String theory as a generalization of gauge symmetry

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This is a brief review of the relation between string theory and usual gauge theories including Einstein's gravity and Yang-Mills theory. In particular, we would like to explain how string theory extends or generalizes ideas behind gauge symmetry and Einstein's general relativity. Most of the material concerning this article can be found in [1, 2], if no further reference is provided.

1 There is gravity

Let us start with some basic facts about string theory. The first and foremost to say is that string theory includes gravity. More importantly, string theory contains quantum gravity, and is the only theory of quantum gravity which admits a ultraviolet-finite and unitary perturbation theory. On the other hand, there is no consistent perturbation theory for the canonical quantization of Einstein's theory of pure gravity.

Contrary to particles, for which it is extremely hard to find a well-behaved (causal, unitary, renormalizable) interaction with the gravitons, strings are almost always accompanied by gravity. As each excited state of a string can be reinterpreted as a particle, the massless spin-two excitation of a closed string is identified with the graviton. This excitation exists in all five superstring theories (type I, type IIA, type IIB, heterotic $SO(32)$ and heterotic $E_8 \times E_8$) as well as the bosonic string theory.

A string can have (infinitely) many vibration modes. The graviton is a massless spin-2 osillation mode of a closed string. Each oscillation mode should be matched with a certain particle in spacetime. Each particle is associated with a vertex operator defined in the 1+1 dimensional quantum field theory living on the string worldsheet. Interactions among particles are determined in string theory by correlation functions of vertex operators. A Feynman diagram for a spacetime particle scattering process corresponds to a path integral of the worldsheet theory. Unlike particle theory, where interaction vertices and propagators are built in a Lagrangian, all ingredients of Feynman diagrams are fixed in string theory by the definition of a free string.

There are two complimentary approaches to see how gravitational interaction is determined in string theory. The first approach is to compute the Feynman diagram of a scattering process involving gravitons, in the flat trivial background. From the result one can construct order by order a field theory involving the metric to reproduce the scattering amplitudes.

The second approach is to consider a string propagating in a perturbative deformation of the flat background. The spacetime metric appears in the kinetic term of the string worldsheet action. The requirement of (quantum) conformal invariance then imposes a strong constraint on the metric. The constraint involves the metric and its derivatives, and can be viewed as the equation of motion of the metric. In this approach, the equation of motion for the spacetime metric is in fact a self-consistency condition. This is a remarkable feature of string theory which is not unique to gravity. Dynamics and kinematical constraints are unified in string theory.

Of course, the results of the two approaches agree with each other. They also agree with the Einstein equation at low energies for weak coupling (Newton) constant. At high energies (compared with the energy scale of the string tension), string theory modifies Einstein's theory, but with general covariance intact.

2 There is a bigger gauge symmetry

In addition to the graviton, strings have other oscillation modes including many other gauge fields. For open strings, there is a massless spin-1 oscillation mode corresponding to a gauge field in spacetime.

Non-Abelian gauge symmetry also arises in string theory in various ways. One possibility is to associate a so-called Chan-Paton factor to each endpoint of an open string. The massless spin-1 oscillation mode is then labelled by two indices, and the corresponding spacetime gauge field is promoted to a matrix. The result is that the $U(1)$ gauge symmetry is enhanced to a non-Abelian gauge symmetry. This happens for open strings in type I string theory, and the gauge group has to be $SO(32)$ for self-consistency. Another possibility is to add currents corresponding to a global symmetry on the string worldsheet, or to compactify the spacetime in a particular way so that new massless spin-1 oscillation modes appear.

Similarly, when there are multiple coincident D-branes on top of each other, an open string stretched between two D-branes naturally acquire labels specifying the two D-branes. For coincident N D-branes, there is an $U(N)$ gauge symmetry on the D-brane worldvolume.

In addition to the massless spin-1 oscillation mode, there are (infinitely) many other oscillation modes corresponding to gauge fields of higher spins. It is believed that these gauge fields are massive due to a spontaneous symmetry breaking mechanism, and the symmetry may be restored in the high energy limit. String extends general covariance and usual gauge symmetry to a much larger symmetry. There are higher spin gauge fields of arbitrarily many Lorentz indices. Furthermore, as the symmetry is so highly intertwined, it is tempting to conjecture that its self-consistency uniquely fixes the dynamics of the whole theory.

