OPTIMALITY OF INVESTMENT UNDER IMPERFECTLY ENFORCEABLE FINANCIAL CONTRACTS

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We investigate the optimality of aggregate investment and its policy implications under an environment in which financial contracts are imperfectly enforceable. We show that too much investment occurs when the ratio of own capital to debt is smaller than the ratio of project returns in terms of future values across periods, and too low investment occurs otherwise. A subsidy (tax) on the risk-free interest income can close the over- (under-) investment gap, but this policy may not be welfare improving. (JEL E62, E44, G14)

I. INTRODUCTION

A well-known implication of asymmetric information in credit markets, as studied by Stiglitz and Weiss (1981), is that when the quality of projects is private information, low-quality projects will drive high-quality projects out of the market, resulting in credit rationing. This means there is underinvestment compared with the optimal level of investment given full information. In contrast, De Meza and Webb (1987) reach an entirely different result using a framework in which the environment and information structure are very similar to Stiglitz and Weiss. They show that asymmetric information causes good projects to draw in bad and leads to too much investment than is socially efficient.[1] As pointed out in their article, it is their specification of the structure of stochastic project returns that leads to this strikingly different result.[2] Thus, it raises a question regarding the optimality of investment in an environment in which the structure of stochastic project returns plays no role.

Apart from this difference, a common feature of the two articles, together with a large body of literature, is that the financial contracts are perfectly enforceable. However, borrowers may default either because their investment returns are below what were promised to the lenders or because they can profit more by running away without repaying their debts. Absconding without repaying debt is feasible only when the lenders have limited control of the investment returns or the savings of the borrowers. Akerlof and Romer (1993) distinguish this aspect of moral hazard, dubbed as "looting," from the pursuit of highly risky investments to "gamble for resurrection." In this case lenders and the courts are not able to enforce repayments from the borrowers.[3]

1. See Proposition 2 in De Meza and Webb (1987) for details.

2. In De Meza and Webb (1987), the set of project return contains only two realizations—high and low—which is the same across entrepreneurs. Entrepreneurs differ in their probabilities of success. Therefore, "good" entrepreneurs have higher expected returns than the "bad." In Stiglitz and Weiss (1981), the expected return is constant across entrepreneurs. "Bad" entrepreneurs can have a very high realization of return with a low probability of success, whereas the "good" have rather smoother returns across states.

3. Page 2 of Akerlof and Romer (1993) says, "Poor accounting, lax regulation, or low penalties for abuse give owners an incentive to pay themselves more than their firms are worth and then default on their debt obligations." In particular, looting is more likely when looters can count on the government to bear the losses.

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In this article we study the optimality of aggregate investment in a nonstochastic environment with private information and limited enforcement. We show that an over- (under-) investment occurs if a borrower's capital–debt ratio is smaller (larger) than the ratio of cross-period cash flows. In other words, too much investment occurs when the cash flows of investment returns are accrued relatively quickly or when the stake in a borrower's project is too low. This implies that when entrepreneurs prefer projects that yield a faster stream of returns, it is more likely to result in over-investment because it is either easier for entrepreneurs to be solvent or because this attracts borrowers who want to pocket a higher profit and run away without repaying. Moreover, over-investment is more likely to arise where entrepreneurs have a low level of net worth relative to their debt, which may be due to a lenient collateral requirement or lax lending practice.

The results herein have policy implications for economies in which enforcement is costly due to either primitive screening and monitoring technology or inefficiency in the court system. We show that a subsidy (tax) on risk-free interest income can close the over- (under-) investment gap, but in contrast to De Meza and Webb (1987), this policy tends to reduce social welfare.

The rest of the article is organized as follows. Section II outlines the environment of the model. Section III states the decision rules under limited enforceability and then derives the condition for over- or underinvestment. Section IV analyzes the policy implications of this model, and section V concludes.

