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A SEARCH MODEL OF MONEY AND CIRCULATING PRIVATE DEBT WITH APPLICATIONS TO MONETARY POLICY*

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A search-theoretic model is used to examine the coexistence of money and circulating private debt. Money is still valued even though there coexists credit which circulates among agents and dominates in the rate of return. When there coexist multiple equilibria, the equilibrium with credit Pareto dominates the one without credit if money supply is not extremely plentiful. This article also provides some predictions about the effects of monetary policies. A policy of open market operations whereby government discounts less for second-hand debt decreases the value of money, credit, and interest rate prevailing in the market. This policy can also improve welfare by making credit trade feasible when the only equilibrium entails no credit without intervention.

1. INTRODUCTION

Both credit and money can support exchange when there is an absence of double coincidence of wants. Why is money valued facing the competition from an asset which is used as a medium of exchange and bears a positive rate of return? What makes credit an imperfect substitute for currency even though it has characteristics (such as it can circulate) which make it a close substitute? To explain the coexistence of money and circulating private debt, we need a model in which it is somehow difficult to carry out exchange.[2] Search-theoretic models, which permit us to be explicit with the transaction patterns and exchange process, are suitable for representing trading frictions such as the absence of double coincidence of wants.

Using a search-theoretic model with money, bargaining, and credit, Shi (1996) shows that money coexists with credit which yields a higher rate of return. [3] However,

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[2] Townsend (1989) studies an economy with private information and spatial separation in which currency is used among strangers while credit can be used among agents in an enduring relationship. Townsend and Wallace (1987) explain the existence of circulating private securities in a model of intertemporal trade in spatially and informationally separated markets. Bernhardt (1989) shows that enduring trade relationships are valued and loans are optimal in a small close-knit economy.

[3] Previous search-theoretic models on money include Kiyotaki and Wright (1989, 1991, 1993); previous search-theoretic models on money and bargaining include Trejos and Wright (1995)

he imposes a restriction to preclude circulation of private debt, which makes credit an obviously inferior means in conducting transactions compared to fiat money. Aiyagari et al. (1996) present a form of legal restriction to show the coexistence of money and interest-bearing default-free government securities, but private debt is ruled out by assumptions. To consider more sensibly the effects of monetary policy such as open market operations, we need to incorporate private debt which change hands among agents. In this paper, I extend Shi's (1996) model to explicitly consider circulation of private debt in order to capture the following features in a modern economy: credit arises endogenously; money is used as a medium of exchange, a means to repay the debt, and to buy second-hand debt; and private debt can be cleared through third parties.[4]

In this economy the trading frictions that give rise to money as a medium of exchange also create the role for credit. I demonstrate that, while the characteristics of credit (such as it can alleviate the double coincidence problem and debt instruments circulate among agents) make it a close substitute for money, it is not a perfect substitute. Money is still valued even though credit dominates in the rate of return. Credit is inferior to money since in this economy monetary repayment is the only means to retire the debt and repayment takes time. I find that monetary equilibria exist with and without credit. When there coexist multiple equilibria, the equilibrium with active credit trade can entail higher welfare if the stock of money is not extremely large. Credit and money can support more exchange than either instrument alone; i.e., the coexistence of money and credit increases efficiency. It is also shown how the purchasing power of money, interest rate, and discounts on second-hand debt depend on the details of the model such as whether liquidity is ample and how impatient people are.

I then proceed to examine the effects of monetary policies in this economy. There are several studies on government policy in monetary search models. For example, Aiyagari and Wallace (1997) and Li and Wright (1998) model government transaction policy regarding what government accepts in transaction and examine its effects on private agents' trading strategies.[5] For a theoretical consideration, I want to explore additional policy implications for search-theoretic models by studying an economy with money and circulating private debt. I formulate government agents as in Aiyagari and Wallace (1997) and Li and Wright (1998) by including a class of agents, called government agents, who are subject to the same trading frictions as private agents, but who perform some exogenously specified trading policies. Their role is to conduct the open market operations by buying and selling second-hand securities and issuing debt at prices specified exogenously.

The objective here is to show how monetary policies affect the acceptance of private debt and welfare. I also study how these policies affect the value of money,

and Shi (1995); and previous search-theoretic models of credit include Diamond (1990), but Shi (1996) provides the first model with money, bargaining, and credit.

[4] Some of the features have been explored in overlapping generations models (see, e.g., Freeman, 1996a, 1996b; Green, 1997).

[5] Some more examples include Ritter (1995), who studies a model where the role of government is to issue fiat money in the first place, and Green and Weber (1996), who study a model where the role of government agents is to detect and confiscate counterfeit notes.

interest rates, and discounts on second-hand debt. It is found that when credit does not exist without government intervention, monetary policies can induce the existence of equilibrium with active credit trade and thus increase welfare. One such policy is an open market operation that discounts less for second-hand debt than the market rates. This policy drives down the value of money and interest rate prevailing on the market. Since debtors' cost to acquiring money to repay the debt is determined by the value of money, this policy makes repayment of debt incentive compatible and thus credit arrangements become feasible.

The rest of the article is organized as follows. Section 2 presents the basic model. Section 3 discusses the existence and properties of monetary equilibrium where credit trade is feasible. In section 4, I introduce government into the basic model and discuss the effects of various monetary policies. Section 5 concludes with a summary of the main results, a discussion of some historical evidence, and suggestions for future work.

2. THE BASIC MODEL

I use the frameworks in Shi (1996) and Aiyagari et al. (1996) to motivate the basic model.

2.1. *The Environment*. Time is discrete and the horizon is infinite. There is a $[0, 1]$ continuum of infinitely lived agents. Specialization, which motivates gains from trade but also makes trade difficult, is modeled here as follows. There are N distinct, perfectly divisible but perishable goods (or services) at each date and N types of agents with equal population, where $N \geq 4$. Each type is specialized in consumption and production: a type i agent consumes good i and produces good $i+1$ (modulo N), for $i = 1, 2, \ldots, N$. Note that, to rule out a double coincidence of wants in any meeting between two agents, we need only $N \geq 3$, while a stronger assumption is imposed here to rule out the possibility to repay debts with goods.[6] When agent i consumes q units of his consumption good he enjoys utility $u(q)$; when he produces q units of his production good he suffers disutility $c(q)$. We normalize $c(q) = q$ with no loss of generality. Production is instantaneous. The utility function is defined on $[0, \infty)$, is strictly increasing and twice differentiable, and $u(0) = 0$, $u'(0) = \infty$, and $u''(q) < 0$ for all $q > 0$. Also, there is a $\hat{q} > 0$ such that $u(\hat{q}) = \hat{q}$. Each agent i maximizes expected discounted utility with a discount rate r.

There are no centralized market places for trading goods or assets. Agents enter a trading process characterized by bilateral random matching. In each period, trading partners arrive to an agent according a Poisson process with a constant rate $\beta > 0$. The meeting technology exhibits constant returns to scale; i.e., the number of agents with whom a given agent is matched is the arrival rate times the total number of agents involved in exchange. Agent's trading history is private information to the agent. However, agent's type and asset holding, such as holding no asset, a unit of money, or private debt, is public information.

