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Herd Behavior, Bubbles and Social Interactions in Financial Markets

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Abstract

This paper studies herd behavior, bubbles and social interactions in financial markets through the asset pricing models with heterogeneous interacting agents. The relationship between social interactions, herd behavior and bubbles is examined. It is found that herd behavior arises naturally when there are strong enough social interactions among individual investors. In addition, an extremely small bubble may cause a sufficiently large number of traders to engage in herd behavior when the social interactions among traders are strong.

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1 Introduction

There is a strong evidence of herd behavior among traders in the financial markets indicating that agents are influenced in their decision-making process in the markets by what others around them are doing. For example, Hong, Kubik and Stein (2004) cite empirical evidence by using Health and Retirement Study data that show that social interactions help to increase the participation rate of the stock market. Scharfstein and Stein (1990) also point out that reputation concerns and "share the blame" effects are possible reasons that drive professional money managers to follow the herd.

Moreover, the empirical evidence of the existence of social interactions among investors in the financial market can be found in the survey in Shiller and Pound (1989). The survey evidence in Shiller and Pound (1989) indicates that since investors lack any clear sense of objective evidence regarding the prices of speculative assets, the process by which their opinions are derived may be especially social. In particular, investors form their decisions based on the decisions of their colleagues.

Herding behavior has also been demonstrated in the trading decisions of institutional investors and in recommendation decisions of stock analysts in Welch (2000). It is found in Welch (2000) that the tendency of analysts to follow the prevailing consensus is not stronger when that consensus proves to be correct than when it is wrong. Thus, the consensus on herding is consistent with models in which analysts herd based on little information.

The asset pricing models with heterogeneous interacting agents have recently been proposed to deal with these important social and economic phenomena, for example, Brock and Hommes (1997), Lux (1998), Chang (2007) and Alfarano and Milakovic (2009). The agent-based model is typically implemented to model interacting agents with heterogeneous beliefs. Traders with heterogeneous beliefs, that is, groups of investors with different expectations about future prices, are also examined in a structural asset pricing model by Friedman (1953); DeLong, Shleifer, Summers and Waldmann (1990); Brock, Lakonishok and LeBaron (1992); Hommes (2001, 2006); LeBaron (2006); and Brock and Hommes (1997, 1998). An important question raised by these papers is whether the "irrational" traders can survive in the financial market when trading alongside the "rational" traders. Brock, Lakonishok and LeBaron (1992) have shown empirically that simple technical trading strategies applied to the Dow Jones index may outperform several efficient market hypothesis stochastic finance models such as the random walk model or a GARCH-model. Moreover, DeLong, Shleifer, Summers and Waldmann (1990) have shown that a constant fraction of "noise traders" may on average earn a higher expected return than "rational" traders in a finite horizon financial market, and may survive in the market with positive probability.

Vaglica, Lillo, Moro and Mantegna (2008) also show that the heterogeneous agents are key ingredients for the emergence of some aggregate properties which characterize the financial market. Moreover, Gerasymchuk, Panchenko and Pavlov (2010) consider the performance of the strategies to be available only locally through some local networks due to the different speeds of transformation of the information. Four types of networks, including a fully connected network, a regular lattice, a small world, and a random network, are presented in their paper to investigate the asset price dynamics with local interactions under heterogeneous beliefs. Gerasymchuk, Panchenko and Pavlov (2010) demonstrate that, due to a different speed of information transmission in different networks, the network structure influences asset price dynamics in terms of the region of stability and volatility. In Gerasymchuk, Panchenko and Pavlov (2010), there is no direct imitation among traders and coordination arises when traders adopt the same rule dynamically. By allowing uninformed noise traders to directly imitate each other, Tedeschi, Iori and Gallegati (2012) show on the other hand, that the notion that noise traders quickly go bankrupt and are eliminated from the market is unrealistic in the presence of herding and positive feedback. Tedeschi, Iori and Gallegati (2012) also show that chartists and fundamentalists under-perform noise traders when imitation is high.

Brock and Hommes (1998) introduce a simple asset pricing model with heterogeneous beliefs based on the model presented by Lucas (1978). The Adaptive Belief System (ABS) presented in Brock and Hommes (1998) is able to create extremely rich asset price dynamics under the hypothesis of heterogeneous expectations among traders. On the other hand, the interacting agents model in financial markets has also been discussed in some recent papers in order to explain the herd behavior in the financial market and to capture the complexity of asset price dynamics. For example, Kaizoji (2000), Leombruni, Palestrini, and Gallegati (2003), and Chiarella, Gallegati, Leombruni, and Palestrini (2003). Moreover, Chang (2007) investigates the dynamics of a simple present value asset pricing model with both social interactions and heterogeneous beliefs. Investors revise their beliefs in each period according to two features: the "fitness measure",[1] which is the past realized profit, and the exogenous "social interactions measure", which is the interaction among traders' expectations of the mean choice level in the economy. Chang (2007) found that the endogenous type of social interactions arises naturally when there exist both exogenous social interactions and heterogeneous beliefs among traders. Furthermore, it is shown in Chang (2007) that the characteristics of

[1]See also Blume and Easley (1992, 1993).

the steady state and the dynamic behavior of asset prices are determined by the strength of the endogenous type of social interactions.