II. THE ENVIRONMENT

Consider a three-period economy, indexed by $t = 0, 1$, and 2. There are a continuum of risk-neutral agents with a population normalized to be unity and many competitive intermediaries. The gross risk-free interest rate $\bar{r}$, yielded from a safe asset, is assumed to be fixed. In each period there is only a single consumption good. At date 0 each agent is endowed with w units of goods, $w > 0$. Agents differ in their entrepreneurial ability indexed by e, $e \in [0, 1]$. Entrepreneurial abilities are independent across agents, and for each agent, e is distributed according to the

probability density function $g(\cdot)$ and probability distribution function $G(\cdot)$. A higher e means better entrepreneurship or productivity, which is perfectly correlated with the agent's investment return.

At date 0 each agent has access to an investment technology that takes I units of goods and yields a certain $q_1 e$ units of output at date 1 and $q_2 e$ at date 2 for an agent with entrepreneurial ability e. The amount of investment I is strictly greater than the agent's endowment, thus external financing is necessary. Each potential entrepreneur takes an amount of loan $B = I - w$ from her bank. If a project is terminated at date 1, then the liquidation value of the project is δ, $\delta < I$.

Information Structure and Contracting Problem

There are two frictions in this model. First, we assume that borrowers can choose to run away with investment returns without repaying their loan obligations. Absconding borrowers, however, cannot take the project with them, and creditors alone do not have the required skill to operate these projects. Hart and Moore (1998) study the foreclosure right of debt contracts under the same assumptions. In a model where reputation effect does not work, the only way banks can secure their loan repayment is to threaten to liquidate projects. Because borrowers have no incentive to pay anything at date 2, they are required to repay at date 1. If a borrower default, then the bank seizes the asset (the funded project) and liquidates it.

Second, an individual's entrepreneurial ability is nonverifiable to outsiders. If the entrepreneurial ability is publicly observable, then lenders would be able to figure out which borrowers will run away, and thus the nonenforceability problem can be resolved. When the entrepreneurial ability is private information, the direct financing is not viable even if it is feasible. The function of banks is to pool funds and to maintain zero expected profit by smoothing away those who will default through diversification. Banks are able to seize and liquidate the projects that borrowers leave behind to compensate their losses.

According to our specification, the information structure outlined above corresponds to a nonstochastic case in Hart and Moore

(1998).[4] They show that all debt contracts that satisfy lenders' break-even constraint with equality are optimal.

The Optimal Investment under Full Information and Perfect Enforceability

As a benchmark, we first consider the optimal level of investment when entrepreneurial ability is publicly observable and contracts are enforceable. In this case, all projects with nonnegative net present value, $Qe \geq \bar{r}^2 I$, will be financed, where $Q \equiv \bar{r}q_1 + q_2$ is the future value of a project's total returns. Let $e^F \equiv \bar{r}^2 I/Q$ be the minimum level of entrepreneurial productivity at which the project yields a zero net present value. We assume that $q_1 e + \delta - \bar{r}I < 0$ for any $e < e^F$, which means it is never optimal for the economy to undertake a project and then liquidate it at date 1. This is equivalent to requiring that the project's liquidation value must be low enough,

$$(1) \qquad \delta < \bar{r}q_2 I/Q.$$

Those agents with ability exceeding e^F become entrepreneurs, whereas the rest are depositors. Thus the optimal level of investment is $(1 - G(e^F))I$. Because there will be no default or early liquidation, the gross loan interest rate should be equal to the opportunity cost of funds, which is $r = \bar{r}$.

A notable aspect of this benchmark result differing from those cases below is that since debts are enforceable, a project may be financed even when the borrower's date 1 output is less than his or her debt obligation, $q_1 e < B\bar{r}$. The borrower is allowed to postpone the rest of unpaid debt at date 2. This occurs when $e^F < B\bar{r}/q_1$. Thus, for those entrepreneurs with ability $e \in [e^F, B\bar{r}/q_1]$, they will repay $q_1 e$ at date 1 and the rest $(B\bar{r} - q_1 e)$ at date 2, leaving $(Qe - B\bar{r})$ to him- or herself. Recall that we have assumed $Qe \geq \bar{r}^2 I$. Thus, $Qe - B\bar{r} > \bar{r}^2 w$, such that borrowers' participation is assured.

