4. Surface exchange processes

On both sides of the interface, a very thin layer exists where the molecular diffusion coefficients play a major role in the transport. The interface is consequently a significant barrier to the transport from ocean to atmosphere and vice versa, with little or no turbulent transport of scalar quantities across it.

The sea surface microlayer covers the upper 10 to 1000 µm deep boundary layer of the ocean where important physical, chemical, and biological processes take place. As turbulence is damped close to the surface molecular transport processes take over the transfer of momentum, heat and mass from the upper ocean to the sea surface. As characteristic features molecular sublayers extend from the surface to depths of about 1000 µm (viscous sublayer), 500 µm (conductive or thermal sublayer), and 50 µm (diffusive sublayer). The conductive sublayer is also referred to as the cool skin. The structures of the molecular sublayers are complex due to the variability of wind stress acting on the sea surface, due to heat, radiative and gas fluxes crossing these layers, and due to rainfall. A schematic vertical section through the ocean is shown in figure below. The logarithmic scale ranging from the diameter of a molecule to the maximum depth of the world ocean underlines the importance of the top millimetre of the sea.
Castro et al. (2003) defined six different processes or regimes for refinement of renewal theory, most of them analogous to Thorpe (1995). These include free convection, forced convection driven by wind shear stress, saturated shear, capillary waves, microscale wave breaking, and residual subsurface turbulence from breakers or large-scale breaking.

Skin layer thickness

The existence of a sublayer just below the air-sea interface in which the transfer of heat is primarily by molecular conduction has been verified by Khundzhua et al. (1977). They made measurements of the vertical temperature gradient across the interface of the Black Sea and found that for wind speed up to 6–8 ms\(^{-1}\) there is a layer of 0.2–0.6 mm thickness of in which the temperature profile is linear. Khundzhua et al. also found that below a depth of a few millimeters the temperature was nearly uniform down to 0.3 meter.

Sauders (1967) hypothesized the existence of a thin quasi-laminar region in the ocean and adjacent to its surface within which the frictional component of a wind stress is communicated viscously. The mean thickness of this region \(d\) is determined only by the viscous stress, kinematic viscosity, and the water density. A dimensional argument leads to the relation

\[
\delta = \frac{\lambda \nu}{\sqrt{\tau / \rho_w}} = \frac{\lambda \nu}{u_{*w}}
\]  

(4.0.1)

The total cooling at the air-sea interface is given by:

\[
- Q = R_n - H_s - H_l
\]

(4.0.2)
The temperature gradient at the interface is defined by molecular conductive processes:

\[ Q = k \frac{\partial T}{\partial z} \bigg|_0 \]  

(4.0.3)

From (4.0.3), the temperature difference across the thermal skin layer can be presented as:

\[ T_s = T_b - \Delta T \]

\[ \Delta T = \frac{Q\delta}{k} = \frac{\lambda Qv}{k u_{ws}} \]  

(4.0.4)

Numerous studies of proportionality constant, \( \lambda \), have been performed over the ocean: Wu (1971), Hasse (1971), Grassl (1976), Paulson and Simpson (1981), Fairall et al. (1996). Sauders (1967) suggested that the \( \lambda \) is from 5 ~ 10, except under calm wind conditions. This estimation was from a few observations with temperature difference of 0.2 ~ 0.3°C.

Wu (1971) suggested a constant value of 11.6 for \( \lambda \), and estimated that the viscous-sublayer is approximately double of the thermal skin layer. Fairall et al. (1996) developed a parameterization of \( \lambda \) for moderate- and calm-wind regimes for which most measurements were available. The complex functional form for \( \lambda \) takes into account the free convection and forced convection mechanism. For strong wind regime, \( Q \) increases with wind intensity and \( \Delta T \) reaches a small constant value. Artale et al. (2002) used a step function in the parameterization of \( \lambda \) for all range of wind conditions.

