Example 3.1

The pressure of a gas in a piston-cylinder varies with volume according to (a) $pV = C$; (b) $pV^2 = C$. The initial pressure is 400 kPa, the initial volume is 0.02 m^3, and the final volume is 0.08 m^3. Determine the work for both processes.

Solution

Given: Gas pressure in a piston-cylinder varies as a function of volume.

Find: The work done in the expansion process.

Sketch and Given Data:

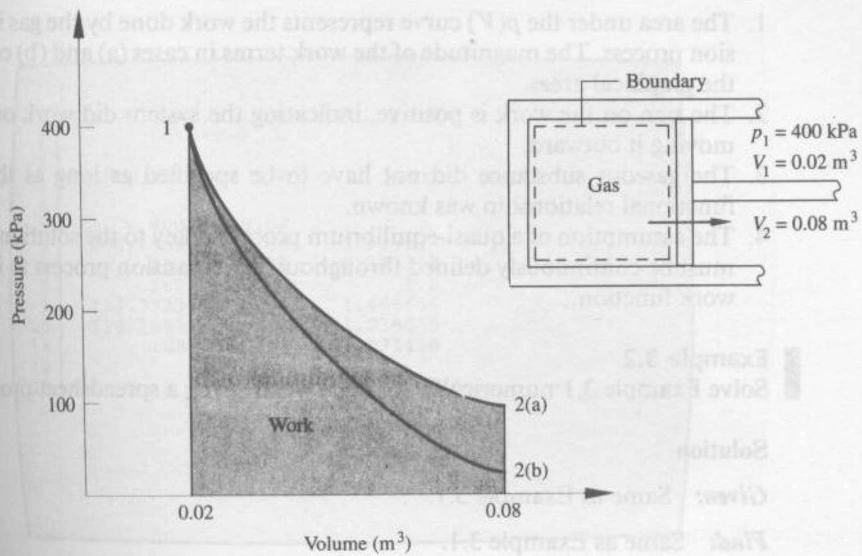

Figure 3.5

Assumptions:

1. The gas undergoing the expansion is a closed system.
2. The expansion is in quasi-equilibrium as the pressure is continuously defined during the expansion process.

Analysis: The work is determined by integrating equation (3.16) for the functional relationship for $p(V)$ for cases (a) and (b).

Case (a):

$$W = \int_1^2 p\,dV = C\int_1^2 \frac{dV}{V} = C\ln\left(\frac{V_2}{V_1}\right)$$

$$C = pV = p_1V_1$$

$$W = p_1V_1\ln\left(\frac{V_2}{V_1}\right) = \left(400\,\frac{\text{kN}}{\text{m}^2}\right)(0.02\text{ m}^3)\ln\left(\frac{0.08}{0.02}\right) = 11.09\text{ kJ}$$

Case (b):

$$W = \int_1^2 p\,dV = C\int_1^2 \frac{dV}{V^2} = C\left[\frac{1}{V_1} - \frac{1}{V_2}\right].$$

$$C = p_1V_1^2 = p_2V_2^2$$

$$W = p_1V_1 - p_2V_2 = (400\text{ kN/m}^2)(0.02\text{ m}^3) - (25\text{ kN/m}^2)(0.08\text{ m}^3)$$

$$W = 6\text{ kJ}$$

Comments:

1. The area under the $p(V)$ curve represents the work done by the gas in the expansion process. The magnitude of the work terms in cases (a) and (b) correspond to the graphical areas.
2. The sign on the work is positive, indicating the system did work on the piston, moving it outward.
3. The gaseous substance did not have to be specified as long as the pressure's functional relationship was known.
4. The assumption of a quasi-equilibrium process is key to the solution, as pressure must be continuously defined throughout the expansion process to integrate the work function. ■

Example 3.5

An adiabatic tank similar to the one that Joule used in determining the mechanical-thermal energy equalities contains 10 kg of water. A 110-V, 0.1-A motor drives a paddle wheel and runs for 1 h. Determine the change of specific and total internal energy of the water.

Solution

Given: An adiabatic tank containing water receives work from a motor-driven paddle.

Find: The change of the water's specific and total internal energy.

Sketch and Given Data:

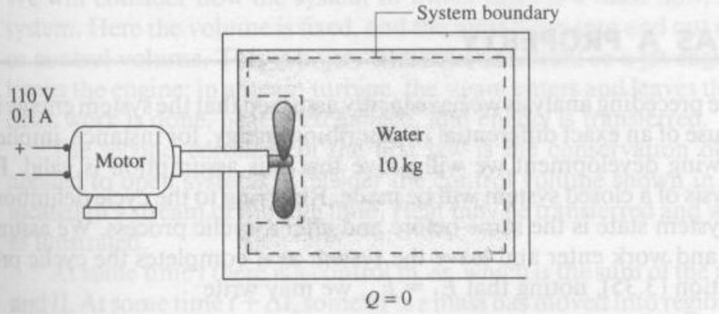

Figure 3.16

Assumptions:

1. The water is a closed system.
2. The process is adiabatic, so the heat transfer is zero.
3. All the electrical work is converted into paddle work.
4. Changes in system kinetic and potential energies are zero.

Analysis: The first law for a closed system is

$$Q = \Delta U + \cancelto{0}{\Delta \mathrm{K.E.}} + \cancelto{0}{\Delta \mathrm{P.E.}} + W$$

$$Q = \Delta U + W$$

The work term is only paddle work, W_p, which is done on the system (negative), and the system is adiabatic ($Q = 0$).

$$0 = \Delta U - W_p$$

$$\Delta U = m(u_2 - u_1) = W_p$$

The paddle work needs to be determined from the power consumption data.

$$W_{electrical} = \mathcal{V}_i = (110 \text{ V})(0.1 \text{ A}) = 11 \text{ W} = 11 \text{ J/s}$$

$$W_{electrical} = (\dot{W}_{electrical})(t) = (11 \text{ J/s})(3600 \text{ s}) = 39.6 \text{ kJ}$$

$$\Delta U = 39.6 \text{ kJ}$$

$$\Delta u = \Delta U/m = 39.6 \text{ kJ}/10 \text{ kg} = 3.96 \text{ kJ/kg}$$

Comments:

1. Understanding the term *adiabatic*, meaning the heat transfer is zero, is pivotal to the problem solution.
2. Paddle work only goes into a system and hence is negative.
3. It is important to distinguish between specific and total internal energy, making sure the first-law equation is dimensionally correct. ■

Example 3.7

A nozzle is a device that converts fluid thermal energy, enthalpy, into kinetic energy. The kinetic energy may be used to drive a mechanical device such as a turbine wheel, thus converting the fluid energy into mechanical work. A nozzle receives 0.5 kg/s of air at a pressure of 2700 kPa and a velocity of 30 m/s and with an enthalpy of 923.0 kJ/kg, and the air leaves at a pressure of 700 kPa and with an enthalpy of 660.0 kJ/kg. Determine the exit velocity from the nozzle for (a) adiabatic flow; (b) flow where the heat loss is 1.3 kJ/kg.

Solution

Given: Air flows through a nozzle, a device that converts fluid thermal energy into kinetic energy.

Find: The air's exit velocity from the nozzle.

Sketch and Given Data:

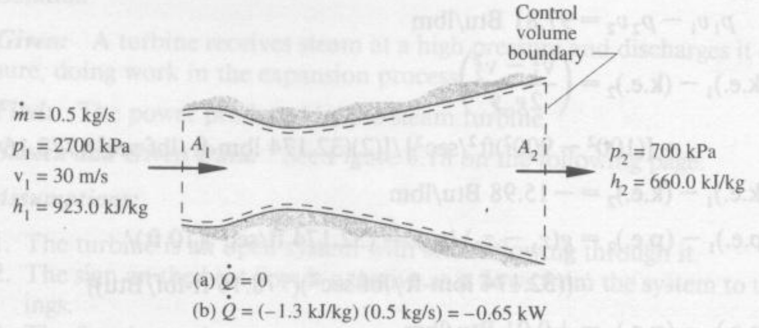

Figure 3.19

Assumptions:

1. The flow rate is steady, the system open.
2. The work crossing the system control volume is zero.
3. The change in potential energy from inlet to exit can be neglected.

