Government Transaction, Inflation, and Unemployment

Te-Tsun Chang

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Introduction

- Berensten, Menzio and Wright (2011)
- Labor Search (Mortensen and Pisarides 1994) + Monetary Search (Kiyotaki and Wright 1993)
- US: Positive-Sloped Phillips Curve
- Karanassou, Sala, and Snower (2003), Franz (2005), and Schreiber and Wolters (2007): the Phillips curve is negatively sloped in European countries.
Government Transaction

- Friedman (1977): In the modern world, governments are themselves producers of servers sold on the market: from postal services to a wide range of other items...
- The size of Gov’t affect prices and allocation?
- Government size or Government Transaction v.s Unemployment and Inflation?
- Some evidences
Literature Review

- **Lagos and Wright 2005 (LW) + Mortensen and Pisarides 1994 (MP):** Berensten, Menzio, and Wright (2011); Lucy Q. Liu (2009)

- **LW + RBC Labor:** Rocheteau, Rupert, and Wright (2007); Dong (2010)

- **MP + New Keynesian:** Gertler, Sala, and Trigari (2008); Gertler and Trigari (2009); Trigari (2009)

- **Shi Model (Large Household):** Shi (1998); Wang and Shi (2006)
Li and Wright (1998)

- government agents behave in an exogenous way regarding which objects they accept in trade and at what price
- Government agents’ transaction policy affects the set of equilibria.
Model Structure

- Li and Wright (1998) + BMW
- agents: firms $f$, households $h$, and government agents $g$
- $h \in [0, 1]$, the measure of $g$ is $\psi$; $f$ is arbitrarily large
- Each period consists of three subperiods.
- People go through three rounds of trades in one period
- subperiod: Labor mkt (MP mkt), Goods mkt (KW mkt), Arrow-Debreu mkt (AD mkt)
Some Notations

- **Value Functions:**
  - MP: $U^j_e(z)$
  - KW: $V^j_e(z)$
  - AD: $W^j_e(z)$.

  where $j \in \{h, f\}$; $e \in \{0, 1\}$ and $z \in [0, \infty)$ is the real balance.

- In the MP market, $e = 1$ if an agent is matched and $e = 0$ otherwise.
Some Notations in AD

- $z = m/p$,
- $m$ is the dollars an agent bring to the AD market
- $p$ is the current price level
- $\rho$: the reciprocal of the inflation rate in AD.
Government

- $M$: in the form of lump-sum transfers $\pi M$ in the AD market
- $\pi$: the growth rate of money (= inflation rate).
- $\hat{M} = (1 + \pi)M$: the evolution of the money stock
- $w^g$: wages for bureaucrats
- $b$: UI benefits
- $T$: lump-sum taxes

Gov’t:

$$\psi \lambda_{g,b} \rho d^b + \psi w^g + bu = T + \frac{\pi M}{p} + \psi \lambda_{g,s} \rho d^s,$$  \hspace{1cm} (1)

where $\lambda_{g,b}$ and $\lambda_{g,s}$ are the probabilities to complete a trade.
Household: AD

\[
W^h_e(z) = \max_{x, \hat{z}} \{ x + (1 - e)l + \beta U^h_e(\hat{z}) \}
\]

s.t. \( x + \hat{z} = ew + (1 - e)b + F - T + z, \)

FOC: \( \beta \frac{\partial U^h_e(\hat{z})}{\partial \hat{z}} = 1, \)

Envelope Condition: \( \frac{\partial W^h_e(\hat{z})}{\partial \hat{z}} = 1. \)
Household: KW

\[ V_e^h(z) = \alpha_h \{ v(q) + W_e^h [\rho(z - d)] \} \]

\[ + \alpha_p^h \{ v(q^s) + W_e^h [\rho(z - d^s)] \} + (1 - \alpha_h - \alpha_p^h) W_e^h (\rho z). \]

*(q, d)*: terms of trade between *h* and *f*.

*(q^s, d^s)*: terms of trade between *h* and *g*

*v(q)*: utility from trade in KW;

*\alpha_h*: probability of a buyer to meet firms

*\alpha_p^h*: probability of a buyer to meet government agents
Household: MP

\[ U^h_1(z) = \delta V^h_0 + (1 - \delta) V^h_1, \]
\[ U^h_0(z) = \lambda^h V^h_1 + (1 - \lambda^h) V^h_0, \]

\(\delta\): job destruction rate
\(\lambda^h\): job creation rate

If match function is \(N(u, v)\), \(\lambda^h = N(u, v)/u\), \(v\) is the vacancy
Firm: MP

