Origins of concentric cylinders viscometry*

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Synopsis

The history of the concentric cylinders apparatus for measuring the shear viscosity of liquids, and its attribution to Maurice Couette, have been explored. Examination of the Nineteenth Century literature has revealed that the concept goes back to Stokes and later Margules, the design and execution of the apparatus, apparently independently, to Perry, Couette, Mallock, and Schwedoff. Mallock's and Schwedoff's measurements were the most accurate and were within 1% of the viscosities derived from Poiseuille's measurements on the basis of no slip at the tube walls and cylinder surfaces. Measurement of fluid viscosity was closely linked to the adoption of the no-slip boundary condition at solid-fluid interfaces. © *2005 The Society of Rheology.*

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I. INTRODUCTION

Concentric cylinders are widely employed to measure shear viscosities of "complex liquids," i.e., liquids whose microstructures require more than microseconds, and typically milliseconds or longer, to equilibrate locally within a flow. Familiar examples are polymer solutions, certain surfactant solutions, concentrated suspensions of colloidal particles, and many composite colloidal and polymeric materials in chemical, food, and other technologies.

Liquid contained in a narrow annulus between two coaxial cylinders, one or both of which rotate, experiences a nearly uniform shear rate. In the simplest situation, one cylinder is stationary and the other is set in motion with either a constant velocity or constant torque. To arrive at the viscosity, the torque on the stationary element or the angular velocity of the moving element is measured. The ratio of the torque to the angular velocity is by a definition the shear viscosity of the liquid up to a constant.

$$\mu = 2\pi R^2 \tau \Bigg/ 4\pi\left[\left(\frac{\alpha^2 R^2}{\alpha^2 - 1}\right)(\Omega_o - \Omega_i)\right], \qquad (1)$$

where R is the radius of the inner cylinder, α is the ratio of the radius of the outer cylinder to that of the inner cylinder, Ω_o and Ω_i are, respectively, the angular velocities of the outer and inner cylinders, and τ is the shear stress per unit length of the inner cylinder.

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Today, this flow is widely referred to as “Couette flow,” and the method “Couette rheometry,” after Maurice Marie Alfred Couette, who is regarded to have first constructed such an apparatus and made accurate measurements [Couette (1888, 1890)]. Two authoritative reviews in the 1930s [Dryden *et al.* (1932), Graetz and Stöckl (1931)] established beyond doubt that at least two other researchers independently designed and built similar devices at around the same time, if not earlier. However, neither of these reviews is complete. Nor is Donnelly’s (1991) recent account of the history of concentric cylinder flow. Here we recount and evaluate the Nineteenth Century work of Mallock, Schwedoff, and Perry, as well as of Couette, with the concentric cylinder apparatus.

II. NO-SLIP BOUNDARY COUNDITION

A little background is germane. The story of the determination of fluid viscosity is closely linked with the adoption of the no-slip boundary condition at a solid-liquid interface. Navier (1823), in his celebrated memoir on the laws of fluid motion, introduced and explained two coefficients of the internal resistance to slipping, one within the liquid and the other between the solid and the liquid [Navier (1823), p. 416]. Moreover, he did not explicitly use the word “viscosity” to denote the first. He also derived an expression for the fluid velocity in a circular tube by assuming perfect slip at the wall. (The flow rate would then vary with the cube of the tube radius, which he did not specifically mention.) In 1839, Hagen reported experiments in which the liquid flow rate in circular tubes varied with slightly more than the fourth power of the tube radius [Hagen (1839)]. Sutera and Skalak (1993) established that in the same year as Hagen, Poiseuille submitted his experimental measurements on tube flow of liquids to the French Academy. In those measurements he clearly demonstrated that at sufficiently low liquid velocities, the flow rate through circular tubes varies with the fourth power of the tube radius. His result contradicted theoretical treatments available in 1839, and it was probably the reason it was not accepted until he repeated the experiments with additional liquids in the presence of a designated committee! By 1847, his experiments and measurements of liquid viscosities were fully communicated [cf. Sutera and Skalak (1993)] and thereafter became the standards to match. However, Poiseuille did not himself calculate the absolute viscosity of liquids, for that would have been tantamount to adopting the no-slip boundary condition at the tube wall.

