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Resource rivalry and endogenous lobby

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Abstract

We present a two-sector model to depict the determination of trade preference. The model highlights lobby as a rivalry between sectors in competition for resources where the outcome of the lobby race is determined by each sector's ability to generate rent at a given welfare cost to the general population. We investigate the relation between the structure of trade protection and the resource endowment.

JEL Classification: F13; D72

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1. Introduction

It is increasingly recognized that government interventions in trade are as certain as government impositions of tax. There is also increasing interest among economists to offer an interpretation for this almost universal phenomenon. This can be seen in the voluminous literature known as the political economy of trade (for a survey, see Helpman, 1995).[1] One of the most brilliant models is offered by Grossman and Helpman (1994), which depicts the formation of trade policy as the result of an interaction

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[1] Besides the research cited by Helpman, more traditional literature includes Tullock (1967); Bhagwati (1968, 1980, 1982); Becker (1983, 1985).

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between the government and interest groups. The government is concerned with the welfare of the general population as well as political contributions that can be obtained from interest groups to finance election campaigns to keep itself in power. Interest groups, knowing the government's need for political support, make contributions to influence the choice of trade policies. In short, it is the government's need for political support that makes trade intervention inevitable. The government offers 'trade protection for sale' to raise funds to keep itself in power. Trade protection is almost a necessary evil in any democracy.

If that is the case, then it is interesting to ask whether income levels will make a difference to the level and the structure of trade protection. Will a country's trade regime come closer to free trade as its per-capita income rises? If so, then at least we may expect all countries ultimately to convert to free trade if their incomes are high enough. We should therefore be patient and understand developing countries' resistance to trade liberalization programs such as those championed by World Trade Organization. It is also interesting to note that some countries give preference to the export sector, while others favor the import-competing sector and that the preference may also change over time. For example, the focus of US trade policy was to protect weak sectors (importing) before the 1980s, and the focus was shifted to assisting strong sectors (exporting) in securing access to foreign markets. How is this related to the structure of the economy?

The purpose of this paper is to offer some answers to these questions. We present a two-sector model in line with Grossman and Helpman's concept of making political contributions for policy favors.[2] The model highlights the rivalry between sectors in competition for policy favors in a game-theoretic setting. Compared to the Grossman and Helpman (1994) paper in which a multi-sector model is adopted to describe the nature of equilibrium, the simplicity of the two-sector model offers several advantages. It allows us to explicitly identify the degree of political contribution as the one that maximizes the lobbyist's net income while keeping the rival lobby at bay. It allows us to examine the relations between the resource endowment and the structure of protection. An industry may be forced to lobby even if it knows that free trade is a superior outcome. In fact, the rivalry between sectors makes free trade an infeasible political choice. Whether a government adopts a policy in favor of its export sector or import-competing sector, depends, among other things, on factor intensity and the size of the respective sectors. There is no guarantee that rising incomes will turn a protectionist into a free-trade believer. Growth can make a country deviate away from, or move closer to free trade. In particular, balanced growth, in the sense that both sectors grow at the same rate, will not change the structure of protection, whereas unbalanced growth will.

The rest of the paper is organized as follows. The basic model is set up in Section 2. Section 3 characterizes the equilibria of our lobbying game, and compares them with the equilibria in a menu auction which was originally studied by Grossman and Helpman

[2] A similar framework is also applied in some other research. See Cadot et al. (1997); Mitra (1999); Olarreaga (1999); Goldberg and Maggi (1999).

(1994). Then comparative statistics is analyzed in Section 4. The last section concludes with our discussion.

2. The model

We consider a small open economy with two goods and three factors. The population of L people provide inelastic labor and among them, two capitalists own specific factors to produce goods 1 and 2, respectively. We use K_i to denote the quantity of the specific factor for good i.

2.1. Consumption

Consumers are assumed to have the Leontief utility function:[3]

$$u(x_1, x_2) = \min\{x_1, x_2\},$$

where x_i denotes the consumption of good i. Let good 1 be the numeraire good, p the relative price of good 2 in the domestic market, and I the combined disposable income of all consumers. The aggregate demands for these two goods are:

$$x_1 = x_2 = \frac{I}{1+p}. \tag{1}$$

Let y_i denote the domestic output of good i. The disposable income is:

$$I = y_1 + py_2 - T(p), \tag{2}$$

where $T(p)$ is the tax raised (or subsidy paid) to maintain trade protection. When the domestic price p is set different from the international price p^*, the government has a tax revenue (or a subsidy payment) of:

$$T(p) = -(p - p^*)(x_2 - y_2). \tag{3}$$

When $p > (<)\, p^*$, there is an import tariff (subsidy) for good 2 if $x_2 > y_2$; and an export subsidy (tax) for good 2 if $x_2 < y_2$. (1)–(3) imply:

$$x_1 = x_2 = \frac{y_1 + p^* y_2}{1 + p^*}. \tag{4}$$

To simplify the analysis, in the following, we shall assume the tax rate τ to be invariant with income:

$$\tau = \frac{T(p)}{(y_1 + py_2)}. \tag{5}$$

2.2. Production

Each good is assumed to be monopolized by the specific factor owner, whose production function is:

$$y_i = K_i L_i^{b_i}, \quad b_i \in (0,1), i = 1, 2, \tag{6}$$

[3] Hence, their demand functions and indirect utility functions could be aggregated.

where L_i denotes the labor input. Though the production technology appears to be increasing returns to scale, this paper does not exploit this property, since K_i is exogenously given in our model. Facing the domestic product price p and wage rate w, capitalist i, who has a Leontief preference, chooses the labor employment to maximize his net real income (in terms of a pair of goods 1 and 2):

$$\pi_i(p) = \begin{cases} \max\limits_{L_1} \dfrac{(1-\tau)}{1+p}[K_1 L_1^{b_1} - wL_1] & \text{for } i = 1, \\ \max\limits_{L_2} \dfrac{(1-\tau)}{1+p}[pK_2 L_2^{b_2} - wL_2] & \text{for } i = 2. \end{cases}$$

In the labor market, the equilibrium wage equates the value of the marginal product of labor in two sectors[4]:

$$w = b_1 K_1 L_1^{b_1 - 1} = b_2 p K_2 L_2^{b_2 - 1}; \tag{7}$$

and the market clear condition for labor is:

$$L = L_1 + L_2. \tag{8}$$

2.3. The government

For any domestic price p set by the government, the equilibrium wage rate and labor allocation are determined by (7) and (8). The output of each industry $y_i(p)$ and the capitalist's real income $\pi_i(p)$ are then in turn determined. Because the capitalist's well-being hinges upon domestic product price p, each considers tendering offer $C_i (i = 1, 2)$ to lobby for a favorable price. To facilitate future analysis, the unit of C_i is set to be a pair of two goods. Following Grossman and Helpman (G&H), we assume the government to be concerned about both the political contributions from lobbyists and social welfare. The latter concern becomes important in view of future elections. From (4), summing up individuals' utilities, the social welfare V is:[5]

$$V(p; p^*) = \frac{y_1(p) + p^* y_2(p)}{1 + p^*}. \tag{9}$$

Let a denote the weight the government gives to social welfare. Taking the weight given to political contributions as unity, the government's utility is:

$$G = \max\,[C_1 + aV(p_1; p^*), C_2 + aV(p_2; p^*), aV(p^*; p^*)]; \quad a > 0.$$

The lobby will be formulated as a two-stage game. At the first stage, two capitalists simultaneously propose a package of a domestic price and the political contribution to be tendered: (p_i, C_i), $i = 1, 2$. The government then evaluates their proposals. We shall solve the game for its subgame perfect equilibrium (SPE), which consists of no weakly dominated strategies.

[4] In general, the wage will not be equal to the value of the marginal product of labor when the labor market is a duopsony. However, our analysis remains valid if one considers a lobbyist in the model as a representative agent of a few suppliers who own the same specific factor and form a lobby group.

[5] G&H instead assume wage to be fixed, and their welfare is defined in terms of consumers' surplus derived from quasilinear utility functions.

