Agency Costs, Net Worth and Business Fluctuations

Bernanke and Gertler (1989, AER)

1 Introduction

- Many studies on the business cycles have suggested that financial factors, or more specifically the condition of firm and household balance sheets ("solvency" or "credit-worthiness"), matter for the dynamics of macroeconomic activity. For example, Mishkin (1978) and Bernanke (1983) argue that the weakness of borrowers' balance sheets contributed to the severity of the Great Depression.

- This paper presents a model to analyze the role of borrowers' balance sheets (specifically the borrower's net worth) in the business cycles

  - Real Business Cycle (RBC)
  - OLG (abstract from long-term contracts)
  - Information asymmetry – costly state verification (CSV, Townsend (1979))

- Given asymmetric information between borrowers and lenders, the optimal financial arrangement entails agency cost
  $\Longrightarrow$ The cost of external fund > the cost of internal fund.

- Borrower's net worth $\downarrow$
  $\Longrightarrow$ agency cost $\uparrow$ (periods of financial distress (at which time borrower's net worth is low) is also times of high agency cost of investment).

- In aggregate the result that borrower's net worth and agency cost are inversely correlated implies

  - Since borrower's net worth is pro-cyclical, agency cost is counter-cyclical.
    $\Longrightarrow$ leading to investment fluctuation and cyclical persistence (financial accelerator).

- Shocks to borrower's net worth, independent of technological shock in RBC, is an initiating source of real fluctuations (financial factors matter).
  (e.g. debt deflation (I. Fisher (1933)): an unanticipated fall in price level, or a fall in relative price of borrower's collateral leads to an increase in indebtedness in real terms and a decline in borrower's net worth).

- The main goal of this paper is to draw a connection between the condition of borrower's net worth and the agency costs and to demonstrate how this connection may play a role in the dynamics of business cycles.

2 The Model

There are overlapping generations of two-period live agents, and a countable infinity of agents in each generation (so that all variables are expressed in per capita, rather than aggregate, quantities).

– a "generation" = the entry and exit of firms from credit markets

– a "period" = the length of a typical financial contract

	Population	Endowment
Entrepreneurs	η	L^e
Lenders	$1-\eta$	L

Assume $\eta L^e + (1-\eta) L = 1$.

Borrowers are heterogeneous. Entrepreneurs differ in the cost of investment (or efficiency), indexed by $\omega \sim unif\,[0,1]$, where entrepreneurs with low ω have a lower cost of investment (higher efficiency).

Two types of goods:

Capital good – fully depreciated in one period (no growth).

Output good – perishable.

Three technologies

(1) storage: output goods can be stored with a gross rate of return $r \geq 1$.

(2) output goods production (per capita) employs capital goods (labor is normalizedto unity)

$$y_t = \widetilde{\theta}_t f(k_t),\ \widetilde{\theta}_t\tilde{}iid,\ E\left(\widetilde{\theta}_t\right) = \theta,\ f(0) > 0,$$

where $\widetilde{\theta}_t$ is aggregate productivity shock.

(3) investment technology

An entrepreneur with efficiency ω must invest $x(\underset{(+)}{\omega})$

$$\begin{array}{ccccc} x(\omega) & & \kappa_i & & \\ t & \longrightarrow & t+1 & & ,\ i = 1, 2, ...n. \\ \text{output good} & & \text{capital good} & & \end{array}$$

Probability of outcome κ_i is π_i (idiosyncratic shock).

The distribution of investment outcome is identical ex ante across projects, i.e., $\sum_i \pi_i \kappa_i = \kappa$ and is not affected by any action or effort of the individual (no moral hazard problem).

To introduce asymmetric information, the realization of investment project outcome is assumed to be private information. Outsiders can employ an audit (monitoring) technology to learn the result, within costs γ units of *capital good.*

Assume that random auditing (mixed strategy) is feasible, i.e., lenders can pre-commit to auditing with some probability.

