Letter to the Editor: Comment on "Origin of concentric cylinder viscometry" [J. Rheol. 49, 807–818 (2005)]. The relevance of the early days of viscosity, slip at the wall, and stability in concentric cylinder viscometry

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I. INTRODUCTION

The paper by Dontula *et al.* (2005) intends to recount and evaluate 19th century work and to "tell the whole story." Despite our great respect for the JOR and other papers by the authors we have the strange impression of reaching a parting of the ways since nearly every line calls for our attention in this manuscript, as in Dontula (1999). The main problems are over the scientific presentation of the no-slip and slip boundary condition, as well as with the citations, which most unfortunately contradict both our unpublished notes and the documents available to us. We also believe that the authors have not properly presented the filiation and continuity which exist between the research of past centuries and present day research in rheology. Finally, old cases are raised in their manuscript which were wisely settled by scientists of the time.

The period considered in this Letter to the Editor will be limited also to the same 19th century period. Some of the findings and details which were not included, and did not fit in, when writing Piau *et al.*'s (1994) biography of Couette will be included.

A short presentation of friction slip and no-slip appears to be necessary. Then, the history of the birth of concepts such as viscosity and friction slip and of their first measurements will be tackled, showing clearly the precedents. After some terminology clarification, remarks will be made on the different experimental contributions to concentric cylinder viscometry over the 1880–1900 period. Our conclusions will follow.

II. SLIP AND NO-SLIP

A. The physics of an interphase and continuum mechanics

The question of slip with friction was a crucial aspect of 19th century research. Slip phenomena and friction at the wall are under the control of the interphase which develops at the wall, involving complex and heterogeneous boundaries, attraction/repulsion forces, or a lack of cohesion. Details of the interphase depend on the physicochemical, mechanical, and geometric properties of both the bulk and wall materials, as well as on stress, temperature, and time scale boundary conditions.

Rheologists need two sets of constitutive equations: one for the phase in the bulk (such as a stress-rate of strain Newton model $\mathbf{T}=2\mu\mathbf{D}$) and one for the interphase (such as a Navier model $\boldsymbol{\tau}=E\mathbf{u}_s$ where $\boldsymbol{\tau}$ is the wall shear stress, $\mathbf{u}_s$ is the slip velocity, and E is Navier's friction coefficient). Each set is a mathematical model written for a physical couple of two elements: the matter at hand and the kind of boundary conditions considered [Piau (1998)].

Understanding, modelling, and controlling slip are still crucial aspects of modern rheology, and they are most important for many suspensions and colloids, as well as for polymers and for lubricants. Simpler fluids such as oil and water may also slip. Indeed, recent techniques seem to allow a friction coefficient for water of $E \approx 10^4$ Pa s.m^{-1} to be measured, and the influence of roughness and of the wetting character of the surface on E to be shown [Cottin-Bizonne *et al.* (2002); Lauga *et al.* (2005)].

B. Beyond the limits of continuum mechanics

It is when the frontiers of continuum mechanics have been trespassed, rather than within any friction slip constitutive equation for an interphase, that the misleading quotation by Dontula *et al.* (2005) of Chauveteau (1982) and Stone *et al.* (2004) in an attempt to define limiting flow scales for no-slip regimes can be understood. Moreover, it is self-evident that no such flow scale limits should be suggested independently and before the materials considered have been specified, as Dontula *et al.* unfortunately proceed to do.

Following Piau (1991) it is advisable to check that the length scale of the flow (or the gap in a rheometer) is larger than about 50 times the mesoscopic length scale of the material (defined after considering both the size and the distance of matter elements) before starting to apply the usual continuum mechanics and rheometry techniques. Constantinescu (1969) explicitly developed gas lubrication continuum theory for Knudsen number values smaller than 0.01. Dissociated human red blood cell suspensions show a change in their apparent viscosity when the smooth tube diameter is smaller than about 400 microns, i.e., fifty times eight microns, the diameter of the cells. A plot of "viscosity" values measured by Chauveteau (1982) versus the ratio of the pore or tube diameter (measured or derived from the permeability for the three different porous media) to the particle length would illustrate again the role of the factor 50 rule, as Chauveteau's findings were obtained with molecules the hydrodynamic diameter of which could be nearly as big or even bigger than that of the tubes.