MP:

\[ U_1^f (z) = \delta V_0^f + (1 - \delta) V_1^f, \]
\[ U_0^f (z) = \lambda_f V_1^f + (1 - \lambda_f) V_0^f. \]

\[ \lambda_f = \mathcal{N}(u, v)/v \]
Firm: KW mkt

KW:

\[ V_0^f = 0 \]
\[ V_1^f = \alpha_f W_1^f [y - c(q), \rho d] + \alpha_p^f W_1^f [y - c(q^b), \rho d^b] \]
\[ + (1 - \alpha_f - \alpha_p^f) W_e^h (y, 0). \]

\( y \): output in a match
\( c(q) = q \): transformation cost
Firm: AD mkt

\[ W^f_1(x, z) = x + z - w + \beta U^f_1. \]

\[ W^f_0 = \max\{k, \beta U^f_0\}. \]
Equilibrium

- Goods mkt: Nash bargaining $\rightarrow (q, d) = (g^{-1}(\rho z), z)$

- Labor mkt: Nash bargaining $\rightarrow w = \eta[\beta(1-\delta)](b+l)+(1-\eta)[\beta(1-\delta-\lambda_h)]R \over 1-\beta(1-\delta)+\eta\beta\lambda_h$

- Steady state condition: $(1-u)\delta = N(u, v)$

- $\alpha_h = {S \over B+S+G} = {1-u \over 2-u+\psi}$, $\alpha^p_h = {G \over B+S+G} = {\psi \over 2-u+\psi}$

- $\alpha_f = {B \over B+S+G} = {1 \over 2-u+\psi}$, $\alpha^p_f = {G \over B+S+G} = {\psi \over 2-u+\psi}$
LW curve: From Household’s Problem

\[ q^s = q, d^s = d: \]

\[ i = \frac{1 - u + \psi}{2 - u + \psi} \left( \frac{\nu'(q)}{g'(q)} - 1 \right), \]

Define: \( i = \frac{1}{\beta \rho} - 1 \).

\( q^s \) too small:

\[ i = \frac{1 - u}{2 - u + \psi} \frac{\nu'(q)}{g'(q)} - \frac{1 - u + \psi}{2 - u + \psi} \]
MP curve: From Firms’ Problem

\[ q^b = q, \ d^b = d: \]

\[ k = \frac{\eta \frac{N(u,v)}{v} \{ y - b - l + \frac{1+\psi}{2-u+\psi} [g(q) - q] \}}{r + \delta + (1 - \delta) \frac{N(u,v)}{u}}. \]

\[ q^b \neq q: \]

Consider government agents make a take-it-or-leave-it offers to firms, \( \rho d^b = q^b: \)

\[ k = \frac{\eta \frac{N(u,v)}{v} \{ y - b - l + \frac{1}{2-u+\psi} [g(q) - q] \}}{r + \delta + (1 - \delta) \frac{N(u,v)}{u}}. \]
Without government agents
With government agents

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LW

MP
Results

- LW curve:
  As $q^s = q$, $\psi \uparrow \implies$ LW shifts to the right
  As $q^s$ small enough, $\psi \uparrow \implies$ LW shifts to the left

- MP curve:
  if $q^b = q$, $\psi \uparrow \implies$ MP shifts to the left
  if $\rho d^b = q^b$, $\psi \uparrow \implies$ MP shifts to the right
\[ \nu(q) = Aq^{1-q}/(1 - a), \quad N(u, v) = Zu^{1-\sigma}v^\sigma \]

- \( b = w/2 \)

- Hagedorn and Manovskii (2008): \( (b + l)/y = 0.95 \)
Calibrations

Table: Key parameter values

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<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\beta$</td>
<td>discount factor</td>
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<tr>
<td>$l$</td>
<td>value of leisure</td>
<td>0.504</td>
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<td>$A$</td>
<td>KW utility weight</td>
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<td>$a$</td>
<td>KW utility elasticity</td>
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<td>$\delta$</td>
<td>job destruction rate</td>
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<td>$k$</td>
<td>vacancy posting cost ($10^{-4}$)</td>
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<td>$Z$</td>
<td>MP matching efficiency</td>
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<td>$\sigma$</td>
<td>MP matching $v$ elasticity</td>
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<td>$\eta$</td>
<td>MP firm bargaining share</td>
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<td>$\theta$</td>
<td>KW firm bargaining share</td>
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Case I: $q^s = q^b = q$

Table: $u$

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<td>0.048</td>
<td>0.046</td>
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<tr>
<td>$i = 0.071$</td>
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Table: \( q \)

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<td>0.12</td>
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<tr>
<td>( i = 0.071 )</td>
<td>0.091</td>
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<tr>
<td>( i = 0.074 )</td>
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<td>0.092</td>
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Case II: $q^s \neq q, \rho d^b = q^b$

Table: $u$

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Conclusion

- The presence of government agents changes the set of equilibria.
- The size of government matters for the slope of Phillips curve.