3. The SPE of the lobbying game

In our setting, the lobby race is triggered by a rivalry over limited resources, namely, labor. The profit of capitalist i can be derived as:

$$\pi_i(p) = V(p)s_i(p)(1-b_i), \quad i = 1, 2, \tag{10}$$

where $1 - b_i$ is capitalist i's share of the sector's gross product, and s_i is sector i's share in GDP:

$$s_1(p) = \frac{y_1}{y_1 + py_2}, \qquad s_2(p) = 1 - s_1(p). \tag{11}$$

In (10), the free-trade term p^* maximizes $V(p)$, but not $s_i(p)$. Fig. 1 illustrates how the domestic price p determines labor allocation, and hence the sector's share in GDP. If we raise the domestic price p in Fig. 1, capitalist 1 will receive less labor L_1, and hence produce less y_1. At the same time, L_2 and y_2 both increase. The total effect is:

Lemma 1. *$s_1(p)$ decreases in p, and $s_2(p)$ increases in p.*

In the following, we shall first explain intuitively how capitalist i designs his package (p_i, C_i) of policy proposal, and then present the technical details. To promote (p_i, C_i), i has to keep government's utility, i.e. $C_i + aV(p_i)$, high enough; and at the mean time, to enhance his own utility, $\pi_i(p_i) - C_i$. In view of this, C_i serves as a pure transfer between i and the government and p_i should be chosen to maximize the joint utilities W_i:

$$\max_p W_i(p) = \pi_i(p) + aV(p), \quad i = 1, 2. \tag{12}$$

From (10), we can rewrite the joint utilities as:

$$W_i(p) = V(p)[(1-b_i)s_i(p) + a] \tag{13}$$

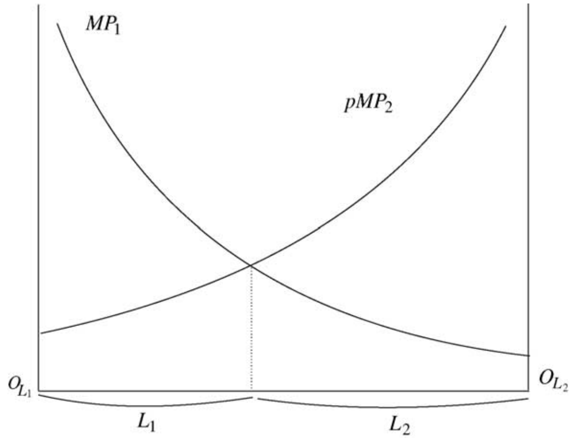

Fig. 1. Labor allocation. Note: MP_i denotes the marginal product of labor in sector i.

Let $\bar{p}_i$ denote the price that maximizes W_i. The following lemma characterizes $\bar{p}_i$. Its proof is given in the Appendix A.

Lemma 2. (*a*) $\bar{p}_2 > p^* > \bar{p}_1$. (*b*) $\pi_1(p^*) > \pi_1(\bar{p}_2)$ *and* $\pi_2(p^*) > \pi_2(\bar{p}_1)$.

It turns out that the party whose $\bar{p}_i$ yields a higher sum of utilities of all players in the game, including two capitalists and the government, is the successful lobby. Let $\dot{W}_i$ denote this sum:

$$\dot{W}_i \equiv \pi_1(\bar{p}_i) + \pi_2(\bar{p}_i) + aV(\bar{p}_i); \quad i = 1, 2. \tag{14}$$

The difference between $\dot{W}_i$ and $W_i(\bar{p}_i)$ is i's opponent's utility. This is important to i, because if his proposed price $\bar{p}_i$ does not harm the opponent's utility much, the opponent will not react by a strong counter-lobby. The following two propositions provide the necessary and sufficient conditions for SPE involving no weakly dominated strategies. Their proofs are given in the Appendix A.

Proposition 1. *Let G_i (G_j) denote the government's payoff if she adopts i's (j's) equilibrium proposal. When i wins and j loses in equilibrium, the SPE with no weakly dominated strategy played satisfies the following conditions*:

$$G_i = G_j, \tag{15}$$

$$p_i = \bar{p}_i, \tag{16}$$

$$p_j = \bar{p}_j, \tag{17}$$

$$C_i = C_j + aV(\bar{p}_j) - aV(\bar{p}_i), \tag{18}$$

$$C_j \geq \pi_j(\bar{p}_j) - \pi_j(\bar{p}_i), \tag{19}$$

$$\dot{W}_i \geq \dot{W}_j, \tag{20}$$

$$\text{Let } k \equiv \min\{\dot{W}_i - \dot{W}_j, \pi_j(\bar{p}_i)\} \geq 0.$$

$$C_j \begin{cases} \leq \pi_j(\bar{p}_j) - \pi_j(\bar{p}_i) + k, & \text{if } \dot{W}_i - \dot{W}_j < \pi_j(\bar{p}_i) \\ < \pi_j(\bar{p}_j) - \pi_j(\bar{p}_i) + k, & \text{otherwise.} \end{cases} \tag{21}$$

Proposition 2. *When $\dot{W}_i \geq \dot{W}_j$, $j = 3 - i$, the following strategy profile is an SPE with no weakly dominated strategy played*: (*a*) (p_i, C_i) *and* (p_j, C_j) *are proposals that satisfy* (16)–(19)*and* (21). (*b*) *Considering any proposals* (p_i, C_i) *and* (p_j, C_j), *the government maintains* $p = p^*$, *if* $aV(p^*) > \max\{C_i + aV(p_i), C_j + aV(p_j)\}$. *Otherwise, it shall accept i's* (*j's*) *proposal when* $C_i + aV(p_i) \geq (<) C_j + aV(p_j)$.

In equilibrium, not only is the losing party's profit lower than that in free trade (Lemma 2(b)), the winning lobbyist may also end up with a lower profit after paying his political donations, and the government turns out to be the sole winner.[6] The following corollary shows when this will occur. Its proof is given in the Appendix A.

Corollary 1. *Let i and j denote the winning and losing sector, respectively. When $b_i = b_j = b$, or when $b_i > b_j$, either lobbyist ends up with a lower profit than what he earns in free trade.*

When the precondition of the above corollary does not hold, the winning lobby does have a chance to earn more than under free trade. For instance, consider the case in which $b_1 = 3/4$, $b_2 = 1/4$, $a = 1$, $p^* = 1$, $L = 100$ and $K_2 = 10$. Suppose the political gifts are at their minimum values, i.e. $k = 0$ in (21). It can be shown that capitalist 2 is the winning lobby when K_1 is either as small as 1 or as large as 161. In the former (latter) case, capitalist 2 earns more (less) profits than under free trade.

3.1. Menu auction

This section relates our bidding game to the menu auction studied by G&H. In G&H's model, lobbyist i proposes a contribution schedule $f_i(p)$ that indicates how his political donation will vary with the domestic price to be chosen. Considering these menus, the problem of the government is:

$$\max_p \; f_1(p) + f_2(p) + aV(p).$$

Proposition 3. *For any SPE in our game with proposals $\{(\bar{p}_i, C_i)\}_{i=1,2}$ which involve no weakly dominated strategies, there exists a corresponding SPE in the menu auction, with $\{f_i(p)\}_{i=1,2}$ constructed as follows*:

$$f_i(p) = \begin{cases} C_i, & \text{if } p = \bar{p}_i \\ 0, & \text{otherwise} \end{cases} \quad \text{for } i = 1, 2. \tag{22}$$

Proof. See the Appendix A. □

Though our game is closely related to the menu auction as Proposition 3 demonstrates, the literature of menu auction focuses on other equilibria than those depicted in Propositions 1 and 2. In their pioneering study, Bernheim and Whinston (B&W, 1986, pp. 4–6) point out there are multiple equilibria in a menu auction, and they propose the truthful Nash equilibrium (TNE) may be focal. In a TNE, the contribution schedule $f_i(p)$ reflects the true preference of bidder i, and it is shown that: (i) Bidder i's best response correspondence always contains a truthful strategy, so there is no loss to i to restrict himself to playing truthfully. (ii) TNE are coalition-proof. G&H (1994, p. 841) instead consider bidders to use differentiable contribution schedules. In their model, this leads to the same

[6] In this case, to lobbyists, it is a game even worse than the contest analyzed by Posner (1975), in which firms compete to become a monopolist, and the rent-seeking expenses just break even with the expected rent.

measure of trade protection as the TNE predicts, and when evaluating the political contributions, in face of multiple (differentiable) equilibrium schedules, G&H restrict the analysis to the TNE.

The contribution schedules derived from the SPE of our game are neither differentiable nor truthful. If we follow B&W's or G&H's line of thinking, when holding a menu auction, the government does not expect schedules in (22) to be submitted. In the following, we shall show schedules in (22) yield a higher payoff to the government than the TNE. If the government ever has a choice between our game and the menu auction, it is to its advantage to auction off protection in our way to realize an unlikely result in the menu auction.

Let $\bar{p}$ denote the domestic price realized in the TNE in a menu auction, and $\bar{n}_i$ the net payoff of i in equilibrium, $\bar{n}_i \equiv \pi_i(\bar{p}) - f_i(\bar{p})$. Theorem 2 by B&W characterizes the TNE with the following conditions:

$$\bar{p} = \operatorname{argmax}_p \hat{W}(p) = \pi_1(p) + \pi_2(p) + aV(p), \tag{23}$$

and $(\bar{n}_1, \bar{n}_2)$ lies on the Pareto frontier of the following set:

$$\begin{aligned}\{(n_1, n_2) \in R_2^* | n_1 &\leq \pi_1(\bar{p}) + \pi_2(\bar{p}) + aV(\bar{p}) \\ &- (\pi_2(\bar{p}_2) + aV(\bar{p}_2))(\equiv x), n_2 \leq \pi_1(\bar{p}) + \pi_2(\bar{p}) + aV(\bar{p}) \\ &- (\pi_1(\bar{p}_1) + aV(\bar{p}_1))(\equiv y), n_1 + n_2 \leq \pi_1(\bar{p}) + \pi_2(\bar{p}) + aV(\bar{p}) \\ &- aV(p^*)(\equiv z)\},\end{aligned} \tag{24}$$

where $\bar{p}_i$ is the price that maximizes $W_i(p)$ in (12).