Sequence of events

At the beginning of t capital goods κ_i are realized from investment at $t-1$

– To audit or not to audit

– Date t aggregate shock $\widetilde{\theta}_t$ realized, output goods y_t are produced

– Financing contracting and entrepreneurs invest $x(\omega)$ at the end of t

– At the beginning of $t+1$ capital goods κ_i are realized from investment at t

– To audit or not to audit

– Date $t+1$ aggregate shock $\widetilde{\theta}_{t+1}$ realized, output goods y_{t+1} are produced.

Thus, incentive constraints relevant to decisions in t need depend only on expected values of functions of $\widetilde{\theta}_{t+1}$.

Investments undertaken in a given period have mutually independent outcome, so that there is no aggregate uncertainty about the quantity of capital produced.

i_t = the number of investment projects undertaken in t per capita.

h_t = the fraction of projects initiated in t that are monitored (bankrupt).

For any given t, next period stock of capital per capita is given by

$$k_{t+1} = (\kappa - h_t\gamma)i_t.$$

Assume $\theta f'(0)\kappa > rx(0) + \gamma$, $\theta f'(\kappa\eta) > rx(1)$.

(It guarantees that it's profitable for some but not all entrepreneurs to operate.)

Preferences

– Entrepreneurs are risk neutral who consume only when old (*IF entrepreneurs may choose to consume and IF they live infinitely...*).

– Lenders $U(Z_t^y) + \beta E_t(Z_{t+1}^0)$

Note that both entrepreneurs and lenders are risk neutral w.r.t. $t+1$ consumption. This allows us to concentrate on the role of the agents' wealth in mitigating agency costs rather than on the issues of risk-sharing.

Young agents work when they are young. The market wage rate is w_t.

Savings (*net worth*)

– Entrepreneur $S_t^e = w_t L^e$

– Lender $S_t = w_t L - Z_t^y(r)$

3 Equilibrium under Perfect Information ($\gamma = 0$)

$\widehat{q}_{t+1}$ = expected relative price of capital in $t+1$

Define $\overline{\omega}$ to be the efficiency level of the entrepreneur who is indifferent between investment and storage (the marginal borrower):

$$\widehat{q}_{t+1}\kappa = rx(\overline{\omega}), \tag{1}$$

where $rx(\overline{\omega})$ is the opportunity cost of the marginal borrower (because production of capital is at $t+1$ from investment at time t).

Given $\widehat{q}_{t+1}$, those entrepreneurs with $\omega < \overline{\omega}$ earn a positive profit.

Note that $\overline{\omega}$ depends only on $\widehat{q}_{t+1}$, in particular, it does not depend on entrepreneurs' net worth S_t^e.

Recall that i_t denotes the number of projects undertaken in t, and ω is distributed uniformly, i_t equals the fraction of entrepreneurs whose efficiency levels are higher than $\overline{\omega}$ ($\omega < \overline{\omega}$)

$$
\begin{aligned}
i_t &= \eta\overline{\omega}, \\
k_{t+1} &= \kappa i_t = \eta\kappa\overline{\omega}. \qquad (2)
\end{aligned}
$$

By (1), (2), and (2),

$$
\widehat{q}_{t+1} = \frac{1}{\kappa} r x(\underset{(+)}{\frac{k_{t+1}}{\kappa\eta}}), \qquad \text{(SS)}
$$

which is capital goods supply curve. It is positively sloped because a higher expected value of $\widehat{q}_{t+1}$ raises the number of entrepreneurs who can profitably invest ($\overline{\omega}$ increases), so that a larger share of savings is devoted to capital formation.

For the capital goods demand,

$$
\widehat{q}_{t+1} = \theta f'(k_{t+1}) \qquad \text{(DD)}
$$

$\widehat{q}_{t+1}$ = expected price of capital = rental rate (since capital is fully depreciated in one period).

Since (SS) and (DD) are independent of period t state variables, $\widehat{q}_t$ and k_t are constant over time. Quantities of output goods (y_t) fluctuate in proportional to (serially uncorrelated) productivity shock $\widetilde{\theta}_t$. In this case, the aggregate investment demand $\eta \int_0^{\overline{\omega}} x(\omega)d\omega$ is fixed over time.

