# Wideband energy harvesting based on mixed connection of piezoelectric oscillators 

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#### Abstract

An approach for wideband energy harvesting together with power enhancement is proposed by integrating multiple piezoelectric oscillators with mixed parallel-series connection. This gives rise to the feasibility of shifting the operation frequency band to the dominant frequency domain of ambient excitations. There are two types of connection patterns discussed here: the p-type (s-type) is the parallel (series) connection of all sets of oscillators where some of them may be connected in series (parallel). In addition, the standard interface circuit used for electric rectification is adopted here. The analytic estimates of output power are derived and explicitly expressed in terms of different matrix formulations for these two connection patterns. They are subsequently validated and are found in good agreement with numerical simulations and experimental observations. Finally, the experimental results from the mixed connection of 4 piezoelectric oscillators show that the peak power of each array is about 3.4 times higher than that generated by a single piezoelectric oscillator. In addition, the bandwidth of the array capable of switching connection patterns is around 2.8 times wider than that based on a single array configuration. Hence, the effective bandwidth is enlarged without the loss of peak power.


Keywords: array of piezoelectric oscillators, mixed parallel-series connection, p-type and s-type patterns, standard interface, wideband energy harvesting
(Some figures may appear in colour only in the online journal)

## 1. Introduction

The requirement for self-powered microsensors, coupled with advancement in lower-power microelectronics, has motivated researching harvesting energy from vibration due to the ubiquitous presence of ambient excitations [1, 51]. Among many kinds of vibration-to-electricity converters, piezoelectric transduction has received great attention due to advantages of high power density, no external voltage source requirement, and the ease of implementation in microsystems [5, 22, 29, 31, 48]. Hence, extensive research efforts on the employment of piezoelectric elements for energy harvesting have witnessed a dramatic rise for the last decade. These include developing various structures suitable for extracting mechanical energy from ambient excitations [6, 9-11, 42, 52-54, 57, 76], researching new materials for enhancing electromechanical coupling [4, 8, 43, $49,61]$, and designing interface circuits for maximum power

[^0]transfer to the load [7, 26, 28, 34, 36, 38, 56, 58, 64, 67, 68]. In addition, various finite element models are developed for systemlevel designs $[17,19,41,73,78]$, including the recent advancement in direct simulations of electrically rectified energy harvesters [66].

The majority of these investigations focus on harnessing energy from resonant vibration of a single piezoelectric harvester. While this approach enjoys great success at many aspects, resonant vibration for energy harvesting is efficient only in a narrow bandwidth. It therefore motivates various research efforts for developing techniques to increase the operating bandwidth of harvesters. These incorporate tuning frequency $[14,23,30,33,63]$, designing multi-modal structures [12, 46, 50, 65, 71, 77], and improving bandwidth by nonlinearity $[16,20,27,60]$. In addition, there is an increasing attention for using arrays of piezoelectric oscillators for enlarging bandwidth by tuning the resonance of each oscillator to cover the frequency range of excitation. For


Figure 1. Arrays of 4 piezoelectric oscillators with different connection patterns. (a) is the type of $1\|2\|(3+4)$ while (b) is the pattern of $(1 \| 2)+3+4$.
example, an early work by Shahruz [55] suggested widening the operational frequency band by employing mechanical band-pass filters made of cantilever beams of different resonances. The inclusion of piezoelectric patches in multifrequency converter arrays was proposed by Ferrari et al [24] and was further improved in power amplitudes by Song et al [59]. Theoretical works for analyzing the electromechanical response of arrays of oscillators were provided by Xue et al [72], Lumentut et al [40], Al-Ashtari et al [2] and Meruane and Pichara [44] later. The employment of arrays in various forms for broadband energy harvesting or power enhancement was subsequently applied to many practical situations [3, 13, 25, 32, 35, 45, 47, 62, 69, 70, 74, 75]. Note that only very few of them mentioned above have considered the effect of interface circuits on harvested power in array configurations. Instead, Lien and Shu [37] and Lin et al [39] have studied the parallel and series connection of piezoelectric oscillators attached to the standard and synchronized switch harvesting on inductor (SSHI) interfaces and provided analytic estimates on harvested power for various cases.

However, a disadvantage using array configurations for wideband improvement is that the peaks of harvested power driven at around the resonance of each oscillator are not uniform within the frequency range of interest. For example, the optimal peak of a parallel (series) connection of oscillators has been shown to be driven at around the largest (smallest) resonance of oscillators [39]. But the rest of peaks of power driven at around the resonances of other oscillators drop significantly and are even lower than those generated by a single oscillator (see figure 11 in [66]). As a result, the effective bandwidth is shortened. Hence, there is an urgent need to devise a way for broadband improvement without the cost of power amplitudes. To resolve it, an idea is proposed here accounting for the fact that the optimal power outputs of parallel and series connection of oscillators are driven at around different extreme resonances of oscillators. It motivates us to develop a multi-array device which is able to switch the connection from parallel to series and vice versa. Thus, it has a tailorable operation frequency band which is capable of being shifted to the dominant frequency domain of ambient excitation sources with multifrequency spectra.

This article presents a methodology for investigating the electromechanical response of an array of oscillators with mixed
parallel-series connection in section 2. The standard interface circuit is considered here. The analytic estimates on harvested power are derived and explicitly expressed for two types of connection patterns. They are subsequently validated both numerically and experimentally in section 3. Finally, section 4 discusses several issues concerning optimal arrangements, optimal loads and switching criterions from various connection patterns of arrays. The conclusions are made in section 5.

## 2. Models

Consider an array of $n$ piezoelectric oscillators whose parameter model under harmonic excitation close to the system resonance is presented by [37, 39]

$$
\begin{gather*}
M_{i} \ddot{u}_{i}(t)+\eta_{i} \dot{u}_{i}(t)+K_{i} u_{i}(t)+\Theta_{i} V_{p_{i}}(t)=F_{i}(t),  \tag{1}\\
-\Theta_{i} \dot{u}_{i}(t)+C_{p_{i}} \dot{V}_{p_{i}}(t)=-I_{i}(t),  \tag{2}\\
F_{i}(t)=\bar{F}_{i} \cos \left(w t-\tau_{i}\right), \tag{3}
\end{gather*}
$$

where $u_{i}$ is the displacement of the $i$ th mass $M_{i}, V_{p_{i}}$ and $I_{i}(t)$ are the $i$ th piezoelectric voltage and current flowing into the specified circuit. The applied force $F_{i}(t)$ acting to the $i$ th oscillator is assumed to be harmonic with $\bar{F}_{i}$ as the magnitude, $w$ as the angular frequency (in radians per second) and $\tau_{i}$ as the given phase shift angle. Besides, above $\eta_{i}, K_{i}, \Theta_{i}$ and $C_{p_{i}}$ are the mechanical damping, stiffness, piezoelectric constant and the capacitance of the $i$ th piezoelectric oscillator. Note that the parameter model based on the single degree of freedom assumption has been validated by Erturk and Inman [21] as long as the correction factor is accounted.

Next, these oscillators are electrically linked with two kinds of connection patterns considered here. The p-type (stype) pattern refers to the case where a part of oscillators connected in series (parallel) are subsequently connected to the rest of oscillators in parallel (series). For example, figures 1(a) and (b) represent the p-type and s-type array patterns, respectively. To see it, let the symbols ' $\|$ ' and ' + ' denote the parallel and series connection of oscillators. Then, figure $1(a)$ is the pattern of $1\|2\|(3+4)$, while figure $1(b)$ is the $(1 \| 2)+3+4$ connection pattern. To avoid complexity and enhance readability, the analysis on deriving the estimate of harvested power is carried out for the patterns shown in


Figure 2. (a) Typical waveforms of the equivalent velocity current $I_{p}^{*}(t)$ and the piezoelectric voltage $V_{p}(t)$. (b) Typical waveforms of the equivalent displacement voltage $V_{p}^{*}(t)$ and the piezoelectric voltage $V_{p}(t)$.
figure 1. The general cases will be briefly discussed later. In addition, the methodology proposed here is based on the equivalent current model $[37,66]$ for the p-type pattern and the equivalent voltage model $[39,66]$ for the s-type pattern. But different from the previous cases that oscillators are completely connected in parallel [37] or in series [39], the mixed electric connection of oscillators requires the careful treatment of the voltage division (branch current) related to displacement and voltage (current) for the p-type (s-type) pattern. Further, the proposed analysis can also be applied to other complicated cases with multi-rank electric connection of oscillators, such as the parallel or series connection of many more different mixed patterns of arrays.

Finally, the electronic interface circuit considered here is the standard one which consists of a full-bridge rectifier followed by a large filtering capacitance $C_{e}$, as demonstrated in figure 1 . The terminal load is replaced by a resistor $R_{L}$ and $V_{c}$ is the DC voltage across it. It is assumed that the rectifying bridge is perfect here.