The main purpose of this paper is to examine the resulting price dynamics, the existence of herd behavior, and the existence of bubbles in the asset pricing model with social interactions and heterogeneous beliefs. Thus, this paper utilizes the well-known framework of Brock and Hommes (1998) and the modification by Chang (2007) to examine the relationship between social interactions, herd behavior and bubbles in a stylized financial market with heterogeneous agent beliefs. It is found that herd behavior among traders occurs naturally when the exogenously given social interactions are strong enough. Furthermore, an extremely small deviation from the fundamental asset value, i.e. a "bubble", can cause a sufficiently large number of traders to engage in herd behavior when the exogenously given social interactions among traders are strong. Therefore, under the framework of Brock and Hommes (1998), this paper demonstrates the relationship between social interactions, herd behavior and bubbles in a stylized financial market resulting in heterogeneous agent beliefs analytically.

The strategy for this paper is as follows. An asset pricing model with social interactions and heterogeneous beliefs, which retains the basic features of the model by Brock and Durlauf (2001a, 2001b), Brock and Hommes (1998) and Chang (2007), is developed in Section 2. Section 2 also contains a description of the model's steady states along with some local stability results. The main results of the paper are presented in Section 3. In particular, Section 3 contains a detailed discussion of the relationships between social interactions, herd behavior, and bubbles. Section 4 concludes the paper.

2 Basic Asset Pricing Model with Social Interactions and Heterogeneous Beliefs

By following Brock and Hommes (1998) and Chang (2007), the basic asset pricing model with social interactions and heterogeneous beliefs is developed in this section. It is assumed that there are two belief types emerging in the financial market when the asset price deviates from its fundamental value. One is referred to as noise traders who believe that the trend will continue, and the other group is referred to as arbitrageurs who believe that the asset price will eventually be driven back to its fundamental values.

Following Brock and Hommes (1998), the Adaptive Belief System is developed as follows. There are two assets, one a risky asset and the other a risk-free asset. The risk-free asset is perfectly elastically supplied at gross interest rate R, where $R > 1$. Therefore, the dynamics of wealth is given by

$$W_{t+1} = RW_t + (p_{t+1} + d_{t+1} - Rp_t)z_{ht} \tag{2.1}$$

where W_t is the total wealth in period t, $\{d_t\}$ is a stochastic dividend process of the risky asset, p_t is the price (ex dividend) per share of the risky asset at time t, and z_{ht} denotes the number of shares of the risky asset purchased at date t for type h investors.

Let E_t and V_t be the conditional expectation and conditional variance based on a publicly available information set containing current and past prices and dividends. In other words, they are the conditional expectation and conditional variance based on the information set containing $\{p_t, p_{t-1}, \cdots ; d_t, d_{t-1}, \cdots\}$. Moreover, let E_{ht} and V_{ht} be the "belief" of an investor of type h regarding the conditional expectation and conditional variance, respectively.

Therefore, assuming that $V_{ht}(p_{t+1} + d_{t+1} - Rp_t) \equiv \sigma^2$ for simplicity, beliefs with regard to the conditional variance of excess returns are the same for all investors and are also constant over time. This assumption is made for tractability following Brock and Hommes (1998).

Each type of investor is assumed to be a myopic mean variance maximizer. That is, the investor of type h solves

$$Max_{z_{ht}}\{E_{ht}W_{t+1} - (a/2)V_{ht}(W_{t+1})\} \tag{2.2}$$

where a is the measure of risk aversion, which is assumed to be equal for all traders.[2]

The demand for shares z_{ht} for a type h investor is given by

$$z_{ht} = E_{ht}(p_{t+1} + d_{t+1} - Rp_t)/a\sigma^2 \tag{2.3}$$

Moreover, let z_{st} represent the supply of shares per investor and n_{ht} be the fraction of investors of type h at time t. Therefore, equating demand with

[2]It is possible that rational arbitrageurs are risk-averse, and noise traders are risk-loving. Therefore, as pointed out by one of the referees, the assumption of homogeneous risk-aversion is unreasonable in the current paper. By relaxing the assumptions of homogeneous degrees of risk-aversion and homogeneous expected conditional variance, Chiarella and He (2002) test the robustness of the results of Brock and Hommes (1998). They also investigate the effects of different memory lengths on the dynamics. Chiarella and He (2002) found that Brock and Hommes's (1998) results are robust to this generalization. However, they do point out that the resulting dynamic behavior with this generalization is considerably enriched and exhibits some significant differences. Thus, how to incorporate the heterogeneous beliefs and risk in a simple asset pricing model with social interactions in order to investigate the resulting herd behavior and bubbles in financial markets will be an important extension of the current paper.

supply gives

$$z_{st} = \sum_h n_{ht}\{E_{ht}(p_{t+1} + d_{t+1} - Rp_t)/a\sigma^2\} \tag{2.4}$$