Witten's cubic string field theory [5] manifestly exihibits the gauge symmetry of bosonic open string theory. The action of the cubic string field theory is

$$S = \langle\left(\frac{1}{2}\Psi * (Q\Psi) + \frac{1}{3}\Psi * \Psi * \Psi\right)\rangle, \tag{1}$$

where $\Psi[X(\sigma)]$ is the string wave function (a state of the string worldsheet theory in the formulation using BRST quantization) and Q is the BRST charge for conformal symmetry. The product labelled by $*$ defines an associative algebra for the string states, which essentially tells us how to glue two strings together into one string. This action has the gauge symmetry

$$\Psi \to Q\Lambda + \Psi * \Lambda - \Lambda * \Psi. \tag{2}$$

We see from this formula that the string wave function is itself a gauge field for a huge gauge symmetry.

The string wave function Ψ can be represented as a generic state in the Hilbert space of the string worldsheet theory in BRST quantization. In terms of an expansion of creation operators,

$$\begin{aligned}\Psi &= \int d^d k \left[\phi(k) + iA_\mu(k)\alpha^\mu_{-1} + \alpha(k)b_{-1}c_0 + iB_\mu(k)\alpha^\mu_{-2} - B_{\mu\nu}(k)\alpha^\mu_{-1}\alpha^\nu_{-1} + \right. \\ &\quad \left. \beta_0(k)b_{-2}c_0 + \beta_1(k)b_{-1}c_{-1} + i\beta_\mu(k)\alpha^\mu_{-1}b_{-1}c_0 + \cdots\right] c_1|0;k\rangle, \qquad (3)\end{aligned}$$

where α^μ_n are the bosonic operators representing fluctuations of the spacetime coordinate X^μ, and c_n, b_n belong to the ghost sector. The coefficients $\phi(k), A_\mu(k), \alpha(k), B_\mu(k)$, etc. are Fourier transforms of spacetime fields. Their gauge transformations are given by

$$\delta A_\mu = \partial_\mu \epsilon^0, \qquad (4)$$

$$\delta\alpha = \frac{1}{2}\partial^2\epsilon^0, \qquad (5)$$

$$\delta B_{\mu\nu} = -\partial_{(\mu}\epsilon^0_{\nu)} - \frac{1}{2}\eta_{\mu\nu}\epsilon^1, \qquad (6)$$

$$\delta B_\mu = -\partial_\mu\epsilon^1 + \epsilon^0_\mu, \qquad (7)$$

$$\delta\beta_\mu = \frac{1}{2}(\partial^2 - 2)\epsilon^0_\mu, \qquad (8)$$

$$\delta\beta_0 = \frac{1}{2}(\partial^2 - 2)\epsilon^1, \qquad (9)$$

$$\delta\beta_1 = -\partial^\mu\epsilon^0_\mu - 3\epsilon^1. \qquad (10)$$

Obviously, A_μ is the massless spin-1 guage field mentioned above. $B_{\mu\nu}$ is a symmetric rank-2 gauge potential. At higher mass levels one finds gauge fields at higher ranks. Except A_μ, the other gauge fields are massive, signaling a spontaneous symmetry breakdown.

The physical meaning of the huge gauge symmetry of string theory has not yet been thoroughly explored. People also suspect that there is another huge hidden (global) symmetry which is spontaneously broken, and that one should study the high energy limit where the symmetry may be restored [6]. The 2 dimensional string theory is the best understood toy model of strings. It has the w_∞ symmetry and the symmetry algebra is strong enough to determine all scattering amplitudes. It has been conjectured by Gross and his collaborators [6] that the full symmetry may uniquely determine the dynamics for higher dimensional strings as well. It is possible that the huge global symmetry has the same origin as the huge gauge symmetry of string theory, or that its existence is a necessity for self-consistency due to the higher spin gauge symmetries [7].

While the general covariance of general relativity is embedded in a bigger gauge symmetry, we do not seem to fully comprehend the physical notion which generalizes the equivalence principle. What are the gendenken experiments analogous to the elevator in free fall?

3 Geometry is induced

Before the advant of string theory, spacetime is put in by hand as a stage in which particles interact and events take place. In string theory, spacetime can be a derived notion. From the viewpoint of the string worldsheet action, the spacetime coordinates of the string are scalar fields on the worldsheet. Properties of spacetime are determined by properties of these scalar fields. Of course one can also view spacetime coordinates of a point particles as a scalar field on the worldline. The crucial difference is that while string worldsheet theory determines the dynamics of spacetime geometry, the particle worldline theory does not.

As spacetime is a derived notion, the geometrical structure of spacetime is something that should be extracted from the theory. We have already mentioned above how the dynamics of spacetime metric is determined in string theory, yet Riemannian geometry is not the only geometrical structure the spacetime can possess.