4. Note that the only difference is that in our article, borrowers are heterogeneous in entrepreneurial ability. However, this does not alter the basic information structure, because the project returns are perfectly correlated with entrepreneurial ability, which is also nonverifiable as in Hart and Moore.

III. THE EQUILIBRIUM UNDER LIMITED ENFORCEABILITY

Given private information of entrepreneurial ability and limited enforcement of repayment, we analyze the occupational choice and default decision for an agent with characteristics (w, e). If some borrowers default, then a loan's rate of return is required to be greater than $\bar{r}$. Let π be the fraction of default to a bank, $\pi \in [0, 1)$. Given the loan contract (B, r), a representative bank earns an expected profit $E(\Pi^B) = Br(1 - \pi) + \delta \pi - B\bar{r}$. Given that the banking sector is competitive, equilibrium requires that the expected profit of each bank is zero. After rearranging, the zero profit condition is

$$(2) \qquad (1 - \pi)r + \pi\delta/B = \bar{r}.$$

Because the equilibrium rate of interest must be greater than $\bar{r}$ when the fraction of default is positive, (2) implies $\delta < B\bar{r}$, meaning that liquidating a project is unprofitable for the bank. Equations (1) and (2) result in a restriction for δ:

$$(3) \qquad \delta < \min\{B\bar{r}, \bar{r}q_2 I/Q\}.$$

An agent will choose one of the following strategies at date 0: (S1) invest at date 0 and not default at date 1; (S2) invest at date 0 and default at date 1; or (S3) not invest at date 0 and be a depositor. The appendix outlines the conditions under which a certain strategy is selected by an agent with ability e.

According to the decision rules, this may lead to a different equilibrium with a different relative magnitude of cash flows q_1 and q_2. In what follows, we consider the simple case in which the cash flows across periods are equal, specifically, $q_1 = q_2 = q$. The other cases, $q_1 < q_2$ and $q_1 > q_2$, can be characterized in the same manner.

The Equilibrium under Equal Cash Flows

Given that date 1 and date 2 cash flows are equal, the decision rules can be presented graphically as in Figure 1. The area, $\{(e, r) \mid e \geq B\bar{r}r/q$ and $e \geq \bar{r}(Br + w\bar{r})/Q, \forall r\}$, represents the combination of entrepreneurial ability and interest rate such that an agent will decide to invest and not default, where $Q = (1 + \bar{r})q$. Some borrowers may default, because they are either insolvent, that is,