Lenders's maximization problem

$$
\begin{cases}
Max \quad U(c_t) + Ec_{t+1} \\
s.t \quad c_t + S_t = w_t L \\
\qquad\qquad Ec_{t+1} = rS_t
\end{cases}
$$

$$\Longrightarrow \quad -U'(\omega_t L - S_t) + r = 0,$$
$$\Longrightarrow \quad S_t = \omega_t L - \phi(\underset{(-)}{r}).$$

4 Equilibrium with Private Information ($\gamma > 0$)

We first derive the optimal financial contract in a partial equilibrium setting, and then embed the financial contracts in a general equilibrium model.

(1) In a given period, $S^e, \widehat{q}, r$ are taken as given

(2) Sssume $x(\omega) > S^e$ for all ω, so that all entrepreneurs need to borrow external funds $(x(\omega) - S^e)$

$\Longrightarrow$ lenders' expected return $r\left[x(\omega) - S^e\right]$

(3) Allow for stochastic monitoring. An implication of permitting random auditing is that the optimal contract will not be in the form of a debt contract.

(4) Consider 2 states of investment outcomes

$$\begin{cases} \text{Bad}, & \kappa_1 & \text{with prob.} & \pi_1 \\ \text{Good}, & \kappa_2 & \text{with prob.} & \pi_2 \end{cases}$$

where $\kappa_2 > \kappa_1$, $\pi_1\kappa_1 + \pi_2\kappa_2 = \kappa$.

4.1 The Contracting Problem

The optimal financial contract selects auditing probability p and consumption plan for the entrepreneurs c.

Let

a = announced state, $a \in \{1, 2\}$

p_a = auditing probability given the announced state, $p_a \in [0, 1]$

$c(a, t)$ = entrepreneurs' consumption if announced state is a, true state is t, and auditing does not occur.

$c^a(a, t)$ = entrepreneurs' consumption if announced state is a, true state is t, and auditing occurs.

Truth-telling (incentive) constraint for entrepreneurs

$$p_1 c^a(1,1) + (1-p_1)c(1,1) \geqslant p_2 c^a(2,1) + (1-p_2)c(2,1), \quad \text{(IC1)}$$
$$p_2 c^a(2,2) + (1-p_2)c(2,2) \geqslant p_1 c^a(1,2) + (1-p_1)c(1,2). \quad \text{(IC2)}$$

Without auditing, the true state is not revealed. Thus, no matter which state occurs, the lender charges a constant repayment. This says, the returns to lender, $\widehat{q}\kappa_t - c(a,t)$ is independent of which state occurred,

$$\widehat{q}\kappa_1 - c(a,1) = \widehat{q}\kappa_2 - c(a,2) \geqslant 0, \forall a \in \{1,2\}.$$

Thus, we have

$$\widehat{q}\kappa_1 - c(1,1) = \widehat{q}\kappa_2 - c(1,2), \quad \text{(IC3)}$$
$$\widehat{q}\kappa_1 - c(2,1) = \widehat{q}\kappa_2 - c(2,2). \quad \text{(IC4)}$$

Expected utilities of entrepreneurs

$$\begin{aligned} EU^e &= \pi_1[p_1 c^a(1,1) + (1-p_1)c(1,1)] \\ &\quad + \pi_2[p_2 c^a(2,2) + (1-p_2)c(2,2)]. \end{aligned}$$

Expected utilities of lenders

$$\begin{aligned} EU^l &= \pi_1\{p_1[\widehat{q}\kappa_1 - \widehat{q}\gamma - c^a(1,1)] + (1-p_1)[\widehat{q}\kappa_1 - c(1,1)]\} \\ &\quad + \pi_2\{p_2[\widehat{q}\kappa_2 - \widehat{q}\gamma - c^a(2,2)] + (1-p_2)[\widehat{q}\kappa_2 - c(2,2)]\}. \end{aligned}$$

The optimal contract solves

$$\begin{aligned} &\underset{\{c,p\}}{max} EU^e \\ &s.t \quad EU^l \geq r(x(\omega) - S^e), \\ &\qquad \text{(IC1) - (IC4) (IC constrains)}, \\ &\qquad c^a \geq 0,\ c(1,1) \geq 0 \text{ (limited liability constraints)}, \\ &\qquad 0 \leq p_i \leq 1. \end{aligned}$$

Competition will force lenders to choose the same contracts and drive the extract surplus down to only the market return rate.