![Fig 4.0.1 Sauders’ proportionality constant \( \lambda \) and thermal skin layer thickness as a function of wind speed (Tu and Tsuang, 2004). Plots are the corresponding values for the Wu (1971) (rhombus), Fairall et al.(1996) (squares) and Artale et al. (2002) (triangles).](image)

**Computing SST from satellite data**

Among the main concerns in the temperature difference across the cool skin of the ocean has been the interpretation of satellite-derived sea surface temperatures as
equivalent to in-situ bulk temperature measurements under clear-sky conditions, the only case where space-borne infrared imagery can be used to monitor the sea surface. While common in-situ measurements of sea surface temperature are representative for the upper decimeters or meters of the bulk water infrared radiometers receive radiation from the upper few micrometers only. Typical temperature differences across the conductive sublayer are of the order 0.3 K. However, the actual value varies with heat, radiative, and momentum fluxes in the upper ocean and is also modified by rainfall so that a range of variability can be expected from -1 K to 1 K.

We now understand the driving forces that cause fluctuations in the skin and bulk temperatures and we are able to parameterize the difference between these two temperatures as functions of net air–sea heat flux and wind speed. Unfortunately these parameterizations require knowledge of net air–sea heat flux to define $\Delta T$ and that is much more difficult to observe than the skin or bulk SSTs themselves. It may be in the future that we can use measured $\Delta T$ to compute the net air–sea heat flux. This will require an accurate knowledge of the ocean wind speed but we have new microwave measurement techniques that can measure ocean wind speed and direction with considerable accuracy. What is needed is an increase in the samples of skin and coincident bulk SSTs along with standard meteorological observations so that we can either further augment our models or confirm them as they stand.

As a result of the presently unresolved complexity of computing $\Delta T$ it is not yet possible to directly compute bulk SST from radiative measurements of skin SST. It is possible to use indirect methods to estimate the bulk SST from the skin SST but the results do not show a general improvement. Acknowledging these limitations in the definition of the relationship between skin and bulk SSTs we presently advocate the parallel computation of both skin and “bulk SST” (the traditional NOAA/NESDIS NLSST) from infrared satellite data. Both of these algorithms employ a split-window technique but the two approaches will use different methods for the calculation of the algorithm coefficients for the individual infrared channels and their difference. The traditional approach to computing “pseudo-bulk” SST takes a set of edited global drifting and moored buoy SSTs as a basis set. The corresponding satellite infrared radiances are then regressed on the buoy temperatures to yield channel and split-window coefficients for the “pseudo-bulk” SST. For the skin temperature the general practice (Schluessel et al. 1987) is to use radiative transfer simulations based on a global set of marine radiosonde profiles and the lowest atmospheric temperature in taken to be the skin SST. In the absence of widespread and continuous in situ measurements of skin SST, this is the only approach we can use. This method is quite sensitive to the radiosonde dataset employed for these calculations. It is very
important to have enough radiosonde profiles to be able to include information on all of the different atmospheric conditions that can occur.

If a suite of in situ skin SST measurements were available, we could drop this use of atmospheric simulations and regress the satellite radiances against the coincident measurements of skin SST. Initially these data are needed to demonstrate that by having in situ skin SST measurements we can improve the accuracy of satellite infrared SST measurements. Since both the in situ and satellite measurements are infrared measurements of the sea surface the accuracy of the satellite measurements with respect to the in situ observations is expected to improve. The remaining errors will be in the atmospheric correction for the skin SST. Eliminating the bulk-skin SST error will focus the efforts of the SST research community on this atmospheric correction problem. Once it is established that in situ skin SSTs are needed, these skin SST calibration/validation data must be transmitted to shore on a real-time basis to make it possible to compute the skin SST algorithm coefficients for the daily production of global skin and bulk (NLSST) SST fields.

Fig. 4.0.2: Global distribution of the temperature difference across the cool skin of the ocean during night in August 1987.