In such circumstances it must be recalled that it is out of the question to fill the Euclidian space of monophasic continuum mechanics with the usual kinematic field quantities defined as arithmetic mean values over a small domain. When field parameters lose any physical significance, and models for discrete systems should be developed instead, an empirical calculus remedy, known under the name of "apparent slip", has been introduced for a long time, independently of any interphase consideration and possibly of any physical understanding. It merely helps to go on fitting monophasic continuum mechanics tools and models, and mainly consists in relaxing the no-slip condition at the wall in the calculations made, with the benefit of an adjustable parameter.

III. PRECEDENCES AND HISTORY OF CONCEPTS, MODELS, AND TECHNIQUES

A. No-slip and slip with friction at the wall boundary conditions

Coulomb (1784) clearly distinguished the concept of friction with slip of the liquid at the wall (which he named adherence) from the concept of viscosity forces (which he named fluid cohesion). He proved [Coulomb (1801)] that using clean tin surfaces and surfaces covered with mutton fat or with sandstone particles had no influence on disk oscillation damping in water. He also checked that changing the hydrostatic pressure level had no influence either. After this inspiring study he concluded that there was complete discrepancy between solid friction and liquid friction at the wall.

The views developed by Coulomb, and followed by Navier (1823), Stokes (1846), Warburg (1870) (see Sec. III C below), Maxwell, Margules, or Couette, among others, are simply up to date for a rheologist. It is through their careful discussions and various experiments with a series of fluids and apparatuses that the no-slip wall boundary condition has been fully justified and adopted for the particular case of classical fluid mechanics when mainly water, oil, and dense gases are considered within large enough vessels, with rough enough surfaces.

It is Navier (not Stokes) who is credited with the Navier friction slip equation: $\boldsymbol{\tau} = E\mathbf{u}_s$, Navier's friction coefficient E, Navier length $b=\mu/E$, and the first equation for the rate of flow of a viscous fluid through a cylindrical pipe [Couette (1888b), p. 262]: $q = \pi R^4 f(1+4\mu/ER)/8\mu$, where f is the pressure gradient, R the tube radius, and μ the shear viscosity. It is the same formula that was rediscovered by de Gennes (1979), and which allows a simple evaluation of the limiting tube diameter $2R=800b$ where a linear Navier slip still changes the rate of flow by 1%.

B. First concentric cylinder apparatus and pendulums

Coulomb (1784) built the first concentric cylinder apparatus we are aware of [Brillouin (1907)], to measure liquid friction. The inner cylinder was 58.6 mm long, with a diameter of 42.9 mm. It was made either of copper or of lead. The outer cylinder diameter was 128.6 mm. The inner cylinder was suspended to a torsion wire and oscillated. The first electric motor was to be invented much later and the source of energy used by Coulomb was stored elastic energy of the wire.

It appears that after Coulomb most of the experimentalists and theoreticians studied disk, cylinder, and sphere pendulum oscillations, including Stokes (1849, read 1843), Stokes (1851, read 1850) in which the Stokes formula for the drag of a sphere attached to a pendulum was published, Meyer (1861), Maxwell (1866), Margules (1881), and Couette (1887).

C. Pipe flow. Some no-slip and accuracy aspects

The second technique used historically from 1840 onwards to measure viscosity [Hagen (1839); Poiseuille (1846)—deposited Acad. Sci. Paris (1839)] is pipe flow [Sutera and Skalak (1993)]. Warburg (1870) was the first to show that clean mercury did not slip during flow within 2–2.6 mm diameter glass tubes. This result was unexpected since mercury does not wet the wall and a difference in slip conditions with the flow of water along clean glass was expected at this time.

Neumann (1883) and Couette independently calculated correct expressions for pressure losses. Couette performed new measurements, and was able to deal with entrance effects in his experimental data. He also checked the no-slip condition at the wall using paraffin walls among other wall materials.

Tube flow is still generally considered as the most accurate technique for obtaining water Newtonian viscosity data.

D. Continuously rotating devices

Couette was the first (1888a–1890) to be able to build a concentric cylinder constant speed viscometer and to obtain significant data sets following the suggestion made by Margules (1881). The outer cylinder was rotated using an electric motor (a recent technology) and bearings. He measured water, air, and colza-oil viscosity on a range of shear rates (50–350 s^{-1}, 170–1800 s^{-1}, and 100–770 s^{-1}, respectively). He clearly identified flow stability and no-slip conditions.