Stokes (1845) in his memoir was probably the first to argue that the liquid next to a solid wall is at rest. But he found that his formulas (which he does not report) did not agree with the experiments of Bossut and Dubuat from 70 years earlier [cf. Stokes (1845), p. 96], which, it is today clear, fell in the turbulent regime of tube flow. However, two pages later he later mentions [cf. Stokes (1845), p. 98]:

> Dubuat found by experiment that when the mean velocity of water flowing through a pipe is less than one inch in a second, the water near the inner surface of the pipe is at rest. If these experiments may be trusted, the conditions to be satisfied in the case of small velocities are those which first occurred to me, and which are included in those just given by supposing $\nu = \infty$.

Stokes’ ν is now known as the slip coefficient (or its inverse). He then derived an expression for the velocity profile of flow in a tilted pipe of radius a [cf. Stokes (1845), pg. 105]:

$$w = \frac{g\rho \sin \alpha}{4\mu}(a^2 - r^2) + U, \tag{2}$$

where U was the velocity close to the pipe surface. Unfortunately, Stokes was not aware of Poiseuille's more accurate measurements, or the no-slip boundary condition would have been adopted earlier. Hence, he suggested that the flow between coaxial cylinders in relative angular motion with respect to each other be used to elucidate the "friction in liquids" and also the boundary condition at a solid wall [Stokes (1845), p. 104]. {Many years later, Margules apparently independently suggested using the same flow to measure the "coefficients of friction and gliding" in 1881 [Margules (1881)]}. One year later, in his 1846 report to the British Association, Stokes interpreted Coulomb's oscillating-disk experiments as supporting his arguments for no slip at a solid wall, but he did not advocate this boundary condition unequivocally [Stokes (1846)]. Five years later, Stokes (1851) verified the no-slip boundary condition at solid-fluid boundaries in his celebrated study on the motion of pendulums [Stokes (1851), see pp. 14–5, 103).

It was not until the late 1850s and early 1860s that an exact expression of the liquid velocity and flowrate in laminar flow in a circular tube was published—independently by three researchers. Jacobson [(1860), p. 91] derived the formula, was probably the first to call it "Poiseuille's law," and verified it with an extensive set of his own experiments. He credited the derivation to Neumann. In March 1860, Hagenbach published the same result [Hagenbach (1860), pp. 394–739], and commented on Navier's error in a footnote. In 1863, Mathieu derived the same formula [Mathieu (1863)]. Each of the earlier adopted no slip at the solid-liquid boundary. However, when in April 1860 Helmholtz published the formula for tube flow, he included a slip coefficient because he had compared his results with some inaccurate early experiments. This issue of slip appears not to have been resolved in that era, because even Lamb, in the first edition of his treatise *Hydrodynamics* in 1879 [Lamb (1879)], reproduced Helmholtz's formula for tube flow, and mentioned that when slip at the walls was neglected, the resulting formula agreed with Poiseuille's experimental results, in particular the fourth-power dependence of flow rate on tube diameter.

It was in this context that Couette, Mallock, Schwedoff, and Perry published the results of their experiments with concentric cylinder devices in the next dozen years, and adopted the no-slip condition to extract the absolute viscosity. Interestingly, when Thorpe and Rodger delivered their famous Bakerian Lecture in 1894 on the relation between the chemical nature of liquids and their viscosities, they did not so much as mention the issue of slip or the concentric cylinder device [Thorpe and Rodger (1894)]. And from his second edition (1895) onward, Lamb held it possible that there is no slip at solid boundaries "in all ordinary cases" [Lamb (1895)] He specifically cited not only Poiseuille's results, but also Whetham's (1890a, 1890b) conclusion from glass, silvered, and copper tubes slightly less than a millimeter in diameter "that no slip occurs, at any rate with solids that are wetted by the liquid." Poiseuille's tube diameters went down to 14 μm, and provided a much more severe test.].

Today no-slip is accepted as the wall boundary condition for the flow of all small molecule fluids that wet the boundaries. For example in a recent review of microfluidic flows, [Stone *et al.* (2004), p. 388] conclude that "… the no-slip boundary condition remains an excellent approximation for flows at scales above tens of nanometers." Measurements have been reported, however, of "apparent viscosities" of certain polymer solutions flowing through cylindrical pores in membranes, of four different diameters between 2.5 and 11 μm, lower than viscosities measured in larger tubes and gaps at very low rates of shear [Chauveteau (1982), also cf. Aubert and Tirrell (1982)].