A global distribution of the monthly mean nocturnal skin effect is shown in Figure 4.0.2 for August 1987. The parameters needed to parameterize the skin effect have been derived from satellite measurements of the Advanced Very High Resolution Radiometer, flown on the NOAA polar orbiters and from the Special Sensor Microwave/Imager, flown as part of the DMSP satellites (Schlüssel, 1996). The map reveals a high spatial variability of the skin effect even after averaging over an entire month. Lowest values are found in the northern Pacific Ocean and in the Arctic Ocean where warm and moist air is advected over cold water and concurrent
deep cloud cover minimizes the energy flux to the atmosphere. In those areas the bulk versus skin temperature differences vanish or even revert their sign showing slightly negative values. Maximum differences are found in subtropical areas and in the storm tracks of the southern oceans due to high energy fluxes leaving the ocean.

Reference:


4.1 The structure of the interface and adjacent layers

The profiles in the molecular sublayers
A surface renewal model which allows small parcels of air or water adjacent to the interface to be replaced intermittently by air or water from the turbulent layers away from the interface.

Heat flux and the related profile

Consider a parcel from the mixed layer that has been moved adjacent to the sea surface. It initially had a uniform temperature equal to the bulk temperature. As it is exposed to the surface, which has a different temperature, thermal conduction takes place and, assuming that the horizontal gradients are negligible in comparison to the vertical gradients,

\[
\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}, \tag{4.1}
\]

where \( \kappa \) is thermal diffusivity. \( T(z,0) = T_b, \quad T(0,t) = T_o \)

\[
T(z,t) = (T_b - T_o) \text{erfc} \left( \frac{z}{2\sqrt{\kappa t}} \right) + T_o, \tag{4.2}
\]

\[
Q(0,t) = -\kappa \frac{\partial T}{\partial z} \bigg|_{z=0} = -\kappa \frac{T_b - T_o}{(2\kappa t)^{1/2}} \tag{4.3}
\]

Defining a distribution function \( \phi(t) \), which represents the fractional area of the surface containing fluid elements that have been in contact with the interface for a time \( t \). The average temperature and heat flux are then given by

\[
T(z) = \int_0^T \phi(t) T(z,t) \, dt \tag{4.4}
\]

\[
Q(0) = \int_0^T \phi(t) Q(0,t) \, dt \tag{4.5}
\]

Suppose each element has the same chance to be replaced, this may be expressed by

\[
\frac{d\phi}{dt} = -\frac{1}{t_c} \phi(t), \tag{4.6}
\]

where \( t_c \) is the characteristic residence time of fluid parcels at the surface

\[
\phi(t) = \phi(0) \exp(-t/t_c) \tag{4.7}
\]

If we require \( \int_0^T \phi(t) \, dt = 1 \), then we find that \( \phi(0) = 1/t_c \).
Such that

$$\phi(t) = \frac{1}{t} \exp\left(-\frac{t}{t_0}\right)$$  \hspace{1cm} (4.8)

From the above, the temperature profile in the molecular sublayer is solved to be

$$\frac{T - T_b}{T_0 - T_b} = \exp\left[-\frac{z}{\sqrt{kL}}\right]$$  \hspace{1cm} (4.9)

$$Q(0) = -\frac{\kappa}{\sqrt{kL}}(T_0 - T_b) = -\frac{\kappa}{z_0} \rho (T_0 - T_b)$$  \hspace{1cm} (4.10)

Other profiles

In a similar manner, we can derive the following

$$\frac{u - u_b}{u_0 - u_b} = \exp\left[-\frac{z}{\sqrt{\nu L}}\right]$$  \hspace{1cm} (4.11)

$$\frac{q - q_b}{q_0 - q_b} = \exp\left[-\frac{z}{\sqrt{\delta L}}\right]$$  \hspace{1cm} (4.12)

where \(z_u = \sqrt{\kappa L}\), \(z_d = \sqrt{\nu L}\), \(z_q = \sqrt{\delta L}\).