Mookherjee and Png (1987) show that the entrepreneur's optimal consumption when he is audited and found to be lying is zero: $c^a(1,2) = c^a(2,1) = 0$ (penalty for cheating).

Since entrepreneurs can hide capital but cannot create capital from nothing, they will not report high outcomes given low outcomes. Thus,we can ignore (IC1) constraint of entrepreneurs.

Rewrite $c(i,i) = c(i) \quad , i = 1, 2$.
Substituting (IC3) into (IC2),

$$p_2 c^a(2) + (1 - p_2)c(2) \geqslant (1 - p_1)\left[c(1) + \widehat{q}\left(\kappa_2 - \kappa_1\right)\right]. \tag{IC2'}$$

Now rewrite entrepreneurs' expected utilities

$$\begin{aligned} EU^e &= \pi_1[p_1 c^a(1) + (1 - p_1)c(1)] \\ &\quad + \pi_2[p_2 c^a(2) + (1 - p_2)c(2)]. \end{aligned}$$

Now rewrite lenders' expected utility

$$\begin{aligned} EU^l &= \pi_1\{\widehat{q}\kappa_1 - p_1\widehat{q}\gamma - [p_1 c^a(1) + (1 - p_1)c(1)]\} + \\ &\quad \pi_2\{\widehat{q}\kappa_2 - p_2\widehat{q}\gamma - [p_2 c^a(2) + (1 - p_2)c(2)]\} \\ &= \widehat{q}\kappa - \widehat{q}\gamma(\pi_1 p_1 + \pi_2 p_2) - EU^e. \end{aligned} \tag{EUL}$$

Lenders' participation constraint requires,

$$EU^l = \widehat{q}\kappa - \widehat{q}\gamma(\pi_1 p_1 + \pi_2 p_2) - EU^e \geqslant r(x(\omega) - S^e).$$

Since this constraint must be binding, the expected utility of entrepreneurs becomes

$$EU^e = -r(x(\omega) - S^e) + \widehat{q}\kappa - \widehat{q}\gamma(\pi_1 p_1 + \pi_2 p_2). \tag{EUE}$$

Thus, the maximization of EU^e is equivalent to minimizing the expected auditing cost

$$\widehat{q}\gamma(\pi_1 p_1 + \pi_2 p_2).$$

WLOG $c^a(2) = c(2)$.

Because $c^a(2)$ and $c(2)$ always come with the convex combination $p_2 c^a(2) + (1 - p_2)c(2)$. But this is not the case for $c^a(1)$ and $c(1)$. See (IC2').

Now we are left with $c^a(1)$, $c(1)$, $c(2)$, p_1, and p_2 to decide. The maximization problem becomes

$$\begin{aligned} &\underset{\{c,p\}}{max} \pi_1 [p_1 c^a(1) + (1 - p_1)c(1)] + \pi_2 c(2) \equiv EU^e \\ &s.t \quad \widehat{q}\kappa - \widehat{q}\gamma(\pi_1 p_1 + \pi_2 p_2) - EU^e \geqslant r(x(\omega) - S^e). \\ &\qquad\quad c(2) \geqslant (1 - p_1) \left[c(1) + \widehat{q}\, (\kappa_2 - \kappa_1) \right]. \end{aligned}$$

In the HW, you are asked to solve for the equilibrium p_1, p_2, $c(1)$, $c^a(1)$, $c(2)$.

