Microeconomics I - Solutions to Exercise 1

1. (1) $y = \frac{x}{\sqrt{x-1}}$,

$$\frac{dy}{dx} = \frac{\sqrt{x-1} - x\frac{1}{2}(x-1)^{-\frac{1}{2}}}{x-1} = \frac{x-2}{2\sqrt{(x-1)^3}}.$$

(2) Since $y = \ln[e^x(2x+1)] = x + \ln(2x+1)$, we have

$$\frac{dy}{dx} = 1 + \frac{2}{2x+1} = \frac{2x+3}{2x+1}.$$

(3) $y = x^5 + \sqrt[4]{x^3} - 2\sqrt{x}$,

$$\frac{dy}{dx} = 5x^4 + \frac{3}{4\sqrt[4]{x}} - \frac{1}{\sqrt{x}}.$$

(4) Since $y = (3x-1)(4x+5)(2x-7) = (12x^2+11x-5)(2x-7)$,

$$\frac{dy}{dx} = (12x^2+11x-5)2 + (2x-7)(24x+11) = 72x^2 - 124x - 87.$$

(5) $y = \frac{2+x}{3+2x-x^2}$,

$$\frac{dy}{dx} = \frac{(3+2x-x^2) - (2+x)(2-2x)}{(3+2x-x^2)^2} = \frac{4x+x^2-1}{(3+2x-x^2)^2}.$$

2. (1) $Y = 25 \cdot x_1^{0.6} x_2^{0.4}$,

$$\frac{\partial Y}{\partial x_1} = f_1 = 25 \cdot 0.6 x_1^{-0.4} x_2^{0.4} = 15\left(\frac{x_2}{x_1}\right)^{0.4}.$$

$$\frac{\partial Y}{\partial x_2} = f_2 = 25 \cdot 0.4 x_1^{0.6} x_2^{-0.6} = 10\left(\frac{x_1}{x_2}\right)^{0.6}.$$

(2) $Y = A[\alpha x_1^{-\rho} + \beta x_2^{-\rho}]^{-\frac{1}{\rho}}$,

$$\frac{\partial Y}{\partial x_1} = f_1 = -A\frac{1}{\rho}\left[\alpha x_1^{-\rho} + \beta x_2^{-\rho}\right]^{-\frac{1}{\rho}-1}\left(-\rho\alpha x_1^{-\rho-1}\right)$$
$$= A\alpha x_1^{-(\rho+1)}\left[\alpha x_1^{-\rho} + \beta x_2^{-\rho}\right]^{-\frac{1+\rho}{\rho}}.$$

$$\frac{\partial Y}{\partial x_2} = f_2 = -A\frac{1}{\rho}\left[\alpha x_1^{-\rho} + \beta x_2^{-\rho}\right]^{-\frac{1}{\rho}-1}\left(-\rho\beta x_2^{-\rho-1}\right)$$
$$= A\beta x_2^{-(\rho+1)}\left[\alpha x_1^{-\rho} + \beta x_2^{-\rho}\right]^{-\frac{1+\rho}{\rho}}.$$

(3) $y = x_1^2 + 5x_1\sqrt{x_2} + x_2$,

$$\frac{\partial Y}{\partial x_1} = f_1 = 2x_1 + 5\sqrt{x_2}.$$

$$\frac{\partial Y}{\partial x_2} = f_2 = 1 + \frac{5x_1}{2\sqrt{x_2}}.$$