The surface layer similarity theory

Assuming horizontal homogeneity (typically within 50m depth or so)

$$du/dt + f k x u - \rho_0^{-1} \partial \tau_H \partial z = -\rho_0^{-1} \nabla p$$, where \(u\) is horizontal momentum,

$$\tau_H = -\rho_0 \left[ u' w' - \nu \partial u/\partial z \right] = -\rho_0 u' w' + \mu \partial u/\partial z.$$  \(\nu = \mu/\rho_0\)

$$f k x u_g + \rho_0^{-1} \nabla p = 0$$

$$du/dt = - f k x (u - u_g) + \rho_0^{-1} \partial \tau_H \partial z$$.

$$\rho_0 \delta z \left[dv/dt + f k x (v - v_g)\right] = \tau_H(z + \delta z) - \tau_H(z)$$

As \(\delta z \to 0\), \(\tau_H(z + \delta z) \approx \tau_H(z)\), so that the turbulent flux can be considered approximately constant within the thin layer.

A thin, lowest sub-layer of the PBL for which \(\tau_H(z + \delta z) \approx \tau_H(z)\) is called the surface layer. The layer may also be called constant flux. Usually the depth of this sublayer is order of 100 m or less. The PBL may entirely consist of the surface layer (typically over a cold surface) or may be much deeper than the surface layer (typically
over a warm surface).

In the following, we present a similarity theory for the surface layer following
Monin and Obukov (1954).

**Neutral surface layer (\(\overline{w\theta'} = 0\))**

Let us first consider the case in which there is no buoyancy production of
turbulence kinetic energy. If we choose the x-axis in the direction of the Reynolds
stress, \(-\rho_0 \overline{u'w'}\), the magnitude of the stress may be written as \(-\rho_0 \overline{u'w'} > 0\).
We define the frictional velocity, \(u_*\)

\[
\left| \tau_{\mu} / \rho_0 \right| = \left| \overline{u'w'} + v \partial u / \partial z \right| = -\overline{u'w'} + v \partial u / \partial z \sim \text{constant} \equiv u_*^2
\]

At the surface \((z = 0)\), \(\overline{u'w'} = 0\) if the surface is smooth

\[
\partial u / \partial z = u_*^2 / v \quad u_{\mu} = u_*^2 v^{-1}z \quad \leftarrow \text{viscous sublayer} \sim \text{mm} \quad (4.13)
\]

In the surface layer above the viscous sublayer where

\[
\left| \overline{u'w'} \right| = \left| \mu \partial u / \partial z \right| \quad \leftarrow \text{inertia sublayer}
\]

Since we are considering neutral case (no buoyancy force is involved in generation of
turbulence, \(\theta'w' = 0\)), by hypothesizing that the vertical shear of the mean flow is
determined by \(u_*\) and \(z\) only, then

\[
\partial u / \partial z = k^{-1} u_*/(z+z_0), \quad (4.14)
\]

where \(k = 0.35 \sim 0.4\) is called the von Karman’s constant. \(z_0\) is a roughness length,
which is added to account for the fact that \(\partial u / \partial z\) is still finite when \(z=0\).

Integrating (4.14) gives

\[
(u - u_0) / u_* = k^{-1} \ln [(z+z_0)/z_0] \quad (4.15)
\]

which is the well-known logarithmic wind profile.

**The matching of surface layer to molecular sublayers**

Eq (4.13) and (4.14) \((z=0) \rightarrow k^{-1} u_* / z_0 = u_*^2 / \nu\)

\(z_0\) (smooth surface) \(= \nu / k u_*\)

Substitute \(z_0\) in eq (4.15)

\[
(u - u_0) / u_* = k^{-1} \ln (k u_* z N + 1) \quad (4.16)
\]
Experimental results obtained by Nikuradse (1933) suggest that

\[
\frac{(u-u_0)}{u_*} = k^{-1} \ln (\frac{u_* z}{\nu}) + 5.5
\]