Assuming there is a zero supply of outside shares for simplicity, ($z_{st} = 0$) for all t. Thus,

$$Rp_t = \sum_h n_{ht}E_{ht}(p_{t+1} + d_{t+1}) \tag{2.5}$$

Therefore, the fundamental solution $p_t^\star$ can be defined as the benchmark of the rational expectations solution as follows:

$$Rp_t^\star = E_t(p_{t+1}^\star + d_{t+1}) \tag{2.6}$$

Let x_t be the deviation in the asset price from the fundamental value, that is,

$$x_t = p_t - p_t^\star$$

Moreover, the traders' beliefs with regard to the conditional expectation are assumed to take the following form:

$$E_{ht}(p_{t+1} + d_{t+1}) = E_t(p_{t+1}^\star + d_{t+1}) + f_{h,t} \tag{2.7}$$

where $f_{h,t}$ is the belief of a type h investor that is conditional on current and past prices and dividends. That is, an investor's belief is some deterministic function of current and past deviations from the fundamental value.

Let $n_{h,t}$ and $f_{h,t}$ represent the fraction of type h investors and their belief type at the beginning of period t before x_t has been observed. Rearranging (2.5) we have

$$Rx_t = \sum_h n_{h,t}f_{h,t} \tag{2.8}$$

Notice that $n_{h,t}$ represents the fraction of type h investors at the beginning of period t before x_t has been observed. Thus x_t is determined by $n_{h,t}$ and $f_{h,t}$ at the beginning of period t. After x_t has been observed, the profit for period t, $\pi_{h,t}$, is also determined and so realized profits for period t for both types can be calculated by investors using all relevant information. Traders make their belief-type choice based on the performance measure, and in the process examine the choices of other people. Therefore, the new fraction of

$n_{h,t+1}$ is based upon all traders' choices of beliefs. Next, x_{t+1} is determined by $n_{h,t+1}$ and $f_{h,t+1}$ at the beginning of period $t+1$, and so on.

In order to calculate the realized profit for trader h as the "fitness function", the realized excess return (R_{t+1}) from period t to period $t+1$ must be obtained. It is given by

$$R_{t+1} = x_{t+1} - Rx_t + \delta_{t+1} \tag{2.9}$$

where $\delta_{t+1} = p^\star_{t+1} + d_{t+1} - E_t(p^\star_{t+1} + d_{t+1})$ and $E_t(\delta_{t+1}) = 0$ for all t. Therefore, the realized profit for a type h investor that is conditional upon the time t information set can be represented as

$$\pi_{ht} = R_t z_{h,t-1} = (x_t - Rx_{t-1} + \delta_t)\frac{(f_{h,t-1} - Rx_{t-1})}{a\sigma^2} \tag{2.10}$$

The belief of a noise trader is assumed to be of the simple linear form

$$f_t \quad = \quad kx_{t-1} + b \quad k \geq 0 \quad b \geq 0$$

where k is the trend and b is the bias of the trader type. So from (2.7) the belief type of noise traders is given by

$$E_{it}(p_{t+1} + d_{t+1}) \quad = \quad E_t(p^\star_{t+1} + d_{t+1}) + kx_{t-1} + b$$

The linear form of the traders' belief is regarded as the idealization of overreacting securities analysts or overreacting investors. This type of belief setting can also be found in De Bondt and Thaler (1985).[3]

The belief of arbitrageurs will also be of the simple linear form

$$f_t \quad = \quad -kx_{t-1} - b$$

Thus, all arbitrageurs are rational arbitrageurs and they are contrarians to noise traders since they believe that the asset price will eventually be driven back to the fundamental value. This assumption is also employed by DeLong, Shleifer, Summers and Waldmann (1990).[4] From the assumption above, the belief type of rational arbitrageurs is given by

$$E_{it}(p_{t+1} + d_{t+1}) \quad = \quad E_t(p^\star_{t+1} + d_{t+1}) - kx_{t-1} - b$$

[3]The belief type of the noise trader used in this paper is equivalent to the term "trend chaser" with a belief bias in Brock and Hommes (1998). Notice that the term "noise trader" may have a different scope than a "trend chaser" with a belief bias in Brock and Hommes (1998). See for example, DeLong, Shleifer, Summers and Waldmann (1990).

[4]The belief type of arbitrageurs used in this paper is equivalent to the term "contrarian" with a belief bias in Brock and Hommes (1998). Notice that the term "arbitrageurs" may be used differently in the finance literature. For example, in Hull (2011), "arbitrageurs" is used to denote the traders who are involved in "locking in a risk-less profit by simultaneously entering into transactions in two or more markets."

Following Durlauf (2001a, 2001b), we assume that investor i's utility at time t is given by

$$U(\omega_{it}) = \pi_{t-1}(\omega_{it}) + S(\omega_{it}, \overline{m}^e_{it}) + \epsilon(\omega_{it}) \tag{2.11}$$

where ω_{it} is a binary belief type choice with support $\{-1, 1\}$, and $\pi_{t-1}(\omega_{it})$ is the past realized profit associated with the individual's belief choice. In other words, the fitness measure associated with a belief type is past realized profit. $\overline{m}^e_{it}$ denotes the conditional probability measure that agent i places on the choices of others at the time of making his own decision. That is, $\overline{m}^e_{it} = \frac{1}{N-1}\sum_{j,j\neq i} m^e_{i,j,t}$ where $m^e_{i,j,t}$ is the subjective expectation from the perspective of agent i about the choice of agent j at time t. $S(.)$ is the social utility associated with the choices, and $\epsilon(\omega_{it})$ is a random utility term which is I.I.D. across agents and is extreme-value distributed. It is assumed that agent i knows $\epsilon(\omega_i)$ at the time of his decision. We assume that $S(\omega_{it}, \overline{m}^e_{it}) = J\omega_{it}\cdot\overline{m}^e_{it}$,[5] where J represents the degree of dependence across agents, that is, J represents the strength of exogenous social interactions.