The optimal auditing probability is

$$p_1 = \frac{r(x(\omega) - S^e) - \widehat{q}\kappa_1}{\pi_2\widehat{q}\,(\kappa_2 - \kappa_1) - \pi_1\widehat{q}\gamma}.$$

To see why net worth S^e and p_1 are inversely related, suppose $S^e \uparrow$

$\Rightarrow$ entrepreneurs obtain more returns in good state (Note that $EU^e = \pi_2 c(2) = \pi_2(1-p_1)\widehat{q}\,(\kappa_2 - \kappa_1)$)

$\Rightarrow$ more stake at risk in good state

$\Rightarrow$ higher risk to falsely claim bad state

$\Rightarrow$ IC constraint is satisfied easier

$\Rightarrow$ Probability of auditing can be lowered.

For $p_1 > 0$, we must have $r(x(\omega) - S^e) > \widehat{q}\kappa_1$, i.e.,

$$S^e < x(\omega) - \frac{\widehat{q}}{r}\kappa_1 \equiv S^*, \forall\omega.$$

We also require $p_1 < 1$ (otherwise $c(2) = 0 \Longrightarrow EU^e = 0$),

$$r(x(\omega) - S^e) - \widehat{q}\kappa_1 < \pi_2\widehat{q}\,(\kappa_2 - \kappa_1) - \pi_1\widehat{q}\gamma.$$

This says,

$$S^e > x(\omega) - \frac{\widehat{q}}{r}\,(\kappa - \pi_1\gamma) \equiv S^{**}, \forall\omega.$$

In this case, the expected utility of an entrepreneur is (compare with (9))

$$\begin{aligned} EU^e &= \pi_2 c(2) = \pi_2(1-p_1)\widehat{q}\,(\kappa_2 - \kappa_1) \\ &= \alpha[\widehat{q}\kappa - r(x(\omega) - S^e) - \pi_1\widehat{q}\gamma]. \end{aligned} \tag{3}$$

where

$$\alpha = \frac{\widehat{q}\,(\kappa_2 - \kappa_1)}{\pi_2\widehat{q}\,(\kappa_2 - \kappa_1) - \pi_1\widehat{q}\gamma} > 1.$$

$$\Longrightarrow \frac{\partial EU^e}{\partial S^e} = \alpha r > r \quad \text{when collateralization is incomplete.}$$

$\Rightarrow (\alpha - 1)r =$ external finance premium.

Rate of return to inside funds is greater than that to external funds, because additional inside funds not only replace outside funds but also reduce expected agency costs.

4.2 Three types of Entrepreneurs (Good, Fair, and Poor)

Define $\underline{\omega}$ and $\overline{\omega}$ $s.t$ ($\underline{\omega}$ and $\overline{\omega}$ are increasing $\widehat{q}$)

$$\begin{cases} \widehat{q}\kappa - rx(\underline{\omega}) - \widehat{q}\pi_1\gamma = 0, \\ \widehat{q}\kappa - rx(\overline{\omega}) = 0, \end{cases}$$

(i) for $\omega < \underline{\omega}$, project expected net return > 0, even when $p_1 = 1 \Rightarrow$ good entrepreneur

(ii) for $\underline{\omega} < \omega < \overline{\omega}$, project expected net return > 0, only if $p_1 = 0 \Rightarrow$ fair entrepreneur

(iii) for $\omega > \overline{\omega}$, project expected net return < 0, even when $p_1 = 0 \Rightarrow$ poor entrepreneur

Note that for $\omega < \underline{\omega}$, $S^{**} < 0$.

S^* and S^{**} are both decreasing in $\widehat{q}$. That is, the classification of entrepreneurs is conditional on $\widehat{q}$.

(9) and (3) shift with different levels of ω. Thus, with different ω, entrepreneurs face different S^*.

Thee are three types of entrepreneurs:

(1) Good Entrepreneurs ($\omega < \underline{\omega}$, and $S^{**} < 0$, thus $EU^e > rS^e$, for all S^e)

For $S^e < S^*$, the marginal return to investing in a project is greater than the return to holding inventories $\Rightarrow p_1 > 0$.