[6] This simplifies discussion on the credit arrangements. Allowing repayment in goods yields slightly different results, see Shi (1996).

2.2. *Fiat Money and Credit*. There are potentially two types of assets in this economy: fiat money and IOUs. All assets are storable and indivisible. Each agent has a storage capacity of one unit of some asset. The assumption on asset indivisibility and the unit upper bound on individual asset holdings is in the interest of tractability.

The economy begins with a proportion of M agents each holding one unit of money (called *money holders*) and $1 - M$ agents each with a production opportunity. These agents are evenly distributed across the N types. Note that types are identical except as regards what they consume and produce. This symmetry makes it sensible to look for equilibria which are symmetric across types.

In order for there to be private IOUs in a steady state, it has to be created and retired. The credit arrangement in this random-matching framework is modeled as follows. When two *producers* (agents who can produce and have no money or any contractual agreement) meet and one can produce the other's consumption good, an IOU can be created in exchange for good. An IOU is retired as the issuer (called *debtor*) gains a unit of some asset to repay the debt.

To be more specific, consider a meeting between producers of type i and $i + 1$. Given the assumed specialization in production and consumption, agent i produces what agent $i + 1$ consumes while agent $i + 1$ produces good $i + 2$, which is not agent i's consumption good. This is called a *single-coincidence meeting.* In this case, agent $i + 1$ can issue an IOU to agent i, which promises to pay one unit of some asset in the future, in exchange for some amount of good $i + 1$. If both agrees to trade, agent i produces for agent $i + 1$ and the latter consumes it and issues an IOU. Agent i becomes a *creditor* and agent $i + 1$ a debtor.

To induce repayment, assume that there is a technology that allows agents to consume only if they posses a particular object, and this object can be surrendered to creditors for use as collateral.[7] Hence, as long as future consumption yields a positive value to the debtor, he will repay the debt and regain his collateral as soon as possible. Assume that each agent has only one object that can be used as collateral so that he cannot issue multiple IOUs. This implies that credit limit is one.

In this random-matching economy, in order for debtors to find creditors and repay the debt, we assume some communication technology that enables debtors to trace those who hold their IOUs. For example, assume each agent has a pager that merely allows communication between debtor and the agent who holds his IOU (and collateral), but not others. This precludes credit chains to develop in this economy (i.e., agents cannot be a creditor and debtor at a time). It is also assumed that the technology does not allow agents to communicate the locations of other agents and, consequently, the swap of IOUs is ruled out. Note that since it is not feasible for agents to swap IOUs, monetary repayment is the only means to retire debt.

In this economy, there are no restrictions on the exchange of second-hand debt. When a creditor meets a suitable money holder or producer, he can sell off his debt

[7] Following Diamond's (1982) story about search models being like island economies where the consumption good is called a "coconut," Shi (1996) called the object needed for consumption a "knife." Alternatively, we can assume a legal system enforcing costlessly repayment of debt as in Diamond (1990), or assume that creditor and debtor stay together and search.

TABLE 1
EXCHANGE PATTERNS IN A MONETARY EQUILIBRIUM WITH CREDIT*

(Consumers)	Producer (Agents Who Produce)	Creditor	Debtor	Money Holder
Producer	newly issued IOUs in exchange for goods (q_c)†	X	X	X
Creditor	second-hand IOUs in exchange for goods (q_{sc})	X	X	X
Debtor	X	X	X	X
Money holder	money in exchange for goods (q_m)	money in exchange for second-hand IOUs plus goods (q_{mc})	money in exchange for goods (q_d)	X

* X denotes no trade in the meeting.
† The quantity inside the parentheses is the amount of goods produced in the exchange.

contract for money or some amount of consumption good.[8] The agent who buys the debt contract becomes new creditor. When debtor acquires money, he finds the new creditor and repay the debt. Hence, unlike Shi (1996) ruling out creditors from exchange and circulation of debt instruments, we allow IOUs to change hands and to be settled through third parties.

2.3. *Exchange and Bargaining*. The sequence of actions within a period occurs as follows. Each agent begins a period holding one unit of some asset or nothing. Assume that each period consists of two subperiods.[9] In the first subperiod agents meet pairwise at random. Agents in pairwise meetings bargain and if it results in exchange, then production and consumption occurs. The second one is the repayment subperiod in which debtors who acquire money at the first subperiod make repayment and clear the debt. Then agents begin the next period. Assume that the act of clearing debt takes only a short moment before the next period begins and so we can ignore the discount factor for the repayment subperiod.

In this economy all trades require at least a single coincidence of wants. Potentially there are credit trade, monetary trade, and trade of second-hand IOUs. A credit trade takes place in a single-coincidence meeting between two producers. A monetary trade takes place between a money holder and producer or debtor. There are two types of trade on second-hand IOUs: one between a money holder and creditor, and the other between a creditor and producer. The exchange patterns in a monetary economy with circulating private debt is summarized in Table 1.

In a credit trade an IOU is issued in exchange for q_c units of good. In a monetary trade between a money holder and producer, the former gives one unit of money in exchange for q_m units of good. In a trade between a money holder and debtor,

[8] I do not consider the problem that the cost of examining collateral prevents transactions of second-hand debt.

[9] I am indebted to a referee for suggesting the setup of two subperiods to make the repayment mechanics more explicit and precise.

an amount of good, q_d, is produced in exchange for money. The debtor then repays his debt, regains collateral, and becomes a producer. The creditor gets repayment and becomes a money holder. In this way an IOU is retired and the creditor–debtor relationship is terminated.

As for the determination of the price of second-hand debt, note that for a creditor to accept monetary repayment, money must be at least as valuable as IOUs. Hence, a second-hand IOU is sold for money at a discount. In a trade between a money holder and creditor, the creditor produces q_{mc} units of good as a discount to exchange his IOU for money. If in a single-coincidence meeting the money holder can produce the creditor's consumption good, there is no trade since the storage capacity allows only one unit of an asset at a time.

When a creditor meets a producer who produces his consumption good, the former can sell his IOU for q_{sc} amount of production. Note that the creditor simply exchanges his second-hand IOU rather than issue new debt since the communication technology prevents credit chains to develop. What is the price for second-hand IOUs in this exchange? Notice that while agents buy second-hand IOUs in the first subperiod, the debtors may acquire money and repay the debt in the second subperiod. However, the earliest time for a newly issued IOU to retire is the second subperiod of the *next* period. It is obvious that the expected time for repayment of second-hand IOUs is shorter than that of newly issued IOUs. Thus, second-hand IOUs, as will be shown in more detail below, manage to charge a higher price than the newly issued IOUs.[10]

Any other single-coincidence meetings result in no trade. For example, when a creditor or money holder can produce a producer's consumption good, there is no trade due to the upper bound of unity on asset holdings. The assumption that agents cannot communicate the locations of other agents rule out the possibility that a debtor produces to acquire an IOU and swap with his creditor.

It is worth emphasizing that if swap of IOUs is feasible, there can exist a pure credit equilibrium—private debt is paid off by private debt. Circulating private debt overcomes trading frictions to such an extent that money is driven out of circulation. I derive this result in Appendix A.