FIGURE 1
Equal Cash Flows $q_1 = q_2 = q$

| Invest and pay back | Invest, solvent and default | Invest, insolvent and default |
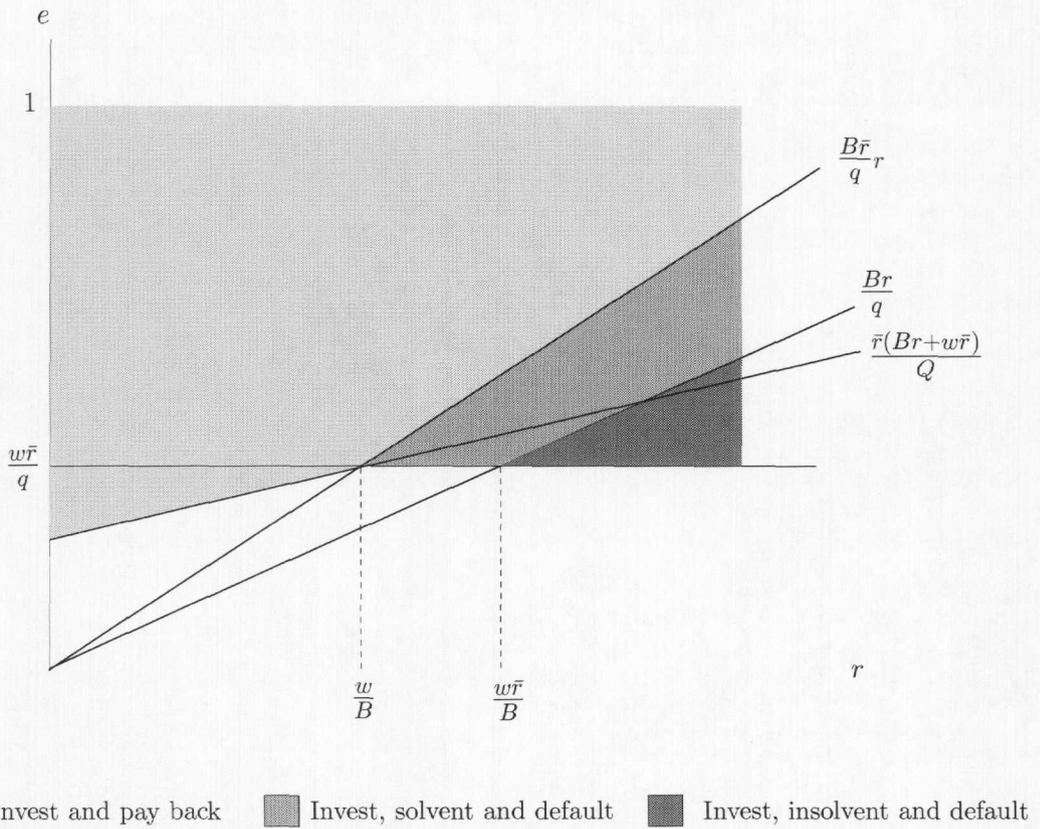
$\{(e, r) \mid e \geq w\bar{r}/q$ and $e \leq Br/q, \forall r > w\bar{r}/B\}$, or because they are better off taking the money and running away even though they are solvent, that is, $\{(e, r) \mid e < B\bar{r}r/q, e \geq w\bar{r}/q$, and $e \geq Br/q, \forall r > w/B\}$. The agents located in the rest of the area become depositors. The intersection of the three lines, $B\bar{r}r/q$, $w\bar{r}/q$, and $\bar{r}(Br + w\bar{r})/Q$, is at point $r = w/B$, which is the inside capital–debt ratio. This ratio will turn out to be important in determining whether there is overinvestment or underinvestment.

From Figure 1 if the interest rate is such that $r \leq w/B$, then there will be no default, and thus the equilibrium interest rate must be $r = \bar{r}$. In this case, the aggregate investment is $(1 - G(e^F))I$, which is equivalent to the optimal level of investment.

Suppose instead that the equilibrium interest rate is such that when $r > w/B$, then the agents' ability within the region $\{e \mid w\bar{r}/q < e \leq B\bar{r}r/q$ for $r \geq w/B\}$ will default. The fraction of default is therefore

$$(4) \qquad \pi = [G(B\bar{r}r/q) - G(B\bar{r}/q)]$$
$$/[1 - G(B\bar{r}/q)].$$

In the following we assume that the probability density function $g(e)$ is uniformly distributed on the support $[0, 1]$. The equilibrium interest rate can then be solved according to (4) and the bank's zero profit condition (2):

$$(5) \qquad r^* = [(\bar{r}\delta + q)]/(2B\bar{r}) \pm 1/2\sqrt{\Delta},$$

where $\Delta = [(\bar{r}\delta + q)^2 - 4\bar{r}^2 B(q - w\bar{r} + \delta w/B)]/B^2\bar{r}^2 > 0$ is assumed to hold.[5] We

5. We may in fact have imaginary roots. We do not dwell on this detail and simply assume that the parameter structure is such that both roots are real.

thus denote r_h to be the larger solution and r_l to be the smaller one in (5). Because a borrower can shop around to select the best offer by the banks, it is straightforward to show that the lowest zero-profit loan interest rate is an equilibrium.

Note that the amount of aggregate investment is insensitive to a change in the loan interest rate for the given range $r > w/B$. This is because the amount of aggregate investment $[1 - G(w\bar{r}/q)]I$ is governed by the participation decision of those marginal defaulters and depositors, weighing date 1 output against the borrower's opportunity cost. A change in the loan interest rate only affects the fraction of default but does not affect the amount of aggregate investment. This property has an important implication for policies that are intended to affect the aggregate investment.

