Risk Diversification

Will diversification remove all the risk of your portfolio? Consider a portfolio P in which equal proportion of funds is invested in each of N assets. the total returns of the portfolio is

$$R_P = \frac{1}{N}\left(R_1 + R_2 + ... + R_N\right).$$

We assume that the variances of each security are equal, $\sigma_1^2 = \sigma_2^2 = ... = \sigma_N^2 = \overline{\sigma^2}$, and covariances of each pair of securities are the same,

$$cov(R_1, R_2) = ... = cov(R_1, R_N) = ... = cov(R_{N-1}, R_N) = \overline{cov}.$$

(or we can consider $\overline{\sigma^2}$ to be the average variance of individual asset, and $\overline{cov}$ the average covariance of individual assets.)

The total variance of the portfolio is

$$\sigma_P^2 = N \bullet \frac{\overline{\sigma^2}}{N^2} + (N^2 - N) \bullet \frac{\overline{cov}}{N^2} = \frac{1}{N}\overline{\sigma^2} + (1 - \frac{1}{N})\overline{cov}.$$

When N approaches infinity, i.e., the investor diversify the portfolio as much as possible,

$$\sigma_P^2 \longrightarrow \overline{cov}.$$

When there are more and more assets, the number of covariances grow faster than the number of variances. Thus, the variance of a perfectly divdersified portfolio reflects the covariances among its individual assets.

Some asset returns move in different directions and can be cancelled out. This part of risk is called unique risk, or non-systemic risk, which can be diversified. It is clear that part of the risk in a protfolio can not be diversified away. This part of risk is referred as the market risk, or systemic risk, which is equal to the average covariance of individual assets in the portfolio ($\overline{cov}$).