00 today.

c. Yes, there is a way to create a synthetic call option with identical payoffs to the call option described above. In order to do this, the company will need to buy gold and borrow at the risk-free rate. The amount of gold to buy is based on the delta of the option, where delta is defined as:

Delta = (Swing of option) / (Swing of price of gold)
Since the call option will be worth 元 250 if the price of gold rises and 元 0 if it falls, the swing of the call option is 元 250 (= 元 250 – 0). Since the price of gold will either be 元 4,000 or 元 3,250 at the time of the option's expiration, the swing of the price of gold is 元 750 (= 元,4000 – 3,250). Given this information the delta of the call option is:

Delta = (Swing of option) / (Swing of price of gold)
Delta = (元 250 / 元 750)
Delta = 1/3 or .3333

Therefore, the first step in creating a synthetic call option is to buy 1/3 of an ounce of gold. Since gold currently sells for 元 3,500 per ounce, the company will pay 元 1166.67 (= 1/3 × 元 3,500) to purchase 1/3 of an ounce of gold. In order to determine the amount that should be borrowed, compare the payoff of the actual call option to the payoff of delta shares at expiration:

Call Option
If the price of gold rises to 元 4,000: Payoff = 元 250
If the price of gold falls to 元 3,250: Payoff = 元 0

Delta Shares
If the price of gold rises to 元 4,000: Payoff = (1/3)(元 4,000) = 元 1,333.3
If the price of gold falls to 元 3,250: Payoff = (1/3)(元 3,250) = 元 1,083.3

The payoff of this synthetic call position should be identical to the payoff of an actual call option. However, buying 1/3 of a share leaves us exactly 元 1,083.3 above the payoff at expiration, whether the price of gold rises or falls. In order to decrease the company's payoff at expiration by 元 1,083.3, it should borrow the present value of 元 1,083.3 now. In three months, the company must pay 元 1,083.3, which will decrease its payoffs so that they exactly match those of an actual call option. So, the amount to borrow today is:

Amount to borrow today = 元 $1,083.3 / 1.1699^{1/4}$
Amount to borrow today = 元 1,041.7

d. Since the company pays 元 1,166.67 in order to purchase gold and borrows 元 1,041.7, the total cost of the synthetic call option is 元 125.00 (= 元 1,166.67 – 1,041.7). This is exactly the same price for an actual call option. Since an actual call option and a synthetic call option provide identical payoff structures, the company should not expect to pay more for one than for the other.

29. To construct the collar, the investor must purchase the stock, sell a call option with a high strike price, and buy a put option with a low strike price. So, to find the cost of the collar, we need to find the price of the call option and the price of the put option. We can use Black-Scholes to find the price of the call option, which will be:

Price of call option with £120 strike price:

$d_1 = [\ln(£80/£120) + (.10 + .50^2/2) \times (6/12)] / (.50 \times \sqrt{(6/12)}) = -.8286$

$d_2 = -.8286 - (.50 \times \sqrt{6/12}) = -1.1822$
$N(d_1) = .2037$

$N(d_2) = .1186$

Putting these values into the Black-Scholes model, we find the call price is:

$C = £80(.2037) - (£120e^{-.10(6/12)})(.1186) = £2.76$

Now we can use Black-Scholes and put-call parity to find the price of the put option with a strike price of £50. Doing so, we find:

Price of put option with £50 strike price:

$d_1 = [\ln(£80/£50) + (.10 + .50^2/2) \times (6/12)] / (.50 \times \sqrt{(6/12)}) = 1.6476$

$d_2 = 1.6476 - (.50 \times \sqrt{6/12}) = 1.2940$

$N(d_1) = .9503$

$N(d_2) = .9022$

Putting these values into the Black-Scholes model, we find the call price is:

$C = £80(.9503) - (£50e^{-.10(6/12)})(.9022) = £33.11$

Rearranging the put-call parity equation, we get:

$P = C - S + Xe^{-Rt}$
$P = £33.11 - 80 + 50e^{-.10(6/12)}$
$P = £0.68$

So, the investor will buy the stock, sell the call option, and buy the put option, so the total cost is:

Total cost of collar = £80 – 2.76 + .68
Total cost of collar = £77.92

<u>Challenge</u>

30. *a.* Using the equation for the PV of a continuously compounded lump sum, we get:

$PV = \$30,000 \times e^{-.05(2)} = \$27,145.12$

b. Using Black-Scholes model to value the equity, we get:

$d_1 = [\ln(\$13,000/\$30,000) + (.05 + .60^2/2) \times 2] / (.60 \times \sqrt{2}\,) = -.4434$

$d_2 = -.4434 - (.60 \times \sqrt{2}\,) = -1.2919$

$N(d_1) = .3287$

$N(d_2) = .0982$

Putting these values into Black-Scholes:

$E = \$13,000(.3287) - (\$30,000e^{-.05(2)})(.0982) = \$1,608.19$

And using put-call parity, the price of the put option is:

$Put = \$30,000e^{-.05(2)} + 1,608.19 - 13,000 = \$15,753.31$

c. The value of a risky bond is the value of a risk-free bond minus the value of a put option on the firm's equity, so:

Value of risky bond = \$27,145.12 – 15,753.31 = \$11,391.81

Using the equation for the PV of a continuously compounded lump sum to find the return on debt, we get:

$\$11,391.81 = \$30,000e^{-R(2)}$
$.37973 = e^{-R2}$
$R_D = -(1/2)\ln(.37973) = .4842$ or 48.42%

d. The value of the debt with five years to maturity at the risk-free rate is:

$PV = \$30,000 \times e^{-.05(5)} = \$23,364.02$

Using Black-Scholes model to value the equity, we get:

$d_1 = [\ln(\$13,000/\$30,000) + (.05 + .60^2/2) \times 5] / (.60 \times \sqrt{5}\,) = .2339$

$d_2 = .2339 - (.60 \times \sqrt{5}\,) = -1.1078$

$N(d_1) = .5925$

$N(d_2) = .1340$

Putting these values into Black-Scholes:

$E = \$13{,}000(.5925) – (\$30{,}000e^{–.05(5)})(.1340) = \$4{,}571.62$

And using put-call parity, the price of the put option is:

$Put = \$30{,}000e^{–.05(5)} + \$4{,}571.62 – \$13{,}000 = \$14{,}935.64$

The value of a risky bond is the value of a risk-free bond minus the value of a put option on the firm's equity, so:

Value of risky bond = $23,364.02 – 14,935.64 = $8,428.38

Using the equation for the PV of a continuously compounded lump sum to find the return on debt, we get:

Return on debt: $\$8{,}428.38 = \$30{,}000e^{–R(5)}$
$.28095 = e^{–R5}$
$R_D = –(1/5)\ln(.28095) = 25.39\%$

The value of the debt declines because of the time value of money, i.e., it will be longer until shareholders receive their payment. However, the required return on the debt declines. Under the current situation, it is not likely the company will have the assets to pay off bondholders. Under the new plan where the company operates for five more years, the probability of increasing the value of assets to meet or exceed the face value of debt is higher than if the company only operates for two more years.

31. *a.* Using the equation for the PV of a continuously compounded lump sum, we get:

$PV = \$60{,}000 \times e^{–.06(5)} = \$44{,}449.09$

b. Using Black-Scholes model to value the equity, we get:

$d_1 = [\ln(\$57{,}000/\$60{,}000) + (.06 + .50^2/2) \times 5] / (.50 \times \sqrt{5}\,) = .7815$

$d_2 = .7815 – (.50 \times \sqrt{5}\,) = –.3366$

$N(d_1) = .7827$

$N(d_2) = .3682$

Putting these values into Black-Scholes:

$E = \$57{,}000(.7827) – (\$60{,}000e^{–.06(5)})(.3682) = \$28{,}248.84$

And using put-call parity, the price of the put option is:

$Put = \$60{,}000e^{–.06(5)} + 28{,}248.84 – 57{,}000 = \$15{,}697.93$

c. The value of a risky bond is the value of a risk-free bond minus the value of a put option on the firm's equity, so:

Value of risky bond = $44,449.09 – 15,697.93 = $28,751.16
Using the equation for the PV of a continuously compounded lump sum to find the return on debt, we get:

Return on debt: $\$28,751.16 = \$60,000e^{-R(5)}$
$.47919 = e^{-R(5)}$
$R_D = -(1/5)\ln(.47919) = 14.71\%$

d. Using the equation for the PV of a continuously compounded lump sum, we get:

$PV = \$60,000 \times e^{-.06(5)} = \$44,449.09$

Using Black-Scholes model to value the equity, we get:

$d_1 = [\ln(\$57,000/\$60,000) + (.06 + .60^2/2) \times 5] / (.60 \times \sqrt{5}) = .8562$

$d_2 = .8562 - (.60 \times \sqrt{5}) = -.4854$

$N(d_1) = .8041$

$N(d_2) = .3137$

Putting these values into Black-Scholes:

$E = \$57,000(.8041) - (\$60,000e^{-.06(5)})(.3137) = \$31,888.34$

And using put-call parity, the price of the put option is:

$Put = \$60,000e^{-.06(5)} + 31,888.34 - 57,000 = \$19,337.44$

The value of a risky bond is the value of a risk-free bond minus the value of a put option on the firm's equity, so:

Value of risky bond = $44,449.09 – 19,337.44 = $25,111.66

Using the equation for the PV of a continuously compounded lump sum to find the return on debt, we get:

Return on debt: $\$25,111.66 = \$60,000e^{-R(5)}$
$.41853 = e^{-R(5)}$
$R_D = -(1/5)\ln(.41853) = 17.42\%$

The value of the debt declines. Since the standard deviation of the company's assets increases, the value of the put option on the face value of the bond increases, which decreases the bond's current value.

e. From *c* and *d*, bondholders lose: $25,111.66 – 28,751.16 = –$3,639.51
From *c* and *d*, stockholders gain: $31,888.34 – 28,248.84 = $3,639.51

This is an agency problem for bondholders. Management, acting to increase shareholder wealth in this manner, will reduce bondholder wealth by the exact amount by which shareholder wealth is increased.

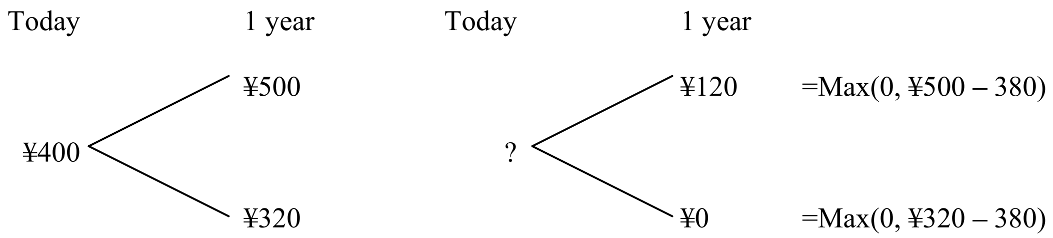
32. *a.* Since the equityholders of a firm financed partially with debt can be thought of as holding a call option on the assets of the firm with a strike price equal to the debt's face value and a time to expiration equal to the debt's time to maturity, the value of the company's equity equals a call option with a strike price of ¥380 billion and 1 year until expiration.

In order to value this option using the two-state option model, first draw a tree containing both the current value of the firm and the firm's possible values at the time of the option's expiration. Next, draw a similar tree for the option, designating what its value will be at expiration given either of the 2 possible changes in the firm's value.

The value of the company today is ¥400 billion. It will either increase to ¥500 billion or decrease to ¥320 billion in one year as a result of its new project. If the firm's value increases to ¥500 billion, the equityholders will exercise their call option, and they will receive a payoff of ¥120 billion at expiration. However, if the firm's value decreases to ¥320 billion, the equityholders will not exercise their call option, and they will receive no payoff at expiration.