### 2.1. P-type pattern: $1||2||(3+4)$

From the connection type shown in figure 1(a),
$V_{p}=V_{p_{1}}=V_{p_{2}}=V_{p_{3}}+V_{p_{4}}, \quad I_{p}=I_{1}+I_{2}+I_{3}, \quad I_{3}=I_{4}$.

Hence, from equations (2) and (4), an equivalent current model is proposed with the formulation provided by [66]

$$
\begin{equation*}
I_{p}^{*}(t)=C_{p}^{*} \dot{V}_{p}(t)+I_{p}(t), \tag{5}
\end{equation*}
$$

where $I_{p}^{*}(t)$ is the equivalent velocity current due to vibration and $C_{p}^{*}$ is the overall capacitance. Both are defined by

$$
\begin{align*}
& I_{p}^{*}=\Theta_{1} \dot{u}_{1}+\Theta_{2} \dot{u}_{2}+\frac{C_{p_{s}}}{C_{p_{3}}} \Theta_{3} \dot{u}_{3}+\frac{C_{p_{s}}}{C_{p_{4}}} \Theta_{4} \dot{u}_{4},  \tag{6}\\
& C_{p}^{*}=C_{p_{1}}+C_{p_{2}}+C_{p_{s}}, \frac{1}{C_{p_{s}}}=\frac{1}{C_{p_{3}}}+\frac{1}{C_{p_{4}}} . \tag{7}
\end{align*}
$$

Under the steady-state condition, the displacement of each oscillator can be set to be

$$
\begin{equation*}
u_{i}(t)=\bar{u}_{i} \cos \left(w t-\theta_{i}-\tau_{i}\right), \tag{8}
\end{equation*}
$$

where $\bar{u}_{i}$ is the magnitude of displacement and $\theta_{i}$ is the unknown relative phase shift. With this formulation, $I_{p}^{*}(t)$ can also be set to be

$$
\begin{equation*}
I_{p}^{*}(t)=\bar{I}_{p}^{*} \sin (w t-\alpha) \tag{9}
\end{equation*}
$$

where $\bar{I}_{p}^{*}$ is the magnitude of $I_{p}^{*}(t)$ and $\alpha$ is the phase shift. Substituting equation (8) into (6) and using the trigonometric relation, $\bar{I}_{p}^{*}$ and $\bar{u}_{i}$ are associated by
$\bar{I}_{p}^{*}=-\sum_{i=1}^{2} w \Theta_{i} \bar{u}_{i} \mathrm{e}^{\mathrm{j}\left(\alpha-\theta_{i}-\tau_{i}\right)}-\sum_{i=3}^{4} w \Theta_{i} \frac{C_{p_{s}}}{C_{p_{i}}} \bar{u}_{i} \mathrm{e}^{\mathrm{j}\left(\alpha-\theta_{i}-\tau_{i}\right)}$,
where $\mathrm{j}^{2}=-1$.
Next, from the operation of the standard interface shown in figure 1, the typical waveforms of $I_{p}^{*}(t)$ and $V_{p}(t)$, under the steady-state excitation of a single signal, are schematically demonstrated in figure 2(a) [37]. Let $t_{i}$ and $t_{f}$ be two time instants such that $t_{f}-t_{i}=\frac{T}{2}$ where $T$ is the period of
mechanical excitation. Both are related to the vanishing points of $I_{p}^{*}(t)$ as shown in figure 2(a). The relation between the velocity current $\bar{I}_{p}^{*}$ and DC voltage $V_{c}$ can be achieved by the consideration of charge conservation as in equation (5). Indeed, from

$$
\begin{equation*}
\int_{t_{i}}^{t_{f}} I_{p}^{*}(t) \mathrm{d} t=\int_{t_{i}}^{t_{f}} C_{p}^{*} \dot{V}_{p}(t) \mathrm{d} t+\int_{t_{i}}^{t_{f}} I_{p}(t) \mathrm{d} t \tag{11}
\end{equation*}
$$

it gives [56, 58]

$$
\begin{equation*}
\frac{2}{w} \bar{I}_{p}^{*}=2 C_{P}^{*} V_{c}+\left(\frac{\pi}{w} \frac{V_{c}}{R_{L}}\right) . \tag{12}
\end{equation*}
$$

Thus, $\bar{I}_{p}^{*}$ and $V_{c}$ are related by

$$
\begin{equation*}
V_{c}=\left(\frac{R_{L}}{\frac{\pi}{2}+w R_{L} C_{p}^{*}}\right) \bar{I}_{p}^{*} . \tag{13}
\end{equation*}
$$

Next, the elimination of $V_{p_{i}}$ from equation (1) can be realized by the consideration of equations (2), (4) and (7). This results in

$$
\begin{align*}
& M_{1} \ddot{u}_{1}(t)+\eta_{1} \dot{u}_{1}(t)+K_{1} u_{1}(t)+\Theta_{1} V_{p}=F_{1}(t),  \tag{14}\\
& M_{2} \ddot{u}_{2}(t)+\eta_{2} \dot{u}_{2}(t)+K_{2} u_{2}(t)+\Theta_{2} V_{p}=F_{2}(t),  \tag{15}\\
& M_{3} \ddot{u}_{3}(t)+\eta_{3} \dot{u}_{3}(t)+K_{3} u_{3}(t)+\frac{\Theta_{3}}{C_{p_{3}}} \\
& \times\left[\left(1-\frac{C_{p_{s}}}{C_{p_{3}}}\right) \Theta_{3} u_{3}(t)-\frac{C_{p_{s}}}{C_{p_{4}}} \Theta_{4} u_{4}(t)+C_{p_{s}} V_{p}\right]=F_{3}(t),  \tag{16}\\
& M_{4} \ddot{u}_{4}(t)+\eta_{4} \dot{u}_{4}(t)+K_{4} u_{4}(t)+\frac{\Theta_{4}}{C_{p_{4}}} \\
& \times\left[-\frac{C_{p_{s}}}{C_{p_{3}}} \Theta_{3} u_{3}(t)+\left(1-\frac{C_{p_{s}}}{C_{p_{4}}}\right) \Theta_{4} u_{4}(t)+C_{p_{s}} V_{p}\right]=F_{4}(t) . \tag{17}
\end{align*}
$$

Consider the balance of generalized energy enforced by the multiplication of $I_{p}^{*}(t)$ to equations (14)-(17). For example, the integration of equation (16) multiplied by $I_{p}^{*}(t)$ over the time period from $t_{i}$ to $t_{f}$ provides

$$
\begin{align*}
\{[ & {\left[K_{3}-w^{2} M_{3}+\frac{\Theta_{3}^{2}}{C_{p_{3}}}\left(1-\frac{C_{p_{s}}}{C_{p_{3}}}\right)\right] } \\
& \left.\times \sin \left(\alpha-\theta_{3}-\tau_{3}\right)+\eta_{3} w \cos \left(\alpha-\theta_{3}-\tau_{3}\right)\right\} \bar{u}_{3} \\
& -\frac{C_{p_{s}}}{C_{p_{3}} C_{p_{4}}} \Theta_{3} \Theta_{4} \sin \left(\alpha-\theta_{4}-\tau_{4}\right) \bar{u}_{4} \\
& -\left[\Theta_{3} \frac{C_{p_{s}}}{C_{p_{3}}} \frac{2 R_{L}}{\left(\frac{\pi}{2}+w R_{L} C_{p}^{*}\right)^{2}}\right] \bar{I}_{p}^{*}=\bar{F}_{3} \sin \left(\alpha-\tau_{3}\right) . \tag{18}
\end{align*}
$$

In addition, we consider differentiating equation (16) with respect to time $t$ and note that $\dot{V}_{p}$ is replaced by $I_{p}^{*}$ and $I_{p}$
through equation (5). Then, the integration of the mentioned formulation from $t_{i}$ to $t_{f}$ gives

$$
\begin{align*}
& \left\{\left[K_{3}-w^{2} M_{3}+\frac{\Theta_{3}^{2}}{C_{p_{3}}}\left(1-\frac{C_{p_{s}}}{C_{p_{3}}}\right)\right]\right. \\
& \left.\quad \times \cos \left(\alpha-\theta_{3}-\tau_{3}\right)-\eta_{3} w \sin \left(\alpha-\theta_{3}-\tau_{3}\right)\right\} \bar{u}_{3} \\
& \\
& \quad-\frac{C_{p_{s}}}{C_{p_{3}} C_{p_{4}}} \Theta_{3} \Theta_{4} \cos \left(\alpha-\theta_{4}-\tau_{4}\right) \bar{u}_{4}  \tag{19}\\
& \\
& \quad-\left[\Theta_{3} \frac{C_{p_{s}}}{C_{p_{3}}} \frac{R_{L}}{\left(\frac{\pi}{2}+w R_{L} C_{p}^{*}\right)}\right] \bar{I}_{p}^{*}=\bar{F}_{3} \cos \left(\alpha-\tau_{3}\right) .
\end{align*}
$$

