

Baumol-Tobin Model of Demand for Money

Assume that purchases must be made with cash. At the start of the period, your income is deposited into a interest-bearing savings account. You must withdraw money from the account to pay for purchases. Let

- PY =beginning of the month nominal income;
- i =(monthly) deposit nominal interest rate;
- $P\delta$ =transaction costs in nominal terms;
- Z =the amount of withdrawal each time.

What is the optimal number of transactions (withdrawals)? The total number of times you have to go to the bank is $n = PY/Z$ and your average cash balance (transaction money demand) is

$$M^d = \frac{1}{2} \times \left(\frac{PY}{n} \times \frac{1}{n} \right) \times n = \frac{PY}{2n} = \frac{Z}{2}.$$

The opportunity cost for holding cash is $i \times Z/2 = i \times PY/(2n)$ and the transaction cost is $P\delta$. Thus the total costs of managing cash is

$$TC = nP\delta + \frac{iPY}{2n}.$$

To minimize the total costs of managing cash, take derivative with respect to n: $P\delta - iPY/(2n^2) = 0$. Thus, we solve for the optimal number of trip to the bank:

$$n = \sqrt{\frac{iY}{2\delta}},$$

and the transaction demand for cash balance is $M^d = \frac{Z}{2} = \frac{PY}{2n} = P\sqrt{\frac{\delta Y}{2i}}$, thus the real balance is

$$\frac{M^d}{P} = \sqrt{\frac{\delta Y}{2i}}.$$

This is the square-root rule for transaction component (cash) of the demand for money. One problem with this formula is that the number of withdrawals should be an integer, but we have in fact treat it as a continuous variable.

We can derive some properties of this money demand function. Firstly, take natural log on the transaction demand for money function

$$\ln \frac{M_t^d}{P_t} = \frac{1}{2} \ln \frac{\delta}{2} + \frac{1}{2} \ln Y_t - \frac{1}{2} \ln i_t,$$

(i) The money demand elasticity of transaction cost = 0.5; (ii) The money demand elasticity of interest rate = -0.5; (iii) The money demand elasticity of income = 0.5.

According to the result of (iii), we know that the transaction demand for money exhibits economies of scale. That is, when income increases, the transaction demand for money increases less proportionally. As a result, the income velocity of money rises.