Therefore, the objective function of investors:

$$U(\omega_{it}) = \pi_{t-1}(\omega_{it}) + J\omega_{it}\cdot\overline{m}^e_{it} + \epsilon(\omega_{it}) \tag{2.12}$$

The investor is a noise trader if he or she chooses $\omega_{it} = 1$, and a rational arbitrageur if he or she chooses $\omega_{it} = -1$. The deterministic asset pricing dynamics with $\delta_t = 0$ are assumed here for all t. Based on (2.10), the "fitness measure" associated with a trader's choice at time t is

$$\pi_{t-1}(\omega_{it} = 1) = \frac{(x_{t-1} - Rx_{t-2})(kx_{t-3} - Rx_{t-2} + b)}{a\sigma^2} \tag{2.13}$$

$$\pi_{t-1}(\omega_{it} = -1) = \frac{-(x_{t-1} - Rx_{t-2})(kx_{t-3} + Rx_{t-2} + b)}{a\sigma^2} \tag{2.14}$$

Therefore, the fraction of trader types is given by

$$n^{(+)}_t = Prob(\omega_{i,t} = 1) = \frac{\exp(\frac{\beta(kx_{t-3}+b)(x_{t-1}-Rx_{t-2})}{a\sigma^2} + \beta J\cdot m_t)}{Z} \tag{2.15}$$

$$n^{(-)}_t = Prob(\omega_{i,t} = -1) = \frac{\exp(\frac{-\beta(kx_{t-3}+b)(x_{t-1}-Rx_{t-2})}{a\sigma^2} - \beta J\cdot m_t)}{Z} \tag{2.16}$$

where β represents the intensity of choice, $n^{(+)}_t$ represents the fraction of noise traders, $n^{(-)}_t$ stands for the fraction of rational arbitrageurs, $Z = \exp(\frac{\beta(kx_{t-3}+b)(x_{t-1}-Rx_{t-2})}{a\sigma^2} +$

[5] Here, $S(.)$ represents proportional spillover-type social interactions; see Brock and Durlauf (2001a, 2001b).

$\beta J \cdot m_t) + \exp(\frac{-\beta(kx_{t-3}+b)(x_{t-1}-Rx_{t-2})}{a\sigma^2} - \beta J \cdot m_t)$.[6] The probability structure is equivalent to the so-called mean field version of the Curie-Weiss model of statistical mechanics. According to the mean field theory, each individual's expectation of the mean choice level is replaced with a common value, that is, $\overline{m}^e_{it} \equiv m_t$ is fixed $\forall i$. The expected value of each of these random variables will be equal to

$$E(\omega_{it}) = \tanh(\frac{\beta(kx_{t-3}+b)(x_{t-1}-Rx_{t-2})}{a\sigma^2} + \beta J \cdot m_t). \tag{2.17}$$

since we require that expectations be rational in a steady state, so that $E(\omega_{it}) = m_t$. Therefore, the Law of Large Numbers can be applied to the large economy behavior under non-cooperative decision-making. In other words, there exists at least one value of m_t such that

$$m_t = \tanh(\beta J m_t + \frac{\beta(kx_{t-3}+b)(x_{t-1}-Rx_{t-2})}{a\sigma^2}) \tag{2.18}$$

where m_t is a self-consistent expectation of the mean of choices across all agents in the large economy. On the other hand, from (2.8)

$$Rx_t = (kx_{t-1}+b)m_t \tag{2.19}$$

Rearranging (2.19)

$$x_t = \frac{m_t(kx_{t-1}+b)}{R} \tag{2.20}$$

The two crucial equations are (2.18) and (2.20).

Thus, from (2.18) and (2.20), the steady state of the dynamic system is

$$x^\star = \frac{m^\star b}{R - km^\star} \tag{2.21}$$

$$m^\star = \tanh(\frac{\beta(kx^\star+b)(1-R)x^\star}{a\sigma^2} + \beta J m^\star) \tag{2.22}$$

Let

$$J_b = \frac{b^2 R(1-R)}{a\sigma^2(R-km^\star)^2}$$

[6] See Brock and Durlauf (2001a, 2001b) and Anderson, Palma and Thisse (1996) for a derivation.

Then

$$m^\star = \tanh(\beta J_m m^\star) \tag{2.23}$$

where $J_m = J + J_b$.