The above result characterizes a different prediction about the trade policy from ours, as demonstrated by the following proposition, the proof for which is given in the Appendix A.

Proposition 4. (*a*) *When* $b_1 = b_2 = b$, $\bar{p}_2 > p^* = \bar{p} > \bar{p}_1$. (*b*) *When* $b_1 < b_2$, $\bar{p}_2 > p^* > \bar{p} > \bar{p}_1$. (*c*) *When* $b_1 > b_2$, $\bar{p}_2 > \bar{p} > p^* > \bar{p}_1$.[7]

In a menu auction, the government's payoff is:

$$G_m = aV(\bar{p}) + (\pi_1(\bar{p}) - \bar{n}_1) + (\pi_2(\bar{p}) - \bar{n}_2).$$

Consider an SPE with no weakly dominated strategy played in our game, and let G_i denote the government's payoff in equilibrium. The following proposition compares the government's payoff in our game and in the menu auction. The proof is provided in the Appendix A.

Proposition 5. $G_m < G_i$.

Comparing (12) and (23), it is easy to see that the total payoff to three players is higher in the menu auction. Proposition 5 still stands, thanks to lobbyists being more generous in donations in our game. This is because in our game a lobbyist bids against one drastic

[7] We thank Professor Grossman for providing this result.

condition proposed by the opponent; while in a menu auction, even if his ideal condition is not fulfilled, there is a spectrum of possible outcomes.

4. Comparative statics

The analysis in Section 3 shows that the domestic price is always lobbied away from the international price in our game. This section will continue to study how this price distortion changes with the political and economic environment. In the following, we shall first study how capitalist i's proposed price $\bar{p}_i$ changes with a, p^*, L, K_1 and K_2. We then study which sector becomes in favor as the domestic resources L, K_1 and K_2 change.

4.1. Individual proposals

Proposition 1 states that capitalist i's proposed price $\bar{p}_i$ maximizes W_i. (9) and (13) together show how the domestic production activities y_1 and y_2 determine V and hence W_i. In our model, the variable inputs that determine y_1 and y_2 are labors L_1 and L_2. Therefore, to find the domestic price p that maximizes joint utilities W_i, we have to first analyze how p affects the labor allocation. In the following, we shall consider the interior solution. Let $z_{,s}$ denote the derivative of z with respect to s. From (7) and (8), it can be derived that:

$$L_{1,p} = -\frac{L_1 L_2}{p[(1-b_1)L_2 + (1-b_2)L_1]} < 0; \quad L_{2,p} = -L_{1,p} > 0.$$

With the Cobb–Douglas production function, we then have:

$$y_{1,p} = -\frac{b_1 b_2 y_1 y_2 p}{b_1(1-b_2)y_1 + b_2(1-b_1)py_2} < 0; \qquad y_{2,p} = -\frac{y_{1,p}}{p} > 0. \tag{25}$$

Because two sectors' lobbied price $\bar{p}_i$'s are derived in the same manner, in the following, we shall discuss the case of sector 1 only. The study naturally extends to sector 2. (9), (13) and (25) imply that to maximize W_1, $\bar{p}_1$ has to satisfy the following FOC:

$$\begin{aligned} W_{1,p}\big|_{p=\bar{p}_1} = {} & \frac{y_1 y_2(b_1 y_1 + b_2 p y_2)}{(1+p^*)[b_1(1-b_2)y_1 + b_2(1-b_1)py_2]} \\ & \times \left\{ -\frac{(y_1 + p^* y_2)(1-b_1)}{(y_1+py_2)^2} + \frac{(1-b_1)y_1 + a(y_1+py_2)}{y_1+py_2} \right. \\ & \left. \times \frac{b_1 b_2(p^*-p)}{p(b_1 y_1 + b_2 p y_2)} \right\}\Bigg|_{p=\bar{p}_1} = 0. \end{aligned} \tag{26}$$

To see the impact of change in the domestic political environment or the international market conditions on the lobbied price $\bar{p}_1$, we differentiate $W_{1,p}$ with respect to a and p^*, respectively. Let $z_{,st}$ denote the second derivative of z with respect to s and t. It can be shown that:

$$W_{1,pa}\big|_{p=\bar{p}_1} > 0, \qquad W_{1,pp^*}\big|_{p=\bar{p}_1} > 0,$$

At the interior solution, $W_{1,pp}\big|_{p=\bar{p}_1} < 0$. It follows immediately that:

Proposition 6. (*a*) *When the government is more concerned about social welfare, both sectors will propose prices closer to the international level, i.e. when a is larger, $\bar{p}_1$ rises, and $\bar{p}_2$ falls.* (*b*) *When the international price of good 2, p^*, becomes higher, both $\bar{p}_1$ and $\bar{p}_2$ become higher.*

The intuition of Proposition 6 is as follows. From Lemma 2, $\bar{p}_2 > p^* > \bar{p}_1$, i.e. each capitalist lobbies to enhance the relative price of his good above the international level. This harms social welfare, and when social welfare becomes a more important concern to the government, both capitalists have to concede to propose a price closer to the international level. On the other hand, for sector i, $\bar{p}_i$ shows the optimal deviation from the international price p^*. When p^* becomes higher, we expect $\bar{p}_i$ to be adjusted in the same direction.

We now turn to see how the lobbied price $\bar{p}_i$ changes when capitals K_1 and K_2 grow. To differentiate $W_{1,p}$ in (26) with respect to K_1, we have:

$$W_{1,pK_1}|_{p=\bar{p}_1}\begin{cases} \leq 0 & \text{if } b_1 > b_2 \text{ and } a \text{ is big enough,} \\ > 0 & \text{otherwise.} \end{cases} \tag{27}$$

This implies that

$$\frac{\partial \bar{p}_1}{\partial K_1} = -\frac{W_{1,pK_1}}{W_{1,pp}}\bigg|_{p=\bar{p}_1}\begin{cases} \leq 0 & \text{if } b_1 > b_2 \text{ and } a \text{ is big enough,} \\ > 0 & \text{otherwise.} \end{cases} \tag{28}$$

The intuition is as follows. From Lemma 1, to lower the domestic price p will increase sector 1's share in GDP, $s_1(p)$. This has a positive effect on W_1 in (13). On the other hand, to lower p further away from the international price p^* reduces the welfare $V(p)$, and hence W_1. How low capitalist 1 should push down the domestic price depends upon the trade-off between these two conflicting forces. At $\bar{p}_1$, the marginal effects of these two forces should be equal. When sector 1's capital K_1 grows, MP_1 curve in Fig. 1 shifts up, and without lowering the domestic price p, we observe that L_1 (L_2) increases (decreases) and sector 1's share $s_1(p)$ increases as a result. This implies that although lowering p still helps to enhance sector 1's share $s_1(p)$, its effect becomes limited. So, we expect $\partial \bar{p}_1/\partial K_1 > 0$. The exceptional case is when $b_1 > b_2$ and a is very large. In light of (12), when $a \to \infty$, because the government cares deeply about social welfare, sector 1 cannot propose allowing the domestic price p to deviate from the international price p^* much, i.e. $\bar{p}_1 \to p^*$. Because $V(p)$ is maximized at $p = p^*$, $\mathrm{d}V(p)/\mathrm{d}p|_{p=p^*} = 0$. That means when $a \to \infty$, $\mathrm{d}V(p)/\mathrm{d}p|_{p=\bar{p}_1} \to 0$: the marginal harm a smaller p brings to $V(p)$ is limited. Thus, capitalist 1 could consider enhancing his share $s_1(p)$ by lowering p, especially when this share enhancement is backed up by a production advantage, i.e. when the productivity of labor is higher in sector 1 ($b_1 > b_2$) and when sector 1's capital K_1 increases.

A similar result holds when the capital in sector 2 increases. To differentiate $W_{1,p}$ in (26) with respect to K_2, we have $W_{1,pK_2} = -W_{1,pK_1}K_1/K_2$. That implies:

$$\frac{\partial \bar{p}_1}{\partial K_2} = -\frac{W_{1,pK_2}}{W_{1,pp}}\bigg|_{p=\bar{p}_1}\begin{cases} \geq 0 & \text{if } b_1 > b_2 \text{ and } a \text{ is big enough,} \\ < 0 & \text{otherwise.} \end{cases} \tag{29}$$

In reality, capital in different sectors changes at the same time. Suppose K_1 and K_2 have the same growth rate: r. For any domestic price p, the labor allocation $L_i(p)$ dictated in (7) and (8) does not change after the growth of K_1 and K_2. This implies the output $y_i(p)$ in (6) grows at the rate r, for $i = 1, 2$. It follows that the welfare $V(p)$ in (9) grows at the same rate, while sectors' shares $s_i(p)$ in (11) stay the same. In total, the joint utility $W_1(p)$ in (13) becomes $1 + r$ times its original value for any p. It follows immediately that:

Proposition 7. *When K_1 and K_2 grow at the same rate, $\bar{p}_1$ and $\bar{p}_2$ remain the same.*

When the growth rate of K_1 is larger than that of K_2, we can reason the change of $\bar{p_1}$ in two steps. First, imagine that K_1 only grows at the same rate as K_2. From Proposition 7, this causes no change to $\bar{p}_1$. Now, consider K_1 continues to grow to its full amount while K_2 stays the same; $\bar{p}_1$ will then change according to (28). The same thought experiment can be made for all other possible cases, and the results are summarized as follows.

