Networks and Market Structures

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Abstract
A new model developed in recent years has emphasized that a network of sellers and buyers, instead of a market, makes an economic exchange event happen. A seller and a buyer must form a relationship link in order to exchange goods. For instance, peddlers have to travel around to meet potential customers to sell goods, or companies have to circularize their mail-order catalogs to make consumers aware. Suppose a fixed number of sellers and buyers existed and intend to exchange goods. We aim to develop a seller-buyer network model, then study agents’ strategies and equilibrium market structures. We will also discuss whether the formation of a monopoly or an oligopoly market mainly caused by high market entrance barriers, or it may just be one possible equilibrium outcome. Finally we will release the condition of a fixed number of players, and study the relationship between the market structures and sellers’ product cost functions.

Keywords: Trading network, Market structure

1 Introduction
How does an economic exchange event happen between a seller and a buyer?\(^1\) The traditional economic analysis offers us an answer by suggesting the existence of “market”. However, what is the nature of a market and how does it function? What kind of market structures form and how do they form? What determines the behaviors of sellers and buyers to form such markets? To answer these questions, we may have to think deeper about the meaning of market and find a new approach to describe sellers’ and buyers’ behaviors.

Empirical evidence shows that an economic exchange often happens only when a seller and a buyer have a relationship, or a “link”. McMillan and Woodruff (1999) find that a longer duration of trading relationship is associated with larger credit among firms in Vietnam. Thus customers identified through business networks receive more credit and networks are used to sanction defaulting customers. Also in many industries, firms devote huge effort and financial\(^1\)In a wide range of markets, sellers and buyers may trade through intermediaries. And not all sellers and buyers should have access to the same intermediaries.
resources to selecting their business partners. Lazerson (1993) shows that in the garment district of Modena (Italy), the majority of artisans assembling clothing for garments works for at least three clients. If links between agents determine a certain trade relationship, then the decision to establish links becomes a very important issue of the agent’s business model.

More and more studies of the industrial organization theory now are taking network approaches as a main analysis tool. In those studies, complex network structures are introduced to restrict the possible relationships between agents. Rochet and Tirole (2002) analyze the issue of interchange fee in the payment card associations and find cooperation among competitors may happen. Goyal and Joshi (2003) find that collaborative links are usually used by firms to make the competitive advantage, and strategically stable networks are often asymmetric. Rey and Tirole (2003) analyze a situation where asymmetric information leads the upstream firms to take Cournot competition to supply input to the downstream firms. However, most of studies discuss the network relationship of sellers/firms, rather than a relationship between sellers and buyers. Kranton and Minehart (2001) first develop a methodology to study seller-buyer networks and show buyers and sellers can form a network that maximizes overall welfare. But there is a lack of discussion in network literature about the cost of links and equilibrium market structures. If we know that the cost of links (like an entrance barrier) can reduce the number of sellers linked to a buyer, or vice versa, then the final number of markets which is formed by at least one seller and one buyer surely relies on the link costs. The fact of a fixed number of sellers and buyers doesn’t necessarily apply to one market structure.

Our study will model people in a two-stage network formation game where sellers and buyers must be connected in order to exchange goods. This model will capture characteristics of a variety of market structures, and show how they are formed when the cost of forming links or the number of agents change.

2 Model

We will set a two-stage game where sellers and buyers must be connected in order to exchange goods. The number of sellers is $n$ and the number of buyers is $m$. Each seller has the capacity to produce $k$ units of goods at zero cost and she can only sell the goods in an integer number to a buyer. All sellers produce the same kind of goods. A seller’s surplus is the total revenue of her sale. All buyers are homogenous and a buyer’s surplus is $S(q) = V(q) - P(q)$. $q$ represents the total quantity of the goods a buyer has purchased and $P(q)$ represents the total price she has paid to the sellers in order to purchase $q$ units of goods. However, a buyer may purchase the goods from several sellers. At stage one, sellers simultaneously decide which buyers to connect. Each connection costs the sellers a very small fixed cost $c$ to form. If there is a link formed between the seller $i$ and the buyer $j$, then we denote $g_{ij} = 1$. And we assign $g_{ij} = 0$ if there is no connection between the seller $i$ and the buyer $j$. A trade network $g$ is the collection of $g_{ij}$. At stage two, sellers can sell their goods to all the linked buyers.
We assume the sellers have all bargain power on pricing the goods, so the seller \(i\) can decide the price \(p_{ij}\) when she sells goods to a connected buyer \(j\) and a quantity of goods \(q_{ij}\), which is the maximal quantity buyer \(j\) is allowed to buy from the seller \(i\). A buyer can decide how many units to buy and who she wishes to trade with. For example, suppose that both seller 1 and seller 2 can sell goods to buyer 1. If seller 1 offers two units of goods at the price of 0.5 and seller 2 offers three units of goods at the price of 0.3, then buyer 1 may decide to buy one unit from seller 1 and three units from seller 2. We can consider that if several buyers are connected to only one seller, then the seller-buyer relationship becomes a monopoly market. If buyers are linked to two sellers, then it is a duopoly market among them. Thus a network \(g\) also represents a market structure where several types of markets may exist. For example, two sellers and three buyers can form a network \(g = \{g_{ij}\} = \{g_{11} = g_{12} = g_{22} = g_{23} = 1; g_{ij} = 0 \text{ otherwise}\}\). This network shows that buyer 1 can only purchase goods from seller 1; buyer 3 can only purchase goods from seller 2, and buyer 2 can purchase goods from both seller 1 and seller 2. Therefore there are two monopoly markets and one duopoly market in this market structure denoted by network \(g\).