It is Margules (1881) (not Stokes) who was the first to calculate the necessary formulas and advocate the use of continuously rotating concentric cylinders for measuring viscosity [Donnelly (1991)]. Stokes (1849 read 1845, sec. 8, pp. 303–304) discovered and corrected Newton's error on the velocity field between rotating cylinders.

Perry F.R.S. (1883) suggested the double-cup geometry, and published drawings of an apparatus which apparently became operational after March, 1892 when he was able to estimate end effects. Perry published in 1893 the data he had obtained on sperm oil viscosity exclusively.

The experimental conditions used by Couette, Perry, and some others are summarized in Table I.

E. Slip measurements

Most of the ways slip and friction measurements can be and are addressed were suggested by Coulomb, Stokes, and Margules. The Coulomb roughness method has been generalized, including in the famous Mooney rheometer. The suggestion by Stokes to use motes to measure velocity and visualize flows in the vicinity of the wall has developed in everyday techniques as well as in more recent fluorescent marking. Margules suggested measuring slip at the wall by changing cylinder diameters, a method which was generalized afterwards to the various sorts of rheometers.

Since progress in electrical and mechanical engineering permitted Couette and others to rotate cylinders easily and continuously, the development of all sorts of engines based on all sorts of physical principles has allowed a variety of motion– controlled or stress-controlled devices to be built. In addition, some experimental devices may be close to tribology set-ups, which sends us back to Coulomb's work on the friction of solids.

F. Transitions

Hagen (1854, cited p. 196 by Brillouin), du Buat (1816), Darcy (1857, cited p. 218 by Brillouin), Reynolds (1883, cited p. 210 by Brillouin), and Couette (1890) all contributed to the identification of the conditions of laminar-turbulent transition. The most celebrated paper was published by Reynolds (1883), when the critical Reynolds number was defined and flow visualizations of legend were obtained with a colored fluid filament. Modern investigations refer to flow becoming unstable between rotating cylinders as Taylor–Couette flow [Donnelly (1991); Chandrasekar (1970)]. Inertial transitions finally impose shear rate limitations on viscometric measurements whatever the instrument is, in addition to other elastic or plastic instabilities. They should not be overlooked in the case of mobile liquids, such as the surfactants and polymer solutions.

IV. TERMINOLOGY: VISCOSITY AND/OR INTERNAL FRICTION

It is all the more surprising to us when Dontula *et al.* in 2005 seem to give bad marks to Navier, Stokes, Margules, or Couette because they used "internal friction" during the 19th century. As regards Couette, he published a 48–page "Notice sur la Viscosité des Liquides" (1888b), where he explained that the word *viscosity* was not so much in use at that time, and needed to be well-defined as several meanings had been given to this word.

For etymologists (Rey (1993)), the adjective *viscous*, introduced from the Low Latin *viscosus* (1256, = "sticky, gluey"), a derivative of *viscum* (mistletoe, and the glue made from mistletoe), applies to a thick liquid which flows with difficulty, and *viscosity* has the same origin. It has been used since in different ways depending on the language considered, possibly with a very pejorative meaning to characterize persons or actions. It may be noticed that glue is related to the Indo-European "gel" (cold), and that rheologists refer to certain materials which may not flow at all as gels. *Viscosity* was also introduced in

TABLE I. Main characteristics of the Couette, Mallock, Perry and Schwedoff set up and data with concentric cylinder rheometers over the 1880–1900 period. More recent Leroux experiments (1925) are also described. N is the number of revolutions per minute and $\dot{\gamma}$ the shear rate. All cylinders presumably made of brass except Schwedoff's (uncertain for Mallock's apparatus) and outer cylinder rotating except for Perry's double cup. The series of Mallock's data with inner cylinder rotating is not shown (only partly available in his paper). Labels (1) and (2) correspond to the two different outer cylinders used by Mallock. The published drawings and speculations by Mallock on a remark by Thomson on river flows give no indication that a glass outer cylinder was used, contrary to Dontula *et al.*'s suggestion. Dimensionless Reynolds number $\mathrm{Re}=\rho V_2 e/\mu$ or (double cup), Taylor number $\mathrm{Ta}=2(\rho V_1 e/\mu)^2(e/R_1)$ where V_2 is the outer cylinder surface velocity, V_1 and R_1 are the surface velocity and radius of Perry's inner rotating cylinder, and e is the gap width. Index c in the table refers to critical values for the appearance of the laminar-turbulent transition (outer cylinder rotation) or Taylor vortices (inner cylinder rotation).