Analysis: Case (a): The first law for an open system with steady flow (subscript 1 refers to inlet state and 2 refers to exit state) is

$$\dot{Q} + \dot{m}[h + \text{k.e.} + \text{p.e.}]_{\text{in}} = \dot{W} + \dot{m}[h + \text{k.e.} + \text{p.e.}]_{\text{out}}$$

$$(\text{k.e.})_2 = (h_1 - h_2) + (\text{k.e.})_1 + [(\text{p.e.})_1 - (\text{p.e.})_2]$$

$$\frac{v_2^2}{2} = (h_1 - h_2) + (\text{k.e.})_1$$

where $\dot{W}$ was eliminated by assumption 2, Δp.e. by assumption 3, and $\dot{Q}$ by adiabatic flow for case (a).

$$\frac{v^2 \text{m}^2/\text{s}^2}{2(1000)\ \text{J/kJ}} = (923.0 - 660)\ \text{kJ/kg} + \frac{30^2 \text{m}^2/\text{s}^2}{2000\ \text{J/kJ}}$$

$$v_2 = 725.9\ \text{m/s}$$

Case (b): The heat loss $q = -1.3$ kJ/kg. The first law for a steady open system is

$$\dot{Q} + \dot{m}[h + \text{k.e.} + \text{p.e.}]_{\text{in}} = \dot{W} + \dot{m}[h + \text{k.e.} + \text{p.e.}]_{\text{out}}$$

Divide by $\dot{m}$ and solve for the kinetic energy out.

$$(\text{k.e.})_2 = q + (h_1 - h_2) + (\text{k.e.})_1 + [(\text{p.e.})_1 - (\text{p.e.})_2]$$

$$\frac{v_2^2}{2} = q + (h_1 - h_2) + (\text{k.e.})_1$$

$$\frac{v_2^2 \text{m}^2/\text{s}^2}{2(1000)\ \text{J/kJ}} = -1.3\ \text{kJ/kg} + (923.0 - 660)\ \text{kJ/kg} + \frac{30^2 \text{m}^2/\text{s}^2}{2000\ \text{J/kJ}}$$

$$v_2 = 724.1\ \text{m/s}$$

Comments:

1. The effect of heat transfer from the nozzle reduces the velocity of the air exiting the nozzle, as that energy cannot be converted into kinetic energy.
2. It is important to maintain correct unit balances in converting specific kinetic energy to kilojoules/kilograms.
3. To obtain work to or from a control volume, shaft rotation is required. There will never be work from a nozzle, as no shaft is present. ∎

Example 3.11
A power plant produces 1000 MW of electricity and uses residual oil as the fuel. The oil has a heating value, the maximum energy liberated in combustion, of 43 000 kJ/kg. The overall efficiency of the power plant is 52%. Determine the instantaneous heat input, the daily fuel consumption, and the dimensions of a cylindrical storage tank ($L = D$) to hold a 3-day supply of oil. The density of the oil is 990 kg/m^3. Determine in addition the amount of heat rejected to the environment.

Solution

Given: A power plant operating on a thermodynamic cycle produces power while consuming energy, heat. The cycle efficiency and the net power produced are given.

Find: The heat input required for the cycle, the fuel necessary in kilograms per second, the size of a tank to hold a 3-day fuel supply, and the heat rejected to the environment.

Sketch and Given Data:

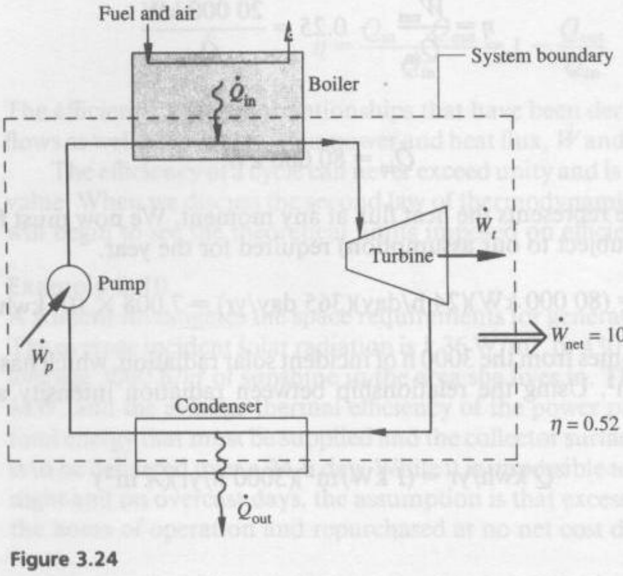

Figure 3.24

Assumptions:

1. The power plant operates on a thermodynamic cycle.
2. All the energy liberated by burning the fuel is converted to heat into the system.

Analysis: From the expression for the cycle efficiency, equation (3.65), the heat input may be determined.

$$0.52 = \frac{1000}{\dot{Q}_{in}}$$

$$\dot{Q}_{in} = 1923 \text{ MW}$$

From equation (3.64) we can determine the heat out.

$$\dot{W}_{net} = \dot{Q}_{in} - \dot{Q}_{out}$$

$$\dot{Q}_{out} = 1923 - 1000 = 923 \text{ MW}$$

The heat input is created by burning fuel, thus

$$\dot{Q}_{in} = \dot{m}_f h_{RP}$$

where $\dot{m}_f$ is the fuel flow rate in kilograms/second and h_{RP} is the heating value of the fuel. Thus, the fuel flow rate may be solved for.

$$1\,923\,000 \text{ kW} = (\dot{m}_f \text{ kg/s})(43\,000 \text{ kJ/kg})$$

$$\dot{m}_f = 44.72 \text{ kg/s}$$

The total mass for a 3-day supply is

$$m_f = (44.72 \text{ kg/s})(3600 \text{ s/h})(24 \text{ h/day})(3 \text{ day}) = 1.159 \times 10^7 \text{ kg}$$

The density relates volume and mass, hence the total volume required is

$$(V_f \text{m}^3) = (m_f \text{kg})/(\rho \text{ kg/m}^3)$$

$$V_f = (1.159 \times 10^7)/(990) = 11\ 708 \text{ m}^3$$

Lastly, the volume of a cylinder is $V = \pi LD^2/4$, thus for the oil tank with $L = D$,

$$11\ 708 = \pi D^3/4 \qquad D = L = 24.6 \text{ m}$$

Comments:

1. The fuel and air are not part of the system; they only provide a means to obtain the heat input.
2. This example illustrates the tremendous mass of fuel that must be transported, stored, and consumed in a power plant with excellent overall efficiency.
3. We do not need to know how the cycle produces the power to determine some of the overall effects, such as total fuel consumption and the heat rejected to the environment.
4. The heat rejected to the environment is usually to a water source, but may be directly to the atmosphere via cooling towers. ■

Example 5.5

A piston-cylinder contains 2 kg of helium at 300°K and 100 kPa. The helium is compressed irreversibly to 600 kPa and 450°K, and 100 kJ of heat is transferred to the surroundings in the process. Determine the final volume and the work done, considering helium to be an ideal gas.

Solution

Given: Helium, an ideal gas, is compressed irreversibly from 300° K and 100 kPa to 450°K and 600 kPa. The process transfers 100 kJ of heat to the surroundings.

Find: The final volume and the work done in the process.