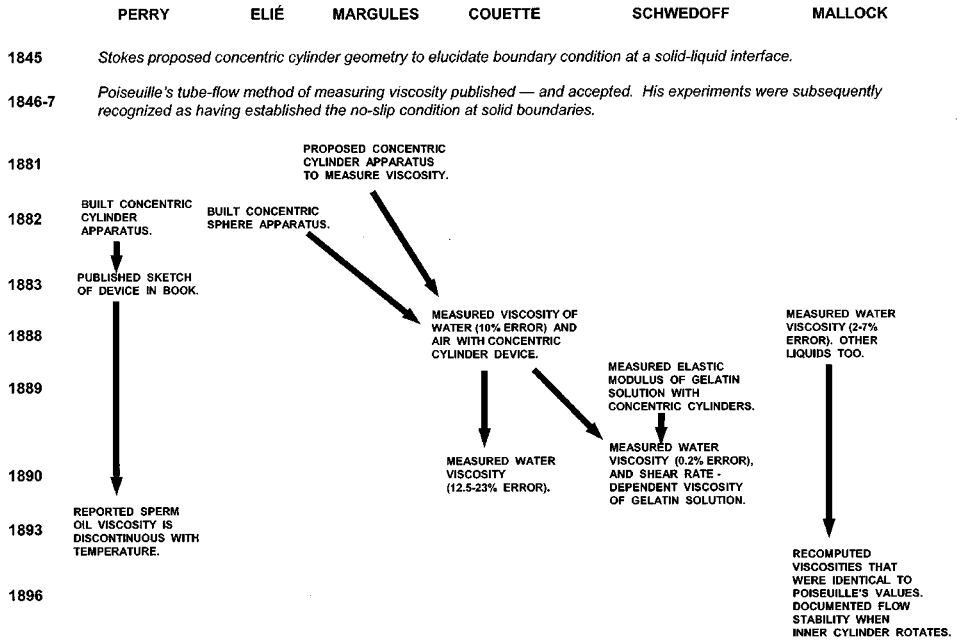

FIG. 1. History of the concentric cylinder apparatus to measure fluid viscosity. Arrows connect those works cited by other authors. Perry, Couette, Mallock, and Schwedoff all built their first concentric cylinder device independent of each other. Perry built the first device; Couette and Mallock reported the first shear viscosity measurements; Schwedoff's viscosity measurements were 0.2% lower than Poiseuille's reported standards; Mallock was the first to build an apparatus in which either cylinder could be turned, and reported that the flow is unstable when the inner cylinder rotates (he was always above the critical Taylor number).

III. COUETTE'S CONTRIBUTION

Couette (1890) studied *both* pressure gradient-driven flow in tubes *and* the rotational flow in an elaborate concentric cylinder apparatus. Piau *et al.*'s (1994) biography is the most complete account available of Couette's personal life and scientific career. Couette's theoretical correction for pressure loss at the tube entrance was more accurate than Hagenbach's (1860) correction, from which it differed by a numerical multiplier (but Couette's formula was identical to Neumann's formula reported 30 years earlier by Jacobson, of which Couette was apparently unaware). He also developed an experimental technique by which these corrections were totally eliminated. Couette's concentric cylinder apparatus was inspired by Margules' (1881) concept and Élie's work with concentric spheres [Élie (1882)] (see Fig. 1). His measurements with the concentric cylinder device in which the inner cylinder was stationary were not accurate. His measured values of shear viscosity of water were higher than Poiseuille's values by 15% [Couette (1890), p. 460]. For instance, on page 460 of his 1890 publication he observed

> …Pour compléter la vérification, j'ai calculé le coefficient de frottement intérieur ε
> de l'eau à 16,7°, en remplacant, dans la formule (13), P/N
> par la valeur moyenne que nous venons de trouver. J'ai obtenu ainsi la valeur
>
> $$\varepsilon = 0{,}01255$$
>
> au lieu que les expériences de *Poiseuille*, qui méritent toute confiance, donnent, pour la même température,

$$\varepsilon = 0{,}01096$$

> La valeur que j'ai trouvée est done trop forte d'environ 15 pour 100. Ce résultat me paraît dû aux imperfections de l'appareil, …