Experimental conditions	**Couette 1888-1890**	**Mallock I 1888**	**Schwedoff 1890**	**Perry 1893**	**Mallock II 1896**	**Leroux 1925**
Initial aim of the study	Navier Eqs. validity transition	Viscosity coeff. measurement	Elasticity, viscosity of fluids	Viscosity coeff. measurement	Stability and transition	Viscosity coeff. measurement
Main test fluid	Water	Water	Water	Sperm oil	Water	Water
Temperature (°C)	12.8–19.1	4, 13.8, and 48	19.1	17.5–81	2–55.5	1.5–44.5
Mean radius (mm)	143.9	48.27	34.8	121.4/109.0 (double cup)	87.88 (1) 81.60 (2)	27.68
Gap e (mm)	2.465	3.81	12.57	10.2/10.2	23.11 (1) 10.55 (2)	4.81
Sample height (mm)	79.06	110.7	30 and 60	54.25–82.75	193.55–213.15	50.49
End effects reduction	2 guard rings 32.6 mm long each	Air bubble	Diff.	Diff.	Air bubble +short ring +mercury	2 guard rings 35.6 mm up 25 mm down
N **range (rpm)**	8.71–453	0.16–15.9	2.5–2.8	8.75–115	2–126	2–2.4
V_2 **range (cm/s)**	13.3–695	0.83–83.3	1.09–1.22	11.6–152 (external wall)	2.1–131 (1) 1.8–114 (2)	0.63–0.77
$\dot{\gamma}$ **range** ($\mathbf{s^{-1}}$)	54–2819	2–219	≈0.9	11.3–149	0.9–56.7 (1) 1.9–123 (2)	1.3-1.6
Re **range (and)** Ta **range (double cup)**	327–17100	32–3175	137–153	Re (outer gap): 6–1800 Ta (inner gap); 103–11470 (17.5 °C) 1630–262 000 (65 °C)	480–30 300 (1) 192–12 090 (2)	30–37
Critical N_c **(rpm)**	56	?	-	50(17.5 °C) <17 (65 °C)	69 (1, 20 °C) 68 (2, 20 °C)	-
Transition Re_c **(or)** Ta_c**/double cup**	2116	?	-	Ta_c: 3400 (17.5 °C) <5800 (65 °C)	16600 (1) 6500 (2)	-
Error evaluation	yes	no	no	no	no	yes
Other fluids data	Air, colza-oil	no	Glycerin, castor oil, gelatin	no	no	no

physics (1710) to describe the state of a pasty fluid, the movements of which are slowed down.

It was only after that a neutral use of the word *viscosity* developed as a substitute for the expressive and physically sound *internal friction*. Both expressions have been used for a long period [Lamb (1932); Prandtl (1949)].

"Viscosity" remains an unclear word in common language, unlike volume and clock time. It still has the ambiguities of its linguistic origins for newcomers: words like *glue, sticky, gel,* and *pasty* seem very far from any air or water-flow property. Hence, after having introduced internal forces and stresses to nonspecialists, we believe that pedagogic circumstances at least do exist where it may still be useful to go on using internal friction and to maintain the use of this terminology as an equivalent to the shorter term of viscosity. Moreover, it may appear to compensate for the loss of physical meaning associated with the walls sometimes confusingly introduced to define viscosity.

V. REMARKS ON THE DIFFERENT EXPERIMENTAL CONTRIBUTIONS TO CONCENTRIC CYLINDER RHEOMETRY (1880–1900)

A. Accuracy

Let us remember first of all that Poiseuille's work on water viscosity tube measurements published in 1843 was known all over the world, and that human nature has to be taken into account. It also happens that today Poiseuille values are no longer the reference values for the viscosity of water. The Poiseuille value for water viscosity at 20 °C *was larger than* 1.009 *mPa s whereas the value accepted throughout most of the world nowadays is close to* 1.002 ± 0.2% [Collings and Bajenov (1983); Barnes (2002)].

Couette observed a systematic (not random) excess deviation of about 12.5% for his absolute air and water viscosity measurements (1888a; 1890). The 22.9% unduly mentioned by Dontula *et al.* (2005) corresponds to eccentricity tests only (see Sec. C below).