Proposition 8. *Consider a case in which K_1 grows at a higher (lower) rate than K_2. (a) $\bar{p}_1$ becomes lower (higher), when $b_1 > b_2$ and a is sufficiently large; and in all other conditions, $\bar{p}_1$ becomes higher (lower). (b) $\bar{p}_2$ becomes lower (higher), when $b_2 > b_1$ and a is sufficiently large; and in all other conditions, $\bar{p}_2$ becomes higher (lower).*

Lastly, we shall study the impact of population growth on the lobbied price $\bar{p}_i$. To differentiate $W_{1,p}$ with respect to L, we have:

$$W_{1,pL}|_{p=\bar{p}_1} = W_{1,pK_1}\frac{K_1(b_1 - b_2)}{L}\begin{cases} \geq 0 & \text{if } b_1 > b_2 \text{ and } a \text{ is small enough,} \\ = 0 & \text{if } b_1 = b_2, \\ < 0 & \text{otherwise.} \end{cases}$$

This along with (27) implies:

Proposition 9. *(a) When $b_1 = b_2$, $\bar{p}_1$ and $\bar{p}_2$ are invariant with the change in labor population L. (b) When $b_1 > b_2$, $\bar{p}_2$ increases with L and $\bar{p}_1$ increases (decreases) with L, if a is sufficiently small (large). (c) When $b_2 > b_1$, $\bar{p}_1$ decreases with L and $\bar{p}_2$ increases (decreases) with L, if a is sufficiently large (small).*

The mechanism behind Proposition 9 is as follows. When $b_1 = b_2 = b$, if the population becomes λ times the original size, for any domestic price p, from (7) and (8), labor input in each sector becomes λ times larger, and output λ^b times larger. It follows that welfare $V(p)$ increases to λ^b times the original level, and sectors' shares $s_i(p)$'s remain unchanged. So, $W_i(p)$ is λ times that before the population growth, $\forall\ p$. The maximum of $W_i(p)$ therefore occurs at the same $\bar{p}_i$. When $b_1 < b_2$, the productivity of labor in sector 1 is lower than that in sector 2, and when the labor supply L increases, because $[(\partial L_1/\partial L)/L_1]/[(\partial L_2/\partial L)/L_2] = (1 - b_2)/(1 - b_1) < 1$, L_1 grows at a smaller rate than L_2. This along with b_1 being smaller implies y_1 increases at a smaller rate than y_2, and if the domestic price p stays the same, sector 1's share $s_1(p)$ shrinks. To mitigate this unfavorable change, capitalist 1 will propose to lower the relative price of good 2, p, further. Similarly, when $b_1 > b_2$, an increase in L enhances sector 1's share $s_1(p)$, and we expect capitalist 1 to be lax on lowering p to enhance the share, and the harm of price distortion to welfare $V(p)$ will be more of a concern to him. So, we expect $\bar{p}_1$ to increase to be closer to the international price. The exceptional case is when a is sufficiently large. As discussed previously, when $a \to \infty$, $\mathrm{d}V(p)/\mathrm{d}p|_{p=\bar{p}_1} \to 0$. Since the marginal effect on $V(p)$ is limited, capitalist 1 will consider enhancing his sector share by lowering p, especially when the share enhancement is supported by an advantage in labor employment, i.e. when $b_1 > b_2$ and L grows.

4.2. Party in favor, political gifts and domestic price

The previous section shows how individual lobbied price changes with the political and economic environment. To understand how the domestic price changes in the end, we have to further study who becomes the winning lobbyist as the environment changes. From Proposition 1, this leads to a comparison of $\dot{W}_1$ and $\dot{W}_2$, and from (14), the government will set the domestic price to be $\bar{p}_1$ when

$$\dot{W}_1 - \dot{W}_2 = V(\bar{p}_1)[a + s_1(\bar{p}_1)(1 - b_1) + (1 - s_1(\bar{p}_1))(1 - b_2)] - V(\bar{p}_2)[a + s_1(\bar{p}_2)(1 - b_1) + (1 - s_1(\bar{p}_2))(1 - b_2)] > 0.$$

Suppose the capital at each sector grows at the same rate: r. From the discussion about Proposition 7, each capitalist will continue lobbying for the same price $\bar{p}_i$, which results in the same share for sector 1, $s_1(\bar{p}_i)$. On the other hand, the welfare $V(\bar{p}_i)$ will grow at the rate r, for $i = 1, 2$. It then follows that the magnitude of $\dot{W}_1 - \dot{W}_2$ above becomes $1 + r$ times the original difference, while its sign remains unchanged. That means the government favors the same capitalist when the capital in each sector grows at a uniform rate. Together with Proposition 7, it implies that:

Proposition 10. *When K_1 and K_2 grow at the same rate, the domestic price stays the same.*

In our model, labor and capital are complementary factors, i.e. $\partial^2 y_i/\partial L_i \partial K_i > 0$, when capital grows, labor becomes more valuable a resource because its productivity is enhanced, and we expect each capitalist to offer more to compete for labor allocation. (18), (19) and (21) characterize the donations in intervals. It can be easily checked that the bounds of these intervals increase to $1 + r$ times the original values when the sectorial capital grows uniformly at rate r. In sum, when the rivalry between two sectors intensifies, we expect the government to receive more political gifts, while no change occurs in the domestic price.

The analysis in the previous section shows that when the sectorial capitals K_1 and K_2 grow at an uneven rate, or when the labor population L grows, there are a few possibilities with which $\bar{p}_1$ and $\bar{p}_2$ might change. So the share $s_1(\bar{p}_i)$ in the above expression also has a few possibilities to change. To avoid this complexity and to see better how the system works, in the following, we shall focus on simplified cases when $b_1 = b_2 = b$. This reduces the condition for $p = \bar{p}_1$ to be:

$$\dot{W}_1 - \dot{W}_2 = [V(\bar{p}_1) - V(\bar{p}_2)](1 + a - b) > 0, \tag{30}$$

i.e. whether $\dot{W}_1$ is larger than $\dot{W}_2$ hinges upon whether $\bar{p}_1$ creates a larger welfare V than $\bar{p}_2$.

Case 1. When $b_1 = b_2 = b$, and labor supply increases from L to λL. Proposition 6(a) states that in this case, $\bar{p}_1$ and $\bar{p}_2$ both stay unchanged, and the discussion following the proposition shows that $V(\bar{p}_i; \lambda L) = \lambda^b V(\bar{p}_i; L)$, for $i = 1, 2$. That means $\dot{W}_1 - \dot{W}_2$ in (30) does not change sign after the labor supply increases. So the winner of the lobbying game is the same person as before, and the domestic price remains unchanged.

Regarding the political gifts, it can be easily checked that the bounds of C_i and C_j become λ^b times the original values. That means when lobbyists are richer because the resource competed for becomes more bountiful, they are more generous in making political donations. Because the marginal product of labor is assumed to be diminishing, the (bounds of) political gifts will increase with labor population in a decreasing rate. On the other hand, the domestic price is not distorted further as the government benefits more.

Case 2. When $b_1 = b_2 = b$, and the capital at sector 1 increases from K_1^0 to K_1^n. Let $\bar{p}_i^o$ and $\bar{p}_i^n$ denote the price lobbied by i, when K_1 is K_1^o and K_1^n, respectively. From Proposition 8, $\bar{p}_i^n > \bar{p}_i^o$, for $i = 1, 2$. The change in the domestic price, along with the change in K_1, brings changes to labor allocation and social welfare. Let $L_1(p, K_1)$ denote the labor allocated to sector 1 when the domestic price is p and sector 1 is equipped with capital K_1. The change in $\dot{W}_1 - \dot{W}_2$ has the same sign as the following term:

$$[V(L_1(\bar{p}_1^n, K_1^n); K_1^n) - V(L_1(\bar{p}_1^o, K_1^o); K_1^o)] \tag{31}$$

$$-[V(L_1(\bar{p}_2^n, K_1^n); K_1^n) - V(L_1(\bar{p}_2^o, K_1^o); K_1^o)]. \tag{32}$$

(31) shows the change of $\dot{W}_1/(1 + a - b)$, and can be decomposed into:

$$V(L_1(\bar{p}_1^n, K_1^n); K_1^n) - V(L_1(\bar{p}_1^o, K_1^n); K_1^n) \tag{33}$$

$$+V(L_1(\bar{p}_1^o, K_1^n); K_1^n) - V(L_1(\bar{p}_1^o, K_1^o); K_1^n) \tag{34}$$

$$+V(L_1(\bar{p}_1^o, K_1^o); K_1^n) - V(L_1(\bar{p}_1^o, K_1^o); K_1^o). \tag{35}$$

In (33), K_1 is fixed at the new level, and the focus is on how the welfare changes when the domestic price is changed from $\bar{p}_1^o$ to $\bar{p}_1^n$. Because the new price is raised closer to the international price, and results in a welfare improvement, (33) is positive. In (35), the labor allocation is fixed. Apparently, welfare increases when there is more capital in Section 1. So, (35) is also positive. The sign of (34) is uncertain. When $K_1 = K_1^n$, welfare V is maximized when $L_1 = L_1(p^*, K_1^n)$. Because $\bar{p}_1^o < p^*$ and $K_1^o < K_1^n$, $L_1(\bar{p}_1^o, K_1^n)$ is greater than $L_1(p^*, K_1^n)$ and $L_1(\bar{p}_1^o, K_1^o)$. The order between $L_1(p^*, K_1^n)$ and $L_1(\bar{p}_1^o, K_1^o)$ is uncertain. When the latter is larger, (34) is negative; otherwise, the sign of (34) is uncertain.