He attributed the evident errors to the guard rings with which he tried to eliminate end effects [g and g' in Fig. 2(d)], and to the eccentricity of the cylinders. Eccentricity between coaxial cylinders does raise the measured torque (cf. Sommerfeld's lubrication theory) and consequently the apparent viscosity, but in order to explain Couette's discrepancies, the eccentricity in his apparatus would have to be in excess of 25% of the radial gap between the cylinders. Furthermore, Couette tabulated results of his experiments on water over 7 months, and his values were between 12.5% and 22.9% larger than Poiseuille's values [Couette (1890), p. 461]. These errors were most likely due to his guard rings. Couette then calibrated his instrument with Poiseuille's values of water viscosity, and thereupon estimated the viscosity of air. His estimate was 10% lower than James C. Maxwell's and Oscar E. Meyer's values, and 1% higher than O. Schumann's values, all with the oscillating-disk method [Couette (1890) pp. 467–846]. Thus, his measurements of air viscosity with the concentric cylinder apparatus were not absolute as [Donnelly (1991), p. 37] claims.

It is curious that Couette used the word "viscosity" sparingly; he preferred to use "coefficient of friction in liquids" (including in the title of his 1890 article), even though across the English Channel the word viscosity had replaced "coefficient of friction" at least 25 years earlier [Kelvin (1865), p. 290].

IV. MALLOCK'S CONTRIBUTION

Unremarked by Thorpe and Rodger (1894), Mallock, working under the aegis of Rayleigh, had independently constructed a concentric cylinder device of which the inner cylinder was stationary, measured the viscosity of water over a range of temperatures, and obtained results that were close to Poiseuille's reported values [Mallock (1888), p. 131]. They were more accurate than Couette's measurements, which were published the same year. In 1896, he recomputed the viscosities after G. G. Stokes pointed out that he had not used quite the right formula; his recomputed viscosities were "very close" to Poiseuille's values. {We recomputed viscosities from Mallock's (1888) data using the formula he gave [Mallock (1896)] and found them to be within 1% of viscosity values listed by Bingham (1922), which in turn were 0.34% lower than those computed from Poiseuille's measurements at 15 °C}. Towards the end of his paper, Mallock pointed out the significance of his own work

> The chief interest of these experiments, beyond that attaching to an independent determination of μ by a new method, lies in the comparatively high velocities at which viscous forces remain the principal cause of resistance.

He concluded his short paper with

> Many experiments were made on the viscosity of fluids other than water, but as I find that the results do not differ materially from those of Poiseuille it is unnecessary to give them here.

Mallock's arrangement to reduce end effects was ingenious and convenient, as much as Couette's guard rings were elaborate. The bottom of the inner cylinder trapped an air bubble that separated the inner cylinder from the liquid beneath it [Fig. 2(c)]. This design reduced the extra torque from the liquid sheared below the level of the inner cylinder. This design is now standard in the ARES rheometer (TA Instruments, New Castle, DE) and other commercial rheometers, though Mallock's contribution has been forgotten and Mercier (1932) is generally credited with the invention [see Mooney and Ewart (1934)].