Sketch and Given Data:

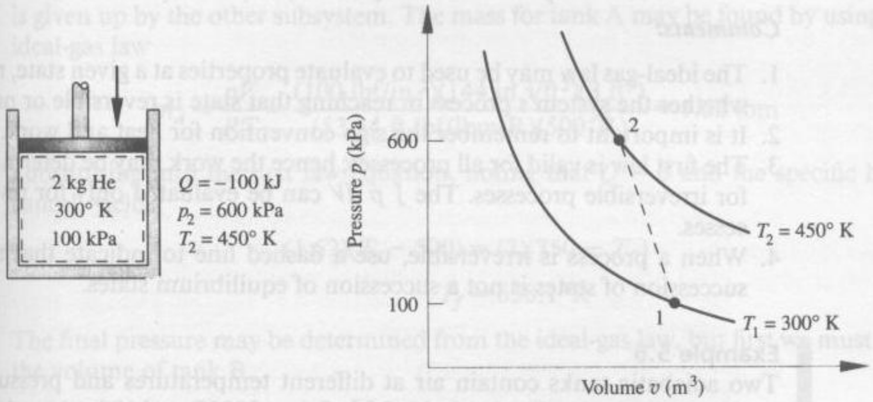

Figure 5.14

Assumptions:

1. The helium is a closed system.
2. The initial and final states are equilibrium states.
3. Helium may be considered an ideal gas.
4. The changes in kinetic and potential energies are zero.

Analysis: The first law for a closed system is

$$Q = \Delta U + \cancel{\Delta K.E.}^{\,0} + \cancel{\Delta P.E.}^{\,0} + W$$

The changes in kinetic and potential energies are assumed to be zero. Solving for the work yields

$$W = Q - \Delta U$$

For an ideal gas the equation of state for internal energy is

$$\Delta U = mc_v(T_2 - T_1) = (2\ \text{kg})(3.1189\ \text{kJ/kg-K})(450°\text{K} - 300°\text{K})$$
$$\Delta U = 935.7\ \text{kJ}$$

Solving for work yields

$$W = -100\ \text{kJ} - 935.7\ \text{kJ} = -1035.7\ \text{kJ}$$

The minus sign on the answer indicates that work was added to the system.

The ideal-gas law may be used to determine the final volume at state 2:

$$V = \frac{mRT}{p} = \frac{(2\ \text{kg})(2.077\ \text{kJ/kg-K})(450°\text{K})}{(600\ \text{kN/m}^2)} = 3.12\ \text{m}^3$$

Comments:

1. The ideal-gas law may be used to evaluate properties at a given state, regardless of whether the system's process in reaching that state is reversible or not.
2. It is important to remember the sign convention for heat and work.
3. The first law is valid for all processes; hence the work may be determined from it for irreversible processes. The $\int p\, dV$ can be evaluated only for reversible processes.
4. When a process is irreversible, use a dashed line to indicate that the system's succession of states is not a succession of equilibrium states. ■

Example 6.2
A rigid 1-m³ tank receives 500 kJ of heat and paddle work delivered for 1 h with a shaft torque of 1 J and a speed of 300 rpm. The tank contains steam initially at 300 kPa and 90% quality. Determine the final system temperature.

Solution

Given: A tank contains wet steam and receives heat and paddle work, increasing the system's energy.

Find: The final system temperature.

Sketch and Given Data:

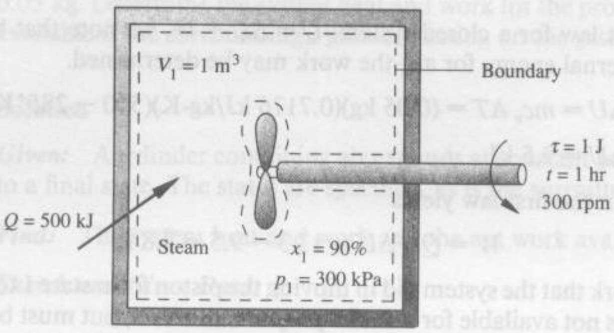

Figure 6.5

Assumptions:

1. The system is constant-volume and closed.
2. The system mechanical work is zero in light of assumption 1.
3. The initial and final states are equilibrium states.
4. The changes in kinetic and potential energies are zero.

Analysis: The first law for a closed system, invoking assumptions 2 and 4, may be written as

$$Q = \Delta U + W_p$$

where W_p is the paddle work added to the system by the rotating propeller.

The properties of steam at state 1 are found by using Table A.6, yielding

$$u_1 = u_f + x_1 u_{fg} = 560.9 + (0.90)(1982.9) = 2345.5 \text{ kJ/kg}$$

$$v_1 = v_f + x_1 v_{fg} = 0.001\,073 + (0.90)(0.604\,78) = 0.5454 \text{ m}^3\text{/kg}$$

$$m = V_1/v_1 = (1 \text{ m}^3)/(0.5454 \text{ m}^3\text{/kg}) = 1.833 \text{ kg}$$

The paddle work may be found from equation (3.20).

$$W_p = \tau\omega t = (1.0 \text{ J/rad})(5 \text{ rev/s})(2\pi \text{ rad/rev})(1 \text{ h})(3600 \text{ s/h})$$

$$W_p = 113\,100 \text{ J} = 113.1 \text{ kJ}$$

Substituting into the first-law equation, noting that work into a system is negative and heat into a system is positive, yields

$$500 \text{ kJ} = \Delta U - 113.1 \text{ kJ}$$

$$\Delta U = 613.1 \text{ kJ} = m(u_2 - u_1) = (1.833 \text{ kg})(u_2 - 2345.5 \text{ kJ/kg})$$

$$u_2 = 2680.0 \text{ kJ/kg}$$

In this situation we know the final state is defined by its internal energy and specific volume, $v_2 = v_1$, for a constant-volume process.

From the superheated steam table we find that the final state's temperature is approximately 220°C. This is found by performing a double interpolation matching both the specific volume and internal energy, a tedious manual task, but one readily accomplished by TK Solver. SATSTM.TK computes a temperature of 221.1°C at a pressure of 410.8 kPa.

Comments:

1. Paddle work is always negative and irreversible: it can only be added to the system, hence negative and the system cannot cause the paddle to rotate, hence irreversible.
2. Double interpolations in the steam table are quite time-consuming when done manually. Solution on the computer is much easier and faster.
3. Mechanical work for a constant-volume process is zero, regardless of any other energy interactions. ■

Example 6.4

A compressor receives 0.75 m³/s of air at 290°K and 101 kPa. The compressor discharges the air at 707 kPa and 435°K. The heat transfer from the control volume is determined to be 2.1 kW. Determine the power required to drive the compressor.

Solution

Given: An air compressor receives air at known inlet and exit conditions and with a known rate of heat transfer from the control volume.

Find: The power to drive the compressor.

Sketch and Given Data:

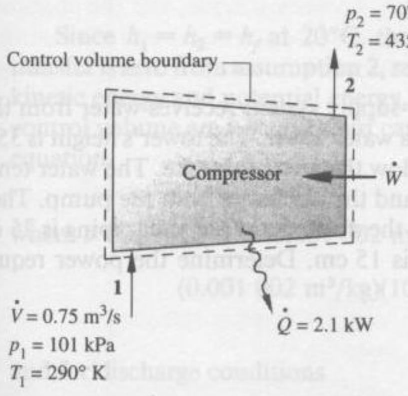

Figure 6.11

Assumptions:

1. Air at these temperatures and pressures may be considered an ideal gas.
2. The flow is steady-state.
3. The changes in kinetic and potential energies may be neglected.