Value of company (in billions)

Equityholders' call option price with a strike of ¥380 (in billions)



If the project is successful and the company's value rises, the percentage increase in value over the period is 25 percent [= (¥500 / ¥400) – 1]. If the project is unsuccessful and the company's value falls, the percentage decrease in value over the period is –20 percent [= (¥320 / ¥400) –1]. We can determine the risk-neutral probability of an increase in the value of the company as:

Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (\text{Probability}_{\text{Fall}})(\text{Return}_{\text{Fall}})$
Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (1 - \text{Probability}_{\text{Rise}})(\text{Return}_{\text{Fall}})$
0.07 = $(\text{Probability}_{\text{Rise}})(.25) + (1 - \text{Probability}_{\text{Rise}})(-.20)$
$\text{Probability}_{\text{Rise}}$ = .60 or 60%

And the risk-neutral probability of a decline in the company value is:

$\text{Probability}_{\text{Fall}} = 1 - \text{Probability}_{\text{Rise}}$
$\text{Probability}_{\text{Fall}} = 1 - .60$
$\text{Probability}_{\text{Fall}}$ = .40 or 40%

Using these risk-neutral probabilities, we can determine the expected payoff to the equityholders' call option at expiration, which will be:

Expected payoff at expiration = (.60)(¥120,000,000) + (.40)(¥0)
Expected payoff at expiration = ¥72,000,000

Since this payoff occurs 1 year from now, we must discount it at the risk-free rate in order to find its present value. So:

PV(Expected payoff at expiration) = (¥72,000,000 / 1.07)
PV(Expected payoff at expiration) = ¥67,289,720

Therefore, the current value of the company's equity is ¥67,289,720. The current value of the company is equal to the value of its equity plus the value of its debt. In order to find the value of company's debt, subtract the value of the company's equity from the total value of the company:

V_L = Debt + Equity
¥400,000,000 = Debt + ¥67,289,720
Debt = ¥332,710,280

b. To find the price per share, we can divide the total value of the equity by the number of shares outstanding. So, the price per share is:

Price per share = Total equity value / Shares outstanding
Price per share = ¥67,289,720 / 500,000
Price per share = ¥134.58

c. The market value of the firm's debt is ¥332,710,280. The present value of the same face amount of riskless debt is ¥355,140,187 (= ¥380,000,000 / 1.07). The firm's debt is worth less than the present value of riskless debt since there is a risk that it will not be repaid in full. In other words, the market value of the debt takes into account the risk of default. The value of riskless debt is ¥355,140,187. Since there is a chance that the company might not repay its debtholders in full, the debt is worth less than ¥355,140,187.

d. The value of Strudler today is ¥400 billion. It will either increase to ¥800 billion or decrease to ¥200 billion in one year as a result of the new project. If the firm's value increases to ¥800 billion, the equityholders will exercise their call option, and they will receive a payoff of ¥420 billion at expiration. However, if the firm's value decreases to ¥200 billion, the equityholders will not exercise their call option, and they will receive no payoff at expiration.

Value of company (in billions)		Equityholders' call option price with a strike of ¥380 (in billions)		
Today	1 year	Today	1 year	
	¥800		¥420	=Max(0, ¥800 – 380)
¥400		?		
	¥200		¥0	=Max(0, ¥200 – 380)

If the project is successful and the company's value rises, the increase in the value of the company over the period is 100 percent [= (¥800 / ¥400) – 1]. If the project is unsuccessful and the company's value falls, decrease in the value of the company over the period is –50 percent [= (¥200 / ¥400) –1]. We can use the following expression to determine the risk-neutral probability of an increase in the value of the company:

Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (\text{Probability}_{\text{Fall}})(\text{Return}_{\text{Fall}})$
Risk-free rate = $(\text{Probability}_{\text{Rise}})(\text{Return}_{\text{Rise}}) + (1 - \text{Probability}_{\text{Rise}})(\text{Return}_{\text{Fall}})$
0.07 = $(\text{Probability}_{\text{Rise}})(1) + (1 - \text{Probability}_{\text{Rise}})(-.50)$
$\text{Probability}_{\text{Rise}}$ = .38 or 38 percent

So the risk-neutral probability of a decrease in the company value is:

$\text{Probability}_{\text{Fall}}$ = $1 - \text{Probability}_{\text{Rise}}$
$\text{Probability}_{\text{Fall}}$ = 1 – .38
$\text{Probability}_{\text{Fall}}$ = .62 or 62%

Using these risk-neutral probabilities, we can determine the expected payoff to the equityholders' call option at expiration, which is:

Expected payoff at expiration = (.38)(¥420,000,000) + (.62)(¥0)
Expected payoff at expiration = ¥159,600,000

Since this payoff occurs 1 year from now, we must discount it at the risk-free rate in order to find its present value. So:

PV(Expected payoff at expiration) = (¥159,600,000 / 1.07)
PV(Expected payoff at expiration) = ¥149,158,879
Therefore, the current value of the firm's equity is ¥149,158,879.