Define

$$
\begin{array}{cc}
\tilde{I}_{1}=w \Theta_{1} \bar{u}_{1} \mathrm{e}^{\mathrm{j}\left(-\theta_{1}-\tau_{1}\right)}, & \tilde{V}_{1}=\frac{\bar{F}_{1}}{\Theta_{1}} \mathrm{e}^{-\mathrm{j} \tau_{1}}, \\
\tilde{I}_{2}=w \Theta_{2} \bar{u}_{2} \mathrm{e}^{\mathrm{j}\left(-\theta_{2}-\tau_{2}\right)}, & \tilde{V}_{2}=\frac{\bar{F}_{2}}{\Theta_{2}} \mathrm{e}^{-\mathrm{j} \tau_{2}}, \\
\tilde{I}_{3}=\left(\frac{C_{p_{s}}}{C_{p_{3}}}\right) w \Theta_{3} \bar{u}_{3} \mathrm{e}^{\mathrm{j}\left(-\theta_{3}-\tau_{3}\right)}, \tilde{V}_{3}=\left(\frac{C_{p_{3}}}{C_{p_{s}}}\right) \frac{\bar{F}_{3}}{\Theta_{3}} \mathrm{e}^{-\mathrm{j} \tau_{3}}, \\
\tilde{I}_{4}=\left(\frac{C_{p_{s}}}{C_{p_{4}}}\right) w \Theta_{4} \bar{u}_{4} \mathrm{e}^{\mathrm{j}\left(-\theta_{4}-\tau_{4}\right)}, \tilde{V}_{4}=\left(\frac{C_{p_{4}}}{C_{p_{s}}}\right) \frac{\bar{F}_{4}}{\Theta_{4}} \mathrm{e}^{-\mathrm{j} \tau_{4}} . \tag{20}
\end{array}
$$

With the help of equations (10) and (20), the combination of equations (18) and (19) results in

$$
\begin{align*}
& \mathrm{j} Z_{1}^{\text {std }} \tilde{I}_{1}+\mathrm{j} Z_{1}^{\text {std }} \tilde{I}_{2}+\left[\left(\frac{C_{p_{3}}}{C_{p_{s}}}\right)^{2}\left(\frac{K_{3}}{w \Theta_{3}^{2}}-\frac{w M_{3}}{\Theta_{3}^{2}}+\mathrm{j} \frac{\eta_{3}}{\Theta_{3}^{2}}\right)\right. \\
& \left.\quad+\frac{1}{w C_{p_{s}}}\left(\frac{C_{p_{3}}}{C_{p_{s}}}-1\right)+\mathrm{j} Z_{1}^{\text {std }}\right] \tilde{I}_{3} \\
& \quad+\left(-\frac{1}{w C_{p_{s}}}+\mathrm{j} Z_{1}^{\text {std }}\right) \tilde{I}_{4}=\tilde{V}_{3}, \tag{21}
\end{align*}
$$

where $Z_{1}^{\text {std }}$ is the equivalent load impedance of the current type for the case of the standard interface circuit and is defined by $[39,66]$

$$
\begin{equation*}
Z_{1}^{\text {std }}=\frac{2 R_{L}}{\left(\frac{\pi}{2}+w R_{L} C_{p}^{*}\right)^{2}}-\mathrm{j} \frac{R_{L}}{\left(\frac{\pi}{2}+w R_{L} C_{p}^{*}\right)} \tag{22}
\end{equation*}
$$

Finally, these steps used for deriving equation (21) can be applied to equations (14), (15) and (17), giving rise to

$$
\begin{align*}
& {\left[\left(\frac{K_{1}}{w \Theta_{1}^{2}}-\frac{w M_{1}}{\Theta_{1}^{2}}+\mathrm{j} \frac{\eta_{1}}{\Theta_{1}^{2}}\right)+\mathrm{j} Z_{1}^{\text {std }}\right] \tilde{I}_{1}} \\
& \quad+\mathrm{j} Z_{1}^{\text {std }} \tilde{I}_{2}+\mathrm{j} Z_{1}^{\text {std }} \tilde{I}_{3}+\mathrm{j} Z_{1}^{\text {std }} \tilde{I}_{4}=\tilde{V}_{1} \tag{23}
\end{align*}
$$

and

$$
\begin{align*}
& \mathrm{j} Z_{1}^{\text {std }} \tilde{I}_{1}+\left[\left(\frac{K_{2}}{w \Theta_{2}^{2}}-\frac{w M_{2}}{\Theta_{2}^{2}}+\mathrm{j} \frac{\eta_{2}}{\Theta_{2}^{2}}\right)+\mathrm{j} Z_{1}^{\text {std }}\right] \tilde{I}_{2} \\
& \quad+\mathrm{j} Z_{1}^{\text {std }} \tilde{I}_{3}+\mathrm{j} Z_{1}^{\text {std }} \tilde{I}_{4}=\tilde{V}_{2} \tag{24}
\end{align*}
$$

and

$$
\begin{align*}
& \mathrm{j} Z_{1}^{\mathrm{std}} \tilde{I}_{1}+\mathrm{j} Z_{1}^{\mathrm{std}} \tilde{I}_{2}+\left(-\frac{1}{w C_{p_{s}}}+\mathrm{j} Z_{1}^{\text {std }}\right) \tilde{I}_{3} \\
& \quad+\left[\left(\frac{C_{p_{4}}}{C_{p_{s}}}\right)^{2}\left(\frac{K_{4}}{w \Theta_{4}^{2}}-\frac{w M_{4}}{\Theta_{4}^{2}}+\mathrm{j} \frac{\eta_{4}}{\Theta_{4}^{2}}\right)\right. \\
& \left.\quad+\frac{1}{w C_{P_{s}}}\left(\frac{C_{p_{4}}}{C_{p_{s}}}-1\right)+\mathrm{j} Z_{1}^{\text {std }}\right] \tilde{I}_{4}=\tilde{V}_{4} . \tag{25}
\end{align*}
$$

These results can be simply expressed in terms of the matrix formulation of generalized Ohm's law. In other words,

$$
\begin{equation*}
\tilde{\mathbf{V}}=\tilde{\mathbf{Z}} \tilde{\mathbf{I}}, \quad \tilde{\mathbf{V}}=\left(\tilde{V}_{\alpha}\right), \quad \tilde{\mathbf{I}}=\left(\tilde{I}_{\beta}\right), \quad \tilde{\mathbf{Z}}=\left(\tilde{Z}_{\alpha \beta}\right) \tag{26}
\end{equation*}
$$

where $\tilde{\mathbf{Z}}$ is the generalized impedance matrix whose components $\tilde{Z}_{\alpha \beta}$ can be explicitly obtained from the coefficients of $\tilde{I}_{\beta}$ in equations (21), (23)-(25) (see equation (55) in the general case). Note that the diagonal terms of $\tilde{\mathbf{Z}}$ are associated to the system parameters like $M_{i}, \eta_{i}, K_{i}, \Theta_{i}, C_{p_{i}}$ and $Z_{1}^{\text {std }}$, while the off-diagonal terms of it mainly depend on the equivalent load impedance $Z_{1}^{\text {std }}$.

Finally, the average harvested power is

$$
\begin{equation*}
P=\frac{V_{c}^{2}}{R_{L}} \tag{27}
\end{equation*}
$$

and from equations (10), (13) and the definition of $\tilde{I}_{i}$ in equation (20), the harvested DC voltage $V_{c}$ is

$$
\begin{equation*}
V_{c}=\left(\frac{R_{L}}{\frac{\pi}{2}+w R_{L} C_{p}^{*}}\right)\left|\tilde{I}_{1}+\tilde{I}_{2}+\tilde{I}_{3}+\tilde{I}_{4}\right| . \tag{28}
\end{equation*}
$$

Each $\tilde{I}_{i}$ can be determined by matrix inversion of generalized Ohm's law defined by equation (26).

### 2.2. S-type pattern: $(1|\mid 2)+3+4$

From the connection type shown in figure 1(b), it gives
$V_{p}=V_{p_{1}}+V_{p_{3}}+V_{p_{4}}, \quad V_{p_{1}}=V_{p_{2}}, \quad I_{p}=I_{1}+I_{2}=I_{3}=I_{4}$.