(4.17)

which is valid for \( \zeta_* \equiv \frac{u_* z}{\nu} \gg 1 \)

If \( u-u_0 = 0 \), Eq (4.17) gives

\[
z_0 \text{ (smooth surface)} = 0.11 \frac{\nu}{u_*}
\]

(4.18)

Matching (4.11) to (4.17)

\[
\frac{u-u_b}{u_0-u_b} \approx \exp\left[-\frac{\zeta}{\xi_u}\right] = \exp\left[-\frac{\zeta_*}{\xi_u}\right], \text{ where } \xi_u \equiv \frac{z u_*}{\nu}, \zeta_u \equiv \frac{z u_*}{\nu}, \zeta_u = \sqrt{\xi_u}
\]

\[
\frac{u-u_b}{u_0-u_b} = \frac{u-u_0+u_0-u_b}{u_0-u_b} = \frac{u-u_0}{u_0-u_b} + 1
\]

\[
\frac{u-u_0}{u_0-u_b} = -1 + \exp\left[-\frac{\zeta_*}{\xi_u}\right]
\]

For \( \frac{\zeta_*}{\xi_u} \ll 1 \), \( \frac{u-u_0}{u_0-u_b} \approx -1 + \exp\left[-\frac{\zeta_*}{\xi_u}\right] \approx -\frac{\zeta_*}{\xi_u} \rightarrow \zeta^* = \xi_u, \text{ } u = u_b \)

\[
u \frac{u-u_0}{u_*} = \xi_u \left[ 1 - \exp\left(-\frac{\zeta^*}{\xi_u}\right) \right]
\]

(4.19)

Now, requiring (4.17) and (4.19) to match for both \( \frac{u-u_0}{u_*} \) and \( \frac{1}{u_*} \frac{\partial u}{\partial \zeta^*} \),

\[
\xi_u \left[ 1 - \exp\left(-\frac{\zeta^*}{\xi_u}\right) \right] = \frac{1}{\kappa} \ln \xi_u^* + 5.5
\]

\[
\exp\left(-\frac{\zeta^*}{\xi_u}\right) = \frac{1}{\kappa \xi_u^*}
\]

The solution is \( \xi_u = 16 \) and \( \zeta_u^* = 47 \).
Figure 4.1  Velocity profile from the molecular sublayer to the surface layer as described by (4.17) and (4.19) (solid line) and laboratory measurements by Reichardt (1940) (open circles). After Liu et al. (1979).

Assuming $u^*=0.2 \text{ m s}^{-1}$, (~5 to 6 m s$^{-1}$ at 10 m height), $\nu_{\text{air}}=1.5\times10^{-5} \text{ m}^2 \text{ s}^{-1}$, the height corresponding to $\zeta^*=47$ is $z = 47\nu_{\text{air}} / u^* = 3.5 \text{ mm}$, and the wind speed at that height is about 3 ms$^{-1}$. Above this height the logarithmic profile extends to several meters above the surface.

In the ocean, the dimensions are different because at the surface $\tau_a = \tau_w = \rho_w u_w^2$. So $u_w = (\rho_a/\rho_w) u_a^* \sim 0.035 u_a^* = 0.007 \text{ ms}^{-1}$. With $\nu_w(20^\circ\text{C}) = 1.05\times10^{-6} \text{ m}^2 \text{ s}^{-1}$, this gives a depth of transmission of about 7 mm below the surface and the difference in surface drift and drift at this level of about 0.11 ms$^{-1}$.