The current value of the company is equal to the value of its equity plus the value of its debt. In order to find the value of the company's debt, we can subtract the value of the company's equity from the total value of the company, which yields:

V_L = Debt + Equity
¥400,000,000 = Debt + ¥149,158,879
Debt = ¥250,841,121

The riskier project increases the value of the company's equity and decreases the value of the company's debt. If the company takes on the riskier project, the company is less likely to be able to pay off its bondholders. Since the risk of default increases if the new project is undertaken, the value of the company's debt decreases. Bondholders would prefer the company to undertake the more conservative project.

33. *a.* Going back to the chapter on dividends, the price of the stock will decline by the amount of the dividend (less any tax effects). Therefore, we would expect the price of the stock to drop when a dividend is paid, reducing the upside potential of the call by the amount of the dividend. The price of a call option will decrease when the dividend yield increases.

b. Using the Black-Scholes model with dividends, we get:

$d_1 = [\ln(£84/£80) + (.05 - .02 + .50^2/2) \times .5] / (.50 \times \sqrt{.5}) = .3572$

$d_2 = .3572 - (.50 \times \sqrt{.5}) = .0036$

$N(d_1) = .6395$

$N(d_2) = .5015$

$C = £84e^{-(.02)(.5)}(.6395) - (£80e^{-.05(.5)})(.5015) = £14.06$

34. *a.* Going back to the chapter on dividends, the price of the stock will decline by the amount of the dividend (less any tax effects). Therefore, we would expect the price of the stock to drop when a dividend is paid. The price of put option will increase when the dividend yield increases.

b. Using put-call parity to find the price of the put option, we get:

$\$84e^{-.02(.5)} + P = \$80e^{-.05(.5)} + 14.06$
$P = \$8.92$

35. $N(d_1)$ is the probability that "z" is less than or equal to $N(d_1)$, so $1 - N(d_1)$ is the probability that "z" is greater than $N(d_1)$. Because of the symmetry of the normal distribution, this is the same thing as the probability that "z" is less than $N(-d_1)$. So:

$N(d_1) - 1 = -N(-d_1)$.

36. From put-call parity:

$$P = E \times e^{-Rt} + C - S$$

Substituting the Black-Scholes call option formula for C and using the result in the previous question produces the put option formula:

$$\begin{aligned}
P &= E \times e^{-Rt} + C - S \\
P &= E \times e^{-Rt} + S \times N(d_1) - E \times e^{-Rt} \times N(d_2) - S \\
P &= S \times (N(d_1) - 1) + E \times e^{-Rt} \times (1 - N(d_2)) \\
P &= E \times e^{-Rt} \times N(-d_2) - S \times N(-d_1)
\end{aligned}$$

37. Based on Black-Scholes, the call option is worth €50! The reason is that present value of the exercise price is zero, so the second term disappears. Also, d_1 is infinite, so $N(d_1)$ is equal to one. The problem is that the call option is European with an infinite expiration, so why would you pay anything for it since you can *never* exercise it? The paradox can be resolved by examining the price of the stock. Remember that the call option formula only applies to a non-dividend paying stock. If the stock will never pay a dividend, it (and a call option to buy it at any price) must be worthless.

38. The delta of the call option is $N(d_1)$ and the delta of the put option is $N(d_1) - 1$. Since you are selling a put option, the delta of the portfolio is $N(d_1) - [N(d_1) - 1]$. This leaves the overall delta of your position as 1. This position will change dollar for dollar in value with the underlying asset. This position replicates the dollar "action" on the underlying asset.