The properties of steady states of the dynamic system can be found in the following proposition:

Proposition 1. *(Steady State)*
The steady states $(m^\star, x^\star)$ of equations (2.18) and (2.20)
(1) If $\beta J < 1$, there exists only one steady state $(0, 0)$.
(2) If $\beta J \geq 1$, there exist one or three or five steady states. Furthermore, at least one steady state will be $(0, 0)$.

Proof of Proposition 1: (See the Appendix)

Thus, the fundamental solution ($x^\star = 0$) will always be one possible steady state in the financial market, based on Propositions 1 even with strong social interactions.

The traders' profit at the steady state is discussed in the following proposition:

Proposition 2. *(Steady State Profit)*
At the steady state, all traders earn zero profit if the steady state asset price is at its fundamental value. Furthermore, rational arbitrageurs make more profit than noise traders if the steady state asset price is above its fundamental value. On the other hand, when the steady state asset price is below its fundamental value, rational arbitrageurs make more profit than noise traders if $x^\star < -\frac{b}{k}$, rational arbitrageurs earn less profit than noise traders if $x^\star > -\frac{b}{k}$, and rational arbitrageurs make the same profit as noise traders if $x^\star = -\frac{b}{k}$.

Proof of Proposition 2: (See the Appendix)

In order to investigate the dynamic stability of (2.18) and (2.20), all traders' expectations of the mean choice level at time t are assumed to be given by $\overline{m}^e_{it} \equiv m_{t-1} \quad \forall i$. In other words, we consider the dynamics of a sequence of economies in which expectations are myopic. Therefore, (2.18) and (2.20) can be rewritten as

$$m_t = \tanh(\beta J m_{t-1} + \frac{\beta(kx_{t-3} + b)(x_{t-1} - Rx_{t-2})}{a\sigma^2}) \tag{2.24}$$

$$x_t = \frac{km_{t-1}x_{t-1} + bm_{t-1}}{R} \tag{2.25}$$

Therefore, the local dynamic characteristics of the steady state can be analyzed by expanding (m_t, x_t) around the steady state value $(m^\star, x^\star)$.

Proposition 3. *(Local Dynamics)*
Assume that all arbitrageurs are rational arbitrageurs. Then, the local dynamic of the steady states $(m^\star, x^\star)$ *of (2.24) and (2.25) can be characterized as:*
(1) When $\beta J < 1$, $(0,0)$ *is a unique and locally stable steady state.*
(2) When $\beta J \geq 1$*:*
(i) If $b = 0$*, there are three steady states. One is* $(0,0)$*, a locally unstable one. The other two steady states are* $(m^\star_+, 0)$ *and* $(m^\star_-, 0)$*, where* $m^\star_+ > 0$ *and* $m^\star_- < 0$*. These two steady states are locally stable if* $|km^\star| < R$ *and locally unstable if* $|km^\star| > R$*.*
(ii) If $b \neq 0$*, there will be one or three or five steady states.* $(0,0)$ *will be one of the steady states in all cases. If there is only one steady state in the market,* $(0,0)$ *is locally stable. Otherwise,* $(0,0)$ *is locally unstable. In the three steady states case, the other two steady states are* $(m^\star_+, \frac{m^\star_+ b}{R - km^\star_+})$ *and* $(m^\star_-, \frac{m^\star_- b}{R - km^\star_-})$*, where* $m^\star_+ > 0$ *and* $m^\star_- < 0$*. These two steady states are locally stable if* $|\frac{\partial \tanh(\beta J_m m)}{\partial m}|$ *evaluated at* $m^\star$ *is less than* 1 *and* $|km^\star| < R$*. Otherwise, they are locally unstable steady states. In the five steady states case, in addition to the three steady states described above, the other two steady states are* $(m^\star_1, \frac{m^\star_1 b}{R - km^\star_1})$ *and* $(m^\star_2, \frac{m^\star_2 b}{R - km^\star_2})$*, where* $m^\star_1 > 0$ *and* $m^\star_2 > 0$*. These two steady states are locally stable if* $|\frac{\partial \tanh(\beta J_m m)}{\partial m}|$ *evaluated at* $m^\star$ *is less than* 1 *and* $|km^\star| < R$*. Otherwise, they are locally unstable steady states.*

Proof of Proposition 3 (See the Appendix)

According to Proposition 3, when the noise traders extrapolate only weakly ($k < R$) and there is no belief bias ($b = 0$), $(m^\star_+, 0)$ and $(m^\star_-, 0)$ will be locally stable steady states under strong exogenous social interactions.[7] Moreover, when the noise traders extrapolate only weakly ($k < R$) and traders have belief bias ($b \neq 0$), there is a positive probability that a unique locally stable steady state $(0,0)$ will exist in the market, even under strong social interactions.

3 Herd Behavior, Bubbles and Social Interactions

Based on the theoretical model developed in the previous section, a comparison between the current model and the existing literature with rational arbitrageurs and noise traders is presented in this section. Moreover, the

[7]Notice that the sufficient condition for $|km^\star| < R$ is $k < R$.

connections among social interactions, herd behavior and price bubbles are explored.