In each single-coincidence meeting agents bargain over the quantity of goods surrendered for one unit of some asset. If bargaining involves the provision of goods, consumption occurs and an IOU is created or assets change hands. I use a simple version of bilateral bargaining approach used in Shi (1995) and Trejos and Wright (1995) by assuming that consumers make take-it-or-leave-it offers to their trading partners who produce goods. This implies that consumers extract the entire trade surplus.

Let V_i, $i = p, m, c, d$, be the expected discounted utility from beginning a period with no asset, a unit of money, credit, and debt, respectively. Let V_{sc} denote the

[10] Of course it is interesting to consider the private information problem associated with debt contracts and how it produces discounts on the resale of second-hand debt. However, this requires keeping track of discounts on IOUs of different ages, which would be very complicated in the present framework.

expected value to a creditor at the second subperiod.[11] Suppose individuals bargain taking as given these values. The implications of the bargaining rule that agents who produce do not gain in a trade give us the following terms of trade:

(1) $$q_m = V_m - V_p$$

(2) $$q_c = V_c - V_p$$

(3) $$q_{mc} = V_m - V_{sc}$$

(4) $$q_{sc} = V_{sc} - V_p$$

In a trade between a money holder and producer, the trade surplus is $V_m - V_p - q_m$ for producer and $u(q_m) + V_p - V_m$ for money holder. The take-it-or-leave-it offer made by the money holder gives producer zero trade surplus, which yields Equation (1). The trade is acceptable to the money holder if and only if

(5) $$u(q_m) + V_p - V_m = u(q_m) - q_m \geq 0$$

Note that (5) is equivalent to the condition $0 \leq q_m \leq \hat{q}$.

In a credit trade, the would-be creditor's surplus is $V_c - V_p - q_c$, which is zero due to the take-it-or-leave-it offer by the would-be debtor. The credit trade yields positive trade surplus if and only if

(6) $$u(q_c) + V_d - V_p \geq 0$$

In a trade between a money holder and creditor, the latter provides his IOU and q_{mc} of production for one unit of money. Since agents who buy second-hand IOUs at the meeting subperiod acquire the expected value of getting repayment in the second subperiod, trade surplus for the money holder is $u(q_{mc}) + V_{sc} - V_m$. Thus, money holders are willing to buy second-hand IOUs if and only if

(7) $$u(q_{mc}) + V_{sc} - V_m \geq 0$$

Zero trade surplus for the IOU-holder implies $q_{mc} = V_m - V_{sc}$.

In a trade between a producer and an IOU-holder, the former produces q_{sc} units of goods for the second-hand IOU. This trade entails the producer an expected value of V_{sc}. Thus, the take-it-or-leave-it offer which makes the producer indifferent from accepting and rejecting implies $q_{sc} = V_{sc} - V_p$. Creditors who sell off their IOUs get to consume q_{sc} units of goods but surrender the expected value of holding IOUs to the second sub-period. Hence, creditors are willing to trade if and only if

(8) $$u(q_{sc}) + V_p - V_{sc} \geq 0$$

[11] Agents acquiring second-hand IOUs at the first subperiod have some chances to change states at the second subperiod. Debtors who get money at the first subperiod will repay the debt and become a producer. It turns out that we can ignore debtors' temporary state of holding money at the first subperiod. Other agents carry the states determined at the end of the first subperiod to the beginning of next period.

In a trade between money holders and debtors, the trade surplus is $V_p - V_d - q_d$ for debtors and $u(q_d) + V_p - V_m$ for money holders. In general, we could also assume that money holders make take-it-or-leave-it offers to the debtors. However, to simplify analysis in the present model I consider the case where money holders propose q_m to all agents that can produce their consumption goods. This can be motivated by an assumption that money holders cannot distinguish between a producer and a debtor. Of course, we need to check whether it is a best response for money holders to propose q_m to those who produce their consumption goods; i.e.,

$$(P_p + P_d)[u(q_m) - q_m] \geq P_d\left[u(V_p - V_d) - q_m\right] \tag{9}$$

is satisfied in equilibrium (see the Appendix for proof). There may exist other equilibria in which money holders use other strategies, but this is not what I pursue here.

Given that money holders propose q_m we need to check whether the debtor is willing to engage in trade; i.e.,

$$V_p - V_d - q_m \geq 0 \tag{10}$$

Note that debtors need to suffer disutility q_m to acquire money and make repayment. The payoff of repaying the debt is the continuing value to participate in the economy, which in turn depends on the expected value of trade. In other words, the penalty on failure to repay the debt is being forced out of trading opportunities. Since monetary repayment is the only means to clear debt, condition (10) is necessary for IOUs to be repaid in this economy. Hence, anything that lowers the cost of acquiring money or raises the value of trade will enhance debtors' incentive to make repayment and thus makes the existence of credit more feasible.

In this model there is direct competition between money and credit. Whenever a monetary trade is possible between a money holder and producer, the former can choose whether to issue an IOU or use money in exchange for goods. If the money holder issues an IOU, he consumes q_c units of good. After consumption he will immediately make repayment. The credit trade thus yields surplus $u(q_c) + V_p - V_m$. However, if he uses money to buy goods, he gets $u(q_m) + V_p - V_m$. Since money is a more valuable asset than credit, agents have no incentive to conduct trade with credit when they have money in hand.

3. MONETARY EQUILIBRIUM

In this article I consider only symmetric stationary equilibrium. Agents choose trading strategies to maximize their expected lifetime utility, taking as given others' strategies and steady-state conditions. Without loss of generality, we normalize $\beta/N = 1$. I start with briefly examining a monetary equilibrium in which credit arrangements are not feasible.

DEFINITION 1. A monetary equilibrium without credit is (V_p, V_m, V_d, V_c) and q_m such that (i) $q_m > 0$; (ii) q_m satisfies (1) and (5); (iii) $V_c = V_d = 0$; (iv) $V_p - V_d - q_m <$

0 and

$$rV_p = M(V_m - V_p - q_m)$$

$$rV_m = (1 - M)\left[u(q_m) + V_p - V_m\right]$$

One can show that for all $M \in (0, 1)$ there exists a unique monetary equilibrium without credit. If people believe that IOUs will not be repaid, they will not engage in a debt contract at the first place. The absence of credit in a monetary equilibrium is a self-fulfilling phenomenon.

3.1. *Monetary Equilibrium with Credit*. Now I examine monetary equilibria where credit arrangement is feasible.

3.1.1. *Steady-state conditions*. Potentially there are four types of agents in this economy: money holder, producer, creditor, and debtor, of which the measure is denoted by M, P_p, P_c, and P_d, respectively. Thus,

$$P_p + P_c + P_d = 1 - M \tag{11}$$

Given unity credit limit and no credit chains,

$$P_c = P_d \tag{12}$$

In a stationary monetary equilibrium with credit, the steady-state condition for the distribution of types is described by

$$MP_d = (P_p)^2 \tag{13}$$

Note that in deriving condition (13) I have incorporated the equilibrium strategies such as agents are willing to engage in credit trade and debtors are willing to trade with money holders and make repayment. Equation (13) equates the outflow and inflow into the fraction who are debtors. The outflow equals the fraction of such agents who meet and trade with money holders and repay the debt. The inflow equals the proportion of producers who have a single-coincidence meeting in which one issues an IOU and becomes a debtor. Note that trade of second-hand IOUs has no effect on the measure of debtors.