Similarly, the change of $\dot{W}_2/(1 + a - b)$ in (32) can be decomposed into the following three effects:

$$V(L_1(\bar{p}_2^n, K_1^n); K_1^n) - V(L_1(\bar{p}_2^o, K_1^n); K_1^n) \tag{36}$$

$$+V(L_1(\bar{p}_2^o, K_1^n); K_1^n) - V(L_1(\bar{p}_2^o, K_1^o); K_1^n) \tag{37}$$

$$+V(L_1(\bar{p}_2^o, K_1^o); K_1^n) - V(L_1(\bar{p}_2^o, K_1^o); K_1^o). \tag{38}$$

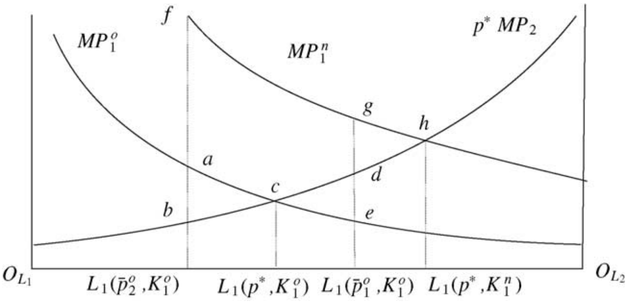

Fig. 2. Labor allocation and deadweight loss. Note: MP_1^o and MP_1^n denotes the marginal product of labor at sector 1, when $K_1 = K_1^o$ and when $K_1 = K_1^n$.

The argument that (33) is positive can be applied to show (36) to be negative, and this implies (33)–(36) is positive.

We now turn to the comparison between (35) and (38). (38) is positive for the same reason that (35) is positive. What concerns us is the difference between these two positive terms. (35)–(38) can be rearranged to be: $[V(L_1(\bar{p}_1^o, K_1^o); K_1^n) - V(L_1(\bar{p}_2^o, K_1^o); K_1^n)] - [V(L_1(\bar{p}_1^o, K_1^o); K_1^o) - V(L_1(\bar{p}_2^o, K_1^o); K_1^0)]$. The latter term evaluates the impact of labor allocation on welfare when K_1 is still K_1^o. From Lemma 2(a), $L_1(\bar{p}_2^o, K_1^o) < L_1(p^*, K_1^o) < L_1(\bar{p}_1^o, K_1^o)$. Either $L_1(\bar{p}_2^o, K_1^o)$ or $L_1(\bar{p}_1^o, K_1^o)$ causes a deadweight loss, and in terms of good 1, it is the area of $\triangle abc$ ($\triangle cde$) when $L_1 = L_1(\bar{p}_2^o, K_1^o)$ ($L_1 = L_1(\bar{p}_1^o, K_1^o)$) as shown in Fig. 2. The former term of (35)–(38) evaluates the deadweight losses resulting from the same labor allocations, but with $K_1 = K_1^n$ in the background. When K_1 increases to K_1^n, more labor than $L_1(p^*, K_1^o)$ should be allocated to sector 1 to achieve efficiency. If $L_1(p^*, K_1^n)$ surpasses $L_1(\bar{p}_1^o, K_1^o)$ as depicted in Fig. 2, the deadweight loss is the area of $\triangle fbh$ ($\triangle gdh$) when $L_1 = L_1(\bar{p}_2^o, K_1^o)$ ($L_1(\bar{p}_1^o, K_1^o)$). A straightforward comparison of these areas shows: $(\triangle fbh - \triangle gdh) - (\triangle abc - \triangle cde) > 0$, and it follows that (35)–(38) > 0. On the other hand, if $L_1(p^*, K_1^n) < L_1(\bar{p}_1^o, K_1^o)$, it can be shown that when $K_1 = K_1^n$, the deadweight loss becomes larger (smaller) than the area of $\triangle abc$ ($\triangle cde$) when $L_1 = L_1(\bar{p}_2^o, K_1^o)$ ($L_1(\bar{p}_1^o, K_1^o)$). In this case, (35)–(38) is also positive.

Lastly, (37) is positive because $L_1(p^*, K_1^n) > L_1(\bar{p}_2^o, K_1^n) > L_1(\bar{p}_2^o, K_1^o)$. If (34) is negative, apparently (34)–(37) is negative. If (34) is positive, to repeat the same graphical analysis for (35)–(38), we find (34)–(37) ends with a comparison of two irrelevant areas. This leaves the sign of (34)–(37) uncertain.

To summarize, our analysis of (33) through (38) shows $\dot{W}_1 - \dot{W}_2$ can either increase or decrease when the capital at sector 1 grows. Fig. 3 presents a simulation result that shows these two possible cases. In the simulation, we set $K_2 = 10$, $p^* = 1$, $a = 1$, $b = 1/2$, $L = 100$, and let K_1 runs from 1 to 15. In this set of data, capitalist 1 cannot lobby successfully when his capital is less than the opponent's, i.e. only when $K_1 \geq K_2$, is $\dot{W}_1 - \dot{W}_2$ positive. Fig. 4 shows how the domestic price p changes when K_1 increases. Before K_1 reaches 10, the government always protects sector 2, and more so when sector 1 becomes stronger, i.e. p increases with K_1. Once K_1 reaches the critical level, the government switches to protect sector 1, and we observe that the domestic price falls sharply below the international price p^*. In reality, the protected sector changes through

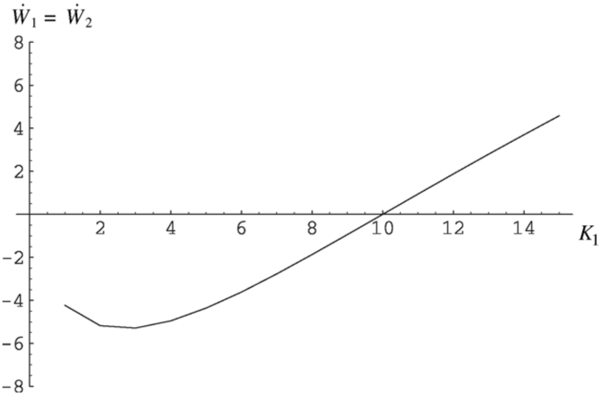

Fig. 3. Simulation result: $\dot{W}_1 - \dot{W}_2$.

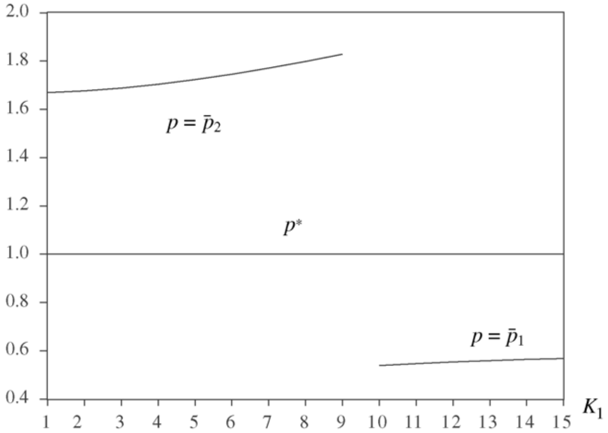

Fig. 4. Simulation result: the domestic price.

time. Our model shows that it can simply be a result of changes in domestic resources. It is worth noting that although in this special case, the government always protects the exporting sector, when $b_1 \neq b_2$, the government may protect either the exporting or the importing sector.[8]

In this case of unbalanced growth, it is uncertain whether the capitalist in the growing sector offers more or less political gifts to lobby. On one hand, labor becomes a more valuable resource to the growing sector, because its productivity is enhanced by more inputs of capital. On the other hand, Eq. (7) shows that the market automatically allocates more labor to the growing sector, thus sector 1 need not distort the domestic price further to lobby for more resources. Actually, Proposition 8 shows that the growing sector will

[8] In another simulation, we set b_1 to be 1/3 and b_2 to be 2/3, while letting all other variables take the same value as in case 2. The government protects sector 1 when $K_1 \geq 12$, but sector 1 is not an exporting sector until K_1 reaches 35.

propose a price closer to the free-trade condition. In view of this, capitalist 1 may offer less political gifts because there is less welfare loss to be compensated. To continue with the above simulation setting, we find the government receives more political gifts before K_1 increases to 14, and less afterwards.