According to Proposition 1, at least one steady state asset price is the asset's fundamental value when all arbitrageurs are rational arbitrageurs. Note that this result is similar to the argument given by Friedman (1953).[8] Friedman's argument consists of two parts. First, rational arbitrageurs will push asset prices towards the fundamental values and, second, irrational traders will eventually be driven from the market. The first part of his argument coincides with the present paper. However, by emphasizing the interactions among investors, the second part of Friedman's argument is less likely to be true in the current model. Recalling the previous section, noise traders can coexist with rational arbitrageurs under weak social interactions ($J < 1/\beta$) at the steady state when all traders' expectations are myopic.[9] With strong social interactions ($J > 1/\beta$), noise traders are possibly driven out by rational arbitrageurs. However, depending on the initial conditions within the economy, rational arbitrageurs can also be driven out of the market by noise traders at the steady state.

By examining the common belief that the market selects for rational investors, Blume and Easley (1992, 1993) find that the link between rationality and fitness is weak. That is, not every rational rule survives and not every irrational rule vanishes relative to any rational rule.[10] The finding in Blume and Easley (1992, 1993) is also supported in the current model. By using past realized profit as the fitness measure, rational arbitrageurs are driven from the market with strong enough social interactions under some specific initial conditions, while noise traders survive in the market even though asset prices are always consistent with rationality.

Based on Proposition 3, regardless of the strength of social interactions, one possible asset price steady state is the asset's fundamental value when all arbitrageurs are rational arbitrageurs. Thus, the "fundamentalist" equilibrium price equation will be reestablished at the steady state if there are investors with rational beliefs in the economy. However, rational arbitrageurs do not always survive at the steady state. Moreover, with weak social interactions ($J < 1/\beta$), it is impossible to find any rational rule which beats all irrational rules since $m^\star = 0$ is the unique equilibrium at the steady state and also both

[8]Friedman (1953, p. 175) argues, "People who argue that speculation is generally destabilizing seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on average sell when currency is low and buy when it is high."

[9]That is, $\overline{m}_{it}^e \equiv m_{t-1} \quad \forall i$

[10]Blume and Easley (1992, 1993) find that the most fit behavior in the financial market is that which maximizes the expected growth rate of wealth share accumulation.

types of investors survive in the long run. However, due to nonlinearity, there is no obvious answer as to whether every irrational rule vanishes relative to some rational rule under strong social interactions ($J > 1/\beta$).

In order to investigate the relationships among social interactions, herd behavior and bubbles, the following definition is offered:

Definition 1. *(Herd Behavior)*
A trader i with belief choice ω_i engages in herd behavior at steady state $(m^\star, x^\star)$ if
(1) ω_i has the same sign as $m^\star$ at the steady state.
(2) $m^\star \neq 0$.

Therefore, a trader that engages in herd behavior has two characteristics. First, his or her belief choice has the same sign as the mean choice level in the market at the steady state. In addition, the steady state mean choice level does not equal zero.

The definition of herd behavior here is somewhat different from the one in Leombruni, Palestrini, and Gallegati (2003), and Chiarella, Gallegati, Leombruni, and Palestrini (2003). There are two major components in the agent's utility function in the current paper, one being the past realized profit as the fitness measure associated with the agent's belief type and the other the social utility associated with the choice of belief type. Thus, although there exist social interactions among all traders, the herd behavior arises only when the agent's belief choice is the same as the belief type of more than 50% of the traders in the market. By contrast, in Leombruni, Palestrini, and Gallegati (2003), and Chiarella, Gallegati, Leombruni, and Palestrini (2003), herd behavior is one of the agents' belief type choices in an economy that is against fundamentalism.

However, the findings in Proposition 2 are consistent with the results in Leombruni, Palestrini, and Gallegati (2003), and Chiarella, Gallegati, Leombruni, and Palestrini (2003), where they show that herd behavior can be a rational strategy since it allows a herd agent to gain excess returns on an asset by exploiting information not contained in the fundamental solution. Based on Proposition 2, rational arbitrageurs will make more profit than noise traders if the steady state asset price is above its fundamental value, and rational arbitrageurs will make more or less profit than noise traders if the steady state asset price is below its fundamental value, depending on the parameter values. Thus, by introducing social interactions into the models, both types of traders can gain excess returns on a risky asset, with some positive probability, based on their expectation schemes.

With this definition we obtain the following observation:

Proposition 4. *(Herd Behavior and Social Interactions)*
Assume that
(1) All traders' expectations of the mean choice level at time t are $m_t^e = m_{t-1}$.
(2) All arbitrageurs are rational arbitrageurs.
(3) The initial state is at the steady state $(m^\star, x^\star) = (0, 0)$.
(4) $k < R$.
Then, assuming an extremely small expectation deviation from the steady state $(0, 0)$,
1. Without exogenous social interactions ($J = 0$), herd behavior occurs at the new steady state with probability 0.
2. With weak exogenous social interactions ($\beta J < 1$), herd behavior occurs at the new steady state with probability 0.
3. With strong exogenous social interactions ($\beta J > 1$), herd behavior occurs at the new steady state with some positive probability. Furthermore, the scale of herd behavior depends on the strength of exogenous social interactions, and the direction of herd behavior depends on the direction of the deviation.