Using (11)–(13) we solve for the measure of each type in a stationary monetary equilibrium with credit,

$$P_c = P_d = \left[4 - 3M - \sqrt{M(8 - 7M)}\right]\Big/8$$

and $P_p = 1 - M - 2P_d$. Note that all the measures are between 0 and 1 for all $M \in (0, 1)$.

3.1.2. *Value functions*. Given terms of trade (1)–(4), we have the following value functions in flow return:

$$rV_p = P_p \max[u(q_c) + V_d - V_p, 0] \tag{14}$$

$$rV_m = (P_p + P_d)\max[u(q_m) + V_p - V_m, 0] + P_c \max[u(q_{mc}) + V_{sc} - V_m, 0] \tag{15}$$

$$rV_d = M(-q_m + V_p - V_d) \tag{16}$$

$$rV_c = P_p \max[u(q_{sc}) + V_p - V_c, V_{sc} - V_c] + M \max(-q_{mc} + V_m - V_c, V_{sc} - V_c) + (1 - P_p - M)(V_{sc} - V_c) \tag{17}$$

These value functions incorporate the implications of the bargaining rule that those who produce goods do not gain in a trade, and the fact that money is a more valuable asset than IOUs. The creditor's expected value in the repayment subperiod is[12]

$$V_{sc} = MV_m + (1 - M)V_c \tag{18}$$

Explanations for the value functions are given as follows. Equation (14) sets the flow value to a producer equal to the probability of a single-coincidence meeting multiplied by the gain of issuing an IOU to consume q_c units of good and become a debtor. Equation (15) sets the flow return of holding money equal to the probability of meeting a producer or a debtor, multiplied by the gain of trading, plus the probability of meeting a creditor, multiplied by the expected utility of changing asset positions. Equation (16) describes the flow return to a debtor, which is the expected value of meeting a money holder, producing to gain money and repaying the debt. Equation (17) sets the flow return to a creditor equal to the gain of selling his IOU to a producer and money holder, plus the gain of moving to the repayment subperiod if he did not sell off his IOUs in the first subperiod. Equation (18) describes the value to a creditor at the beginning of second subperiod, which is the probability of the debtor getting money to repay the debt multiplied by the gain of acquiring repayment, plus the continuation value if he did not get repayment.

From (1)–(4) and (18) we get

$$q_{mc} = (1 - M)(q_m - q_c) \tag{19}$$

$$q_{sc} = q_c + M(q_m - q_c) \tag{20}$$

Note that Equation (20) implies $q_{sc} > q_c$ in equilibrium: second-hand IOUs are sold at a higher price than the newly issued IOUs.

[12] In general, $V_{sc} = [MV_m + (1 - M)V_c]/(1 + r_\epsilon)$, where r_ϵ is the discount rate. To make the analysis simple, we assume that repayment takes only a short moment before the next period begins and thus $r_\epsilon \to 0$.

DEFINITION 2. A monetary equilibrium with active credit trade is a vector of value functions $V = (V_p, V_m, V_d, V_c, V_{sc})$, quantities of trade $Q = (q_m, q_c, q_{mc}, q_{sc})$, and distribution of agents $P = (P_p, P_c, P_d)$ such that $q_m > 0$, $q_c > 0$, $q_m \geq q_c$, and

(i) given Q and P, V satisfies (14)–(18);
(ii) incentive constraints (5)–(10) are satisfied;
(iii) given V, Q satisfies (1)–(4);
(iv) P satisfies (11)–(13).

The algorithm to find a monetary equilibrium with credit is as follows. I substitute incentive constraints and terms of trade into value functions and then substitute the value functions into the equation $q_m - q_c = V_m - V_c$ and (2) to obtain two equations, $f(q_c, q_m) = 0$ and $g(q_c, q_m) = 0$, where

$$\begin{aligned}
f(q_c, q_m) &= (P_p + P_d)u(q_m) - \left[1 + r - M(P_c + P_p)\right]q_m \\
&\quad + P_c u[(1 - M)(q_m - q_c)] - P_p u[q_c + M(q_m - q_c)] \\
&\quad + \left[1 + r - P_d - M(P_c + P_p)\right]q_c \\
g(q_c, q_m) &= \left[M(r + M + 2P_p) - P_p M(r + M + P_p)\right]q_m \\
&\quad + P_p(r + M + P_p)u[q_c + M(q_m - q_c)] - P_p(r + M)u(q_c) \\
&\quad - (r + M + P_p)\left[r + M + P_p(1 - M)\right]q_c
\end{aligned}$$

An equilibrium is a solution to the system of equations $f(q_c, q_m) = 0$ and $g(q_c, q_m) = 0$ which satisfy incentive constraints (5)–(10). That is, the remaining work for finding an equilibrium is to show that the solutions $q_m, q_c \leq \hat{q}$ satisfy

condition 1: $q_m \geq q_c > 0$

condition 2: $u(q_c) > q_m(r + P_p)/P_p$

and Equation (9). Condition 2 comes from substituting value functions into (10). One can show that if (10) is satisfied, so is constraint (6). This implies that if debtors are willing to repay the debt, agents are willing to engage in credit trade.

THEOREM 1. *Given* $M \in (0, 1)$, *there are solutions to* $f(q_c, q_m) = 0$ *and* $g(q_c, q_m) = 0$ *which satisfy condition* 1.

PROOF. See the Appendix.

Note that Theorem 1 does not specify if there are multiple solutions. To find monetary equilibria with active credit trade amounts to checking if the solutions $q_m, q_c \leq \hat{q}$ in Theorem 1 satisfy condition 2 and (9). Since I am not able to provide general conditions for the existence of equilibrium, I draw the following result from a large number of numerical examples.[13]

Given that the rate of time preference r is not too big, there exists a solution $q_m, q_c \in (0, \hat{q}]$ *to* $f = 0$ *and* $g = 0$ *which satisfies condition 2 and (9) if* $M \geq M_0$.

This result implies that liquidity must be ample and agents not too impatient for private debt to exist and circulate.

3.1.3. *Rate of return on credit.* In this economy, the fact that private debt circulates and is primarily used as a medium of exchange makes it a close substitute for money; however, credit dominates money in rate of return. The rate of return on money is zero since one produces q_m to get one unit of money which he can use to acquire q_m after a random duration of time. To get an IOU which promises monetary repayment in the future, one needs to produce q_c. After a random duration of time ("maturity"), say, t_d, he gets repayment of one unit of money which he can use to acquire production of q_m. Following Shi (1996), I define the interest rate here by $1 = (q_m/q_c)e^{\rho t_d}$; i.e.,

$$\rho = \frac{1}{t_d} \ln\left(\frac{q_m}{q_c}\right)$$

From Theorem 1 we know that, if there exists a monetary equilibrium with credit, $q_m \geq q_c$, and so the interest rate is positive for all finite maturity. Credit dominates money in rate of return in this economy. The reason is that money is used to repay the debt and repayment takes time. Notice that, as I have shown before, if swap of IOUs was possible, private debt can be repaid by private debt and money would have no value. Under this circumstance, private debt would be more valuable than money.