Case 3. $b_1 = b_2 = b$, K_1 grows at a higher rate than K_2. Suppose K_2 grows to be $K_2(1 + r_1)$ and K_1 grows to be $K_1(1 + r_1 + r_2)$, where $r_2 > 0$. First consider the halfway case: K_i grows to be $K_i(1 + r_1)$, for $i = 1, 2$. From the previous analysis of case 2, the favored party remains unchanged in this half-way case. Now, fix capital at sector 2 to be $K_2(1 + r_1)$ and increase capital at sector 1 to $K_1(1 + r_1 + r_2)$. We find ourselves back to case 2 above. It is uncertain whether $\dot{W}_1 - \dot{W}_2$ changes sign, and the identity of the favored party is uncertain.

Lastly, we would like to bring to attention some caveat for the case in which $b_1 \neq b_2$. Case 2 above leaves us the impression that in a symmetric case ($b_1 = b_2$, and $p^* = 1$), the sector with bigger muscle, i.e. equipped with more capital, wins the lobby. What happens if two sectors are endowed with the same capital, say $K_1 = K_2 = 10$, and the terms of trade at the international market lean toward neither good, i.e. $p^* = 1$, but labor in sector 1 is more productive, i.e. $b_1 > b_2$? In this case, sector 1 need not be the one with bigger muscle to win the lobby. When $b_1 \approx b_2$, sector 1 does have an edge. For instance, if $b_1 = 2/3$, $b_2 = 1/3$, $a = 1$ and $L = 100$, capitalist 1 is the winner of the game. However, if $b_1 \gg b_2$, the capital's share of profit in sector 1 is much less than that in sector 2, and we expect capitalist 1 to be less motivated to lobby. If we change b_1 and b_2 in the above data set to be 3/4 and 1/4, respectively, then capitalist 2 becomes the winning lobbyist.

5. Conclusion

In a two-sector model like ours, the sector that is more able to create rent at a lower cost to consumers will win the race for policy favor. At least part of the rent will be transferred to the government as political contributions, however. The winner of the lobby race will not necessarily gain from such a trade preferences, as the political contributions needed to win the race may be higher than the rent, but the winner will necessarily be better off than letting the rival sector dictate the policy. The lobby competition between sectors, driven by the fear that the rival sector will win the favor of the government, makes free trade an infeasible policy choice. Political contribution is tantamount to a bribe to prevent the government from hurting the sector's interest. The simplicity of the two-sector model, combined with the single-offer auction, means that the lobbyist knows that his policy proposal will be adopted if the contribution he offers is accepted.

As usual, rent-seeking activity in our model is unproductive. In general, the sector that commands more intra-marginal gains from resource reallocation while carrying a smaller weight in the consumption bundle will be the winner in the lobby race. The consumption bias is assumed away in our model by a symmetric Leontief utility function. Resource endowment hence plays a central role in the outcome of the lobby game and in the determination of the structure of protection. A country having asymmetric resource endowments between the sectors will adopt a policy closer to free trade than a country having symmetric endowments of resources between the sectors. Since the former also has a higher tendency to trade if the world consumption pattern is similar, the policy choice resulting from our model will reinforce rather than impede the tendency to trade. Thus a country endowed with asymmetric resources is naturally more open to trade partly because of its need for foreign resources, and partly because it engenders a political process leaning toward freer trade. Political contribution in an asymmetric economy also tends to be small because the smaller sector presents little threat to the dominant sector in the lobby competition. A government with a long-term planning horizon may, however, wish to help the smaller sector grow in order to extract more contributions from the larger sector.

The model is intended to depict the policy choice of an incumbent government that values political contributions but is also concerned about the welfare of the general population. It is essentially a mix of the interest group model and the national interest model as described by Baldwin (1989). It ignores the political competition among the candidates who are running for office. If the welfare function of the government portrayed in our model in fact depicts the probability of being elected, then there is no chance for the repressed sector to turn to the opposition by proposing an alternative policy formula with commensurate political contributions, because the opposition will never win the election with this platform and the attached political funds. Therefore, trade policy will never be a debatable issue in the campaign because every party knows which sector should be favored and "taxed" with political gifts to stay in power once it is elected. Only if the winning formula for election changes, e.g., the voters become more aware of their well-being or the effectiveness of campaign funds in swinging votes decreases, will the structure of protection change.

Acknowledgements

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Appendix A

Proof of Lemma 2.

(a) In view of (13), $W_1(p)$ hinges upon the magnitudes of $V(p)$ and $s_1(p)$. The social welfare $V(p)$ is maximized when $p = p^*$. On the other hand, Lemma 1 shows $s_1(p)$ decreases in p. Therefore, $W_1(p)$ must be maximized at $\bar{p}_1 < p^*$. $\bar{p}_2 > p^*$ can be proved similarly.

(b) Because $s_1(p)$ decreases in p and $\bar{p}_2 > p^*$, $s_1(p)$ is larger when p is set to be p^* than $\bar{p}_2$. This, along with $V(p^*) > V(\bar{p}_2)$, establishes that $\pi_1(p^*) > \pi_1(\bar{p}_2)$, where $\pi_1(\cdot)$ is defined in (10). Similarly, $\pi_2(p^*) > \pi_2(\bar{p}_1)$. $\square$

The following lemma is used in the Proof of Propositions 1 and 2.

Lemma 3. (a) $\forall\ p > 0$, $\pi_i(p) > 0$, $i = 1, 2$. (b) $\lim_{p\to\infty}\pi_1(p) = \lim_{p\to 0}\pi_2(p) = 0$.

Proof.

(a) From (9)–(11), we have to show $y_i(p) > 0$ to establish $\pi_i(p) > 0$. With Cobb–Douglas technology, $\forall\ p > 0$, as $L_1(L_2)$ approaches 0, $MP_1(pMP_2)$ in Fig. 1 goes to infinity. So, the labor allocation is always an interior solution. This implies $\forall\ p > 0$, $y_i(p) > 0$, for $i = 1, 2$.
(b) From Fig. 1, we observe $\lim_{p\to\infty}L_1(p) = 0$. It implies that $\lim_{p\to\infty}s_1(p) = 0$. On the other hand, the social welfare is bounded: $V(p) \le V(p^*)$, $\forall\ p > 0$. The above two conditions and (10) and (11) imply $\lim_{p\to\infty}\pi_1(p) = 0$. Similarly, one can show $\lim_{p\to 0}\pi_2(p) = 0$.

□

Proof of Proposition 1. Consider an SPE involving no weakly dominated strategy, in which *i*, the winner, proposes (p_i, C_i), and *j*, the loser, proposes (p_j, C_j). We first establish that

$$C_i > 0.$$

If $C_i = 0$, the only possible case in which the government will decide in *i*'s favor is $p_i = p^*$. Given that *i* proposes $(p^*, 0)$, *j* has a better proposal than (p_j, C_j) which causes *i* to lose. From Lemma 2(a), $\varepsilon \equiv (W_j(\bar{p}_j) - W_j(p^*))/2 > 0$. If *j* instead proposes $(\bar{p}_j, aV(p^*) - aV(\bar{p}_j) + \varepsilon)$, the government will switch to accept *j*'s offer, and *j*'s payoff will be higher than $\pi_j(p^*)$, his payoff when losing to *i*, by ε. This contradicts the notion that losing is *j*'s best response to *i*'s equilibrium strategy. We now study the 7 listed conditions in turn. Condition (15): Because *i* is the winner, $G_i \ge G_j$. Suppose $G_i = G_j + \delta, \delta > 0$, then *i* can win with another package, $(p_i, C_i - \varepsilon)$ where $0 < \varepsilon < \min\{C_i, \delta\}$, and become better off. This contradicts (p_i, C_i) being *i*'s best response to *j*'s equilibrium proposal. So, we must have $G_i = G_j$.

Condition (16): Given *j*'s proposal, suppose *i* wins with (p_i, C_i) where $p_i \ne \bar{p}_i$. Define $C_i' \equiv C_i + aV(p_i) - aV(\bar{p}_i)$. We shall show $(\bar{p}_i, C_i')$ is a better response to *j*'s equilibrium strategy than (p_i, C_i). We first establish that C_i' is feasible, i.e. $C_i' \ge 0$. This is obvious when $aV(p_i) \ge aV(\bar{p}_i)$, because $C_i > 0$. We only have to consider the case when $aV(p_i) < aV(\bar{p}_i)$. From Lemma 2(a),

$$aV(\bar{p}_i) < aV(p^*). \tag{A.1}$$

Therefore, $aV(p_i) < aV(p^*)$, and for *i* to persuade the government to lower the social welfare, he must donate more to compensate for this loss:

$$C_i \ge aV(p^*) - aV(p_i). \tag{A.2}$$

(A.1) and (A.2) together imply $C_i' > 0$.