Analysis: The first law for a steady-flow open system is

$$\dot{Q} + \dot{m}[h_1 + (\text{k.e.})_1 + (\text{p.e.})_1] = \dot{W} + \dot{m}[h_2 + (\text{k.e.})_2 + (\text{p.e.})_2]$$

The mass flow rate of the air is not known but may be calculated from the ideal-gas equation of state written as

$$p\dot{V} = \dot{m}RT$$

thus,

$$\dot{m} = (101\ \text{kPa})(0.75\ \text{m}^3/\text{s})/[(0.287\ \text{kJ/kg-K})(290°\text{K})]$$

$$\dot{m} = 0.91\ \text{kg/s}$$

Using assumptions 1 and 3 the first law becomes

$$\dot{W} = \dot{Q} + \dot{m}(h_1 - h_2) = \dot{Q} + \dot{m}c_p(T_1 - T_2)$$

where the ideal-gas equation of state for enthalpy has been used.

$$\dot{W} = -2.1\ \text{kW} + (0.91\ \text{kg/s})(1.0047\ \text{kJ/kg-K})(290 - 435°\text{K})$$

$$\dot{W} = -134.7\ \text{kW}$$

Comments:

1. The heat and work flux terms are negative, indicating heat leaving the control volume and power being required to compress the air.
2. One can verify the use of the ideal-gas model by checking that the generalized compressibility factor is unity for the conditions in this problem. ■

Example 7.2

A six-cylinder engine with a 4×4-in. bore and stroke operates on the Carnot cycle. It receives 51 Btu/cycle of heat at 1040°F and rejects heat at 540°F while running at 300 rpm. Determine the mean effective pressure, hp, and heat flow out of the engine.

Solution

Given: An engine operating on the Carnot cycle between fixed temperature limits and receiving a certain amount of heat.

Find: The mean effective pressure and power produced.

Sketch and Given Data:

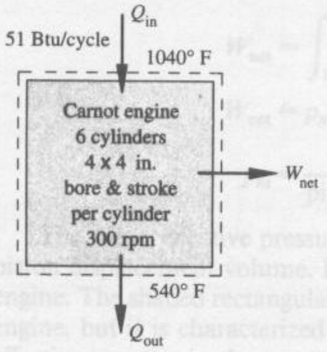

Figure 7.7

Assumption: The system is shown on the sketch and operates on the Carnot cycle.

Analysis: The temperature limits and heat added per cycle allow us to calculate the work produced per cycle. Knowing the work and information about the engine displacement volume allows calculation of the mean effective pressure. The heat rejected is the difference between the heat added and work produced per cycle. The efficiency for this engine is

$$\eta_{th} = \frac{T_H - T_C}{T_H} = \frac{1500 - 1000}{1500} = 0.333$$

The work per cycle may be determined from the efficiency to be

$$W_{net} = (0.333)Q_{in} = (0.333)(51 \text{ Btu/cycle}) = 17 \text{ Btu/cycle}$$

The total piston displacement volume is equal to the displacement of each cylinder multiplied by the number of cylinders.

$$V_{PD} = (6 \text{ cylinders})\left(\frac{\pi}{4}\right)\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right)^2\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 0.1745 \text{ ft}^3$$

The mean effective pressure is

$$p_m = \frac{W_{net}}{V_{PD}} = \frac{(17 \text{ Btu})(778.16 \text{ ft-lbf/Btu})}{(0.1745 \text{ ft}^3)(144 \text{ in.}^2/\text{ft}^2)}$$

$$p_m = 526 \text{ lbf/in.}^2$$

The power produced is found by multiplying the net work per cycle by the number of cycles per minute.

$$\dot{W}_{net} = (17 \text{ Btu/cycle})\ (300 \text{ cycle/min})(0.02358 \text{ min-hp/Btu})$$

$$\dot{W}_{net} = 120.5 \text{ hp}$$

The heat flow from the engine is found in an analogous manner. The heat rejected per cycle is

$$Q_{out} = W_{net} - Q_{in} = 17 - 51 = -34 \text{ Btu}$$

The heat flow rate out is

$$\dot{Q}_{out} = (-34 \text{ Btu/cycle})(300 \text{ cycles/min}) = -10{,}200 \text{ Btu/min}$$

Comments:

1. The net work of 17 Btu/cycle represents the total work of all six cylinders completed in one cycle or revolution of the engine.
2. Conversion between ft-lbf and Btu is important.
3. The heat rejected represents the energy that would be dissipated to the atmosphere by an actual automobile's radiator and exhaust. ■

Example 7.3

A Carnot engine uses 0.05 kg of air as the working substance. The temperature limits of the cycle are 300°K and 940°K, the maximum pressure is 8.4 MPa, and the heat added per cycle is 4.2 kJ. Determine the temperature, pressure, and volume at each state of the cycle.

Solution

Given: A Carnot engine, the mass of air it operates on, temperature limits, heat added, and maximum pressure.

Find: The pressure, temperature, and volume at each state point around the cycle.

Sketch and Given Data:

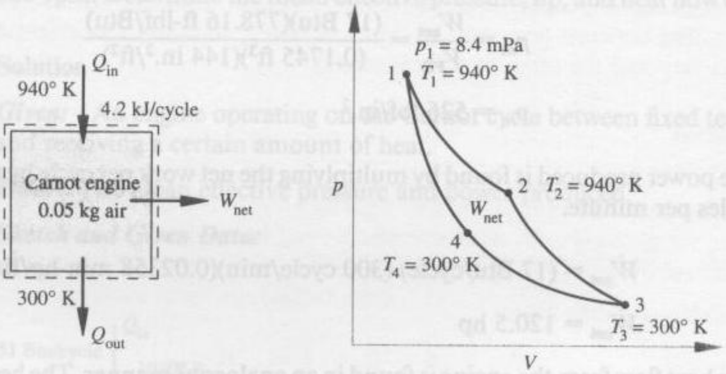

Figure 7.8

Assumption: Air is an ideal gas.

Analysis: Determine the cycle state points by using the ideal-gas equations of state and the expressions for the processes forming the cycle.

State 1:

$$T_1 = 940°\text{K} \qquad p_1 = 8.4 \text{ MPa} \qquad V_1 = \frac{mRT_1}{p_1}$$

$$V_1 = (0.05 \text{ kg})(0.287 \text{ kJ/kg-K})(940°\text{K})/(8400 \text{ kN/m}^2) = 0.001\ 606 \text{ m}^3$$

State 2: The process from state 1 to state 2 is constant-temperature heat addition; thus,

$$Q_{1\text{-}2} = p_1 V_1 \ln \{V_2/V_1\}$$

$$4.2 \text{ kJ} = (8400 \text{ kN/m}^2)(0.001\ 606 \text{ m}^3)(\ln \{V_2/V_1\})$$

$$\ln \{V_2/V_1\} = 0.3113 \qquad V_2 = 0.002\ 193 \text{ m}^3$$

$$p_2 = \frac{mRT_2}{V_2} = (0.05 \text{ kg})(0.287 \text{ kJ/kg-K})(940°\text{K})/(0.002\ 193 \text{ m}^3)$$

$$p_2 = 6150 \text{ kPa} \qquad T_2 = 940°\text{K}$$

State 3: Using the relationships for a reversible adiabatic process from state 2 to state 3, the values of pressure and volume at state 3 may be determined.

$$\frac{V_3}{V_2} = \left(\frac{T_2}{T_3}\right)^{1/(k-1)} = \left(\frac{940}{300}\right)^{2.5}$$

$$V_3 = 0.038\ 11 \text{ m}^3$$

Similarly for pressure

$$\frac{p_3}{p_2} = \left(\frac{T_3}{T_2}\right)^{k(k-1)} = \left(\frac{300}{940}\right)^{3.5}$$

$$p_3 = 112.9 \text{ kPa}$$

State 4: The heat out may be calculated from the thermal efficiency. Knowing this and the process, constant-temperature heat rejection, the values at state 4 may be determined. However, it is also possible to use the process relationships from state 1 to state 4 in a manner analogous to going from state 2 to state 3.

$$\left(\frac{V_4}{V_1}\right) = \left(\frac{T_1}{T_4}\right)^{1/(k-1)} = \left(\frac{940}{300}\right)^{2.5}$$

$$V_4 = 0.027\ 91 \text{ m}^3$$

$$p_4 = \frac{mRT_4}{V_4} = (0.05 \text{ kg})(0.287 \text{ kJ/kg-K})(300°\text{K})/(0.027\ 91 \text{ m}^3)$$

$$p_4 = 154.2 \text{ kPa}$$

Comment: One need not proceed in one direction around the cycle to find the cycle state points.