Given that maturity is random I use the notion of expected interest rate, $E\rho \approx M \ln(q_m/q_c)$, to check how interest rate is affected by parameters.[14] The rate of time preference r affects interest rate through the relative price of credit and money, not maturity. Figure 1 (drawn at $M = 0.8$) illustrates the effects of the rate of time preference r on the value of money, credit, and interest rate. From Figure 1 we see that as r increases, q_m and q_c decrease but interest rate increases. As agents become more impatient, the value of money and credit is lower and interest rate is higher.

Money supply affects interest rate through the relative price of credit and money, and maturity. Figure 2 (drawn at $r = 0.01$) illustrates the effects of money supply on the value of money, credit, interest rate, and welfare. An increase in money supply lowers the value of money, credit, and interest rate. As liquidity becomes more ample, the repayment of debt is faster and hence interest rate goes down.

[13] $u(q) = \sqrt{q}$ is used in numerical examples. Given $r = 0.01$, condition 2 is violated when $M \leq 0.64$ and equilibria with credit do not exist. When $r = 0.005, M \leq 0.67$, and $r = 0.05, M \leq 0.57$, condition 2 is violated and so equilibria with credit do not exist.

[14] An IOU is repaid when the debtor meets a suitable money holder, which follows a Poisson process with a rate M. Thus, the expected maturity is $Et_d = 1/M$. For simplicity we use the approximation $E(1/t_d) = 1/Et_d$, which gives us the simple form of expected interest rate defined above.

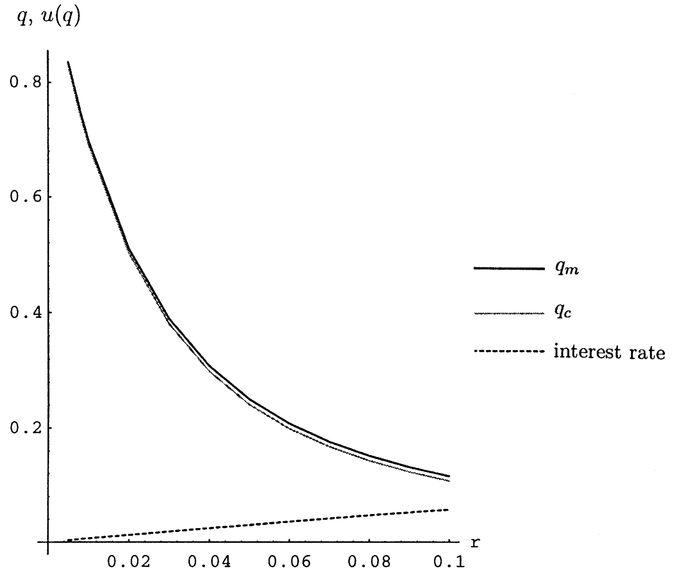

FIGURE 1

3.1.4. *Welfare*. Whenever there exists a monetary equilibrium with active credit trade, there also exists a monetary equilibrium without credit. Let W denote the welfare criterion where

$$W = P_p V_p + M V_m + P_d V_d + P_c V_c$$

The welfare criterion can be interpreted as long-run expected utility of a representative agent, not conditional on the current status. When there are multiple equilibria welfare is compared according to the welfare criterion W.

From Figure 2 we see that welfare in both types of equilibria increases as money supply is increased up to some threshold. If the stock of money is not extremely large ($M < \overline{M}$) welfare in the equilibrium with credit is higher than that in the equilibrium without credit. The coexistence of credit and money can support more exchange than either instrument alone and hence improves welfare. It is also found that the value of money in the equilibrium with credit is lower than that in the one without credit. The intuitive reason is as follows. In a monetary equilibrium with credit, producers have higher reservation value since they can make credit trade. Besides, money holders have more trading opportunities in the equilibrium with credit because they can buy second-hand IOUs. Hence, even though money holders acquire less production in the trade with producers, the expected lifetime utility is higher due to higher frequency of trade. After trade, money holders become producers and so it turns out that a lower quantity exchanged in monetary exchange actually improves welfare.

If the stock of money is extremely large ($M > \overline{M}$), welfare in the equilibrium without credit becomes higher than that in the equilibrium with credit. In this case welfare in both equilibria is decreasing in money supply (see Figure 2). Note that an increase

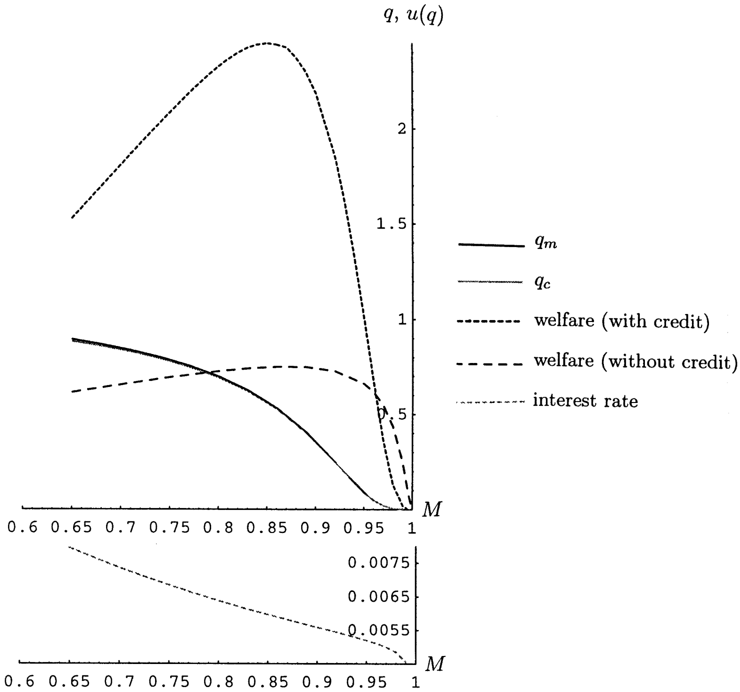

FIGURE 2

in the fraction of agents holding money reduces the number of producers. When the loss in reducing frequency of consumption outweighs the gain in overcoming trade frictions, an increase in money supply reduces welfare. That is, when money supply is extremely plentiful, the benefit of private debt in reducing the double coincidence of wants problem cannot compensate its cost in reducing the amount of production and, as a result, the feasibility of credit reduces welfare.

3.1.5. *Remarks*. Before I proceed to discuss government policy, I give comments on some features of the model that may be thought of being crucial to the coexistence of money and credit. One may suspect that coexistence would not survive if it was feasible to develop credit chains in this economy. To consider credit chains, one needs to keep track of information regarding a particular contractual relationship, such as, the number of agents in a chain and how many agents will be removed from this debt contract if someone in the chain makes repayment. This would make the analysis intractable. My conjecture is that, as long as money is one means to repay the debt, and there is no credit chain involving all types of agents to such an extent that a pure credit economy becomes feasible, the result of coexistence of money and credit should survive.

Another feature concerns the upper bound of unity on individual asset holdings. Note that money and private debt are subject to the same storage capacity constraint in this economy. Since agents' lack of purchasing power may sometimes encounter the opportunities for trade, they have incentive to issue IOUs to make purchases and so credit trade is feasible if repayment can be enforced. Thus, even agents were allowed to accumulate assets, they would conduct credit trade when they do not have enough purchasing power in a meeting. On the other hand, the relaxation of the assumption on asset holdings would not rule out money either, as long as money is one means to clear debt.