A similar matching can be achieved for the temperature profile

$$\frac{\Theta - \Theta_0}{\Theta_*} = \zeta_0 \left[ 1 - \exp\left( -\frac{\zeta}{\zeta_0} \right) \right]$$

and the logarithmic profile in the turbulent layer

Figure 4.2  Temperature profile from the molecular sub-layer to the surface layer as described by (4.20) and logarithmic relation (solid line) and laboratory measurement by Deissler and Eian (1952). After Liu et al. (1979).
The match gives
\[ \frac{\Theta - \Theta_0}{\theta_*} = 2.18 \ln(\zeta^*) + 3.8 \]  
(4.20)

\[ \frac{\Theta - \Theta_0}{\theta_*} = \xi_u [1 - \exp(-\frac{\zeta^*}{\xi_u})], \xi_u = 13, \zeta_u^* = 45 \]  
(4.20)

where \( \xi_u = \frac{T_h - T_0}{\theta_*} = \frac{z_0 u_*}{\kappa}, \theta_* = -\frac{w' \theta'}{u_*} = -\frac{w' T'}{u_*} \)

Humidity observations are not available so close to the interface. The match of the molecular sublayer to the logarithmic turbulent layer cannot be checked. However, if we assume that scalar quantities behave similarly, it is possible to formulate a model for the transfer of humidity as well as trace gases across the interface. This will be discussed below.

**Transition from smooth to rough flow**

Roughness Reynolds number: \( Rr \equiv u_* z_0 / \nu \)

\[ Rr < 0.13 \text{ for flow to be smooth, } z_0 = 0.11 v/u_* \]

\[ Rr > 2.5 \text{ for flow to be rough, } z_0 = a u_*^2 / g \text{ (Charnock 1955) } \]  
(4.21)

0.011 < \( a < 0.018 \)

\[ Rr = 2.5 = u_* z_0 / \nu = u_* a u_*^2 / g \nu \to u_*^3 / g \nu = 167 \text{ (} a = 0.015 \text{) } \]

\( u_* = 0.29 \text{ m s}^{-1} \) and \( z_0 = 1.3 \times 10^{-4} \text{ m} \)

\[ Rr = 0.13 = u_* z_0 / \nu = u_* a u_*^2 / g \nu \to u_*^3 / g \nu = 8.7 \text{ (} a = 0.015 \text{) } \]

\( u_* = 0.11 \text{ m s}^{-1} \) and \( z_0 = 1.8 \times 10^{-5} \text{ m} \)

Wu (1980), 0.17 < \( Rr < 2.33, \text{ } 9.2 < u_*^3 / g \nu < 126 \text{ (} a = 0.0185 \text{) } \)

and \( 0.11 \text{ m s}^{-1} < u_* < 0.265 \text{ m s}^{-1} \)

\[ z_0 = z_0 \text{ (smooth)} + z_0 \text{ (trans)} + z_0 \text{ (rough)} \]  
(4.22)

In the above discussion, the following are being neglected

1. presence of small water waves (surface tension & kinematic viscosity of the water)
2. intermittent nature of the turbulent wind

The roughness of the sea surface will further be considered below.
Figure 4.3  The roughness length as a function of $u_*$ from smooth to rough flow (solid). Asymptotic relations (straight dashed lines).

We may speculate that a surface renewal model will continue to be valid close to the interface, going from smooth to rough flow. Assuming what

$$t_* \propto z_0 / u_* \rightarrow t_* \propto \nu / u_*^2$$ for smooth flow,

$$t_* \propto \left( \nu / g u_* \right)^{0.5}$$ for transitional flow, \hfill (4.23)

$$t_* \propto (u_*/g)$$ for rough flow.

Although the transition layer increases in thickness with increasing $u_*$. There is as yet no way to verify experimentally that profiles of the form (4.19) are valid near the surface because the wave height also increases with windspeed. It is very difficult to determine wind profiles at heights lower than the height of the dominant waves.

Summary

The logarithmic profile is valid only for $z/z_0 \gg 1$, which means that in (4.14) $z_0$ in the numerator may be negected. Eq. (4.14) and (4.21) give

$$(u-u_0) / u_* = k^{-1} \ln (z/z_0) = k^{-1} \ln (zg/ u_*^2) + B$$ \hfill (4.24)

where $B = k^{-1} \ln a$, $B=10.5$ for $a=0.105$. This equation gives a reasonable description of the wind profile over water under neutral conditions for windspeeds $u > 8 \text{ m s}^{-1}$.