Example 7.5

A home requires 5.0 kW to maintain an indoor temperature of 21 °C when the outside temperature is 0 °C. A heat pump is used to provide the energy. Determine the minimum power required.

Solution

Given: A heat pump providing energy for home heating with temperature limits and heat required noted.

Find: The minimum power required to operate the heat pump.

Sketch and Given Data:

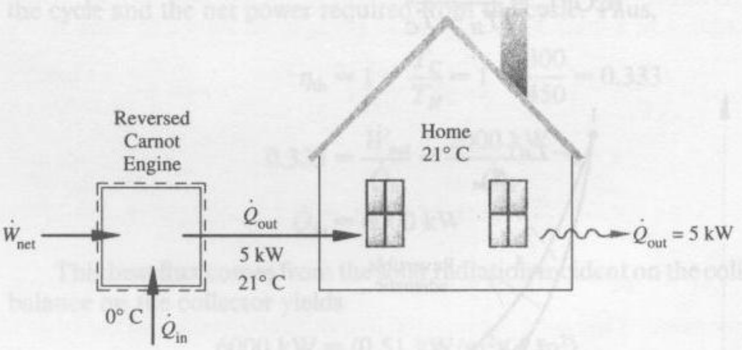

Figure 7.11

Assumption: The surroundings and the house are the low- and high-temperature heat reservoirs, respectively.

Analysis: The reversed Carnot cycle requires the least power to move heat from one temperature to another. The house requires 5.0 kW of heat out of the cycle to maintain itself at 21 °C. If this heat flow differs from 5.0 kW, the house's temperature will increase or decrease. The reversed cycle is the system being analyzed; thus, the heat out of the cycle is the desired effect even though this becomes the heat into the house. The $(COP)_h$ for the reversed Carnot cycle is

$$(COP)_h = \frac{\dot{Q}_{out}}{\dot{W}_{net}} = \frac{T_H}{T_H - T_C}$$

$$(COP)_h = \frac{5 \text{ kW}}{\dot{W}_{net}} = \frac{294}{294 - 273} = 14$$

$$\dot{W}_{net} = 5/14 = 0.357 \text{ kW}$$

Comments:

1. Absolute temperatures must be used in the calculations.
2. The power required in an actual heat pump will be greater due to irreversibilities.

Example 8.1

Calculate the change of entropy of 3 kg of air that changes state from 300°K and 100 kPa to 800°K and 500 kPa.

Solution

Given: Air changing state with both states completely identified.

Find: The change of entropy of the air.

Sketch and Given Data:

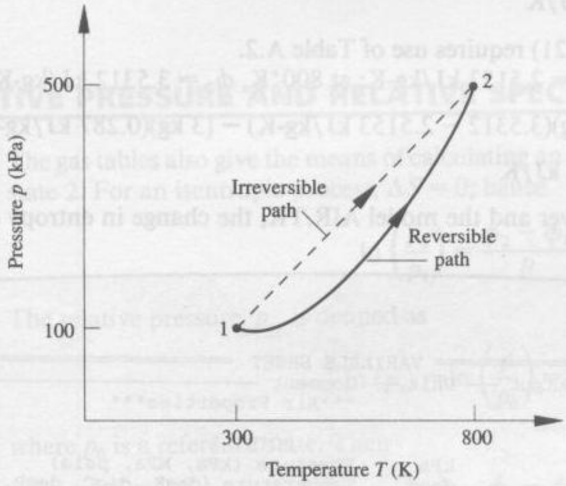

Figure 8.3

Assumption: Air behaves as an ideal gas.

Analysis: The change of entropy of an ideal gas is given by equations (8.19), (8.20), and (8.21). In equation (8.21) the variation of specific heats with temperature is included. Solve for the air volume at each state using the ideal-gas equation of state.

$$V_1 = \frac{mRT_1}{p_1} = (3\text{ kg})(0.287\text{ kJ/kg-K})(300°\text{K})/(100\text{ kN/m}^2)$$

$$V_1 = 2.583\text{ m}^3$$

$$V_2 = \frac{mRT_2}{p_2} = (3\text{ kg})(0.287\text{ kJ/kg-K})(800°\text{K})/(500\text{ kN/m}^2)$$

$$V_2 = 1.378\text{ m}^3$$

The change of entropy using equation (8.19) yields

$$S_2 - S_1 = mc_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right)$$

$$S_2 - S_1 = (3\text{ kg})(0.7176\text{ kJ/kg-K}) \ln\left(\frac{800}{300}\right) + (3\text{ kg})(0.287\text{ kJ/kg-K}) \ln\left(\frac{1.378}{2.583}\right)$$

$$S_2 - S_1 = 1.57\text{ kJ/K}$$

From equation (8.20)

$$S_2 - S_1 = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{p_2}{p_1}\right)$$

$$S_2 - S_1 = (3\text{ kg})(1.0047\text{ kJ/kg-K}) \ln\left(\frac{800}{300}\right) - (3\text{ kg})(0.287\text{ kJ/kg-K}) \ln\left(\frac{500}{100}\right)$$

$$S_2 - S_1 = 1.57\text{ kJ/K}$$

Using equation (8.21) requires use of Table A.2.

At 300°K, $\phi_1 = 2.5153$ kJ/kg-K; at 800°K, $\phi_2 = 3.5312$ kJ/kg-K.

$$S_2 - S_1 = (3\text{ kg})(3.5312 - 2.5153\text{ kJ/kg-K}) - (3\text{ kg})(0.287\text{ kJ/kg-K}) \ln(5)$$

$$S_2 - S_1 = 1.66\text{ kJ/K}$$

Using TK Solver and the model AIR.TK, the change in entropy can be easily calculated.

```
==================== VARIABLE SHEET ====================
St Input- Name- Output --- Unit --- Comment
                                    ***Air Properties***

                                          POINT 1
   100    P1               kPa      Pressure (kPa, MPa, psia)
   300    T1               degK     Temperature (degK, degC, degR, degF)
          v1    .86073     m3/kg    Specific Volume (m3/kg, ft3/lbm)

                                          POINT 2
   500    P2               kPa      Pressure (kPa, MPa, psia)
   800    T2               degK     Temperature (degK, degC, degR, degF)
          v2    .45985     m3/kg    Specific Volume (m3/kg, ft3/lbm)

                                     CHANGE (POINT 2-POINT 1)
          DELh  521.67     kJ/kg    Enthalpy (kJ,kg, BTU/lbm)
          DELu  378.15     kJ/kg    Internal Energy (kJ/kg, BTU/lbm)
          DELs  .55616     kJ/kg-K  Entropy (kJ/kg-K, B/lbm-R)
```

The change in entropy for the 3 kg is thus

$$S_2 - S_1 = (3\text{ kg})(0.55616\text{ kJ/kg-K}) = 1.66848\text{ kJ/K}$$

Comments:

1. In problems where there is a significant temperature variation, as there is in this example, the variation of specific heat with temperature has a significant effect in calculating the change of entropy. In this case the difference is nearly 6%.
2. When the initial and final states of a system are given, the entropy change may be computed whether or not the process the system underwent was reversible. Entropy is a system property and as such does not depend on the path taken by the system in changing state, and a reversible path may be chosen. This is implicit in the derivation of the expression for change of entropy. ■

Example 13.1

An engine operates on the air-standard Otto cycle. The conditions at the start of compression are 27°C and 100 kPa. The heat added is 1840 kJ/kg. The compression ratio is 8. Determine the temperature and pressure at the end of each process in the cycle, the thermal efficiency, and the mean effective pressure.