Then, from equations (2) and (29), an equivalent voltage model is given by

$$
\begin{equation*}
\dot{V}_{p}^{*}(t)=\dot{V}_{p}(t)+\frac{1}{C_{p}^{*}} I_{p}(t), \tag{30}
\end{equation*}
$$

where $V_{p}^{*}$ is the equivalent displacement voltage due to vibration and $C_{p}^{*}$ is the overall capacitance. Both are defined by

$$
\begin{equation*}
V_{p}^{*}=\frac{\Theta_{1}}{C_{p_{1}}+C_{p_{2}}} u_{1}+\frac{\Theta_{2}}{C_{p_{1}}+C_{p_{2}}} u_{2}+\frac{\Theta_{3}}{C_{p_{3}}} u_{3}+\frac{\Theta_{4}}{C_{p_{4}}} u_{4}, \tag{31}
\end{equation*}
$$

$$
\begin{equation*}
\frac{1}{C_{p}^{*}}=\frac{1}{C_{p_{1}}+C_{p_{2}}}+\frac{1}{C_{p_{3}}}+\frac{1}{C_{p_{4}}} . \tag{32}
\end{equation*}
$$

Under the steady-state condition, the displacement of each oscillator can be set to be the form given by equation (8). Thus, from equation (31), $V_{p}^{*}$ can be set to be

$$
\begin{equation*}
V_{p}^{*}(t)=\bar{V}_{p}^{*} \cos (w t-\alpha) \tag{33}
\end{equation*}
$$

where $\bar{V}_{p}^{*}$ is the magnitude of $V_{p}^{*}(t)$ and $\alpha$ is the phase shift. Substituting equation (8) into (31) and using the trigonometric relation, we have

$$
\begin{equation*}
\bar{V}_{p}^{*}=\sum_{i=1}^{2} \frac{\Theta_{i}}{C_{p_{1}}+C_{p_{2}}} \bar{u}_{i} \mathrm{e}^{\mathrm{j}\left(\alpha-\theta_{i}-\tau_{i}\right)}+\sum_{i=3}^{4} \frac{\Theta_{i}}{C_{p_{i}}} \bar{u}_{i} \mathrm{e}^{\mathrm{j}\left(\alpha-\theta_{i}-\tau_{i}\right)} . \tag{34}
\end{equation*}
$$

Next, from the characteristics of the standard interface circuit shown in figure 1, the typical waveforms of $V_{p}^{*}(t)$ and $V_{p}(t)$, under the steady-state excitation of a single signal, are schematically illustrated in figure 2(b) [39]. Let $t_{i}$ and $t_{f}$ be two time instants such that the difference of these two is equal to one half of the period of mechanical excitation. In addition, both are related to the extreme values of $V_{p}^{*}(t)$ as illustrated in figure 2(b). Similar to the previous approach, the relation between the DC voltage $V_{c}$ and the equivalent displacement voltage $\bar{V}_{p}^{*}$ can be obtained by considering the principle of charge conservation. Indeed, the time integration of equation (30) from $t_{i}$ to $t_{f}$ gives $[56,58]$

$$
\begin{equation*}
2 \bar{V}_{p}^{*}=2 V_{c}+\frac{1}{C_{p}^{*}}\left(\frac{\pi}{w} \frac{V_{c}}{R_{L}}\right), \tag{35}
\end{equation*}
$$

which in turn provides

$$
\begin{equation*}
V_{c}=\left(\frac{w R_{L} C_{p}^{*}}{\frac{\pi}{2}+w R_{L} C_{p}^{*}}\right) \bar{V}_{p}^{*} . \tag{36}
\end{equation*}
$$

Next, using equations (2) and (29) to eliminate $V_{p_{i}}$ from equation (1) gives

$$
\begin{align*}
& \frac{\mathrm{d}}{\mathrm{~d} t}\left[M_{1} \ddot{u}_{1}(t)+\eta_{1} \dot{u}_{1}(t)+K_{1} u_{1}(t)\right] \\
& \quad+\Theta_{1}\left[\frac{\Theta_{1} \dot{u}_{1}(t)+\Theta_{2} \dot{u}_{2}(t)}{C_{p_{1}}+C_{p_{2}}}-\frac{I_{p}}{C_{p_{1}}+C_{p_{2}}}\right]=\dot{F}_{1}(t), \tag{37}
\end{align*}
$$

$$
\begin{align*}
& \frac{\mathrm{d}}{\mathrm{~d} t}\left[M_{2} \ddot{u}_{2}(t)+\eta_{2} \dot{u}_{2}(t)+K_{2} u_{2}(t)\right] \\
& \quad+\Theta_{2}\left[\frac{\Theta_{1} \dot{u}_{1}(t)+\Theta_{2} \dot{u}_{2}(t)}{C_{p_{1}}+C_{p_{2}}}-\frac{I_{p}}{C_{p_{1}}+C_{p_{2}}}\right]=\dot{F}_{2}(t), \tag{38}
\end{align*}
$$

$$
\frac{\mathrm{d}}{\mathrm{~d} t}\left[M_{3} \ddot{u}_{3}(t)+\eta_{3} \dot{u}_{3}(t)+K_{3} u_{3}(t)\right]
$$

$$
\begin{equation*}
+\Theta_{3}\left[\frac{\Theta_{3} \dot{u}_{3}(t)}{C_{p_{3}}}-\frac{I_{p}}{C_{p_{3}}}\right]=\dot{F}_{3}(t) \tag{39}
\end{equation*}
$$

$$
\begin{align*}
& \frac{\mathrm{d}}{\mathrm{~d} t}\left[M_{4} \ddot{u}_{4}(t)+\eta_{4} \dot{u}_{4}(t)+K_{4} u_{4}(t)\right] \\
& \quad+\Theta_{4}\left[\frac{\Theta_{4} \dot{u}_{4}(t)}{C_{p_{4}}}-\frac{I_{p}}{C_{p_{4}}}\right]=\dot{F}_{4}(t) . \tag{40}
\end{align*}
$$

The balance of generalized energy is enforced by the multiplication of $V_{p}^{*}(t)$ to equations (37)-(40). For example, the time integration of equation (37) multiplied by $V_{p}^{*}(t)$ from $t_{i}$ to $t_{f}$ provides

$$
\begin{align*}
& {\left[\left(\begin{array}{l}
\left.K_{1}-M_{1} w^{2}+\frac{\Theta_{1}^{2}}{C_{p_{1}}+C_{p_{2}}}\right) \\
\left.\quad \times \sin \left(\alpha-\theta_{1}-\tau_{1}\right)+\eta_{1} w \cos \left(\alpha-\theta_{1}-\tau_{1}\right)\right] \bar{u}_{1} \\
\quad+\frac{\Theta_{1} \Theta_{2}}{C_{p_{1}}+C_{p_{2}}} \sin \left(\alpha-\theta_{2}-\tau_{2}\right) \bar{u}_{2} \\
\quad+\left[\left(\frac{\Theta_{1}}{C_{p_{1}}+C_{p_{2}}}\right) \frac{2 w R_{L} C_{p}^{* 2}}{\left(\frac{\pi}{2}+w R_{L} C_{p}^{*}\right)^{2}}\right] \bar{V}_{p}^{*}=\bar{F}_{1} \sin \left(\alpha-\tau_{1}\right)
\end{array} .\right.\right.}
\end{align*}
$$

In addition, the integration of equation (37) over the time period from $t_{i}$ to $t_{f}$ provides

$$
\begin{align*}
& {\left[\left(K_{1}-M_{1} w^{2}+\frac{\Theta_{1}^{2}}{C_{p_{1}}+C_{p_{2}}}\right)\right.} \\
& \left.\quad \times \cos \left(\alpha-\theta_{1}-\tau_{1}\right)-\eta_{1} w \sin \left(\alpha-\theta_{1}-\tau_{1}\right)\right] \bar{u}_{1} \\
& \quad+\frac{\Theta_{1} \Theta_{2}}{C_{p_{1}}+C_{p_{2}}} \cos \left(\alpha-\theta_{2}-\tau_{2}\right) \bar{u}_{2} \\
& \quad-\left[\left(\frac{\Theta_{1}}{C_{p_{1}}+C_{p_{2}}}\right) \frac{\pi C_{p}^{*}}{\pi+2 w R_{L} C_{p}^{*}}\right] \bar{V}_{p}^{*}=\bar{F}_{1} \cos \left(\alpha-\tau_{1}\right) \tag{42}
\end{align*}
$$