Couette's 1888a data gave a relative value for the viscosity of air equivalent to 1.794×10^{-5} Pa s at 20 °C when water is used by Couette as a calibration fluid. The best values reported by others from tube flow and from oscillating disc techniques at that time were in the range $1.78 - 1.98 \times 10^{-5}$ Pa s at 20 °C over the same period. According to Schlichting (1968), the exact value should have been 1.80×10^{-5} Pa s at 20 °C, though a significant dispersion is observed between various sources and air humidity was not measured by Couette.

Couette also checked that colza-oil viscosity was independent of the shear rate value. Though he did not calculate the viscosity himself, we can deduce from his data a relative result for the viscosity value 0.10 Pa s at 20 °C which is correct. He was able to use his rotating apparatus to obtain viscosity relative values that were accurate to within less than a few percent, and correctly quantify the critical Reynolds number of the laminar-turbulent transition for air and for water which is good.

Couette believed eccentricity was the main reason for the systematic deviations in the concentric cylinder apparatus and checked for the existence of a minimum when changing the eccentricity [Couette (1890a), p. 454]. He said how eccentricity could be compensated and why he did not proceed to improve the apparatus.

Couette successively tried each of the three flow families: oscillating, continuously rotating, and pipe flow apparatuses. He soon reduced his contribution to the first to a theoretical one, took a great experimental interest in the second one, and came back to pipe flow techniques when needed for their accuracy. It seems that he had already understood that pendulum oscillations and rotating cylinders would never compete with the tube flow. For instance, using tubes at 10 °C, Couette found 1.303 mPa s (1890a, p. 503)

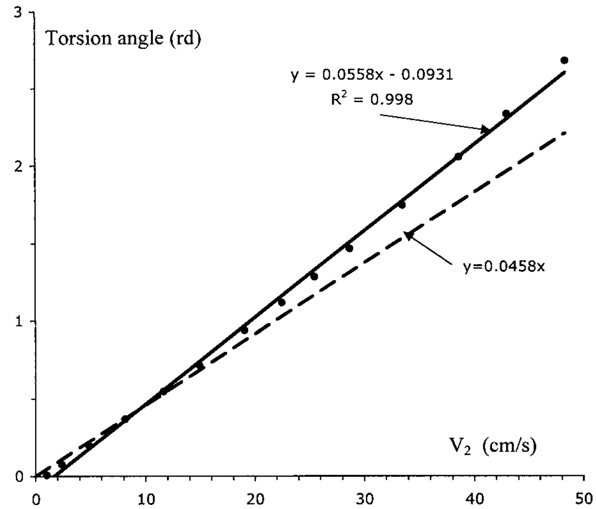

FIG. 1. Linear fit in the 0.8–48 cm/s interval of the curve giving the torsion angle vs the outer cylinder surface velocity V_2, at 13.8 °C [data from a discretization of Mallock's curve supposed to be drawn through his not shown data (1888, Diagram 1)]. The so-called tangent to the original Mallock quadratic curve whose slope gives the viscosity value is also drawn (dashed line).

for water viscosity and Poiseuille 1.309, while the values given by Collings and Bajenov (1983, Table 2) range from 1.304 to 1.307±0.2%. Couette could be confident when using water as a calibration fluid.

Either with the concentric cylinder apparatus or with tubes, Couette obtained data sets which were considered and proved to be the best at the time as regards the (absolute and relative) values of viscosity. However, Couette's major priorities for his thesis work were Navier–Stokes equation validity and no-slip condition relevance, as well as laminar-turbulent transition, rather than just the accuracy of some measurement.

Mallock systematically encountered problems in working the apparatuses and interpreting his own data. His first results (1888) are not really convincing. They show continuous curves where a transition with a slope change is expected. Moreover, viscosity values "very close to Poiseuille ones" are obtained from these results in 1896, after two successive adjustments due to erroneous formulas, using so poor a method as to find tangents at the origin of badly drawn mean curves with zero offsets; in fact, a linear fit of his 1888 curves in the 0.8–48 cm/s interval for V_2 ($Re < 2000$) would give more than 20% deviation at 13.8 °C (cf. Fig. 1). Then, with the new version of his apparatus, about 70% and 25% deviations "he could not explain" are reached by Mallock at 10–14 °C (1896, Diagram 10, curves a,b). His conclusions about the laminar turbulent transition are troublesome [Taylor (1923), p. 328].