Optimality of Investment

Because the two solutions to (5) are the possible equilibrium rates of interest when the fraction of default is positive, any solution lower than the risk-free rate $\bar{r}$ cannot be an equilibrium. We check the relative magnitude between the smaller root r_l and $\bar{r}$:

$$(6) \qquad r_l - \bar{r} \propto (\delta - B\bar{r})(w - B\bar{r}).$$

According to constraint (3), we know $\delta - B\bar{r} < 0$, and thus the smaller root r_l is lower than $\bar{r}$ if $w/B > \bar{r}$; that is, if the inside capital–debt ratio of the borrower is greater than $\bar{r}$. When this occurs, the equilibrium rate of interest is indeed the larger root r_h. On the other hand, when $w/B < \bar{r}$, the equilibrium interest rate is r_l.

Because the aggregate investment is $(1 - G(w\bar{r}/q))I$ and the optimal level is $(1 - G(e^F))I$, it is straightforward to check that there is overinvestment if

$$(7) \qquad\qquad w/B < \bar{r}$$

and vice versa. This says that overinvestment occurs, at which the equilibrium interest rate is r_l, if the inside capital–debt ratio of the borrower is less than $\bar{r}$, and underinvestment occurs, at which the equilibrium interest rate is r_h, if otherwise.

In the case of overinvestment where the equilibrium interest rate is $r^* = r_l$, a fraction $[G(Br/q) - G(w\bar{r}/q)]$ of borrowers is drawn into business so as to exploit the benefit from absconding with date 1 output. In the case of underinvestment where the equilibrium rate is $r^* = r_h$, the fraction of default is even larger, because $G(Br_h/q) > G(Br_l/q)$.

IV. POLICY IMPLICATIONS

As discussed, policies that directly affect the loan rate of interest will have no effect on the level of aggregate investment and thus will not affect the participation decision of those marginal depositors and defaulters. According to (7), over-investment arises when the inside capital-debt ratio of a borrower is lower than the risk-free rate. To close the gap, the government may subsidize on the risk-free interest income. To see this, let s be the subsidy rate, and note that the investment gap from optimality is

$$(8) \quad D = (1 - G(w\bar{r}(1+s)/q))I$$
$$- (1 - G(e^F))I$$
$$= -\bar{r}BI[1/(1+\bar{r}) + s][w/B - \bar{r}_s]/q,$$

which is positive when there is over-investment, where $\bar{r}_s = \bar{r}[1 + s(1+\bar{r})]^{-1}$. It can be checked that D is decreasing in the subsidy rate, $dD/ds < 0$, and thus the gap becomes smaller when the subsidy rate increases. The investment gap drops to zero when the subsidy rate is set at $s = (B\bar{r} - w)/w(1 + \bar{r})$, which is strictly positive by (7).

We then check the fraction of default under this interest rate policy. Note that the bank's zero-profit condition now becomes

$$(9) \qquad (1 - \pi_s)r_s + \pi_s\delta/B = (1+s)\bar{r},$$

where π_s and r_s are the fraction of default and loan rate under subsidy, respectively. According to (4), we have

$$(10) \qquad \pi_s = [G(B\bar{r}[1+s]r_s/q)$$
$$- G(w\bar{r}[1+s]/q)]$$
$$/[1 - G(w\bar{r}[1+s]/q)].$$

Using (9) and (11) and imposing $\pi_s = 0$, we can solve for the corresponding subsidy rate $s = (w - B\bar{r})/B\bar{r}$, which is negative by (7). This implies that a tax (rather than a subsidy) on risk-free interest income is needed to eliminate the default problem.

The intuition for why a subsidy on risk-free interest income can close the over-investment gap is that the subsidy raises the opportunity cost of those marginal entrepreneurs, inducing them to switch to become depositors and thus reducing investment. However, the subsidy raises the banks' cost of funds and thus the loan rate, leading to a higher fraction of default. This contrasts with De Meza and Webb's clear-cut result that an interest rate policy can unambiguously restore optimality. The welfare implication of this interest rate policy is thus obscure.