Define

$$
\begin{array}{cc}
\hat{V}_{1}=\left(\frac{\Theta_{1}}{C_{p_{1}}+C_{p_{2}}}\right) \bar{u}_{1} \mathrm{e}^{\mathrm{j}\left(-\theta_{1}-\tau_{1}\right)}, & \hat{Q}_{1}=\left(\frac{C_{p_{1}}+C_{p_{2}}}{\Theta_{1}}\right) \bar{F}_{1} \mathrm{e}^{-\mathrm{j} \tau_{1}}, \\
\hat{V}_{2}=\left(\frac{\Theta_{2}}{C_{p_{1}}+C_{p_{2}}}\right) \bar{u}_{2} \mathrm{e}^{\mathrm{j}\left(-\theta_{2}-\tau_{2}\right)}, & \hat{Q}_{2}=\left(\frac{C_{p_{1}}+C_{p_{2}}}{\Theta_{2}}\right) \bar{F}_{2} \mathrm{e}^{-\mathrm{j} \tau_{2}}, \\
\hat{V}_{3}=\left(\frac{\Theta_{3}}{C_{p_{3}}}\right) \bar{u}_{3} \mathrm{e}^{\mathrm{j}\left(-\theta_{3}-\tau_{3}\right)}, & \hat{Q}_{3}=\left(\frac{C_{p_{3}}}{\Theta_{3}} \bar{F}_{3} \mathrm{e}^{-\mathrm{j} \tau_{3}},\right. \\
\hat{V}_{4}=\left(\frac{\Theta_{4}}{C_{p_{4}}}\right) \bar{u}_{4} \mathrm{e}^{\mathrm{j}\left(-\theta_{4}-\tau_{4}\right)}, & \hat{Q}_{4}=\left(\frac{C_{p_{4}}}{\Theta_{4}}\right) \bar{F}_{4} \mathrm{e}^{-\mathrm{j} \tau_{4}} . \tag{43}
\end{array}
$$

With the help of equations (34) and (43), the combination of equations (41) and (42) results in

$$
\begin{align*}
& {\left[\left(\frac{C_{p_{1}}+C_{p_{2}}}{\Theta_{1}}\right)^{2}\left(K_{1}-M_{1} w^{2}+\mathrm{j} w \eta_{1}\right)+\left(C_{p_{1}}+C_{p_{2}}\right)\right.} \\
& \left.\quad-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}}\right] \hat{V}_{1} \\
& \quad+\left[\left(C_{p_{1}}+C_{p_{2}}\right)-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}}\right] \hat{V}_{2}-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{3} \\
& \quad-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{4}=\hat{Q}_{1} \tag{44}
\end{align*}
$$

where $Z_{2}^{\text {std }}$ is the equivalent load impedance of the voltage type for the case of the standard interface circuit and is defined by $[39,66]$

$$
\begin{equation*}
\frac{1}{Z_{2}^{\text {std }}}=\frac{2 w^{2} R_{L} C_{p}^{* 2}}{\left(\frac{\pi}{2}+w R_{L} C_{p}^{*}\right)^{2}}+\mathrm{j}\left(\frac{\pi w C_{p}^{*}}{\pi+2 w R_{L} C_{p}^{*}}\right) \tag{45}
\end{equation*}
$$

Finally, the steps used for deriving equation (44) can be applied to equations (38)-(40), and this gives

$$
\begin{align*}
& {\left[\left(C_{p_{1}}+C_{p_{2}}\right)-\frac{1}{\mathrm{j} w Z_{2}^{\mathrm{std}}}\right] \hat{V}_{1}} \\
& \quad+\left[\left(\frac{C_{p_{1}}+C_{p_{2}}}{\Theta_{2}}\right)^{2}\left(K_{2}-w^{2} M_{2}+\mathrm{j} w \eta_{2}\right)\right. \\
& \left.\quad+\left(C_{p_{1}}+C_{p_{2}}\right)-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}}\right] \hat{V}_{2} \\
& \quad-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{3}-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{4}=\hat{Q}_{2} \tag{46}
\end{align*}
$$

and

$$
\begin{align*}
&- \frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{1} \\
&-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{2} \\
&+\left[\left(\frac{C_{p_{3}}}{\Theta_{3}}\right)^{2}\left(K_{3}-w^{2} M_{3}+\mathrm{j} w \eta_{3}\right)+C_{p_{3}}-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}}\right] \hat{V}_{3}  \tag{47}\\
&-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{4}=\hat{Q}_{3},
\end{align*}
$$

and
$-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{1}-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{2}-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}} \hat{V}_{3}$
$+\left[\left(\frac{C_{p_{4}}}{\Theta_{4}}\right)^{2}\left(K_{4}-w^{2} M_{4}+\mathrm{j} w \eta_{4}\right)+C_{p_{4}}-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}}\right] \hat{V}_{4}=\hat{Q}_{4}$.

Furthermore, these results can be simplified in terms of the matrix formulation of charging on capacitance. In other words,

$$
\begin{equation*}
\hat{\mathbf{Q}}=\hat{\mathbf{C}} \hat{\mathbf{V}}, \quad \hat{\mathbf{Q}}=\left(\hat{Q}_{\alpha}\right), \quad \hat{\mathbf{V}}=\left(\hat{V}_{\beta}\right), \quad \hat{\mathbf{C}}=\left(\hat{C}_{\alpha \beta}\right) \tag{49}
\end{equation*}
$$

where $\hat{\mathbf{C}}$ is the generalized capacitance matrix whose components $\hat{C}_{\alpha \beta}$ can be explicitly obtained from the coefficients of $\hat{V}_{\beta}$ in equations (44), (46)-(48) (see equation (60) in the general case). Note that the diagonal terms of $\hat{\mathbf{C}}$ are associated to the system parameters like $M_{i}, \eta_{i}, K_{i}, \Theta_{i}, C_{p_{i}}$ and $Z_{2}^{\text {std }}$, while the off-diagonal terms of it mainly depend on the equivalent load impedance $Z_{2}^{\text {std }}$.

Finally, the average harvested power is

$$
\begin{equation*}
P=\frac{V_{c}^{2}}{R_{L}} \tag{50}
\end{equation*}
$$

and from equations (34), (36) and the definition of $\hat{V}_{i}$ in equation (43), the harvested DC voltage $V_{c}$ is

$$
\begin{equation*}
V_{c}=\left(\frac{w R_{L} C_{p}^{*}}{\frac{\pi}{2}+w R_{L} C_{p}^{*}}\right)\left|\hat{V}_{1}+\hat{V}_{2}+\hat{V}_{3}+\hat{V}_{4}\right| . \tag{51}
\end{equation*}
$$

Each $\hat{V}_{i}$ is obtained by inverting the matrix formulation of charging on capacitance defined by equation (49).

### 2.3. General formulations

2.3.1. P-type. Suppose there are $m$ oscillators connected in series and they are subsequently connected in parallel with the rest $(n-m)$ oscillators. Without loss of generality, consider the pattern of $1\|2\| \cdots\|(n-m)\|$ $[(n-m+1)+(n-m+2)+\cdots+n]$. The analytic estimate of harvested power generated by this array structure can be derived using the equivalent current model proposed in section 2.1. Indeed, define

$$
\begin{align*}
& \tilde{V}_{\alpha}=\frac{\bar{F}_{\alpha}}{\Theta_{\alpha}} \mathrm{e}^{-\mathrm{j} \tau_{\alpha}}, \quad \alpha=1, \cdots, n-m \\
& \tilde{V}_{\beta}=\left(\frac{C_{p_{\beta}}}{C_{P_{s}}}\right) \bar{F}_{\bar{F}_{\beta}} \mathrm{e}_{\beta}^{-\mathrm{j} \tau_{\beta}}, \quad \beta=n-m+1, \cdots, n, \tag{52}
\end{align*}
$$

and

$$
\begin{align*}
& \tilde{I}_{\alpha}=w \Theta_{\alpha} \bar{u}_{\alpha} \mathrm{e}^{\mathrm{j}\left(-\theta_{\alpha}-\tau_{\alpha}\right)}, \quad \alpha=1, \cdots, n-m \\
& \tilde{I}_{\beta}=\left(\frac{C_{p_{s}}}{C_{p_{\beta}}}\right) w \Theta_{\beta} \bar{u}_{\beta} \mathrm{e}^{\mathrm{j}\left(-\theta_{\beta}-\tau_{\beta}\right)}, \quad \beta=n-m+1, \cdots, n, \tag{53}
\end{align*}
$$

where $C_{p_{s}}$ is the overall capacitance of the series connection of the last $m$ oscillators; i.e.,

$$
\begin{equation*}
\frac{1}{C_{p_{s}}}=\frac{1}{C_{p_{(n-m+1)}}}+\frac{1}{C_{P_{(n-m+2)}}}+\cdots+\frac{1}{C_{P_{n}}} \tag{54}
\end{equation*}
$$