Solution

Given: An engine operating on the air-standard Otto cycle with known compression ratio, initial conditions, and heat added.

Find: The cycle state points, the thermal efficiency, and the mean effective pressure.

Sketch and Given Data:

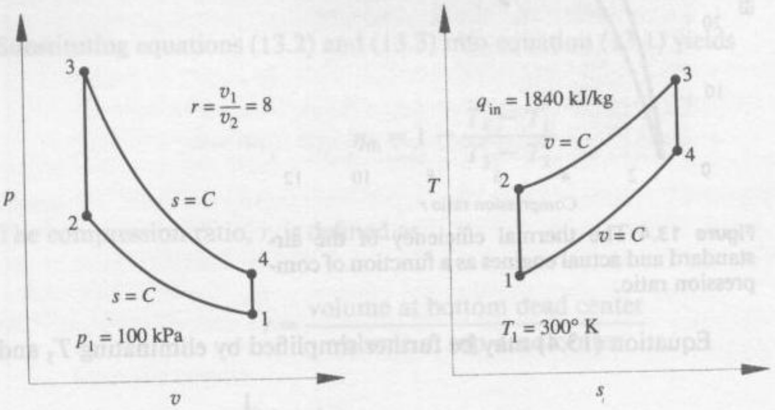

Figure 13.5

Assumptions:

1. The air in the piston-cylinder is a closed system.
2. The air is an ideal gas.
3. The changes in kinetic and potential energies may be neglected.

Analysis: The temperature and pressure at state 1 are given: $T_1 = 300°K$, $p_1 = 100$ kPa. Follow the processes around the cycle. The process from state 1 to state 2 is isentropic, so the reversible adiabatic relationships for an ideal gas may be used.

$$pV^k = C \text{ and } V_1/V_2 = r = 8$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$T_2 = (300°\text{K})(8)^{0.4} = 689.2°\text{K}$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{k}$$

$$p_2 = (100 \text{ kPa})(8)^{1.4} = 1837.9 \text{ kPa}$$

The process from state 2 to state 3 is constant volume, and we have noted that the heat transfer is equal to the change of internal energy from first-law analysis.

$$q = u_3 - u_2 = c_v(T_3 - T_2)$$
$$1840 \text{ kJ/kg} = (0.7176 \text{ kJ/kg-K})(T_3 - 689.2°\text{K})$$
$$T_3 = 3253.5°\text{K}$$
$$p_3 = p_2\left(\frac{T_3}{T_2}\right) = (1837.9 \text{ kPa})\left(\frac{3253.5}{689.2}\right) = 8676.1 \text{ kPa}$$

The process from state 3 to state 4 is isentropic; hence the reversible adiabatic ideal-gas relationships may be used.

$$pV^k = C \quad \text{and} \quad V_3/V_4 = 1/r$$
$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1}$$
$$T_4 = (3253.5°\text{K})\left(\frac{1}{8}\right)^{0.4} = 1416.2°\text{K}$$
$$\frac{p_4}{p_3} = \left(\frac{V_3}{V_4}\right)^{k}$$
$$p_4 = (8676.1 \text{ kPa})\left(\frac{1}{8}\right)^{1.4} = 472.1 \text{ kPa}$$

The thermal efficiency may be found from equation (13.9) for an ideal gas with constant specific heats as in this model:

$$\eta_{\text{th}} = 1 - \frac{1}{(r)^{k-1}} = 1 - \frac{1}{(8)^{0.4}} = 0.565 \quad \text{or } 56.5\%$$

The mean effective pressure is defined as

$$p_m = \frac{w_{\text{net}}}{\text{displacement volume}} = \frac{w_{\text{net}}}{v_1 - v_2}$$

The specific volumes are found from the ideal-gas equation of state.

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287 \text{ kJ/kg-K})(300°\text{K})}{(100 \text{ kN/m}^2)} = 0.861 \text{ m}^3\text{/kg} \qquad v_4 = v_1$$

$$v_2 = \frac{v_1}{r} = (0.861/8) = 0.1076 \text{ m}^3\text{/kg} \qquad v_3 = v_2$$

$$w_{\text{net}} = (\eta_{\text{th}})(q_{\text{in}})$$
$$w_{\text{net}} = (1840 \text{ kJ/kg})(0.565) = 1039.6 \text{ kJ/kg}$$

Hence the mean effective pressure is

$$p_m = \frac{(1039.6 \text{ kJ/kg})}{(0.861 - 0.1076 \text{ m}^3\text{/kg})} = 1379.9 \text{ kPa}$$

Comments:

1. The thermal efficiency of the Otto cycle is high, in part because the maximum temperatures and pressures are also very high in this air-standard cycle. This is because the specific heat variation with temperature is neglected, there is no dissociation, and heat is added at constant volume. In an actual engine none of the preceding is true. The advantages of using the air-standard model lie with its simplicity and its giving a sense of direction to changes that affect an actual engine. We can determine that increasing the compression ratio improves the thermal efficiency, without running experiments to verify this.
2. The mean effective pressure is a useful measure in comparing engines operating on different cycles—the greater the mean effective pressure, the smaller the engine may be for a given work output. For instance, the mean effective pressure in a Carnot cycle is quite low, about one-tenth the value in this example. ■

Sometimes the clearance volume in the piston-cylinder of a reciprocating engine is expressed as a percentage of the total displacement volume; it is called the percentage of clearance, c, and is defined as

$$c = \frac{\text{clearance volume}}{\text{displacement volume}} = \frac{V_2}{V_1 - V_2}$$

Example 13.3

An engine operates on the air-standard Diesel cycle. The conditions at the start of compression are 27°C and 100 kPa. The heat supplied is 1840 kJ/kg, and the compression ratio is 16. Determine the maximum temperature and pressure, the thermal efficiency, and the mean effective pressure.

Solution

Given: An engine operating on the air-standard Diesel cycle with known initial temperature and pressure, compression ratio, and heat added.

Find: The cycle maximum temperature and pressure, the thermal efficiency, and the mean effective pressure.

Sketch and Given Data:

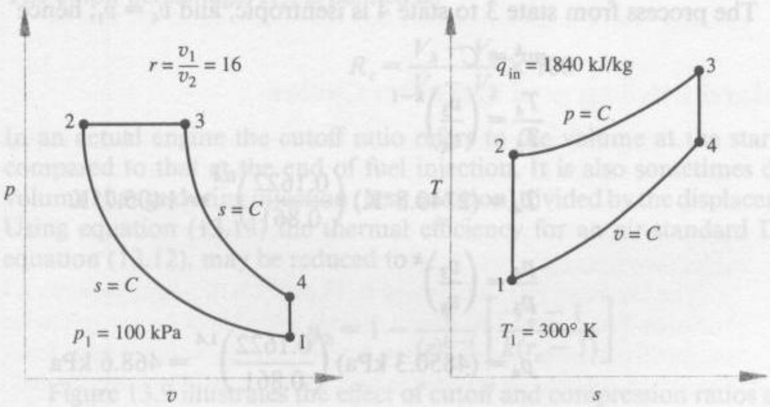

Figure 13.8

Assumptions:

1. The air in the piston-cylinder is a closed system.
2. The air in the system is an ideal gas with constant specific heats.
3. The changes in kinetic and potential energies may be neglected.