By construction, $(\bar{p}_i, C'_i)$ yields the same payoff to the government as (p_i, C_i). So, i can also win with $(\bar{p}_i, C'_i)$. Moreover, i's net payoff, $\pi_i(\bar{p}_i) - C'_i$, will be higher than when winning with (p_i, C_i). This is because

$$[\pi_i(\bar{p}_i) - C'_i] - [\pi_i(p_i) - C_i] = \pi_i(\bar{p}_i) - (C_i + aV(p_i) - aV(\bar{p}_i)) - [\pi_i(p_i) - C_i] > 0,$$

for $p_i \neq \bar{p}_i$. The above reasoning prevents i from proposing a price other than $\bar{p}_i$ in equilibrium.

Condition (17): Suppose $p_j \neq \bar{p}_j$. Let $C'_j \equiv C_j + aV(p_j) - aV(\bar{p}_j).C'_j > 0$, for the same reason that $C'_i > 0$ above. We shall show $(\bar{p}_j, C'_j)$ weakly dominates (p_j, C_j). Let G'_j denote the government's payoff when she accepts $(\bar{p}_j, C'_j)$. By construction, $G'_j = G_j$. Therefore, for any proposal (p'_i, C'_i) by i, either $(\bar{p}_j, C'_j)$ and (p_j, C_j) both beat (p'_i, C'_i), or both lose to it. In the former case, $(\bar{p}_j, C'_j)$ yields a strictly higher payoff to j than (p_j, C_j), for the same logic used when proving condition (16). In the latter case, j gains $\pi_j(p'_i)$ with either proposal.

Condition (18): Condition (18) follows from conditions (15)–(17).

Condition (19): Suppose condition (19) does not hold, then $\exists\, \delta > 0$, such that $\pi_j(\bar{p}_j) - C_j - \delta = \pi_j(\bar{p}_i)$. Given i's equilibrium strategy, if j changes to propose $(\bar{p}_j, C_j + \delta/2)$, the government will accept his proposal, because her payoff will become $G_j + \delta/2$, higher than $G_i (= G_j)$. On the other hand, j's payoff also becomes higher than when proposing $(\bar{p}_j, C_j)$ and losing to i's $(\bar{p}_i, C_i)$, because

$$\pi_j(\bar{p}_j) - \left(C_j + \frac{\delta}{2}\right) = \pi_j(\bar{p}_i) + \frac{\delta}{2} > \pi_j(\bar{p}_i).$$

This contradicts $(\bar{p}_j, C_j)$ being j's equilibrium strategy, and establishes the claim.

Condition (20): Because $C_i > 0$, we must have:

$$\pi_i(\bar{p}_i) - C_i \geq \pi_i(\bar{p}_j). \tag{A.3}$$

Otherwise, from (15), given j's equilibrium strategy, to propose $(\bar{p}_i, 0)$ makes i lose to j, and leaves i a higher payoff than when winning with $(\bar{p}_i, C_i)$. This negates $(\bar{p}_i, C_i)$ to be a best response to j's equilibrium strategy.

On the other hand, from (18) and (19),

$$C_i \geq [\pi_j(\bar{p}_j) + aV(\bar{p}_j)] - [\pi_j(\bar{p}_i) + aV(\bar{p}_i)]. \tag{A.4}$$

(A.3) and (A.4) together imply $\dot{W}_i \geq \dot{W}_j$.

Condition (21): Condition (21) means C_j cannot exceed $\min\{W_i(\bar{p}_i) - W_i(\bar{p}_j), \pi_j(\bar{p}_j)\}$. From (18) and (A.3), we readily have: $C_j \leq W_i(\bar{p}_i) - W_i(\bar{p}_j)$.

Next, suppose $C_j \geq \pi_j(\bar{p}_j)$. We shall show such C_j causes $(\bar{p}_j, C_j)$ to be weakly dominated by $(\bar{p}_j, 0)$, in conflict with our pre-condition. We first establish that:

$$G_j > aV(p^*). \tag{A.5}$$

From (17) and (19),

$$G_j - aV(p^*) \geq \pi_j(\bar{p}_j) + aV(\bar{p}_j) - (\pi_j(\bar{p}_i) + aV(p^*)).$$

From Lemma 2(b), $\pi_j(\bar{p}_i) < \pi_j(p^*)$, so the above inequality can be further developed to be:

$$G_j - aV(p^*) > W_j(\bar{p}_j) - W_j(p^*) > 0,$$

which establishes the claim.

Consider any proposal by i: (p'_i, C'_i). If j proposes $(\bar{p}_j, C_j)$, (A.5) implies that the government will either set p to be $p'_i(\neq \bar{p}_j)$ or $\bar{p}_j$. In the former case, j's payoff is $\pi_j(p'_i)$. If j changes to propose $(\bar{p}_j, 0)$, there will be no change to his payoff, since he will remain a loser. In the latter case, $(\bar{p}_j, C_j)$ causes j's payoff to be negative when $C_j \geq \pi_j(\bar{p}_j)$. If j instead proposes $(\bar{p}_j, 0)$, the domestic price will be either p'_i or p^*, and j's payoff is strictly positive in either case (Lemma 3(a)). Thus, $(\bar{p}_j, 0)$ weakly dominates $(\bar{p}_j, C_j)$. □

Proof of Proposition 2. (b) is a straightforward statement of government's maximizing her utility. Given (b), we shall show proposals in (a) are equilibrium strategies with no weakly dominated proposals. First note that with (p_i, C_i) in (a), the government is better off accepting i's proposal than maintaining the international price. From (18), (19) and (21),

$$C_i = [\pi_j(\bar{p}_j) + aV(\bar{p}_j)] - [\pi_j(\bar{p}_i) + aV(\bar{p}_i)] + k,$$

where k is specified in (21). Let G_i denote the government's utility under i's proposal:

$$G_i \equiv aV(\bar{p}_i) + C_i = \pi_j(\bar{p}_j) + aV(\bar{p}_j) - \pi_j(\bar{p}_i) + k. \tag{A.6}$$

Applying Lemma 1(a) and (b) in turn, we have:

$$G_i > \pi_j(p^*) + aV(p^*) - \pi_j(\bar{p}_i) + k > aV(p^*)$$

which establishes the claim.

Next, we shall turn to explain (i) it is not to j's advantage to outbid i's offer in (a). (ii) (p_j, C_j) depicted in (a) will be rejected. Therefore, j's strategy in (a) is the best response to i's. (iii) (p_j, C_j) in (a) is not weakly dominated.

(i) To outbid i's offer, (p_j, C_j) has to yield a higher utility to the government than $(\bar{p}_i, C_i)$, namely:

$$C_j + aV(p_j) > [\pi_j(\bar{p}_j) + aV(\bar{p}_j)] - [\pi_j(\bar{p}_i) + aV(\bar{p}_i)] + aV(\bar{p}_i) + k. \tag{A.7}$$

When j's offer is accepted, his net payoff becomes $\pi_j(p_j) - C_j$, and in view of (A.7), we have:

$$\pi_j(p_j) - C_j < \pi_j(p_j) - [\pi_j(\bar{p}_j) + aV(\bar{p}_j) - \pi_j(\bar{p}_i) - aV(p_j)] - k.$$

Because $\pi_j(p) + aV(p)$ is maximized at $p = \bar{p}_j$, the above inequality implies:

$$\pi_j(p_j) - C_j < \pi_j(\bar{p}_i),$$

i.e. it is better for j to lose to i.

(ii) Let G_j denote the government's utility when she accepts j's proposal in (a). (18) implies that $G_i = G_j$; and with the government's choice depicted in (b), she will turn down j's offer.

(iii) In the proof of condition (17) in Proposition 1, it has been shown that (p_j, C_j) in (a) weakly dominates any other proposal by j that also yields G_j to the government. In the following, we shall show for any (p'_j, C'_j) that yields the government a different utility $G'_j(= C'_j + aV(p'_j))$, there exists a proposal (p'_i, C'_i) by i, to which (p_j, C_j) in (a) is a strictly better response for j than (p'_j, C'_j).

Case 1. $G'_j > G_j$. From (A.5), maintaining p^* makes the government worse off than accepting (p_j, C_j) in (a) or (p'_j, C'_j). Lemma 3(b) implies that $\exists\,(p'_i, C'_i)$ by i such that $C'_i + aV(p'_i) < aV(p^*)$. Against this proposal, (p'_j, C'_j) yields j a payoff of:

$$\pi_j(p'_j) - C'_j = \pi_j(p'_j) + aV(p'_j) - G'_j < \pi_j(\bar{p}_j) + aV(\bar{p}_j) - G_j = \pi_j(\bar{p}_j) - C_j.$$

That is, (p_j, C_j) in (a) is a better response to (p'_i, C'_i).

Case 2. $G'_j < G_j$. From (21), $\pi_j(\bar{p}_j) - C_j > 0$. Lemma A.1(b) implies that $\exists\, p'_i > 0$ such that

$$\pi_j(\bar{p}_j) - C_j > \pi_j(p'_i). \tag{A.8}$$

From (A.5), we can pick $\varepsilon \in (0, \min\{G_j - G'_j, G_j - aV(p^*)\})$, and have:

$$C'_i \equiv G_j - aV(p'_i) - \varepsilon > G_j - aV(p^*) - \varepsilon > 0.$$

By construction, $G_j > C'_i + aV(p'_i) > \max\{aV(p^*), G'_j\}$. That means, against (p'_i, C'_i), (p_j, C_j) in (a) by j will be accepted, and j's payoff will become $\pi_j(\bar{p}_j) - C_j$. If (p'_j, C'_j) is instead proposed, (p'_i, C'_i) will be accepted, and j's payoff will be $\pi_j(p'_i)$. The setup of (A.8) makes (p_j, C_j) a better response for j than (p'_j, C'_j).