The matrix formulation of generalized Ohm's law introduced in equation (26) still holds if the impedance matrix $\tilde{\mathbf{Z}}$ is replaced by

$$
\begin{align*}
\tilde{Z}_{\alpha \alpha}= & \frac{1}{w \Theta_{\alpha}^{2}}\left(K_{\alpha}-w^{2} M_{\alpha}+\mathrm{j} w \eta_{\alpha}\right)+\mathrm{j} Z_{1}^{\text {std }}, \\
& \alpha=1, \cdots,(n-m), \\
\tilde{Z}_{\beta \beta}= & \frac{1}{w \Theta_{\beta}^{2}}\left(\frac{C_{p_{\beta}}}{C_{p_{s}}}\right)^{2}\left(K_{\beta}-w^{2} M_{\beta}+\mathrm{j} w \eta_{\beta}\right) \\
& +\frac{1}{w C_{p_{s}}}\left(\frac{C_{p_{\beta}}}{C_{p_{s}}}-1\right)+\mathrm{j} Z_{1}^{\text {std }}, \quad \beta=(n-m+1), \cdots, n, \\
\tilde{Z}_{\beta \gamma}= & \frac{-1}{w C_{p_{s}}}+\mathrm{j} Z_{1}^{\text {std }}, \quad \beta, \gamma=(n-m+1), \cdots, n, \beta \neq \gamma, \\
\tilde{Z}_{\lambda \eta}= & \mathrm{j} Z_{1}^{\text {std }}, \quad \lambda \neq \eta, \quad \text { otherwise } . \tag{55}
\end{align*}
$$

Finally, the harvested average power is

$$
\begin{equation*}
P=\frac{V_{c}^{2}}{R_{L}}, \quad V_{c}=\left(\frac{R_{L}}{\frac{\pi}{2}+w R_{L} C_{p}^{*}}\right)\left|\tilde{I}_{1}+\tilde{I}_{2}+\cdots+\tilde{I}_{n}\right| . \tag{56}
\end{equation*}
$$

2.3.2. S-type. On the other hand, suppose the first $m$ oscillators are connected in parallel and are subsequently connected in series with the rest $(n-m)$ oscillators. It is the pattern of the type $(1\|2\| \cdots \| m)+(m+1)+\cdots+n$. The analytic estimate of harvested power generated by this array configuration is able to be derived based on the equivalent voltage model proposed in section 2.2. Indeed, define

$$
\begin{align*}
& \hat{Q}_{\alpha}=\left(\frac{C_{p_{p}}}{\Theta_{\alpha}}\right) \bar{F}_{\alpha} \mathrm{e}^{-\mathrm{j} \tau_{\alpha}}, \quad \alpha=1, \cdots, m, \\
& \hat{Q}_{\beta}=\left(\frac{C_{p_{\beta}}}{\Theta_{\beta}}\right) \bar{F}_{\beta} \mathrm{e}^{-\mathrm{j} \tau_{\beta}}, \quad \beta=m+1, \cdots, n, \tag{57}
\end{align*}
$$

and

$$
\begin{align*}
& \hat{V}_{\alpha}=\left(\frac{\Theta_{\alpha}}{C_{p_{p}}}\right) \bar{u}_{\alpha} \mathrm{e}^{\mathrm{j}\left(-\theta_{\alpha}-\tau_{\alpha}\right)}, \quad \alpha=1, \cdots, m, \\
& \hat{V}_{\beta}=\left(\frac{\Theta_{\beta}}{C_{p_{\beta}}}\right) \bar{u}_{\beta} \mathrm{e}^{\mathrm{j}\left(-\theta_{\beta}-\tau_{\beta}\right)}, \quad \beta=m+1, \cdots, n, \tag{58}
\end{align*}
$$

where $C_{p_{p}}$ is the overall capacitance of the parallel connection of the first $m$ oscillators and is defined by

$$
\begin{equation*}
C_{p_{p}}=C_{p_{1}}+C_{p_{2}}+\cdots+C_{p_{m}} . \tag{59}
\end{equation*}
$$

The generalized matrix formulation of charging on capacitance introduced by equation (49) still holds if the capacitance matrix $\hat{\mathbf{C}}$ is replaced with

$$
\begin{align*}
\hat{C}_{\alpha \alpha}= & \left(\frac{C_{p_{p}}}{\Theta_{\alpha}}\right)^{2}\left(K_{\alpha}-w^{2} M_{\alpha}+\mathrm{j} w \eta_{\alpha}\right) \\
& +C_{p_{p}}-\frac{1}{\mathrm{j} w Z_{2}^{\mathrm{std}}}, \quad \alpha=1, \cdots, m, \\
\hat{C}_{\beta \beta}= & \left(\frac{C_{p_{\beta}}}{\Theta_{\beta}}\right)^{2}\left(K_{\beta}-w^{2} M_{\beta}+\mathrm{j} w \eta_{\beta}\right) \\
& +C_{p_{\beta}}-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}}, \quad \beta=m+1, \cdots, n, \\
\hat{C}_{\alpha \gamma}= & C_{p_{p}}-\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}}, \quad \alpha, \gamma=1, \cdots, m, \alpha \neq \gamma, \\
\hat{C}_{\lambda \eta}= & -\frac{1}{\mathrm{j} w Z_{2}^{\text {std }}}, \quad \lambda \neq \eta, \text { otherwise. } \tag{60}
\end{align*}
$$

Finally, the harvested average power is

$$
\begin{equation*}
P=\frac{V_{c}^{2}}{R_{L}}, \quad V_{c}=\left(\frac{w R_{L} C_{p}^{*}}{\frac{\pi}{2}+w R_{L} C_{p}^{*}}\right)\left|\hat{V}_{1}+\hat{V}_{2}+\cdots+\hat{V}_{n}\right| . \tag{61}
\end{equation*}
$$



Figure 3. Schematic presentation of an equivalent circuit model for three piezoelectric oscillators attached to the standard interface. The electric connection of oscillators is the p-type $(1+2) \| 3$ pattern as in (a) and the s-type $(1 \| 2)+3$ pattern as in (b).


Figure 4. Numerical validation of the analytic estimate of harvested power against frequency evaluated at the optimal load. (a) is the arrangement of arrays of the types: $(1+2+3),(1+2) \| 3$ and $(1\|2\| 3)$. (b) is the arrangement of arrays of the types: $(1+2+3),(1 \| 2)+3$ and (1||2||3).

## 3. Validation

### 3.1. Numerical validation

Consider a model device where three piezoelectric oscillators are involved. It is well known that the parameter model described by equations (1)-(3) can be interpreted from the concept of circuitry [37]. Indeed, a standard $R^{*} L^{*} C^{*}$ equivalent circuit model can be constructed by setting $R_{i}^{*}=\frac{\eta_{i}}{\Theta_{i}^{2}}$ as resistance, $L_{i}^{*}=\frac{M_{i}}{\Theta_{i}^{2}}$ as inductance, $C_{i}^{*}=\frac{\Theta_{i}^{2}}{K_{i}}$ as capacitance and $V_{\text {source }}^{i}=\frac{\bar{F}_{i}}{\Theta_{i}}$ as voltage source $[66,73]$. For example, figures 3(a) and (b) schematically present the equivalent circuit models endowed with the standard interface circuit for the p-type $(1+2)|\mid 3$ and the s-type $(1|\mid 2)+3$ patterns, respectively.

The parameters used for simulation are $M_{1}=0.001(\mathrm{Kg})$, $M_{2}=0.00092(\mathrm{Kg}), M_{3}=0.000865(\mathrm{Kg}), \eta_{1}=\eta_{2}=\eta_{3}=$ $0.0225\left(\mathrm{~N} \mathrm{~s} \mathrm{~m}^{-1}\right), \quad K_{1}=K_{2}=K_{3}=584.1 \quad\left(\mathrm{~N} \mathrm{~m}^{-\uparrow}\right), \quad \Theta_{1}=$ $\Theta_{2}=\Theta_{3}=0.0007\left(\mathrm{~N}^{2}\right.$ Volt $\left.^{-1}\right), \quad C_{p_{1}}=C_{p_{2}}=C_{p_{3}}=10.57$
$(\mathrm{nF}), \bar{F}_{1}=0.005958(\mathrm{~N}), \bar{F}_{2}=0.005645(\mathrm{~N}), \bar{F}_{3}=0.00541$ $(\mathrm{N})$, and $\tau_{i}=0$. The simulation results based on the conventional software PSpice are illustrated in figure 4 where power is plotted against frequency evaluated at the optimal load. The analytic predictions by the proposed estimates are presented by various continuous color lines, while the circuit simulations are marked by distinct colored points. Two remarks are made here. First, both the simulations and predictions are in good agreement. Hence, it is concluded that the proposed analytic estimates are suitable for the performance evaluation of mixed parallel-series connection of multiple piezoelectric energy harvesters and therefore, provide a useful guidance for design analysis. Second, there is a significant power drop in the central range of frequency shown in figure 4(a) since the peak power generated by the array of $(1+2) \| 3$ pattern is overlapped with that generated by the array of $(1||2|| 3)$ pattern. Instead, the arrangement of arrays shown in figure 4(b) is suitable. This raises a question about the ideal array arrangements so that the peak power is


Figure 5. Experimental setup: (a) computer installed with LabVIEW, (b) power amplifier, (c) DAQ for data acquisition, (d) shaker, (e) 4 piezoelectric bimorphs clamped by a fixture, (f) accelerometer and its signal conditioner, (g) resistance substitution box, (h) standard interface circuit together with a set of DPDT switches, (i) DAQ functioned as a signal generator.