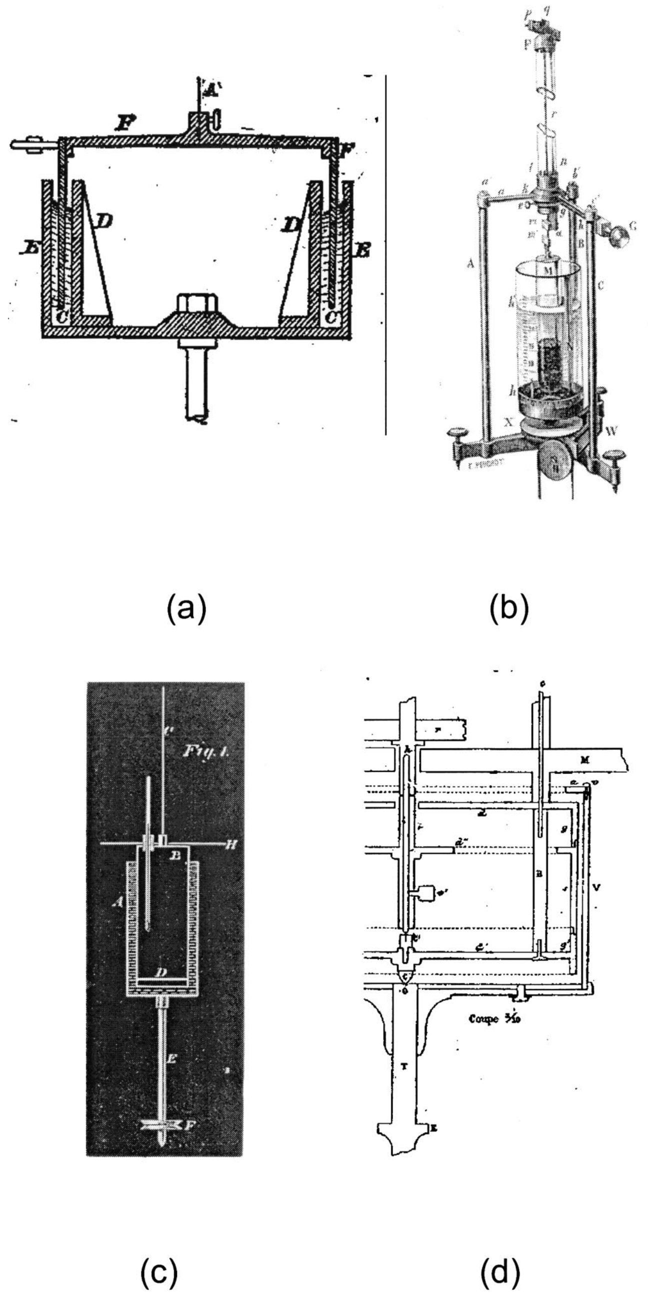

FIG. 2. Concentric cylinder apparatus used to measure liquid viscosities and designed by (a) Perry (1883), (b) Schwedoff (1889), (c) Mallock (1888), and (d) Couette (1890).

Mallock was a careful experimenter; the outer cylinder must have been transparent (he did not explain) because he observed and commented on a minute secondary flow set up between the stationary inner cylinder and the rotating outer cylinder by the bottom of the latter [Mallock (1888), p. 128]

…It was found that at all these speeds the force tending to turn the inner cylinder

> **B** could be represented by the sum of two terms, one varying as the velocity and the other as the square of the velocity; the latter being small compared to the former, even at the highest speed.
> The cause of the square term seems to be that, owing to the action of the bottom of the revolving cylinder, a circulation is set up in the fluid in the annulus, the flow being up the side of the revolving cylinder and down the side of the stationary one, the result being that the fluid having the velocity due to a position near the outer cylinder is by this circulation continuously carried near the inner one, thus making the variation in velocity in the neighborhood of the latter greater than it would otherwise be. As far as could be observed there was no trace of eddies with axes parallel to that of the two cylinders. The proportion between the two terms depends on the ratio between the length of the cylinders, and the breadth of the annulus, the square term become smaller and smaller compared to the other as the ratio increases. Professor J. Thomson [Thomson (1876), (1877)] has pointed out that a circulation having a very similar origin must take place in a stream when flowing round a bend.

Rayleigh communicated this paper to the Royal Society. Mallock then built a new version of his apparatus and studied the stability of the flows with either the inner or the outer cylinder rotating and with a variety of end conditions; Kelvin communicated this paper to the Royal Society [Kelvin (1896)]. Mallock's results went largely unnoticed by those researching viscosity measurements, at least up to the time of Taylor's (1923) landmark experiments and theoretical analysis. Even Leroux (1925), who measured water viscosity with a greatly improved version of Couette's device, made no mention of Mallock's work. Hatschek (1913), who measured the viscosity of suspensions, was among those who earlier had overlooked Mallock.

V. SCHWEDOFF'S CONTRIBUTION

In 1889, Schwedoff reported measurements of the elastic shear modulus of gelatin by tracing the liquid relaxation after an imposed deformation with a concentric cylinder apparatus [Schwedoff (1889)]. In 1890, he reported the viscosity of water, castor oil, glycerin and a gelatin solution with a concentric cylinder device whose outer cylinder was made of glass [Fig. 2(b)] [Schwedoff (1890)]. By that time he was aware of Couette's work [cf. Piau *et al.* (1994)], and so he made clear that he had designed and built his apparatus independently [Schwedoff (1890), p. 38]