To repeat step (iii) above, one can similarly establish for i that (p_i, C_i) in (a) is not weakly dominated. To establish i's strategy in (a) to be a best response to j's in (a), we shall show (i) among all winning offers, the one in (a) yields i the highest profit. (ii) i is better off by winning with such an offer than by losing to j.

(i) Given the opponent's strategy, the best winning offer (p_i, C_i) for i solves the following problem:

$$\max_{p_i, C_i} \pi_i(p_i) - C_i \quad \text{s.t. } C_i + aV(p_i) \ge G_j$$

Because the right-hand-side of the constraint, G_j, is independent of the choice of (p_i, C_i), for any p_i, the best C_i is the one that holds the constraint in equality. Therefore, with (17), (19) and (21), i's problem can be simplified to be:

$$\max_{p_i} \pi_i(p_i) - [aV(\bar{p}_j) + \pi_j(\bar{p}_j) - \pi_j(\bar{p}_i) + k - aV(p_i)]$$

This is the same problem as in (12), so $p_i = \bar{p}_i$; and accordingly, C_i is what stated in (a). With this offer, i's profit is:

$$[\pi_i(\bar{p}_i) + \pi_j(\bar{p}_i) + aV(\bar{p}_i)] - [\pi_j(\bar{p}_j) + aV(\bar{p}_j)] - k$$

From the assumption $\dot{W}_i \ge \dot{W}_j$ and the definition of k in (21), it is larger than $\pi_i(\bar{p}_j)$, i.e. i's profit when losing to j. $\square$

Proof of Corollary 1. We shall show that $\pi_i(\bar{p}_i) - C_i < \pi_i(p^*)$. Because $\bar{p}_j$ maximizes W_j in (12), we have:

$$\pi_i(p^*) > \pi_i(p^*) + \pi_j(p^*) + aV(p^*) - [\pi_j(\bar{p}_j) + aV(\bar{p}_j)].$$

In view of (10), when $b_1 = b_2 = b$,

$$\pi_i(p) + \pi_j(p) + aV(p) = V(p)(1 - b + a),$$

which is maximized when $V(p)$ reaches the maximum, i.e. when $p = p^*$. This observation, along with the above inequality, leads to:

$$\pi_i(p^*) > \pi_i(\bar{p}_i) + \pi_j(\bar{p}_i) + aV(\bar{p}_i) - [\pi_j(\bar{p}_j) + aV(\bar{p}_j)].$$

From (18), (19) and (21), the right-hand-side is exactly $\pi_i(\bar{p}_i) - C_i + k$, where k is a positive number defined in (21).

On the other hand, (10) and (11) imply that:

$$\pi_i(p) + \pi_j(p) + aV(p) = V(p)(1 - b_i + a + (b_i - b_j)s_j(p)).$$

Because $V(p^*) > V(\bar{p}_i)$ and $s_j(p^*) > s_j(\bar{p}_i)$ (from Lemmas 1 and 2(a)), to repeat the logic above, it can be shown again that $\pi_i(\bar{p}_i) - C_i < \pi(p^*)$, when $b_i > b_j$. □

Proof of Proposition 3. Consider any SPE in our game. Let $i(j)$ denote the winner (loser) in this equilibrium. Clearly, with $\{f_k(p)\}_{k=1,2}$ constructed in (22), the government will choose $p = \bar{p}_i$, and her and lobbyists' payoffs stay the same as the equilibrium payoffs in our game. We shall argue that $f_i(p)$ in (22) is a best response to $f_j(p)$ in the menu auction. The reverse can be similarly established.

Given $f_j(p)$, consider a case in which i modifies his menu in (22) to improve his own payoff. Suppose the government still chooses $\bar{p}_i$ after this change, then i can improve his payoff only through a change in $f_i(\bar{p}_i)$. But to increase $f_i(\bar{p}_i)$ reduces i's payoff, and to decrease $f_i(\bar{p}_i)$ makes $\bar{p}_i$ a choice strictly inferior to $\bar{p}_j$ to the government. (See (15).) In sum, there is no way to improve i's payoff with another menu which still keeps $\bar{p}_i$ a chosen price.

Now, consider i submitting a menu $f_i'(p)$ different from (22), and makes $p'(\neq \bar{p}_i)$ the government's choice. If $p' = \bar{p}_j$, then i's payoff is $\pi_i(\bar{p}_j) - f_i'(\bar{p}_j)$ which is in no way higher than $\pi_i(\bar{p}_i) - C_i$, i's payoff when he submits $f_i(p)$ in (22) instead. Because in our game, given j proposes $(\bar{p}_j, C_j)$, if i proposes $(\bar{p}_j, f_i'(\bar{p}_j))$, the government will set $p = \bar{p}_j$ and collect $C_j + f_i'(\bar{p}_j)$. (See (A.5).) This leaves i a payoff of $\pi_i(\bar{p}_j) - f_i'(\bar{p}_j)$, no higher than $\pi_i(\bar{p}_i) - C_i$, since $(\bar{p}_i, C_i)$ is i's best response to j's $(\bar{p}_j, C_j)$ in our game.

If $p' \neq \bar{p}_j$, we have:

$$aV(p') + f_i'(p') \geq aV(\bar{p}_j) + C_j + f_j'(\bar{p}_j).$$

To repeat logic very similar to that above, one can again negate $f_i'(p)$ being a better menu to i than $f_i(p)$ in (22). □

Proof of Proposition 4.

(a) From Lemma 2(a), it suffices to show $\bar{p} = p^*$. When $b_1 = b_2 = b$, (23) becomes:

$$\bar{p} = \text{argmax}_p (1 - b + a)V(p).$$

Clearly, $\bar{p} = p^*$.

(b) When $b_1 < b_2$, (23) can be arranged as:

$$\bar{p} = \text{argmax}_p V(p)[(b_2 - b_1)s_1(p) + (1 - b_2 + a)].$$

To reapply the logic by which we show $\bar{p}_1 < p^*$ in Lemma 2(a), one can establish that $\bar{p} < p^*$. It remains to show $\bar{p} > \bar{p}_1$. We shall establish (i) $\hat{W}(\bar{p}_1) > \hat{W}(p)$, $\forall\ p < \bar{p}_1$, so $\bar{p} \geq \bar{p}_1$. (ii) $d\hat{W}/dp|_{p=\bar{p}_1} > 0$. It then follows $\bar{p} > \bar{p}_1$.

(i) Lemmas 1 and 2(a) imply that $\pi_2(p)$ in (10) is strictly less than $\pi_2(\bar{p}_1)$, $\forall\ p < \bar{p}_1$. Moreover, $\bar{p}_1$ maximizes $\pi_1(p) + aV(p)$. Thus $\hat{W}(\bar{p}_1) > \hat{W}(p)$, $\forall\ p < \bar{p}_1$.

(ii) We already know $d(\pi_1(p) + aV(p))/dp|_{p=\bar{p}_1} = 0$. It remains to show $d\pi_2(p)/dp|_{p=\bar{p}_1} > 0$. From Lemma 1, $ds_2(p)/dp > 0$, and from Lemma 2(a), $dV(p)/dp|_{p=\bar{p}_1} > 0$. In view of (10), the claim is established.

(c) Reasoning similar to that in (b) can be applied to show (c).

□

Proof of Proposition 5. We first show that in (24), $z > x + y$. Only the case $b_1 \leq b_2$ will be considered. Similar logic can be applied to the other case. From (24),

$$z - (x + y) = [W_1(\bar{p}_1) - W_1(\bar{p})] + [W_2(\bar{p}_2) - W_2(p^*)] + [\pi_2(p^*) - \pi_2(\bar{p})]. \tag{A.9}$$

From Proposition 4, when $b_1 \leq b_2$, $\bar{p}_2 > p^* \geq \bar{p} > \bar{p}_1$. This establishes the sum of the first two terms in the right-hand-side of (A.9) to be strictly positive. For the same reason that we establish Lemma 2(b), it can be shown $\pi_2(p^*) \geq \pi_2(\bar{p})$. Thus $z > x + y$.

With this inequality, we have $\bar{n}_1 = x$, $\bar{n}_2 = y$ in (24), and

$$G_m = W_1(\bar{p}_1) + W_2(\bar{p}_2) - (\pi_1(\bar{p}) + \pi_2(\bar{p}) + aV(\bar{p})).$$

From (A.6), we have:

$$G_i - G_m = (\pi_1(\bar{p}) + \pi_2(\bar{p}) + aV(\bar{p})) - (\pi_i(\bar{p}_i) + \pi_j(\bar{p}_i) + aV(\bar{p}_i)) + k > 0.$$

□

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