Table 1. Numerical data of the measured model parameters for these 4 piezoelectric cantilever bimorphs.

|  | 1st | 2nd | 3rd | 4th |
| :--- | :--- | :--- | :--- | :--- |
| $M(\mathrm{~g})$ | 1.091 | 1.151 | 1.091 | 1.091 |
| $K\left(\mathrm{~N} \mathrm{~m}^{-1}\right)$ | 1064.7 | 1144.8 | 1105.7 | 1133.5 |
| $\eta\left(\mathrm{~N} \mathrm{~s} \mathrm{~m}^{-1}\right)$ | 0.0101 | 0.009 | 0.0086 | 0.0073 |
| $\Theta\left(\mathrm{~N} \mathrm{~V}^{-1}\right)$ | 0.000751 | 0.000786 | 0.000787 | 0.000756 |
| $C_{p}(\mathrm{nF})$ | 7.89 | 8.27 | 8.33 | 8.02 |
| $\bar{F}(\mathrm{mN})$ | 2.446 | 2.681 | 2.416 | 2.398 |
| $f_{\mathrm{sc}}(\mathrm{Hz})$ | 157.2 | 158.7 | 160.2 | 162.2 |
| $f_{\mathrm{oc}}(\mathrm{Hz})$ | 162.2 | 163.8 | 165.5 | 167.2 |
| $R_{\mathrm{sc}}^{\text {opt }}(\mathrm{M} \Omega)$ | 0.022 | 0.029 | 0.021 | 0.019 |
| $R_{\mathrm{oc}}^{\text {opt }}(\mathrm{M} \Omega)$ | 1.52 | 1.31 | 1.53 | 1.80 |
| $P^{\mathrm{opt}}(\mu \mathrm{W})$ | 74 | 100 | 84 | 98 |

uniformly distributed in the frequency range of interest. This issue will be discussed in section 4.1.

### 3.2. Experimental validation

The test arrangement is prepared for validating the proposed analytic model and for evaluating the harvesting performance of arrays with mixed patterns of connection. The layout of the experimental setup is shown in figure 5 . The energy harvester device consists of a fixture clamping 4 piezoelectric bimorphs manufactured by Eleceram Technology (Taiwan). A proof mass is bounded to the front of each cantilever beam for enhancing harvested power and tuning resonance. The dimensions of each bimorph are $33 \times 10 \times 0.2 \mathrm{~mm}^{3}$ for the top and bottom piezoelectric layers and $33 \times 10 \times 0.1 \mathrm{~mm}^{3}$ for the substrate made of Cu . The device is mounted on a shaker (Data Physics, V20) controlled by a signal generator from LabVIEW through a power amplifier (Data Physics, PA 300 E ). The acceleration of excitation from the shaker is measured by accelerometer (PCB Piezotronics, 333B42) placed on the top of clamping fixture. A set of Double-Pole Double-Throw (DPDT) switches is used for creating the
desired mixed array configuration. The output DC voltage across the load of different magnitudes of impedance is measured and recorded through the DAQ device (NI 9178 and NI 9229). The equivalent parameters used in the proposed model are identified based on the standard modal testing and are listed in table 1 for each beam [15]. Note that $f_{\text {sc }}$ and $f_{\text {oc }}$ in table 1 are the short circuit and open circuit resonant frequencies of each oscillator. In addition, $R_{\mathrm{sc}}^{\mathrm{opt}}$ and $R_{\mathrm{oc}}^{\mathrm{opt}}$ are the electric loads for generating the optimal power output $P^{\text {opt }}$ operated at around $f_{\mathrm{sc}}$ and $f_{\mathrm{oc}}$, respectively. The existence of two optimal loads for each piezoelectric oscillator will explained in section 4.2.

The device is excited under 0.2 g by a sine sweep signal over the frequency range of $150-175 \mathrm{~Hz}$ through a vibration shaker. Various electric loads ranging from the short-circuit to open-circuit conditions are used for determining the optimal output power. There are 4 array configurations under testing, including the $(1+2+3+4),(1| | 2)+3+4,(1| | 2| | 3)+4$ and $(1||2|| 3|\mid 4)$ patterns. Harvested power against frequency evaluated at the optimal load is shown in figure 6(a) for these 4 array configurations. The experimental measurements are marked by solid color points while the analytic estimates are presented by different continuous color lines. Note that the analysis presented in section 2 does not account for the diode loss. Thus, the voltage drop after rectification is measured at each step to estimate the power dissipated at the diodes for the purpose of comparison. From figure 6(a), the analytic predictions agree fairly well with the experimental observations. In addition, the average of the optimal power output of each single piezoelectric oscillator listed in table 1 under the same excitation level is $89 \mu \mathrm{~W}$. Thus, from figure 6(a), the peak power is around 3.4 times larger than that produced by a single beam in the sense of average. Furthermore, it is distributed roughly uniform within the frequencies ranging from the smallest resonance to the largest resonance of the oscillators, as expected. However, from figure 6(a), the optimal loads are different for each array configuration, causing the inconvenience in the circuit design. Instead, figure 6(b) shows power plotted against frequency evaluated at the fixed load $150 \mathrm{k} \Omega$ whose magnitude is about the average of optimal loads of these 4 array configurations. The comparison between these two figures reveals the small differences in peak power of each connection pattern. Thus, the tuning of optimal loads is avoided in the present case. This issue will be discussed in detail in section 4.2. In addition, the frequency response of the envelope of maximum power for the second oscillator is included in figure 6(b) for the purpose of comparison with that based on the mixed connection pattern. It is presented by the continuous cyan line. The second oscillator is chosen as its optimal power is higher than that of the other oscillators. Note that the exhibition of two identical peaks in power, corresponding to the short-circuit and open-circuit resonances, is the typical frequency response for highly coupled generators [18,57], and the switching between these two peaks can be realized by varying the electric loads.

Finally, a comment is made concerning the resonance tuning of oscillators in an array structure. Let $\Delta f$ be the shift


Figure 6. Comparisons between analytic predictions (color continuous curves) and experimental observations (color solid points). (a) is evaluated at the optimal load for each array but (b) is evaluated at the fixed load for all arrays. The bottom cyan curve in (b) is the theoretical envelope of maximum power for the second oscillator.
between the resonant frequencies of the oscillators. If this frequency shift is large, the enhancement in harvested power is not significant since the electromechanical interaction between oscillators is weak. Instead, if this shift in frequency is small, the bandwidth improvement is not pronounced. A rule of thumb for estimating the limit of $\Delta f$ for a single array configuration is proposed to be

$$
\begin{equation*}
\Delta f<\Delta f^{*}=\frac{f_{\mathrm{oc}}-f_{\mathrm{sc}}}{n-1} \tag{62}
\end{equation*}
$$

where $n$ is the total number of oscillators. If $\Delta f \geqslant \Delta f^{*}$, then some peaks in power driven at around some resonances of oscillators could be smaller than the optimal power of a single oscillator, causing limited improvement in bandwidth. Taken an example of an array of parallel connection of oscillators shown in figure 6(a). From table 1, the average $\Delta f$ is chosen to be around $\Delta f^{*}=1.7 \mathrm{~Hz}$ in the experiment. It is found that the first three peaks in power are all smaller than the optimal power of a single oscillator ( $89 \mu \mathrm{~W}$ in average). As a result, the use of a single array for wideband improvement is not significant. It then motivates developing a multi-array device for wideband amelioration while keeping the optimal peak power as large as possible. This issue of switching array configurations will be discussed in section 4.3.

## 4. Discussions

### 4.1. Ideal arrangements of arrays with mixed connection patterns

One of the requirements for an ideal arrangement of mixing arrays is that the peak power of each array configuration can be maintained evenly over a wider range of frequency.

However, figure 4(a) demonstrates that such a mixing in connection pattern can not be arbitrary. This also motivates how to construct ideal arrangements of arrays with suitable connection patterns for wideband improvement.

