105. Statistics Final Examination 22 June 2017

1. (15 pts.) Let $X_1, \cdots, X_n$ be a sample from an Exponential(λ) distribution.

   (a) Find the likelihood ratio test for testing $H_0 : \lambda = \lambda_0$ versus $H_A : \lambda = \lambda_1$, where $\lambda_1 > \lambda_0$. Explain how to determine the critical point at significance level α.

   (b) Show that the test in part (a) is uniformly most powerful for testing $H_0 : \lambda = \lambda_0$ versus $H_A : \lambda > \lambda_0$.

2. (15 pts.) Let $X_i \sim$ Binomial(n_i, p_i), $i = 1, \cdots, m$, be independent random variables.

   (a) Derive a likelihood ratio test statistic for the hypothesis $H_0 : p_1 = p_2 = \cdots = p_m$ against the alternative that the p_i are not all equal.

   (b) What is the large-sample distribution of the test statistic in part (a)?

3. (15 pts.) Let $X_1, \cdots, X_n$ be a sample from a cdf F, and let F_n denote the ecdf.

   (a) Determine the distribution of $F_n(x)$ and find $E[\,F_n(x)\,]$, $\text{Var}[\,F_n(x)\,]$.

   (b) Find $\text{Cov}[\,F_n(u), F_n(v)\,]$.

4. (15 pts.) Suppose a nonnegative random variable X has hazard function $h(t)$.

   (a) Find the hazard function of aX where a is a positive constant.

   (b) Find the hazard function $h(t)$ when X is uniform$(0, 1)$.

   (c) Find the density function of X when $h(t) = \frac{1}{1+t}$, $t > 0$.

5. (15 pts.) Let $X_1, \cdots, X_n$ be a sample from an Exponential(λ_1) distribution and let $Y_1, \cdots, Y_n$ be an independent sample from an Exponential(λ_2) distribution.

   (a) Determine the expected rank sum of the X's.

   (b) Determine the variance of the rank sum of the X's.

6. (15 pts.) Let $X_1, \cdots, X_n$ be i.i.d. $N(\mu_X, \sigma_X^2)$, and let $Y_1, \cdots, Y_m$ be i.i.d. $N(\mu_Y, \sigma_Y^2)$, where the two samples are independent.

   (a) Find the distribution of s_X^2/s_Y^2, where s_X^2 and s_Y^2 are sample variance.

   (b) Construct a level $100(1-\alpha)\%$ confidence interval for the ratio σ_X^2/σ_Y^2.

   (c) Test the hypothesis $H_0 : \sigma_X = \sigma_Y$ versus $H_A : \sigma_X \neq \sigma_Y$ at significance level α.

7. (30 pts.) Consider the one-way analysis of variance model $Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, $i = 1, \cdots, I$, $j = 1, \cdots, J$, where Y_{ij} is the jth observation for the ith treatment level, μ is the overall mean, α_i is the effect of the ith treatment with $\sum_{i=1}^{I} \alpha_i = 0$, and ε_{ij}'s are independent $N(0, \sigma^2)$ errors.

   (a) Find the mle's of the parameters μ and α_i, $i = 1, \cdots, I$.

   (b) Find the expectation of the sum of squares within and between groups respectively.

   (c) Write down the One-Way Analysis of Variance table.

   (d) Test the null hypothesis $H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_I = 0$ at significance level α.