Now we suppose that the buyer \(j\)’s surplus as follows

\[
S(q) = \frac{1}{2} \left[ 1 - (1 - \sum_{i} q_{ij})^2 \right] - \sum_{i} p_{ij} q_{ij}.
\]  

(1)

By any given \(\langle p_{ij}, q_{ij} \rangle\), the buyer \(j\)’s rational choice will be

\[
\max_{q_{ij}} S(q),
\]

such that

\[
\begin{cases} 
  q_{ij} \leq \bar{q}_{ij} & \text{for } i \in \{i \mid g_{ij} = 1\}, \\
  q_{ij} = 0 & \text{otherwise.}
\end{cases}
\]

We can easily find that buyer will either purchase zero unit or one unit of goods if the price is below \(\frac{1}{2}\). Once there are two sellers selling goods to the same buyer, then the buyer will always buy the goods from the sellers who offers a lower price. Thus we know if there are more than two sellers in a market, a perfect competition will occur and the price of the goods will be zero. Since there is a cost for making a link, we then can prove two propositions below.

**Proposition 1** If the buyer’s surplus is given by 1, then every buyer in an equilibrium trade network will be connected to at most one single seller when \(nk < m\) and exactly connected to one seller when \(nk \geq m\).

**Proof.** We leave all the proofs to the Appendix. ■

**Proposition 2** If the buyer’s surplus is given by 1, then every seller in an equilibrium trade network will connect to \(k\) buyers if \(nk \leq m\).
Now let us assume that the buyer $j$’s surplus as follows

$$S(q) = \frac{1}{2} \left[ 2 - (2 - \sum_i q_{ij})^2 \right] - \sum_i p_{ij} q_{ij}. \quad (2)$$

A buyer’s willing to pay for one unit of goods is 1.5 and 2 for two units of goods. We know if there are three sellers in the market, the seller who offers the highest price will fail to sell the goods. Thus the market will turn to a perfect competition.

**Proposition 3** If the buyer’s surplus is given by 2, then every buyer in an equilibrium trade network will be connected to at most two sellers.

### 3 Perfectly Divisible Goods

In the above section, we assume that goods are indivisible. However, we will release this assumption and study how the equilibrium network structure $g$ will be affected by the capacity $k$, the connection cost $c$, the number of sellers $n$ and the number of buyers $m$. First we will give a case below to show how those parameters affect the equilibrium outcomes.

Consider a network formation game with two sellers and three buyers. The possible network structures are shown in the figures 1 to 6.

![Network Structures](image)

In the above figures, each black spot represents a seller and each white spot represents a buyer. Fig 1 shows that no sellers are linked to a buyer, thus no exchange event will occur. Fig 2 shows that two buyers can purchase goods separately in a monopoly market and one buyer can not find a seller to make a purchase.

An equilibrium strategy for a seller is that she can not be benefited from adding a new link or terminating an existed link. An equilibrium network is resulted by sellers’ equilibrium strategies. In a case where we suppose $k = 1$, $n = 2$, $m = 3$, equilibrium network structures are found as following.

<table>
<thead>
<tr>
<th>Range of $c$</th>
<th>(0.25, 0.111)</th>
<th>(0.076, 0.111)</th>
<th>(0.052, 0.076)</th>
<th>(0, 0.052)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium Structure</td>
<td>Fig. 3</td>
<td>Fig. 4</td>
<td>Fig. 5</td>
<td>Fig. 6</td>
</tr>
</tbody>
</table>

This simple example already shows an asymmetric market structure may occur (Fig. 3 or Fig. 5) even when all sellers (and buyers) are homogenous.
4 Further Study

Further research on market structures could build on to the two stage network formation game introduced here. This research project will advance our understanding in the issues about markets forming and markets interacting. By extending our analysis, we could now understand how the nature of consumers’ demand and suppliers’ capacity affect market structure. We can also add intermediaries into our model. As we know, trades are often occurred in financial markets where much of trade between sellers and buyers is intermediated by a variety of brokers or market makers. Forming links between sellers and buyers becomes a bigger issue for those intermediaries. However, a bigger trade profit has often been made by such networking behavior.

References