Consider a model case where the device consists of six piezoelectric oscillators. The relevant equivalent parameters are similar to those used for model validation as in figure 4 except for the magnitudes of mass and the excitation levels. Suppose the resonant frequencies are sequentially arranged so that the resonance of the first oscillator is smallest while that of the sixth oscillator is the largest. The arrangement of arrays starts with the series connection $(1+2+3+4+5+6)$ and ends with the parallel connection $(1||2|| 3||4|| 5|\mid 6)$ since the peak power has been shown to occur at around the smallest (largest) resonant frequency of oscillators connected in series (parallel) [37, 39]. But there are two different arrangements for sequentially releasing the series connection of oscillators to the parallel connection of oscillators. The first mixing begins with the parallel connection of the last two oscillators with larger resonances; i.e., $1+2+3+4+(5| | 6)$. On the other hand, the second arrangement starts with the parallel connection of the first two oscillators with smaller resonances; i.e., $(1|\mid 2)+3+4+5+6$. The rest of them follows the similar sequential patterns of connection. The simulations based on the analytic estimates reveal that the peak power of each array configuration is not uniformly distributed for the first arrangement, as shown in figure 7(a). The ideal arrangement of mixing connection patterns for broadband purpose is the second arrangement, as illustrated in figure 7(b). Finally, note that both figures 7(a) and (b) are the patterns of the s-type. For the case of the p-type, it can be shown that the ideal mixing patterns start with $1\|(2+3+4+5+6), 1\| 2 \|(3+4+5+6)$, $1||2|| 3|\mid(4+5+6)$, and 1$||2||3||4| \mid(5+6)$, respectively [15].


Figure 7. Power against frequency for two arrangements of mixing connection patterns from 6 piezoelectric oscillators. (a) and (b) show two different ways for sequentially releasing the series connection to the parallel connection of oscillators.

### 4.2. Optimal loads

The switching from one array configuration to the other raises another issue of different optimal loads needed for power enhancement. Indeed, let $R^{\text {opt }}$ be the load for peak power output in the case of a single piezoelectric oscillator. The optimal load for the $n$ identical oscillators connected in parallel (in series) is $\frac{1}{n} R^{\text {opt }}$ ( $n R^{\text {opt }}$ ). Therefore, the magnitude of the optimal load in the case of series connection of all oscillators could be $n^{2}$ times larger than that in the case of parallel connection of oscillators. It will result in the need of implementing a resistive impedance circuit during the process of switching connection. The inclusion of such a circuit could downgrade the performance for broadband improvement. Fortunately, this difficulty can be avoided in the case of strong electromechanical coupling, as explained next.

Let $k_{e}^{2}=\frac{\Theta^{2}}{K C_{p}}$ and $\zeta_{m}=\frac{\eta}{2 \sqrt{K M}}$ be the alternative electromechanical coupling factor and mechanical damping ratio, respectively [56]. The piezoelectric energy harvesting system is in the range of strong electromechanical coupling if $\frac{k_{e}^{2}}{\zeta_{m}}>10[56,57]$. Under this circumstance, there exist two optimal loads $R_{\mathrm{sc}}^{\mathrm{opt}}$ and $R_{\mathrm{oc}}^{\mathrm{opt}}$ for identical peak power, and typically, $R_{\mathrm{sc}}^{\mathrm{opt}} \ll R_{\mathrm{oc}}^{\mathrm{opt}}$. This gives an opportunity for smoothing the magnitudes of optimal loads of different array configurations. Indeed, our simulations based on the analytic estimates had revealed the optimal load of the series connection of oscillators turns out to be the one close to $R_{\mathrm{sc}}^{\mathrm{opt}} \times n$, while that of the parallel connection of oscillators is close to $R_{\mathrm{oc}}^{\mathrm{opt}} / n$ (see figure 11 in [66]). To demonstrate this idea, recall the experiment described in section 3.2. The ratio $\frac{k_{e}^{2}}{\zeta}$ is around 18 for each oscillator, giving rise to two optimal loads as also listed in table 1. It is found that the average of $R_{\mathrm{sc}}^{\text {opt }}$ multiplied by 4 is $91 \mathrm{k} \Omega$. This is very close to the
measured optimal load $100 \mathrm{k} \Omega$ in the case of series connection of oscillators (see the case $(1+2+3+4)$ in figure 6(a)). Next, the average of $R_{\mathrm{oc}}^{\mathrm{opt}}$ divided by 4 is $385 \mathrm{k} \Omega$ which is also close to the measured one $330 \mathrm{k} \Omega$ in the case of parallel connection of oscillators (see the case ( $1||2|| 3|\mid 4$ ) in figure 6(a)). As a result, all the optimal loads of different array patterns are in the same order of magnitude, giving rise to the chance of replacing them by a fixed load. Indeed, figure 6(b) shows power against frequency for various array configurations evaluated at a fixed load $R_{L}=150 \mathrm{k} \Omega$ which is roughly the average of optimal loads. Remarkably, the comparison between figures 6(a) and (b) reveals the little difference in peak power. Hence, the tuning of optimal loads is avoided in the case of strongly coupled electromechanical system.

### 4.3. Broadband improvement by connection switching

The final issue discussed here is that it needs a suitable circuit layout for sensing frequency and triggering the switches of connection for broadband improvement. In the present experiment, the frequency response of piezoelectric voltage $V_{p}$ is monitored through the LabVIEW DAQ. The external excitation frequency is then obtained by the spectral analysis of the feedback signal from $V_{p}$ through the LabVIEW FFT. In addition, the switching from one array configuration to the other is controlled by a set of DPDT switches, as schematically shown in figure 8 . We have devised two switching criterions. The first one initiates the switching at the intersection of power-frequency curves from two adjacent arrays. For example, figure 9 (a) shows the measured power against frequency for 4 different array configurations without connection switching. The magnitudes of harvested power are lower than those in figure 6 since the electric loss from diodes are taken into account. Figure 9(b) shows results allowing switching connection under the optimal mode. As this


Figure 8. Connection patterns controlled by a set of DPDT switches.


Figure 9. Measured power against frequency without switching as in (a), operated at the optimal switch mode as in (b), and operated at the simple switch mode as in (c).

Table 2. Criterion proposed by the simple switch mode.

| $1+2+3+4$ | $(1\|\mid 2)+3+4$ | $(1\|\mid 2 \\| 3)+4$ | $1\\|2\\| 3 \\| 4$ |
| :--- | :---: | :---: | :---: |
| $f<f_{s}+\frac{\Delta f_{p s}}{6}$ | $f_{s}+\frac{\Delta f_{p s}}{6} \leqslant f<f_{s}+\frac{3 \Delta f_{p s}}{6}$ | $f_{s}+\frac{3 \Delta f_{p s}}{6} \leqslant f<f_{s}+\frac{5 \Delta f_{p s}}{6}$ | $f \geqslant f_{s}+\frac{5 \Delta f_{p s}}{6}$ |

approach requires a priori information either from existing tests or simulations results, we have devised another criterion for practical implementation. Indeed, let $f_{s}$ and $f_{p}$ be the resonant frequencies of the series and parallel connection of oscillators, respectively. Then the pattern $(1+2+3+4)$ remains under the frequency range $f<f_{s}+\Delta f_{p s} / 6$ where $\Delta f_{p s}=f_{p}-f_{s}$. Similarly, the criterions for the subsequent connection patterns are listed in table 2 . The harvested power against frequency under this simple switch mode is then illustrated in figure 9(c). From the comparison between figures 9(b) and (c), the simple switch mode works quite well as long as the peak power of each array pattern is distributed evenly within the frequency range of interest.

Finally, taken the measured peak power $83 \mu \mathrm{~W}$ of the second oscillator as the reference point, let the bandwidth of each array configuration is chosen to be the largest frequency range such that the magnitudes of power are higher than this reference power output. Then, from figure 9 , the bandwidth of
mixed arrangements of arrays is about 2.8 times wider than that based on the use of a single array configuration, giving rise to the effective enlargement of bandwidth.

## 5. Conclusions

This article documents both modeling and experimental studies for investigating the electromechanical response of a mixed parallel-series connection of multiple piezoelectric oscillators attached to the standard interface circuit. Such a design offers advantages of power enhancement and tailorable operation frequency band suitable for an environment with multifrequency spectra. Two types of connection patterns are discussed here. The p-type (s-type) pattern is the one where a part of oscillators connected in series (parallel) are subsequently connected to the rest of oscillators in parallel (series). The analytic estimates of harvested power are derived
and explicitly expressed in terms of matrix formulation of generalized Ohm's law for the p-type and of charging on capacitance for the s-type. They are subsequently validated and are found in good agreement with numerical and experimental investigations.

In addition, some relevant issues are discussed. These include the ideal arrangements of arrays for broadband improvement, the feasibility of replacing different optimal loads by a fixed load, and the criterions for triggering the switching of connection. Finally, the experimental results from the mixed connection of 4 piezoelectric oscillators show that the peak power of each array is almost uniformly distributed within the frequency range of interest, and is around 3.4 times higher than that generated by a single piezoelectric oscillator. In addition, the bandwidth of mixed arrangements of arrays is about 2.8 times wider than that based on the use of a single array configuration. As a result, the bandwidth is effectively enlarged without the cost of power amplitudes.

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