Proof of Proposition 4: (See the Appendix)

Based on Proposition 4, the existence of strong exogenous social interactions in the market is a necessary condition for the existence of herd behavior at the new steady state. However, it is not a sufficient condition to generate herd behavior at the new steady state in the financial market.

The bubble is defined as the deviation from the fundamental solution $x^\star = 0$. The relationship between bubbles and social interactions is presented in the next proposition:

Proposition 5. *(Bubbles and Social Interactions)*
Assume that
(1) All traders' expectations of the mean choice level at time t are $m_t^e = m_{t-1}$,
(2) All arbitrageurs are rational arbitrageurs,
(3) The initial state is at the steady state $(m^\star, x^\star) = (0, 0)$,
(4) $k < R$ and $b \neq 0$.
Then, assuming an extremely small bubble occurs at the steady state $(m^\star, x^\star)$,
1. Without exogenous social interactions ($J = 0$), the bubble crashes at the new steady state with probability 1.
2. With weak exogenous social interactions ($\beta J < 1$), the bubble crashes at the new steady state with probability 1.
3. With strong exogenous social interactions ($\beta J > 1$), the bubble crashes at the new steady state when there is a unique steady state. Otherwise, the bubble will stay in the economy, and traders may engage in herd behavior at the new steady state.

Proof of Proposition 5: (See the Appendix).

Based on Proposition 4 and Proposition 5, the relationship between bubbles, herd behavior and social interactions is described in the following observation: when there exist strong exogenous social interactions ($\beta J > 1$) in the economy, an extremely small bubble occurring at the initial steady state will, with some positive probability, result in herd behavior among investors and a price bubble in the economy at the new steady state.

Thus, a bubble will crash in finite times with probability 1 when exogenous social interactions are weak and the noise traders extrapolate weakly, namely, $k < R$. However, when there exists belief bias among traders (i.e., $b \neq 0$) and exogenous social interactions are strong ($\beta J > 1$) in the economy, even the noise traders will extrapolate only weakly, and a bubble may stay at the new steady state and traders will engage in herd behavior with some positive probability.

4 Concluding Remarks

The model presented in this paper explains the social interactions of investors and provides a method for examining closely-related phenomena in financial markets such as herd behavior and bubbles. In particular, it is found that herd behavior can arise naturally when the strength of exogenous social interactions is sufficiently great. Furthermore, an extremely small bubble may cause a sufficiently large number of traders to engage in herd behavior when social interactions among investors are strong.

The most important extension to this paper would be to extract the exact strength of social interactions from economic data. In other words, it would be useful to estimate J from financial data exactly. In addition, extending the belief type selection from a binary choice to a multiple choice framework and relaxing the assumptions of homogeneous degrees of risk aversion and homogeneous expected conditional variance would also constitute an important direction for this paper in the future.

Appendix

Proof of Proposition 1.

Proof. (1) When $\beta J < 1$, since $bR > 0$ and $R > 1$, then $J_b < 0$. Since $\beta J < 1$, $\beta J_m < 1$. Then there exists a unique steady state $(0, 0)$.
(2) When $\beta J \geq 1$, since $bR > 0$ and $R > 1$, then $J_b < 0$, and there exists

a threshold H_R, (which depends on k): (a) If $\beta J_m < 1$, there exists only one steady state $(0,0)$. (b) If $\beta J_m > 1$ and $R > H_R$, there exist three steady states. One of these roots is $(0,0)$, one root is $(m_+, \frac{m_+ b}{R})$, and one root is $(m_-, \frac{m_- b}{R - k m_-})$ where $m_+ > 0$ and $m_- < 0$. (c) If $\beta J_m > 1$ and $R < H_R$, there exist five steady states. One of these roots is $(0,0)$, three roots are $(m_+^1, \frac{m_+^1 b}{R - k m_+^1})$, $(m_+^2, \frac{m_+^2 b}{R - k m_+^2})$, and $(m_+^3, \frac{m_+^3 b}{R - k m_+^3})$, and one root is $(m_-, \frac{m_- b}{R - k m_-})$, where $m_+^1, m_+^2, m_+^3 > 0$ and $m_- < 0$. □

Proof of Proposition 2.

Proof. The profit of each type of trader at the steady state is

$$\begin{aligned} \pi^\star(\omega_{it} = 1) &= \frac{x^\star(1-R)((k-R)x^\star + b)}{a\sigma^2} \\ \pi^\star(\omega_{it} = -1) &= \frac{-x^\star(1-R)((k+R)x^\star + b)}{a\sigma^2} \end{aligned}$$

Thus, $\pi^\star(\omega_{it} = 1) = \pi^\star(\omega_{it} = -1) = 0$ when $x^\star = 0$. Furthermore,

$$\pi^\star(\omega_{it} = 1) - \pi^\star(\omega_{it} = -1) = \frac{x^\star(1-R)(2kx^\star + 2b)}{a\sigma^2}$$