To further investigate the welfare implication of this interest rate policy, we consider a social loss function given by

$$(11) \quad L = \int_{w\bar{r}/q}^{e^F} I\bar{r}^2 dG(e) + \int_{w\bar{r}/q}^{B\bar{r}/q} qe dG(e),$$

where the first term measures the efficiency loss due to over-investment and the second term measures date 2's forgone output, which is lost due to default. Replacing $\bar{r}$ with $(1+s)\bar{r}$ and taking the derivative with respect to s, we are able to derive a sufficient condition for $dL/ds > 0$ is that $dr/ds > I/rB$, meaning the subsidy on risk-free interest income to reduce over-investment is not welfare improving if it causes the loan interest rate to increase too much. This is more likely to happen when the loan interest rate is already at a high level or when the inside capital–debt ratio is relatively low (that is, I/B is low). The latter condition is exactly what leads to over-investment, which means this interest rate policy tends to lower social welfare, because the subsidy that raises the loan interest rate and fraction of default will cause more forgone output than is saved from the efficiency gain due to a decrease in the over-investment gap.

In contrast, when there is underinvestment, a tax on the risk-free interest income will close the gap. Denoting the tax rate as τ, the investment gap from optimality is

$$D = -\bar{r}BI[1+\bar{r}-\tau][w/B - \bar{r}_\tau]/q < 0,$$

where $\bar{r}_\tau = \bar{r}[1 - \tau(1+\bar{r})]^{-1}$. We find that D is increasing in the tax rate, $dD/d\tau > 0$, and therefore the gap can be closed by raising the tax rate, because a tax on risk-free interest income encourages more depositors to take out loans and to invest. Following the above argument, however, this tax policy raises the fraction of default, because these marginal depositors switching to become entrepreneurs will default for sure, thus lowering welfare.

PROPOSITION 1. *Given that the cash flow of date 1 from the project is equal to that of date 2, $q_1 = q_2$, (a) Over-investment will occur when the inside capital–debt ratio of the borrower is less than the risk-free rate of interest; otherwise, underinvestment will result. (b) The government can subsidize on the risk-free interest income to close the investment gap in the case of over-investment and tax in the case of underinvestment. These policies, however, raise the fraction of default and are likely to lower social welfare.*

V. CONCLUDING REMARKS

In this article we essentially provide a counterexample to De Meza and Webb (1987) by investigating the optimality of investment and policy implications in a class of models with private information and limited enforcement. Much of the analysis is done for the case of equal cash flows in which returns from investment projects are the same in both periods. In general, when investment returns are different across periods, a generalization of (7) is that over-investment occurs if the ratio of inside capital to debt is smaller than the ratio of date 1 to date 2 project returns, which is $w/B < \bar{r}q_1/q_2$.[6] When q_1 is larger (smaller) than q_2, over- (under-) investment is more likely to occur than the case when q_1 and q_2 are equal. In practice, the benchmark case (equal cash flows) and the case with increasing cash flows are more plausible. Particularly, when cash flows are rising over time, there will be less looting and a subsidy on interest-rate income is likely to raise welfare.

The model so far concentrates on the feature that banks lack the capability to monitor and enforce borrowers' repayments and thus potential borrowers of differential productivity are all able to finance their projects as long as they decide to borrow. One might wonder how things will be different if we assume that entrepreneurial ability is partially observable. This is in fact equivalent to

6. The analysis of unequal cash flows can be obtained on request from the authors.

the assumption that banks have access to a screening technology with low costs. It would be an interesting extension to allow the banks to imperfectly identify the true productivity of a potential borrower with a cost.