4. GOVERNMENT POLICY

It has been shown that monetary equilibria with credit dominates the one without credit if money supply is not extremely plentiful ($M < \overline{M}$). A resulting implication is that, when the money stock is less than $\overline{M}$, a government policy to induce the existence of credit can improve welfare. In this section I examine whether government policies such as open market operations can insure the existence of monetary equilibrium with credit when there is none without intervention. I also study how monetary policies affect the interest rate on newly issued debt and the discount on second-hand debt. To this end, assume that a fraction of the population γ constitutes a special class of agents called *government agents*. They are in all respects exactly like private agents except that they adopt exogenous trading rules rather than strategies based on maximizing behavior. That is, private agents continue to use the individually maximizing trading strategies, but government agents' trading strategies are specified exogenously by policies.

I consider first a policy that specifies the discount that government agents with money ask in exchange for second-hand securities. This act is a policy regarding the monetary authority exchanging interest-bearing securities with money. Hence, it is called an open market operation, even though there is no centralized market for the exchange of assets in this random-matching economy. I then consider a policy that specifies the quantity that government agents demand when issuing IOUs, and this is called a public debt policy.

Let G_i, denote the measure of government agents who are in state i, where $i = p, c, d$ represent producer, creditor, and debtor, respectively. Let m_p and m_g denote the fraction of private agents and government agents with money, respectively. I show in the Appendix the steady-state conditions and solve for the distribution of types and asset holdings for government agents, which yields $m_p = m_g = M, P_p = G_p, P_c = G_c, P_d = G_d$. The distributions of types and asset holdings are identical for private and government agents. This result is not surprising since the exchange patterns specified in this economy are symmetric across private and government agents, and hence the resulting distribution is symmetric.[15]

4.1. *Open Market Purchases*. Consider first a policy of open market purchase which specifies the quantity, q^p_{mc}, that government agents with money demand in

[15] Alternatively, we could have assumed that, as in Aiyagari et al. (1996), government agents sometimes reject the offer of second-hand IOUs for money. This too will affect the interest rate prevailing on the market. The effect of policy considered here is on the terms of trade, not on the distribution of asset holdings as in Aiyagari et al. (1996).

TABLE 2

EFFECTS OF MONETARY POLICIES IN EQUILIBRIA WITH CREDIT*

	q_m	q_c	q_{mc}	q_{sc}	$M\ln(q_m/q_c)$	W
Without policies ($M = 0.8$)	0.69766	0.69213	0.001107	0.69655	0.006373	2.3295
$q^p_{mc} = 0.001$, $M = 0.8$	0.69761	0.69209	0.001105	0.69651	0.006361	2.3297
$q^p_{mc} = 0.002$, $M = 0.64$	0.90175	0.89065	0.003997	0.89776	0.007938	1.4796
$q^s_{mc} = 0.001$, $M = 0.8$	0.69750	0.69200	0.001106	0.69640	0.006370	2.3301
$q^s_{mc} = 0.002$, $M = 0.8$	0.69874	0.69318	0.001112	0.69763	0.006392	2.3249
$q^s_{mc} = 0.002$, $M = 0.64$	0.90021	0.88902	0.004029	0.89618	0.008006	1.4892
$q^b_c = 0.691$, $M = 0.8$	0.69736	0.69182	0.001108	0.69625	0.006381	2.3316
$q^b_c = 0.880$, $M = 0.64$	0.90081	0.88939	0.004110	0.89670	0.008163	1.4984

* Parameter values are $r = 0.01$ and $\gamma = 0.1$. Note that when $r = 0.01$ and $M \leq 0.64$ the equilibrium with credit does not exist. The numerical examples thus show that policies entail the feasibility of circulating private debt when there is none without intervention.

exchange for second-hand debt. This implies that the value function V_c is changed, since q^p_{mc} may differ from the price prevailing in the private market. The flow value to a creditor becomes

$$
\begin{aligned}
rV_c = {} & P_p \max\left[u(q_{sc}) + V_p - V_c, V_{sc} - V_c\right] + (1 - P_p - M)(V_{sc} - V_c) \\
& + (1-\gamma)m_p \max(-q_{mc} + V_m - V_c, V_{sc} - V_c) \\
& + \gamma m_g \max\left(-q^p_{mc} + V_m - V_c, V_{sc} - V_c\right)
\end{aligned}
$$

Note that private creditors will reject trades with government agents if q^p_{mc} exceeds the market value of q_{mc} since this is the most a private creditor would be willing to trade given take-it-or-leave-it offers.

Suppose that q^p_{mc} is set below the equilibrium value of q_{mc}. Numerical examples show that q^p_{mc} policy decreases the equilibrium value of money, credit, second-hand IOU, and interest rate (see Table 2). As the monetary authority conducts open market purchases which require a lower discount rate on securities, it has the effect of lowering the interest rate prevailing on the market.

Now I show that a policy of open market purchase can improve welfare by making credit trade feasible. Let $F(q_c, q_m) = 0$ and $G(q_c, q_m) = 0$ denote the system of equations to solve (q_c, q_m) under policy q^p_{mc}, and let (q^o_c, q^o_m) denote a solution to $f = 0$ and $g = 0$ without government intervention. Note that (q^o_c, q^o_m) is a monetary equilibrium with credit if it satisfies condition 2. Examples with $u(q) = \sqrt{q}$ show that when (q^o_c, q^o_m) lies outside the convex set defined by condition 2 the solution to $F = 0$ and $G = 0$ may lie within. Hence, monetary policy makes the existence of equilibria with credit possible when the only monetary equilibrium entails no credit without policy intervention.[16]

Notice that condition 2 implies that if q_m is too big, debtors may not be willing to repay the debt. Big q_m means that debtors need to pay a substantial cost to produce in order to get money. The expected utility of repaying debt comes from future consumption value which depends on, among other things, $u(q_c)$. If q_m is too much bigger than $u(q_c)$, it is possible that debtors just default the debt. Knowing this, no one will accept IOUs at the first place. The open market purchases which discount less for second-hand IOUs drives down the value of newly issued IOUs and money in such a way that makes repayment of debt incentive compatible and credit arrangements feasible.

Another question is, can this policy improve welfare in an equilibrium with active credit trade? Numerical examples in Table 2 show that q_{mc}^p policy entails higher welfare for the economy, even though the quantity exchanged in all types of trade is lower. Higher quantity consumed today implies higher disutility to producing in order to get an asset in the future, and thus it does not necessarily improve long-run expected utility.