The profile in the transitional regime, assuming $z_0(\text{trans})=0.1(\nu u_* / g)^{0.5}$, may have the form

$$(u-u_0) / u_* = k^{-1} \ln \left( \nu / g u_* \right)^{0.5} z + 5.8$$ \hfill (4.25)

for wind speeds $3 < u < 8 \text{ m s}^{-1}$.
In the above discussion, the emphasis has been on profiles in the atmospheric surface layer. As soon as waves play a role the situation in the ocean is more complex. The orbital velocities of the waves are usually much larger than the mean drift velocities from the surface down. It is consequently very difficult to measure mean profiles below the interface. The thickness of the surface layer in the ocean is a factor $(p_u/p_w)^{0.5}$ smaller than the thickness of the surface layer in the atmosphere that causes the waves to interact with turbulence through a substantial portion of the ocean surface layer. Beside these geometrical limitations, wave-turbulence interactions, as discussed by Kitaigorodskii and Lumley (1983) and Kitaigorodskii et al. (1983), may be a significant producer of TKE. The shear production of TKE, which in the ocean surface layer is given by $u^3/\kappa z$ and which is about four orders of magnitude smaller than the shear production in the atmospheric surface layer, might be considerably smaller than the turbulence production by wave interaction and by breaking waves. Recent measurements, reported by Agrawal et al. (1992), show that with strong winds the actual dissipation near the surface is often an order of magnitude larger than the mean shear production of TKE. Therefore the basic mechanism that leads to the logarithmic profile may not be present in the ocean surface layer.

4.2 The effect of stratification

*Stratified surface layer (w’θ’ ≠ 0)*

In this case, turbulence can gain (unstable case) or lose (stable case) its kinetic energy through the buoyancy force. The vertical shear of the mean flow should now depend not only on $u_*$ and $z$, but also on the buoyancy production, $g w'\theta'/\theta_0$. Then $\partial u/\partial z = k z u^3/\kappa z$ is not necessarily true as in the neutral case. Generally it should be a non-dimensional combination of $u^3/\kappa z$ and $g w'\theta'/\theta_0$ (dry atm.). This is the basis for the Monin-Obukov similarity theory.

Assuming $g \theta_0^{-1} w' \theta' \sim \text{const}$, then
\[ k z u^{-1} \partial w/\partial z = \phi_m (u_*, z, g \theta_0^{-1} w'\theta') = \phi_m (\zeta) \]  
(4.26)

the only combination to give non-dimensional is
\[ g \theta_0^{-1} w' \theta' / (u^3 z^{-1}) \]  
(4.27)

or (4.27) can be expressed as $\zeta = z/L$  

Monin-Obukhov length  
(4.27)

Note that $u^{-3} / k z = -u' w' \partial u/\partial z$, $L = -u^{-3} / k w' b'$ is the height in the neutral surface layer where the buoyancy flux (~constant) and shear production (decrease with height) are equal.

Define $-w' \theta' = u_\theta$.  
(4.28)

\[ k \theta_0^{-1} \partial \theta/\partial z = \phi_\theta (z/L) \]  
(4.29)
From (4.27) and (4.28), \( \theta^* = \theta_0 \frac{u^*}{k g L} \) \hfill (4.30)

\[
R_i = g \frac{(\partial \ln \theta / \partial z)}{(\partial \theta / \partial z)} \left( \frac{\partial \theta / \partial z}{\partial u / \partial z} \right)^2 = g \theta_0^{-1} \frac{\theta_0 z \phi_h}{k^2 u^* z^2 \phi_m^2} = \frac{u^2 k^2 L^2 z \phi_h}{k^2 u^* z^2 \phi_m^2} = z L^{-1} \phi_h \phi_m^{-2} \hfill (4.31)
\]