APPENDIX

(S1) TO INVEST AT DATE 0 AND NOT TO DEFAULT AT DATE 1

An agent will choose to be an entrepreneur and repay debt at date 1 if

(A-1) $\quad q_1 e > Br \quad$ and $\quad Qe/\bar{r} - Br \geq \max\{q_1 e, w\bar{r}\}$,

where the first inequality means that the agent is solvent at date 1, and the second means that total cash flows net of debt obligation is greater than or equal to the maximum between date 1 cash flow and his or her initial capital. If $q_1 e > w\bar{r}$, the second inequality says that paying back debt and staying in business until the end of date 2 is better than running away with $q_1 e$ at date 1, whereas if $q_1 e < w\bar{r}$, the second inequality guarantees the agent's participation. These conditions can be combined into

(A-2) $\quad e > Br/q_1, e > B\bar{r}r/q_2, \quad$ and

$$e > \bar{r}(w\bar{r} + Br)/Q.$$

(S2) TO INVEST AT DATE 0 AND DEFAULT AT DATE 1

An agent will borrow and invest but default at date 1 if

(A-3) $\quad q_1 e > Br \quad$ and $\quad q_1 e \geq \max\{Qe/\bar{r} - Br, w\bar{r}\}$,

or if

(A-4) $\qquad q_1 e < Br \quad$ and $\quad q_1 e > w\bar{r}$.

The situation (A-3) occurs for an agent who defaults even though he or she is solvent ($q_1 e > Br$). The reason he or she defaults is because total outputs net of debt repayment is lower than the amount he or she can steal ($q_1 e > Qe/\bar{r} - Br$). This strategy is also better than being a depositor ($q_1 e > w\bar{r}$). We name these entrepreneurs looters in the spirit of Akerlof and Romer (1993). The situation (A-4) occurs for an agent who is endowed with an even lower ability, such that he or she cannot afford date 1 debt repayment ($q_1 e < Br$), however, he or she is better off being an entrepreneur than being a depositor, because his or her date 1 cash flow is greater than the opportunity cost ($q_1 e > w\bar{r}$). Therefore, these agents are drawn into the business to crop date 1 cash flow and run away. We call these entrepreneurs outright crooks. The conditions in (A-3) imply

(A-5) $\quad e > Br/q_1 \quad$ and $\quad e > w\bar{r}/q_1 \quad$ and $\quad e < B\bar{r}r/q_2$.

On the other hand, the conditions in (A4) imply

(A-6) $\qquad e < Br/q_1 \quad$ and $\quad e > w\bar{r}/q_1$.

(S3) NOT TO INVEST

An agent prefers being a depositor if

(A-7) $\qquad q_1 e < Br \quad$ and $\quad q_1 e < w\bar{r}$,

or if

(A-8) $\quad q_1 e > Br \quad$ and $\quad w\bar{r} \geq \max\{Qe/\bar{r} - Br, q_1 e\}$.

The conditions in (A-7) state that the agent does not want to invest because he or she will be insolvent at date 1 and also because the amount he or she can run away with is smaller than initial capital. This is equivalent to

(A-9) $\qquad e < Br/q_1 \quad$ and $\quad e < w\bar{r}/q_1$.

The conditions in (A-8) state that even though the agent is solvent at date 1 he or she will not participate anyway, and they imply that

(A-10) $\qquad e > Br/q_1 \quad$ and $\quad e < w\bar{r}/q_1 \quad$ and

$$e < \bar{r}(w\bar{r} + Br)/Q.$$

REFERENCES

Akerlof, G., and P. M. Romer. "Looting: The Economic Underworld of Bankruptcy for Profit." *Brooking Papers on Economic Activity*, 2, 1993, 1–60.

De Meza, D., and D. C. Webb. "Too Much Investment: A Problem of Asymmetric Information." *Quarterly Journal of Economics*, 102(2), 1987, 281–92.

Hart, O., and J. Moore. "A Theory of Debt Based on the Inalienability of Human Capital." *Quarterly Journal of Economics*, 109(4), 1994, 841–79.

———. "Default and Renegotiation: A Dynamic Model of Debt." *Quarterly Journal of Economics*, 113(1), 1998, 1–41.

Stiglitz, J., and A. Weiss. "Credit Rationing in Markets with Imperfect Information." *American Economic Review*, 71(3), 1981, 393–410.