> …Je me suis arrêté à la méthode des deux cylindres concentriques, dont l'intervalle est rempli du liquide à étudier. Proposée d'abord par M. Margules pour l'étude de la viscosité des liquides, cette méthode avait éprové depuis des perfectionnements essentiels entre les mains de M. Couette (1888), qui eut l'heureuse idée d'introduire dans son appareil duex *cylindres de garde*, pour simplifier la théorie de l'expérience. Par hasard, cette méthode se rapproche beaucoup de celle que j'avais adoptée pour mes recherches sur la rigidité des liquides, sans avoir pourtant connaissance des essais de M. Couette dans cette direction. Seulement, au lieu des cylindres de garde, je me suis servi du procédé d'*elimination du fond* que j'ai décrit dans la premiére Partie de ces recherches. (Schwedoffs emphasis)

Schwedoff accounted for the extra torque from the liquid below the inner cylinder and subtracted it; this is the "process of bottom elimination" he refers to in the earlier passage. His measurement of water viscosity was 0.2% lower than Poiseuille's value [Schwedoff (1890), p. 44], more accurate than either Mallock's or Couette's measurements with the concentric cylinder apparatus. In the same work, he determined that the viscosity of a gelatin solution depended on shear rate. Schwedoff cited neither Mallock's nor Perry's work. Neither Graetz and Stöckl (1931) nor Donnelly (1991) cite Schwedoff's work, but [Coleman *et al.* (1966) pp. 88–89] cite both his works, and also comment that Schwedoff's measurements on the gelatin solution were probably the first report of a shear-thinning liquid viscosity in the steady flow between two coaxial cylinders free from "turbulence" or secondary flow.

VI. PERRY'S CONTRIBUTION

Perry's contribution to the development of the concentric cylinder apparatus is the least known. Perhaps that is because he was a teacher (of engineering) first, and only secondarily a researcher. Once a student of James Thomson and an Honorary Assistant of Kelvin at Glasgow, he later taught engineering at the Imperial College of Engineering in Japan (1875–1878), at the City and Guilds of London Technical College in Finsbury (1882–1896), and at the Royal College of Science in Kensington (1896–1913) before retiring (cf. J. C. Poggendorff *Biographisch-literarisches Handworterbuch der exacten Wissenschaften* **3–4**, and obituary notice in 1926 in *Proc. Roy. Soc. London* **A 111**, i–ii). His campaign to improve engineering education methods, and his efforts to bridge the gap between mathematicians and engineers were famous (cf. his obituary notice). Perry also contributed prolifically to the scientific literature while he was teaching (an examination of the *Catalogue of Scientific Papers* compiled by the Royal Society reveals 105 papers coauthored by Perry between 1875 and 1900). Perry's book *Practical Mechanics*, "an attempt to put before non-mathematical readers a *method* of studying mechanics" (Perry's emphasis), contains a drawing of a concentric cylinder apparatus similar to the "double-cup," a current method to measure the viscosity of low viscosity liquids [Macosko (1994)]. He also described the method he used to measure liquid viscosities [Perry (1883), Sec. 242, pp. 248–249, also see Fig. 2(a)]

> …Fig. 143 shows a hollow cylindric body, **F**, supported so that it cannot move sidewise, and yet so that its only resistance to turning is due to the twist it would give the the suspension wire, **A**. **CC** is water or another liquid filling the annular space between the cylindric surfaces **DD** and **EE**, and wetting both sides of **F**. When the vessel **DD EE** is rotated, the water moving past the surfaces of **F** tend to make **F** turn around, and this frictional torque is resisted by the twist which is given to the wire. The amount of twist in the wire gives us, then, a measurement of the viscosity of liquids, and investigations may be made under very different conditions.…

He published his work on "liquid friction" in 1893. He began it by stating

> A piece of apparatus such as is used in this investigation was designed and partly constructed in Japan in 1876; it is described in my book on Practical Mechanics (1883). The specimen actually used by us was constructed at the Finsbury Technical College in 1882, and occasionally used since that time, but no complete sets of observations were attempted till October 1891.

Perry was also concerned about the extra torque from the liquid sheared below the inner cylinder, i.e., end effects [Perry (1893), p. 442], and his double gap design reduced it. Perry also claimed to have measured viscosities of several liquids, but reported only the abrupt fall in viscosity of sperm oil above 40 °C. His paper (one of the reviewers of which was Reynolds) contained no references to Mallock, Couette, or Schwedoff.