Noncommutative geometry is a mathematical notion that generalizes classical geometry [8] A classical manifold is commutative, that is, the algebra of functions on the manifold is a commutative algebra. Mathematicians noticed that, although traditionally one visualizes a manifold as a collection of points (with some topology), the algebra of functions provides an alternative equivalent description to some extent. Hence it is natural to relax the definition of geometry to allow the algebra of functions to be noncommutative. Such a space no longer admits the picture of a set of points, but various geometric structure and quantities can be defined.

In a field theory, the base space can be noncommutative by imposing nontrivial commutation relations for the coordinates. In string theory, the commutativity of spacetime coordinates depends on the quantization of the scalars fields on the worldsheet. In suitable background, noncommutative geometry arises automatically.

In string theory, there are solitonic objects called D-branes. They are submanifolds of spacetime on which open strings end. Turning on a background field called NS-NS B-field, one can quantize the open string and find that the coordinates of the endpoints are noncommutative [9], with the noncommutativity depending on the B-field background. The D-brane field theory is then conveniently defined as a noncommutative field theory [10].

Another way to see the noncommutativity of a D-brane is to consider scattering amplitudes of open strings [12]. In other words, you probe the geometric property of the D-brane worldvolume using open strings. Due to the B-field background, the interactions at low energies are most conveniently described by a field theory living on a noncommutative space. With other background fields (graviphoton) turned on, the spacetime itself can also be noncommutative [13].

In addition to noncommutative geometry, there might be more exotic geometrical notions which we can learn from string theory. In general, one probes the spacetime via strings. The geometry of spacetime is an induced notion derived from decoding string interactions in a certain way. It is conceivable that there can be ambiguity in the decoding process, corresponding to the freedom of making a change of variables (field redefinition). For example, a noncommutative field theory can be reinterpreted as a commutative field theory with higher derivative interactions [11]. We will make more comments below on other ambiguities in defining spacetime properties.

Matrix model provides another way to demonstrate the idea. The BFSS model [14] and the IKKT model [15] are conjectured to be equivalent descriptions of string theory. Spacetime coordinates correspond to $N \times N$ matrices with $N \to \infty$. As matrices the coordinates are generically noncommutative. Interactions in the models would make it impossible to have large extended dimensions in spacetime if there were no supersymmetry to guarantee flat directions in the moduli space. The large scale, (roughly) commutative spacetime around us is not given to us without warrant. The choice of the effective spacetime dimension by our universe can be translated into questions about the free energy of the theory [16].

T-duality (including Mirror symmetry) is another example of the ambiguity in extracting the spacetime structure from string theory. The simplest setting of T-duality is having one spatical direction compactified on a circle. Let the radius be denoted R. It turns out that this theory is equivalent to another string theory with a dual spatial direction compactified on a circle of radius $1/R$ in string units. The two theories dual to each other are different descriptions of the same physical system. The same physical state can have totally different descriptions in the two theories. For instance, a string winding on a circle can be matched with a Kaluza-Klein mode (momentum eigenstate) in the dual theory. But a state is always matched with another state with the same energy. Physicists trying to describe a physical process can adopt either one of the two theories and its language. They may sound very different in words, but the two theories agree when it comes to the prediction of the result of a measurement in terms of pure numbers.

In general, shapes and topology of spacetime can be different between equivalent theories. Even the spacetime dimension is not independent of the choice of description/theory. Superstrings in 10 dimensions are believed to be equivalent to the M-theory in 11 dimensions. Physics is not only observer-dependent, like it already is in Einstein's theory, but is now also theory-dependent. In fact, all five superstring theories are believed to be dual to one another, and also to (a quantum version of) the 11 dimensional theory of supergravity.

4 Summary

To summarize, gauge symmetry is generalized to include gauge fields of arbitrary spins in string theory. Einstein's theory of spacetime is also extended to more general geometric structures. The notion that physics is observer-dependent, which we first learned from the theory of relativity, is magnified to the notion that physics is theory-dependent. It is also believed that Yang's inspiring saying that "Symmetry dictates interaction" is fully honoured by string theory.

An obvious difference between string theory and general relativity or Yang-Mills theory is that the discovery of the latter was motivated by beautiful theoretical notions on symmetry and geometry, but the discovery of string theory was an accident. What is the symmetry principle of string theory? This is one of the most important problems of string theory.

We have so far avoided discussions on holography and cosmological constant in this article. Holographic principle is argued to be a salient feature of quantum gravity [17]. In string theory we have seen remarkable evidences of it [18]. It is generally viewed to be of utmost importance for quantum gravity, but so far our understanding of it remains at a technical level. On the other hand, our understanding of the cosmological constant problem is not even at the technical level. It might be that the secrets of holography and cosmological constant are deeply hidden inside string theory waiting to be discovered, and their revelation will bring us a drastic conceptual breakthrough so that string theory will no longer be called string theory.

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