4.2. *Open Market Sales*. Consider a policy which specifies the quantity, q_{mc}^s, that government agents with second-hand debt supply in exchange for money. The q_{mc}^s policy changes the flow value to a money holder as follows:

$$\begin{aligned} rV_m &= (P_p + P_d)\max[u(q_m) - q_m, 0] \\ &\quad + (1-\gamma)P_c \max[u(q_{mc}) + V_{sc} - V_m, 0] \\ &\quad + \gamma G_c \max\left[u(q_{mc}^s) + V_{sc} - V_m, 0\right] \end{aligned}$$

Unlike q_{mc}^p policy, this policy can have q_{mc}^s greater or less than the market value of q_{mc}. However, a policy with q_{mc}^s such that $u(q_{mc}^s) + q_c - q_m < 0$ is not acceptable to the private agents. Hence, I consider only q_{mc}^s policy such that $u(q_{mc}^s) + q_c - q_m > 0$. Table 2 shows that a policy with q_{mc}^s greater (less) than the market value of q_{mc} increases (decreases) q_m, q_c, q_{mc}, q_{sc}, and interest rate. Similarly, there can exist a monetary equilibrium with active credit trade under policy q_{mc}^s when there is none without policy intervention.

4.3. *Public Debt Policy*. Consider a policy that specifies the quantity, q_c^b, that government agents demand when issuing debt. This policy affects the value to a producer since q_c^b may differ from the market value of q_c for newly issued IOUs. The flow return to a producer now becomes

$$rV_p = P_p \max[u(q_c) + V_d - V_p, 0] + \gamma G_p \max(q_c - q_c^b, 0)$$

[16] Of course, to make the policy effective in this case, the government must be big enough (e.g., see Li and Wright, 1998).

Note that q_c^b must be below equilibrium value of q_c for private producers to agree to trade with government agents. Table 2 shows that the policy q_c^b decreases the equilibrium value of q_m, q_c, and q_{sc} but increases q_{mc} and interest rate.

We conclude this section by summarizing some of the main results: *An open market operation whereby government discounts less for second-hand debt can decrease the value of money, credit, interest rate, and the discounts prevailing in private transactions. A policy of open market operations can improve welfare by making credit trade feasible. The public debt policy whereby government requires higher interest rate for its newly issued debt can decrease the value of money and credit and increase interest rate prevailing in private transactions.*

5. CONCLUSIONS

This article examines the feasibility of circulating private debt in a random-matching economy with a double coincidence of wants problem. I have demonstrated that money is valued even though there coexists credit which circulates among agents and dominates in the rate of return. The coexistence of money and credit improves welfare since there are more trading opportunities available than otherwise. It is also shown that monetary policies can improve welfare by making credit trade feasible when the only equilibrium entails no credit without intervention.

There were some historical episodes in which people issued IOUs and private IOUs got to circulate. For example, Murphy (1978) described an episode in Ireland where banks closed between 1966 and 1976 and personal checks started circulating as a medium of exchange even though people did not know when banks would resume business. It is also known that bills of exchange were commonly used in transactions in north England during late 18th and early 19th centuries. "The creditor drew a bill on the debtor; the debtor (or his agent) accepted it and returned it to the creditor, who either held it till it matured and then presented for payment, or, if he needed ready money, discounted it with some other merchant or banker.... The drawer very often passed it on to meet obligations of his own, and those who received it, in their turns, did the same"(Ashton, 1945: p.25).

These historical episodes have motivated this model to study an economy where potentially every agent has the technology to issue debt and private debt can circulate. Although we call a subset of agents "government" and study the effects of its transaction policies on the value of money, credit, and interest rate, the derived interpretations are not confined to monetary policies only. For example, the transaction policies can be interpreted as those adopted by a group of merchants or institutions such as financial intermediaries that wish to get private debt generally accepted as a means of payment, or influence the interest rate prevailing on the market. This article provides predictions as to under what circumstance these transaction policies may work.[17]

[17] The point here is to suggest that if a group of agents act cooperatively, they can affect the strategies of other participants in the market and the equilibrium results. Of course, if the objective is to study the transaction policies of private institutions, one needs to describe explicitly the rules of behavior such as profit maximizing and incentive constraints.

Some related studies concerning the effect of inside money on allocations include, for example, Cavalcanti and Wallace (1999), Cavalcanti et al. (1999), and Williamson (1999). The major differences of this article from those studies are as follows. In Cavalcanti et al. (1999) only a subset of agents are endowed the technology to issue private debt and private debt is assumed to be perfect substitute for money. In Cavalcanti and Wallace (1999) only a subset of agents have trading histories that are public information and thus they are able to get their debt accepted as a medium of exchange. Williamson (1999) considers an exogenous institution with investment technology which can issue and redeem notes. In this model, every agent has an access to issue debt and private debt can circulate without assumptions on its acceptability or an exogenous institution to guarantee its repayment. The repayment of private debt in this article is driven by debtor's incentive to regain collateral for future consumption.[18]

This article does not model financial intermediaries. However, to consider a relevant institution and how it functions to get private debt circulate, one would like to model financial intermediaries. It would be interesting to consider a model in which the frictions give rise to an endogenous role for money, private debt, and financial intermediation. This should help us investigate the effects on prices and interest rates of policies such as open market operations and legal restrictions on intermediary activities.

APPENDIX A

Here I show that if swap of IOUs is feasible, there can exist a pure credit equilibrium where money has no value and private debt is repaid with private debt. Given the upper bound of unity on asset holdings and credit limit, in a single-coincidence meeting the potential trading situations are as follows. When a producer meets a producer or debtor who produces what he consumes, an IOU is created for q_c units of good. The debtor then takes the newly issued IOU to his creditor and repays the debt. When a creditor meets a producer or debtor who produces his consumption good, he passes his second-hand IOU for q_c units of good. The producer becomes a new creditor and the debtor uses the second-hand IOU to repay his debt. Since the only means to repay the debt is private debt and all debt is unfalsifiable, I confine attention to the case where newly issued IOUs and second-hand IOUs exchange for the same amount of goods.

In a pure credit equilibrium the value functions satisfy

$$rV_p = (P_p + P_d)\left[u(q_c) + V_d - V_p\right]$$

$$rV_d = (P_p + P_c)(-q_c + V_p - V_d)$$

$$rV_c = (P_p + P_d)\left[u(q_c) + V_p - V_c\right]$$

[18] There are some recent studies which emphasize the importance of imperfect memory for the use of money (see, for example, Kocherlakota, 1998; Kocherlakota and Wallace, 1998). In those articles, superior allocations can be achieved without fiat money by certain punishment strategies.

and $V_m = 0$. The take-it-or-leave-it offers which make those who produce indifferent from accepting and rejecting imply $V_d = 0$ and $q_c = V_p$. The steady-state condition is

$$P_p(P_p + P_d) = P_d(P_p + P_c)$$

The above equation equates the outflow and inflow into the fraction who are debtors. The outflow equals the fraction of such agents who meet and trade with producers and creditors and clear the debt with newly issued IOUs and second-hand IOUs, respectively. The inflow equals the proportion of producers who have a single-coincidence meeting in which an IOU is created in exchange for goods. From the steady-state condition, $P_p + P_c + P_d = 1$, and $P_c = P_d$ implied by unity credit limit and no credit chains, we solve for $P_p = P_c = P_d = 1/3$. Hence, a pure credit equilibrium is characterized by

$$q_c = V_p = 4/(2+3r)^2$$

$$V_c = 4(4+3r)/(2+3r)^3$$

$V_d = V_m = 0$ and $P_p = P_c = P_d = 1/3$.