For $S^e \geqslant S^*$, after investing in his own project, for the remaining funds he is indifferent between investment, storage or lending to others $\Rightarrow p_1 = 0$.

(2) Poor Entrepreneurs ($\omega > \overline{\omega}$ and $S' > S^*$, thus $EU^e < rS^e$)

These entrepreneurs become lenders.

(3) Fair Entrepreneurs ($\underline{\omega} < \omega < \overline{\omega}$ and $0 < S^{**} < S' < S^*$)

Entrepreneur's expected return is convex, they (risk-neutral) will enter a lottery so that an entrepreneur who wins will be fully collateralized.

$$\begin{cases} S^*(\omega) & \frac{S^e}{S^*(\omega)} = g(\omega) \\ 0 & 1 - \ g(\omega) \end{cases}$$

Thus, only a fraction $g(\omega)$ of fair entrepreneurs invest.

4.3 Within-Period Equilibrium

In any period t, the inherited per capita capital stock k_t is predetermined. We show how the expected price $\widehat{q}_{t+1}$ and the quantity of new capital k_{t+1} are determined.

(1) Let

$$p(\omega) = \max\left\{\frac{r(x(\omega) - S^e) - \widehat{q}\kappa_1}{\pi_2\widehat{q}(\kappa_2 - \kappa_1) - \pi_1\widehat{q}\gamma}, 0\right\}$$

be the probability that an entrepreneur of type ω is audited when $\omega \leq \overline{\omega}$.

Note that $p(\omega)$ is decreasing in $\widehat{q}$ and S^e; $p(\omega) = 0$ for $S^e \geq S^*(\omega)$.

(2) Due to the "collateralization lottery," fair entrepreneurs ($\underline{\omega} < \omega < \overline{\omega}$) do not face the agency cost of auditing when they invest. But only those who win the lottery are able to invest. Recall that $g(\omega)$ is the fraction of fair entrepreneurs of type ω who can invest. Then,

$$g(\omega) = \frac{S^e}{S^{*(\omega)}} = \min\left\{\frac{rS^e}{rx(\omega) - \widehat{q}\kappa_1}, 1\right\},$$

for $\underline{\omega} < \omega < \overline{\omega}$. Note that $g(\omega)$ is increasing in $\widehat{q}$ and S^e; $g(\omega) = 1$ for $S^e \geq S^*(\omega)$.

(3) Entrepreneurs with $\omega \geq \overline{\omega}$ do not invest.

Total capital formation per capita is

$$k_{t+1} = \eta\left[\kappa\underline{\omega} - \pi_1\gamma\int_0^{\underline{\omega}} p(\omega)d\omega\right] + \eta\kappa\int_{\underline{\omega}}^{\overline{\omega}} g(\omega)d\omega,$$

which is the capital supply curve for the $\gamma > 0$ case.

The capital supply curve now depends on entrepreneurial net worth S^e (which enters into the expressions for $p(\omega)$ and $g(\omega)$). High values of S^e push the capital supply curve rightward.

Finally, capital demand in the $\gamma > 0$ case is identical to the $\gamma = 0$ case.

4.3.1 Comparative Statics

(1) Suppose there is a positive productivity shock (as in RBC). Young entrepreneurs and lenders will accumulate more savings. Higher entrepreneurial net worth (S^e) lowers agency costs and shifts the capital supply curve rightward, raising k_{t+1} and lowering $\widehat{q}_{t+1}$.

Thus, the presence of agency costs introduces a channel through which borrowers' balance sheet matters for investment.

(2) Suppose there is a redistribution of (labor) endowment from entrepreneurs to lenders, i.e., lower L^e and raise L so that $\eta L^e + (1-\eta) L = 1$ still holds. This resembles the effect of "debt-deflation," a situation arises from unindexed debt contracts and/or unexpected deflation redistributes wealth from the debtors to the creditors.

A fall in L^e lowers entrepreneurial net worth (S^e) and raises the agency costs associated with external finance. This shifts the capital supply leftward, lowering k_{t+1} and raising $\widehat{q}_{t+1}$.