Schwedoff (1890) with the declared aim of comparing his results on water with Poiseuille's, uses a geometry similar to Coulomb (1784) for only one very low cylinder velocity (Table I) and one temperature. He gives very few details on his measurements and on their accuracy. However, for mechanical and optical reasons we believe that the precision he reached could not be better than several percent when summing up all the possible error sources in the dimensions and readings of his device which, moreover, is not really easily balanced, and knowing that some errors are doubled with differential methods. In addition, for thermal reasons, systematic water viscosity variations of 3 to 6% at least can be expected. The reasons are that the temperature he used was a mean "ambient" one measured in the atmosphere close to the apparatus. This temperature is frequently different from the true water temperature by 1 or 2 °C at least. Nevertheless, Schwedoff claimed a 0.2% coincidence with the target value for a mean velocity gradient approximately equal to 1 s^{-1}. His results on water viscosity seem to be either too limited or too definite to be noteworthy in the absence of the necessary details.

Schwedoff's contribution to gelatin viscometry was of an exploratory nature only, since he did not notice at all the gel's broken structure and slip under flow conditions (he could have taken advantage of Stokes's recommendation of motes but he did not).

Perry F.R.S. (1893) proposed the double cup geometry with the aim of reducing end effects. It is more an interesting change in design than a genuine innovation. In fact, end effects remained noticeable and he had to apply a differential method as Schwedoff had before. A reduction is known to occur in the range of permitted rotation speeds with the double cup geometry, due to Taylor vortices. No measurements were made by Perry on water. They would need really low cylinder velocities to stay in the steady regime. Data tables are shown (not only for 40 °C) for sperm oil viscosity possibly with some random scatter, and even critical speeds are indicated. They seem to be coherent with the tabulated viscosity values, and with the accepted value of the critical Taylor number 3390 which is good (see Table I). However, erroneous and incomplete conclusions on viscosity variations with temperature were drawn (see Fig. 2) together with expectedly wrong data, which show a discontinuity in the values of oil density at 40 °C. An unexpected failure to perform experiments by oscillating the cup was left unexplained.

B. End effects

Different methods, recalled by Dontula (1999), were used by the contributors to reduce end effects.

Guard rings are a method of extensive interest in physics, and they can indeed be efficient. Couette found a linear (laminar) regime in a rather large interval, and the improved version of Couette's apparatus built by [Leroux (1925, see Table 1)] gave him discrepancies between 0.9 and 2.3% from the true (current) values. Such rings could provide reasonably accurate absolute measurements with a limited end effect correction, though the gap between the guard rings and the suspended cylinder may have shifted Couette's data upwards by about 1.5%. More recently Giesekus [Abdel-Wahab *et al.* (1990)] constructed a new eccentric cylinder rheometer with guard ring equipment which provided reliable data.

The differential method used by Schwedoff (1889) and Perry (1893) to eliminate the significant end effects from their data necessitates several measurements with different wetted heights at the same temperature and the errors are doubled.

The air bubble beneath the inner cylinder had, in the opinion of Mallock (1888, 1896), to be completed with a short ring and a mercury floor in order to enlarge the linear range to 60–70 rpm (revolutions per minute) (1896, Diagram 8). When using these two additional devices, Mallock obtained decreases in the torsion angle, as large as 30% at a medium value (≈28 rpm) of the outer cylinder velocity and 50% at 68 rpm.

C. Eccentricity

Couette intentionally decentered cylinders (1890a) to illustrate and prove the influence of eccentricity, which resulted in a torque increase. He emphasized that the smaller the gap the larger the deviation may be, even for a minor cylinder off-center. In fact, Couette (1890) did obtain a correct formula for the influence of relative eccentricity ε for torque C:

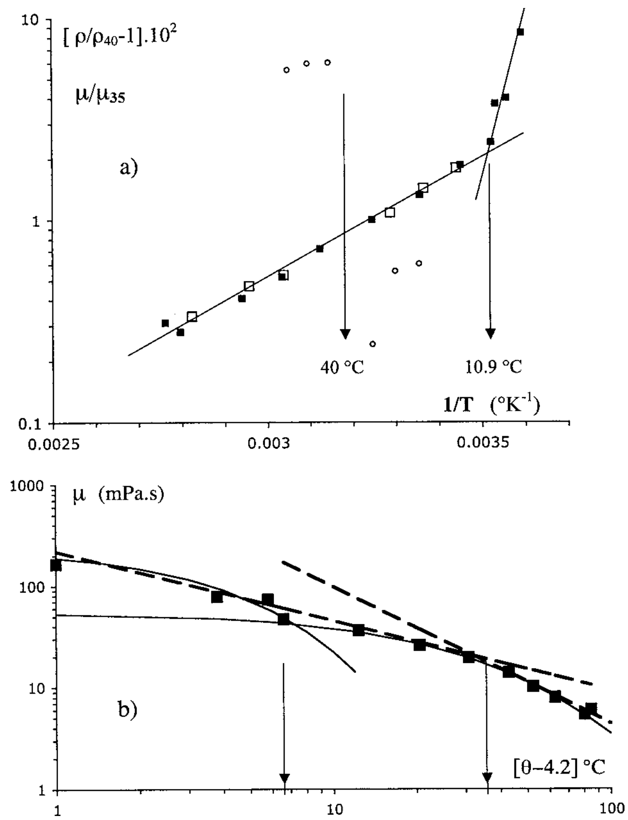

FIG. 2. (a) Viscosity μ taken from Perry's data (1893): Table III (filled squares), Tables IV–XI (empty squares), with respect to 1/T ($°K^{-1}$), with the best two linear fits. The density ρ measurements (empty circles) are also shown. μ_{35} and ρ_{40} were reference values available in Perry's tables. The strong slope change of μ at 10.9 °C might be consistent with the onset of a phase change process or with an experimental problem, Perry did not notice. Contrary to his statement, no abrupt change appears at 40 °C for μ variations. The discontinuity for ρ is questionable. (b) Evaluation of viscosity μ (mPa s) data following the method indicated by Perry, where logarithms are used and $(\theta-4.2)$°C is the temperature difference plotted along the abscissa axis. The equations for the two straight dashed lines were given by Perry (1893, p. 449). The plain curves added correspond to our linear fits defined Fig. 3(a). Vertical arrows still correspond to $\theta=10.9$°C and $\theta=40$°C as in a). It seems that Perry's discussion of his data is biased by artifacts which result from a misuse of logarithmic plots, and that yet unexplained errors appeared in his density measurements.

$$C = \left(1-\varepsilon^2\right)^{-1/2} C(\varepsilon = 0).$$

This formula can also be said to represent the case of short journal bearings shafts with the Sommerfeld condition. A similar formula for long journal bearings with the same $C(\varepsilon=0)$ reads [Frene *et al.* (1990)]:

$$C = \frac{1+2\varepsilon^2}{\left(1+\frac{\varepsilon^2}{2}\right)\left(1-\varepsilon^2\right)^{1/2}} C(\varepsilon = 0).$$

Hence, in order to explain a 10% systematic error one obtains a relative eccentricity of respectively 42% and 22% from the formulas above, which provides upper and lower boundaries. For the real apparatus built by Couette the L/D value was close to 1/2, i.e., close enough to the short bearing case, and between both limits which are reached when

L/D=1/8 and 8, respectively. More complex formulas and/or numerical data would be needed to go into full detail and comment.

It is both the centering and the motion of the parts which contribute to the eccentricity observed in a rotating apparatus. It is a very well-known fact in industry and research that unloaded cylindrical journal bearings are unstable and their motion is orbital around their expected axis position (vertical axis turbines have pad bearings). Details of this motion depend also on the various parts flexibility and lack of balance. We should not be surprised that Couette said he opposed cylinder whirling using obstacles (1890b, p. 22).

Other minor sources not mentioned above for the systematically excessive viscosity values obtained by Couette may be suggested: (a) a slight overestimation of the wire torsion force by counterbalancing it with weights placed in an Atwood machine pan, and neglecting any friction on the pulleys [Couette (1890a), p. 455], (b) the systematic use of three successive oscillations to estimate the steady position, without waiting to reach the steady state [Couette (1890a), p. 454].

Only a relatively very small minority of rheologists will feel interested in absolute measurement techniques nowadays; many people use calibration fluids knowing that for various reasons polymer rheometry accuracy may be frequently poorer than 5% and viscoplastic yield value accuracy is much worse.

VI. CONCLUSION

It is out of the question for rheologists with an interest in concentric cylinders viscometry (hence in viscosity, slip and stability) to overlook the outstandingly visible contributions by Coulomb, Navier, Poisson, Cauchy, Stokes, Poiseuille, Maxwell, Reynolds, Couette, and certain others [already listed in the book by Tanner and Walters (1998) and not mentioned here for the sake of brevity] over the 19th century period considered. This is attested in documents which are too numerous to mention. A fine aspect of the work by Couette is that the physics, the apparatuses, the data, the theory developed, and the calculations made were all under full control.

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