Analysis: Use the ideal-gas relationships and determine the cycle state points. At state 1, $T_1 = 300°K$, $p_1 = 100$ kPa, and

$$v_1 = \frac{RT}{p_1} = \frac{(0.287 \text{ kJ/kg-K})(300°\text{K})}{(100 \text{ kN/m}^2)} = 0.861 \text{ m}^3\text{/kg}$$

The process from state 1 to state 2 is isentropic; hence,

$$pv^k = C \qquad r = \frac{v_1}{v_2} = 16 \qquad v_2 = \frac{0.861}{16} = 0.0538 \text{ m}^3\text{/kg}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} \qquad T_2 = (300°\text{K})(16)^{0.4} = 909.4°\text{K}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{k} \qquad p_2 = (100 \text{ kPa})(16)^{1.4} = 4850.3 \text{ kPa}$$

The process from state 2 to state 3 is constant-pressure with heat addition occurring; hence

$$q = (h_3 - h_2) = c_p(T_3 - T_2)$$

$$1840 \text{ kJ/kg} = (1.0047 \text{ kJ/kg-K})(T_3 - 909.4°\text{K})$$

$$T_3 = T_{\max} = 2740.8°\text{K}$$

$$p_{\max} = p_3 = p_2 = 4850.3 \text{ kPa}$$

$$v_3 = \frac{RT_{\max}}{p} = \frac{(0.287 \text{ kJ/kg-K})(2740.8°\text{K})}{(4850.3 \text{ kN/m}^2)} = 0.1622 \text{ m}^3\text{/kg}$$

The process from state 3 to state 4 is isentropic, and $v_4 = v_1$; hence

$$pv^k = C$$

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{k-1}$$

$$T_4 = (2740.8°\text{K})\left(\frac{0.1622}{0.861}\right)^{0.4} = 1405.7°\text{K}$$

$$\frac{p_4}{p_3} = \left(\frac{v_3}{v_4}\right)^{k}$$

$$p_4 = (4850.3 \text{ kPa})\left(\frac{0.1622}{0.861}\right)^{1.4} = 468.6 \text{ kPa}$$

The process from state 4 to state 1 is constant-volume with heat rejection occurring; the first law yields

$$q = (u_1 - u_4) = c_v(T_1 - T_4)$$

$$q_{\text{out}} = (0.7176 \text{ kJ/kg-K})(300 - 1405.7°\text{K}) = -793.4 \text{ kJ/kg}$$

$$w_{\text{net}} = \sum q = (1840 - 793.4 \text{ kJ/kg}) = 1046.6 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1046.6}{1840} = 0.569 \quad \text{or } 56.9\%$$

The mean effective pressure is

$$p_m = \frac{w_{\text{net}}}{v_1 - v_2} = \frac{(1046.6 \text{ kJ/kg})}{(0.861 - 0.0538 \text{ m}^3\text{/kg})} = 1296.6 \text{ kPa}$$

Comments:

1. Because the piston moves downward as heat is added, the peak temperature and pressure are less than in the Otto-cycle engine.
2. The thermal efficiency could have been calculated using equation (13.12), but in this case the additional data are needed for other parts of the solution process. In general, it is wiser to use the efficiency definition in terms of heat and work, not of temperatures, as the former uses no assumptions and the latter has restrictive requirements.

Example 13.7
A spark-ignition engine produces 224 kW while using 0.0169 kg/s of fuel. The fuel has a higher heating value of 44 186 kJ/kg, and the engine has a compression ratio of 8. The friction power is found to be 22.4 kW. Determine η_{tb}, η_{ti}, η_m, η_b, and η_i.

Solution

Given: A spark-ignition engine with the power it produces, the frictional power, the fuel consumption, and the fuel type.

Find: Various engine efficiencies.

Sketch and Given Data:

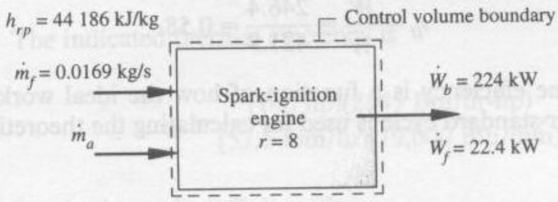

Figure 13.21

Assumption: The theoretical cycle for the spark-ignition engine is the air-standard Otto cycle.

Analysis: Determine the thermal efficiency to be

$$\eta_{tb} = \frac{\dot{W}_b}{\dot{m}_f h_{RP}} = \frac{(224\text{ kW})}{(0.0169\text{ kg/s})(44\ 186\text{ kJ/kg})} = 0.30$$

The indicated power is

$$\dot{W}_i = \dot{W}_b + \dot{W}_f = 246.4\text{ kW}$$

The indicated thermal efficiency is

$$\eta_{ti} = \frac{\dot{W}_i}{\dot{m}_f h_{RP}} = \frac{(246.4\text{ kW})}{(0.0169\text{ kg/s})(44\ 186\text{ kJ/kg})} = 0.33$$

The mechanical efficiency is

$$\eta_m = \frac{\dot{W}_b}{\dot{W}_i} = \frac{224}{246.4} = 0.909$$

The brake engine efficiency requires that we know the theoretical power produced.

From the assumption the theoretical cycle is the air-standard Otto cycle. Hence,

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{k-1}} = 1 - 0.435 = 0.565$$

$$\eta_{\text{Otto}} = \frac{\dot{W}}{\dot{Q}_{\text{in}}} = 0.565 = \frac{\dot{W}}{(0.0169 \text{ kg/s})(44\ 186 \text{ kJ/kg})}$$

$$\dot{W} = 421.9 \text{ kW}$$

The brake engine efficiency is defined as

$$\eta_b = \frac{\dot{W}_b}{\dot{W}} = \frac{224}{421.9} = 0.53$$

and the indicated engine efficiency is

$$\eta_i = \frac{\dot{W}_i}{\dot{W}} = \frac{246.4}{421.9} = 0.58$$

Comment: Engine efficiency is a function of how the ideal work is calculated. Frequently, the air-standard cycle is used for calculating the theoretical work. ■

Example 14.1

An air-standard Brayton cycle has air enter the compressor at 27°C and 100 kPa. The pressure ratio is 10, and the maximum allowable temperature in the cycle is 1350°K. Determine the pressure and temperature at each state in the cycle, and the compressor work, the turbine work, and the cycle efficiency per kilogram of air.

Solution

Given: The initial temperature and pressure, the maximum temperature, and the pressure ratio for an air-standard Brayton cycle.

Find: The cycle state points, the various work terms, and the cycle efficiency.

Sketch and Given Data:

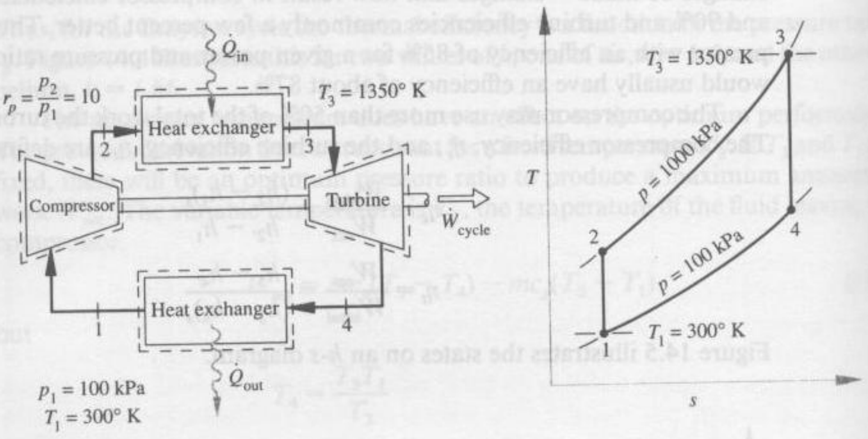

Figure 14.6

Assumptions:

1. Each component is analyzed as a steady-state open system.
2. The processes are for the air-standard Brayton cycle.
3. Air behaves like an ideal gas.
4. The changes in kinetic and potential energies may be neglected.