Then, when $x^\star > 0$, $kx^\star + b > 0$ since $k, b \geq 0$. Thus $\pi^\star(\omega_{it} = 1) - \pi^\star(\omega_{it} = -1) < 0$ since $R > 1$. Therefore, rational arbitrageurs ($\omega_{it} = -1$) make more profit than noise traders ($\omega_{it} = 1$) at the steady state if the steady state asset price is above its fundamental value (i.e., $x^\star > 0$).
On the other hand, when the steady state asset price is below its fundamental value, that is, $x^\star < 0$, then $kx^\star + b > 0$ if $x^\star > -\frac{b}{k}$, $kx^\star + b < 0$ if $x^\star < -\frac{b}{k}$, and $kx^\star + b = 0$ if $x^\star = -\frac{b}{k}$. Thus, rational arbitrageurs make more profit than noise traders if $x^\star < -\frac{b}{k}$, rational arbitrageurs earn less profit than noise traders if $x^\star > -\frac{b}{k}$, and rational arbitrageurs make the same profit as noise traders if $x^\star = -\frac{b}{k}$. □

Proof of Proposition 3.

Proof. Since all arbitrageurs are rational arbitrageurs, all local dynamic characteristics are discussed around steady states:
(1) When $\beta J < 1$, $(m^\star, x^\star) = (0,0)$ is a unique steady state based on Proposition 1. Also $\tanh'(0) = 1$. Therefore, $\beta J_m \tanh'(0) < 1$ since $\beta J < 1$ and $J_b \leq 0$. Thus $(m^\star, x^\star) = (0,0)$ is a unique locally stable steady state.
(2) When $\beta J \geq 1$: (i) When $b = 0$: according to Proposition 3, there are 3 steady states (the same situations as $bR = 0$) and $J_b = 0$. Since $\tanh'(0) = 1$,

therefore $\beta J_m \tanh'(0) = \beta J \tanh'(0) > 1$ and so $(m^\star, x^\star) = (0,0)$ is locally unstable. On the other hand, the other two steady states will be locally stable if $|km^\star| < R$ and locally unstable steady states if $|km^\star| > R$, since $\beta J \tanh'(\beta J m^\star) < 1$ for the other two steady states. See the proof arguments in Brock and Durlauf (2001b). (ii) When $b \neq 0$: since $\frac{\partial \tanh(\beta J_m m)}{\partial m} = (\beta J_m + \beta m J'_m) \tanh'(\beta m J_m)$, where $J'_m = \frac{\partial J_m}{\partial m}$. According to Proposition 3, $(m^\star, x^\star) = (0,0)$ is a unique steady state if $\beta J_m < 1$. Furthermore, $(0,0)$ is locally stable when $(0,0)$ is a unique steady state since $\beta J_m \tanh'(0) < 1$. On the other hand, if there is more than one steady state in the economy, then $\beta J_m > 1$ and $(0,0)$ is a locally unstable steady state since $\beta J_m \tanh'(0) \geq 1$. The other steady states will be locally stable if $|\frac{\partial \tanh(\beta J_m m)}{\partial m}|$ evaluated at $m^\star$ is less than 1 and $|km^\star| < R$. Otherwise, they will be locally unstable steady states. $\square$

Proof of Proposition 4.

Proof. Based on Proposition 1, there is a unique steady state $(m^\star, x^\star) = (0,0)$ if $\beta J < 1$. Moreover, $(0,0)$ is also a locally stable steady state. Therefore, according to the definition of herd behavior, it is impossible to have herd behavior at the new steady state since $m^\star = 0$. On the other hand, based on Proposition 3, there exist one or three or five steady states with strong exogenous social interactions ($\beta J > 1$). Furthermore, $(m^\star, x^\star) = (0,0)$ is a locally unstable steady state if there is more than one steady state in the economy, according to Proposition 3. $|km^\star| < R$ since $k < R$, and thus the other two or four steady states are the locally stable ones if $|\frac{\partial \tanh(\beta J_m m)}{\partial m}|$ evaluated at $m^\star$ is less than 1. Therefore, at least one new steady state will be $m^\star$ which is different from zero when there is more than one steady state in the economy. In other words, herd behavior occurs at the new steady state with some positive probability. Furthermore, with β fixed, $|m^\star| \longrightarrow 1$ if $J \longrightarrow \infty$ and $|m^\star| \longrightarrow 0$ if $J \longrightarrow \frac{1}{\beta}$. Therefore, the scale of herd behavior depends on the strength of the social interactions. Moreover, the sign of the new steady state value $m^\star$ is also determined by the sign of the expectation deviation. $\square$

Proof of Proposition 5.

Proof. (1) According to Proposition 1, $(0,0)$ is the unique stable steady state if $\beta J < 1$. Therefore, the bubble crashes in finite time and goes back to its fundamental value. (2) When $\beta J > 1$, $(0,0)$ will be a locally unstable steady state with some positive probability, according to the proof of Proposition 1. Then if there is an extremely small bubble at the initial locally unstable steady state $(0,0)$, it will cause an extremely small expectation deviation from the steady state $(0, 0)$. Thus, the new steady state will be $(m^\star, \frac{m^\star b}{R - km^\star})$ with some positive probability where $m^\star \neq 0$, according to Propositions 3 and 4. In other words, with some positive probability, the traders will engage in herd behavior and the price bubble will stay at the new steady state when the exogenous social interactions are strong. □

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