APPENDIX B

PROOF OF CONDITION (9). Under incentive constraint (10) for debtors to repay debt, $q_d \geq q_m$. If money holders propose q_d, producers will reject the trade and so the probability of trade is lower. If money holders propose q_m, they get higher probability to trade but less amount of goods to consume. Since q_m is the most that money holders can get to make producers agree to trade, any proposed quantity $q \in (q_m, q_d)$ will be turned down by producers. Hence, any $q \in (q_m, q_d)$ is inferior to q_d. Similarly, any quantity lower than q_m is not to the best interest of money holders. Thus we need only to check whether it is a best response for money holder to propose q_m rather than q_d to all agents who produce their consumption goods. To this end we take into account the relative probability of meeting each of the two types of sellers. If a money holder proposes $q_d = V_p - V_d$ his flow payoff is

$$rV_m = P_d \max\left[u(V_p - V_d) + V_p - V_m, 0\right] + P_c \max[u(q_{mc}) + V_{sc} - V_m, 0]$$

If he proposes q_m his flow payoff is (15). One can show that it is a best response to propose q_m if condition (9) is satisfied. □

APPENDIX C

PROOF OF THEOREM 1. The properties of $f = 0$ and $g = 0$ are as follows. $f(0,0) = 0$, $f(\hat{q}, \hat{q}) = 0$, $f(0, q_m^0) = 0$ where $q_m^0 < \hat{q}$; $g(0,0) = 0$, $g(\hat{q}, q_m') = 0$ where $q_m' > \hat{q}$, and $dq_m/dq_c|_{g=0} > 0$ as $q_c \to \infty$. Also, $f = 0$ and $g = 0$ are both continuous on $[0, \infty)$. Thus, there are solutions to $f = 0$ and $g = 0$. Note that the utility function is defined on $[0, \infty)$. Hence, $f = 0$ locus is above the 45° line for otherwise $u[(1 - M)(q_m - q_c)]$ is not defined. Therefore, the solutions to $f = 0$ and $g = 0$ satisfy $q_m \geq q_c > 0$. □

APPENDIX D

Here I show the steady-state conditions and solve for the distribution of types and asset holdings for government agents. First, $P_p + P_c + P_d = 1 - m_p$, $G_p + G_c + G_d = 1 - m_g$, and $(1-\gamma)m_p + \gamma m_g = M$. Note that unity credit limit and no credit chains imply

$$\gamma G_c + (1-\gamma)P_c = \gamma G_d + (1-\gamma)P_d$$

This equation can be interpreted as the "market clearing condition" that total lending is equal to total borrowing. The steady-state conditions for the distribution of types are described as follows:

$$P_d M = P_p\left[\gamma G_p + (1-\gamma)P_p\right]$$

$$G_d M = G_p\left[\gamma G_p + (1-\gamma)P_p\right]$$

$$m_p\gamma(G_p + G_c + G_d) = (P_p + P_c + P_d)\gamma m_g$$

$$P_c\left[M + \gamma(G_p + m_g)\right] = P_p\left[\gamma G_p + (1-\gamma)P_p + \gamma G_c\right] + m_p\gamma G_c \tag{A.1}$$

The steady-state conditions have similar interpretations as (13). For example, (A.1) equates outflow and inflow into the fraction who are private agents holding IOUs. The outflow equals the fraction of such agents who get repayment of money from debtors or meet government agents who buy second-hand debt. The inflow equals the fraction of private producers who meet other producers and issue IOUs or meet government creditors from whom they buy second-hand debt, plus the fraction of private money holders who buy second-hand debt from government agents. Solving these equations yields $m_p = m_g = M$, $P_p = G_p$, $P_c = G_c$, $P_d = G_d$.

REFERENCES

AIYAGARI, S. R., AND N. WALLACE, "Government Transaction Policy, the Medium of Exchange, and Welfare," *Journal of Economic Theory* 74 (1997), 1–18.

———, ———, AND R. WRIGHT, "Coexistence of Money and Interest-Bearing Securities," *Journal of Monetary Economics* 37 (1996), 397–419.

ASHTON, T. S., "The Bill of Exchange and Private Banks in Lancashire, 1790–1830," *Economic History Review* 1–2 (1945), 25–35.

BERNHARDT, D., "Money and Loan," *Review of Economic Studies* 56 (1989), 89–100.

CAVALCANTI, R., AND N. WALLACE, "A Model of Private Bank-Note Issue," *Review of Economic Dynamics* 2 (1999), 104–36.

———, A. EROSA, AND T. TEMZELIDES, "Private Money and Reserve Management in a Random Matching Model," *Journal of Political Economies* 107 (1999), 929–45.

DIAMOND, P., "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy* 90 (1992), 881–94.

———, "Pairwise Credit in Search Equilibrium," *Quarterly Journal of Economics* 105 (1990), 285–320.

FREEMAN, S., "Clearinghouse Banks and Banknote Over-issue," *Journal of Monetary Economics* 38 (1996a), 101–15.

———, "The Payment System, Liquidity, and Rediscounting," *American Economic Review* 86 (1996b), 1126–38.

GREEN, E. J., "Money and Debt in the Structure of Payments," *Monetary and Economic Studies* 15 (1997), 63–87.

——— AND W. E. WEBER, "Will the New $100 Bill Decrease Counterfeiting?" *Quarterly Review of the Federal Reserve Bank of Minneapolis* (1996).

KIYOTAKI, N., AND R. WRIGHT, "On Money as a Medium of Exchange," *Journal of Political Economy* 97 (1989), 927–54.

——— AND ———, "A Contribution to the Pure Theory of Money," *Journal of Economic Theory* 53 (1991), 215–35.

——— AND ———, "A Search-Theoretic Approach to Monetary Economics," *American Economic Review* 83 (1993), 63–77.

KOCHERLAKOTA, N. R., "Money Is Memory," *Journal of Economic Theory* 81 (1998), 232–51.

——— AND N. WALLACE, "Incomplete Record-Keeping and Optimal Payment Arrangements," *Journal of Economic Theory* 81 (1998), 272–89.

LI, Y., AND R. WRIGHT, "Government Transaction Policy, Media of Exchange, and Prices," *Journal of Economic Theory* 81 (1998), 290–313.

MURPHY, A. E., "Money in an Economy without Banks: The Case of Ireland," *The Manchester School* (1978), 41–50.

RITTER, J. A., "The Transition from Barter to Fiat Money," *American Economic Review* 85 (1995), 134–49.

SHI, S., "Money and Prices: A Model of Search and Bargaining," *Journal of Economic Theory* 67 (1995), 467–98.

———, "Credit and Money in a Search Model with Divisible Commodities," *Review of Economic Studies* 63 (1996), 627–52.

TOWNSEND, R. M., "Currency and Credit in a Private Information Economy," *Journal of Political Economy* 97 (1989), 1323–44.

——— AND N. WALLACE, "Circulating Private Debt: An Example with Coordination Problem," in E. C. Prescott and N. Wallace, eds., *Contractual Arrangements for Intertemporal Trade* (Minneapolis: University of Minnesota Press, 1987).

TREJOS, A., AND R. WRIGHT, "Search, Bargaining, Money and Prices," *Journal of Political Economy* 103 (1995), 118–41.

WILLIAMSON, S. D., "Private Money," *Journal of Money, Credit and Banking,* 31 (1999), 469–91.