Analysis: At state 1 the pressure and temperature are $p_1 = 100$ kPa and $T_1 = 300$°K. Knowing the pressure ratio and the process, isentropic from state 1 to state 2, allows us to find state 2.

$$r_p = \frac{p_2}{p_1} = 10$$

$$p_2 = 1000 \text{ kPa}$$
$$T_2 = T_1(r_p)^{(k-1)/k} = (300°\text{K})(10)^{0.286} = 579.6°\text{K}$$

The process from state 2 to state 3 is constant-pressure, and the maximum temperature occurs at state 3; hence,

$$p_3 = p_2 = 1000 \text{ kPa} \qquad T_3 = T_{\max} = 1350°\text{K}$$

The process from state 3 to state 4 is isentropic, and $p_4 = p_1 = 100$ kPa.

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{(k-1)/k} = (1350)(0.10)^{0.286} = 698.8°\text{K}$$

The first-law analysis for steady state for the turbine, compressor, and high-temperature heat exchanger yields

$$w_c = -(h_2 - h_1) = -c_p(T_2 - T_1)$$
$$w_c = -(1.0047 \text{ kJ/kg-K})(579.6 - 300°\text{K}) = -280.9 \text{ kJ/kg}$$
$$w_t = (h_3 - h_4) = c_p(T_3 - T_4)$$
$$w_t = (1.0047 \text{ kJ/kg-K})(1350 - 698.8°\text{K}) = 654.3 \text{ kJ/kg}$$
$$w_{\text{net}} = \sum w = 373.4 \text{ kJ/kg}$$
$$q_{\text{in}} = (h_3 - h_2) = c_p(T_3 - T_2)$$
$$q_{\text{in}} = (1.0047 \text{ kJ/kg-K})(1350 - 579.6°\text{K}) = 774.0 \text{ kJ/kg}$$
$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 0.482 \quad \text{or } 48.2\%$$

Check:

$$\eta_{\text{th}} = 1 - \frac{1}{(r_p)^{(k-1)/k}} = 0.482$$

Comment: Notice that 43% of the turbine work is used to drive the compressor. The remainder is available as net work. ■

Example 14.4

A gas turbine unit produces 600 kW while operating under the following conditions: the inlet air pressure and temperature are 100 kPa and 300°K; the pressure ratio is 10; the fuel is similar to $C_{12}H_{26}$ and has a ratio of 0.015 kg fuel/kg air; the products of combustion are similar to 400% theoretical air. Calculate the air flow rate, the total turbine work, the compressor work, and the thermal efficiency.

Solution

Given: The power produced from a gas turbine unit, the air inlet conditions, the fuel/air ratio, the fuel type, and the pressure ratio.

Find: Find the air flow rate required, the total turbine and compressor powers, and the thermal efficiency.

Sketch and Given Data:

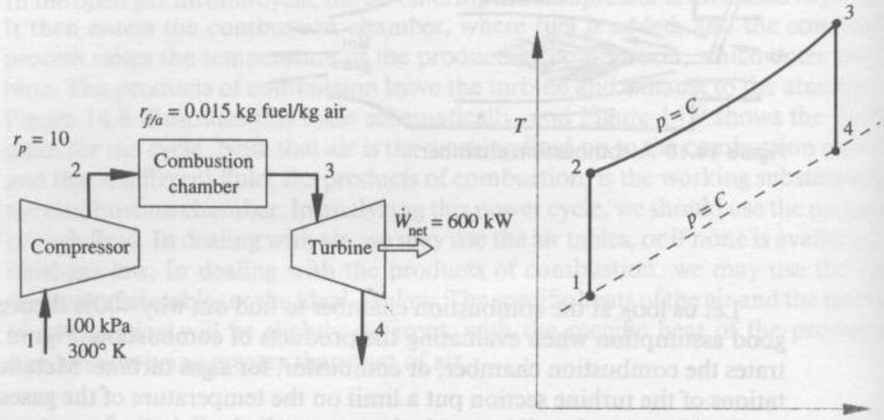

Figure 14.11

Assumptions:

1. Each component is analyzed as a steady-state open system.
2. The processes are for the ideal, open gas turbine cycle.
3. Air and products of combustion behave like ideal gases with variable specific heats.
4. The changes in kinetic and potential energies may be neglected.
5. The compressor, combustion chamber, and turbine are adiabatic.

Analysis: In this problem the air has specific heats that vary with temperature, so Table A.2 is used. At state 1 $h_1 = 300.19$ kJ/kg, and $p_{r_1} = 1.3860$. The first law for an open system applied to the compressor and invoking assumptions 4 and 5 yields

$$\dot{m}_a h_1 = \dot{m}_a h_2 + \dot{W}_c$$

$$\dot{W}_c = -\dot{m}_a(h_2 - h_1)$$

Since no internal efficiencies are given, $h_2 = (h_2)_s$. For an isentropic process,

$$p_{r_2} = p_{r_1}(r_p) = (1.3860)(10) = 13.86$$

$$h_2 = 579.8 \text{ kJ/kg}$$

The adiabatic combustion chamber is the next element in the cycle. It too is an open system, and so the first law applied to it is

$$\dot{m}_a h_2 + \dot{m}_f h_{RP} = (\dot{m}_a + \dot{m}_f) h_3$$

where $\bar{h}_{RP}$ is the enthalpy of combustion. From Table C.3, $h_{RP} = 44\ 102$ kJ/kg.

$$h_2 + r_{f/a}h_{RP} = (1 + r_{f/a})h_3$$

$$(579.8 \text{ kJ/kg}) + (0.015 \text{ kg fuel/kg air})(44\ 102 \text{ kJ/kg fuel})$$
$$= (1.015 \text{ kg products/kg air})h_3$$
$$h_3 = 1241.3 \text{ kJ/kg}$$

The substance leaving the combustion chamber is 400% theoretical air, found in Table A.3. Thus, $p_{r_3} = 220.1$.

The flow through the turbine may be considered isentropic, so

$$p_{r_4} = p_{r_3}\left(\frac{1}{r_p}\right) = 22.01$$

$$h_4 = 662.5 \text{ kJ/kg}$$

Applying the first law for open systems to the turbine while invoking assumptions 4 and 5 yields

$$\dot{m}_a(1 + r_{f/a})h_3 = \dot{W}_c + \dot{W}_{\text{net}} + \dot{m}_a(1 + r_{f/a})h_4$$
$$\dot{W}_c = -\dot{m}_a(h_2 - h_1)$$
$$\dot{W}_{\text{net}} = 600 \text{ kW}$$

Solve for $\dot{m}_a$.

$$(\dot{m}_a \text{ kg/s})(1.015 \text{ kg products/kg air})(1241.3 - 662.5 \text{ kJ/kg}) = 600 \text{ kW} - (\dot{m}_a \text{ kg/s})(579.8 - 300.19 \text{ kJ/kg})$$

$$\dot{m}_a = 1.949 \text{ kg/s}$$

The total turbine power produced is

$$\dot{W}_t = \dot{m}_a(1 + r_{f/a})(h_3 - h_4)$$
$$\dot{W}_t = (1.949 \text{ kg air/s})(1.015 \text{ kg products/kg air})(1241.3 - 662.5 \text{ kJ/kg products})$$
$$\dot{W}_t = 1145 \text{ kW}$$

and the total compressor power is

$$\dot{W}_c = -\dot{m}_a(h_2 - h_1)$$
$$\dot{W}_c = -(1.949 \text{ kg air/s})(579.8 - 300.19 \text{ kJ/kg air}) = -545.0 \text{ kW}$$

The unit's thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{600 \text{ kW}}{(1.949 \text{ kg air/s})(0.015 \text{ kg fuel/kg air})(44\ 102 \text{ kJ/kg fuel})}$$

$$\eta_{\text{th}} = 0.465$$

Comment: The fuel flow rate may be calculated by multiplying the fuel/air ratio by the air flow rate. The size of inlet ducts and fuel tanks can be determined from information provided by first-law analysis. ■