Perry's work went unnoticed until Drew (1901), working in Professor Michelson's laboratory at the University of Chicago, cited his work when he reported measurements on viscosity of water. Interestingly, Drew did not cite Couette or Mallock or Schwedoff.

The concentric cylinder apparatus he used, he explained, was designed and built by his predecessor Johonnott, under the direction of Professor Michelson. Subsequently Gurney, supervised by Professor Michelson and Professor Millikan, reported further work on water and cited Couette, but not Mallock, Schwedoff, or Perry [Gurney (1908)].

VII. CONCLUSIONS

The question of whether a fluid slips at a solid wall, and the determination of fluid viscosity are inseparable. The first issue was slip. Stokes' illuminating research on viscous drag on spheres and slender cylinders [Stokes (1851)] was offset by Piotrowski's (and Girard's) data, which Whetham (1890a, 1890b) finally showed were erroneous, and Helmholtz's analysis of them in terms of slip [Helmholtz (1860)]. Poiseuille's data (1846) and Jacobson's (or Neumann's) analysis of them (1860) might otherwise have settled the issue; they did establish what became a primary technique for measuring fluid viscosity. Even as Lamb (1879) prolonged the ambiguity about slip by giving credence to the Helmholtz–Piotrowski paper, the experimenters who were developing viscosity measurement all adopted the no-slip hypothesis, as had Maxwell as early as 1866. And Maxwell (1866), though fully aware of Helmholtz and Piotrowski, on the basis of his own and other experiments, stated

> I have no doubt that.… there is no slipping.…

The leading alternative technique to Poiseuille's flow emerged in the concentric cylinder apparatus frequently called the "Couette rheometer" after Maurice Couette, who is widely credited with devising the method and making the first accurate measurements. Dryden *et al.* (1932), Graetz and Stöckl (1931), and Donnelly (1991) all show that at least two others developed the same apparatus independently. But none of these reviews tell the whole story. Here we have re-examined the origins of concentric cylinder rheometry and its attribution to Couette.

Stokes and later, Margules (1881), separately *proposed* that the "coefficients of friction and gliding" (both seemed unaware that the no-slip boundary condition at the solid-liquid interface had been conclusively established by Poiseuille's experiments with tube flow) could be measured with a concentric cylinder apparatus. Margules solved for the flow allowing for some slip. Perry built the first concentric cylinder device to measure liquid viscosity in 1882, but he did not publish measurements until 1893. (Élie built a concentric sphere device in 1882 that was inconsequential except for catching Couette's notice.) Couette's measurements of water viscosity (1888, 1890) were between 10% and 23% higher than Poiseuille's reported values. His measurements of air viscosity were not absolute because he calibrated his apparatus with Poiseuille's measurements on water. Mallock's (1888) measurements of water viscosity were more accurate and within 1% of those derived from Poiseuille's measurements. The next year, (1889), Schwedoff reported measurements of the elastic modulus of a gelatin solution with concentric cylinders of his own design. In 1890, he noted that his device was similar to Couette's, and reported the viscosity of water measured with his apparatus that was only 0.2% lower than Poiseuille's values. He also reported the first instance of a shear rate-dependent viscosity with measurements on a gelatin solution, foreshadowing widespread applications of Couette(–Mallock–Schwedoff–Perry) rheometers to rheological characterization of non-Newtonian liquids as surveyed by Coleman *et al.* (1966), [Piau *et al.* (1994)]. A little later Mallock (1896), working under the eyes of Lord Rayleigh and his friend Lord Kelvin, built the first apparatus with which the inner cylinder could be rotated and found, *inter alia*, that the flow between concentric cylinders is unstable when the inner cylinder rotates. The ensuing history of flow stability, from Taylor (1923) onward, is summarized by Donnelly (1991).

It is interesting to note that the original concentric cylinder designers used four different approaches to eliminate end effects [see Dontula (1999) for further discussion]. Three of these are still the methods used today; only Couette's guard rings did not survive. The falling weights which drove rotation in the first designs soon gave way to electric motors. Sensing of torque by mirrors reflecting onto meter sticks was replaced by capacitors but the torsion springs of the original designs remained for over 80 years. Only since the 1970s have springs been replaced by strain gauges and then rebalance transducers and constant torque drag cup motors [Macosko (1994)].

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