Figure. Richardson number as a function of \( \zeta \) (from Businger et al., 1971)

\[ u(z), \theta(z) \rightarrow R_i \rightarrow u^*, \theta^* \rightarrow u'w', w'\theta' \]
$R_i < -0.21$ is necessary to maintain the constant flux layer

**The surface eddy diffusion coefficients**

From (4.26), $u^2 = [(kz/\phi_m)\partial u/\partial z]^2 = u'u'$, $\rightarrow K_m = (k z / \phi_m)^2 \partial u/\partial z$ \hspace{1cm} (4.32)

From (4.26) and (4.29), $u_\theta^2 = [(kz/\phi_m)\partial u/\partial z] \cdot [(kz/\phi_h)\partial \theta/\partial z] = \theta'w'$

$\rightarrow K_h = (kz)^2 / \phi_m \phi_h \partial u/\partial z$ \hspace{1cm} (4.33)

Since both $\phi_m$ and $\phi_h$ are functions of $\zeta$, which is a function of $R_i$, we may formally write

$$K_m = l^2 f_m(R_i) \partial u/\partial z \hspace{1cm} (4.34)$$

$$K_h = l^2 f_h(R_i) \partial u/\partial z \hspace{1cm} (4.35)$$

where $l = kz$, $f_m(R_i) = 1/\phi_m^2$, $f_h(R_i) = 1/\phi_m \phi_h$

**The bulk aerodynamic method**

Integrating (4.26), $u_a - u_o = u^* k^{-1} \int_{z_0} z^{-1} \phi_m (z/L) \, dz$

$\rightarrow \rho_o (u'w)_o = -\rho_o k^2 \left[ \int_{z_0} z^{-1} \phi_m (z/L) \, dz \right]^2 (u_a - u_o)^2$

$$= -\rho_o C_d (u_a - u_o)^2 \hspace{1cm} (4.36)$$

Integrating (4.29), $\theta_a - \theta_0 = \theta^* k^{-1} \int_{z_0} z^{-1} \phi_h (z/L) \, dz$

$\rightarrow \rho_o \theta'w_0 = -\rho_o k^2 \left[ \int_{z_0} z^{-1} \phi_m (z/L) \, dz \int_{z_0} z^{-1} \phi_h (z/L) \, dz \right]^{-1} (u_a - u_o) (\theta_a - \theta_o)$

$$= -\rho_o C_h (u_a - u_o) (\theta_a - \theta_o) \hspace{1cm} (4.37)$$

where subscript 0 denotes $z = z_0$ the roughness parameter $z_0$ is the height of anemometer level ($\sim$10m)

Near Neutral

$z_0/L << 1, \phi_m \sim 1, \phi_h \sim 0.74,$

$\int \phi_m \, z^{-1} \, dz = ln(z/z_0), \int \phi_h \, z^{-1} \, dz = 0.74 \, ln(z/z_0)$

$C_D/C_H \sim [\int \phi_h \, z^{-1} \, dz] / [\int \phi_m \, z^{-1} \, dz] = 0.74$

Assuming $\theta_0 = \theta_s$, one can define the bulk Richardson number

$R_i B = g \, \theta_0^{-1} (\theta_0 - \theta_s) \, z / \nu^2$
\[ \zeta \left[ \int \phi_h z^{-1} \, dz \right] / \left[ \int \phi_m z^{-1} \, dz \right]^2 \]  

(4.38)

For given \( R_i_B \) and \( z/z_0 \), the above formula determines \( \zeta \) and, therefore, \( C_D \) and \( C_H \).

Louis (1979) gives the two bulk coefficients as functions of \( R_i_B \) for selected values of \( z/z_0 \).

4.3 Dynamic interactions between wind and sea surface

4.4 Transport of trace gases across the interface

4.5 The sea surface temperature and the energy budget

4.6 Methods to observe the fluxes in the atmospheric surface layer

Flux balance at the air-sea interface, Surface exchange coefficients, Surface renewal theory, Satellite-measured fluxes